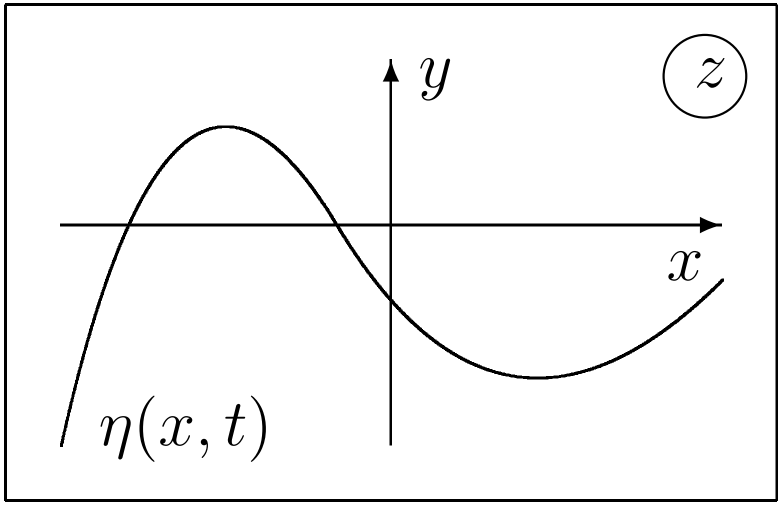
On the Free Surface Hydrodynmics (with surface tension)

Basic Equations



potential irrotational flow

$$\triangle\phi(x,y,t)=0$$

Boundary conditions: $\left[\begin{array}{l} \frac{\partial\phi}{\partial t}+\frac{1}{2}|\nabla\phi|^2+g\eta=\frac{P}{\rho}, \\ \frac{\partial\eta}{\partial t}+\eta_x\phi_x=\phi_y \end{array}\right|$ at $y=\eta(x,t)$.

$$\begin{aligned} \frac{\partial\phi}{\partial y} &= 0, y\to-\infty, \\ \frac{\partial\phi}{\partial x} &= 0, |x|\to\infty. \end{aligned}$$

Hamiltonian and Lagrangian

Hamiltonian H is the total energy of the fluid $H = T + U$

$$
\begin{aligned}
T &= \frac{1}{2}\int_{-\infty}^{\infty} dx \int_{-\eta}^{\eta} (\nabla\Phi)^2 dy\,, \\
U &= \frac{g}{2}\int \eta^2 dx\,.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial\eta}{\partial t} &= \frac{\delta H}{\delta\Psi}\,, \\
\frac{\partial\Psi}{\partial t} &= -\frac{\delta H}{\delta\eta}\,, \\
\Psi(x,t) &= \Phi(x,y,t)\big|_{y=\eta}
\end{aligned}
$$

Action $S = \int L dt$

$$\delta S = 0,$$

Lagrangian

$$L = \int \Psi \frac{\partial\eta}{\partial t} dx - H\,.$$

Explicit Equations

Z - *the conformal mapping*, Φ - *complex velocity potential*.

$$z = x + iy, \qquad \Phi = \Psi + \hat{H}\Psi.$$

$$\begin{aligned} z_t &= iUz_u, \\ \Phi_t &= iU\Phi_u - \hat{P}\left(\frac{|\Phi_u|^2}{|z_u|^2}\right) + ig(z-u). \end{aligned}$$

U is a complex transport velocity:

$$U = \hat{P}\left(\frac{-\hat{H}\Psi_u}{|z_u|^2}\right). \qquad u \longrightarrow w$$

Projector operator $\hat{P}(f) = \frac{1}{2}(1 + i\hat{H})(f)$.

Ovsyannikov,1973

Cubic Equations

$$R = \frac{1}{z_w}, \qquad\qquad \Phi_w = -iVz_w.$$

$$\begin{aligned} R_t &= i\left[UR' - U'R\right], \\ V_t &= i\left[UV' - R\hat{P}(V\bar{V})'\right] + g(R-1) - \\ &\qquad -2\sigma R\hat{P}'(\sqrt{R}'\sqrt{\bar{R}} - \sqrt{\bar{R}}'\sqrt{R}). \end{aligned}$$

Complex transport velocity U is defined via $\hat{P}$

$$U = \hat{P}(V\bar{R} + \bar{V}R).$$

Freak Wave

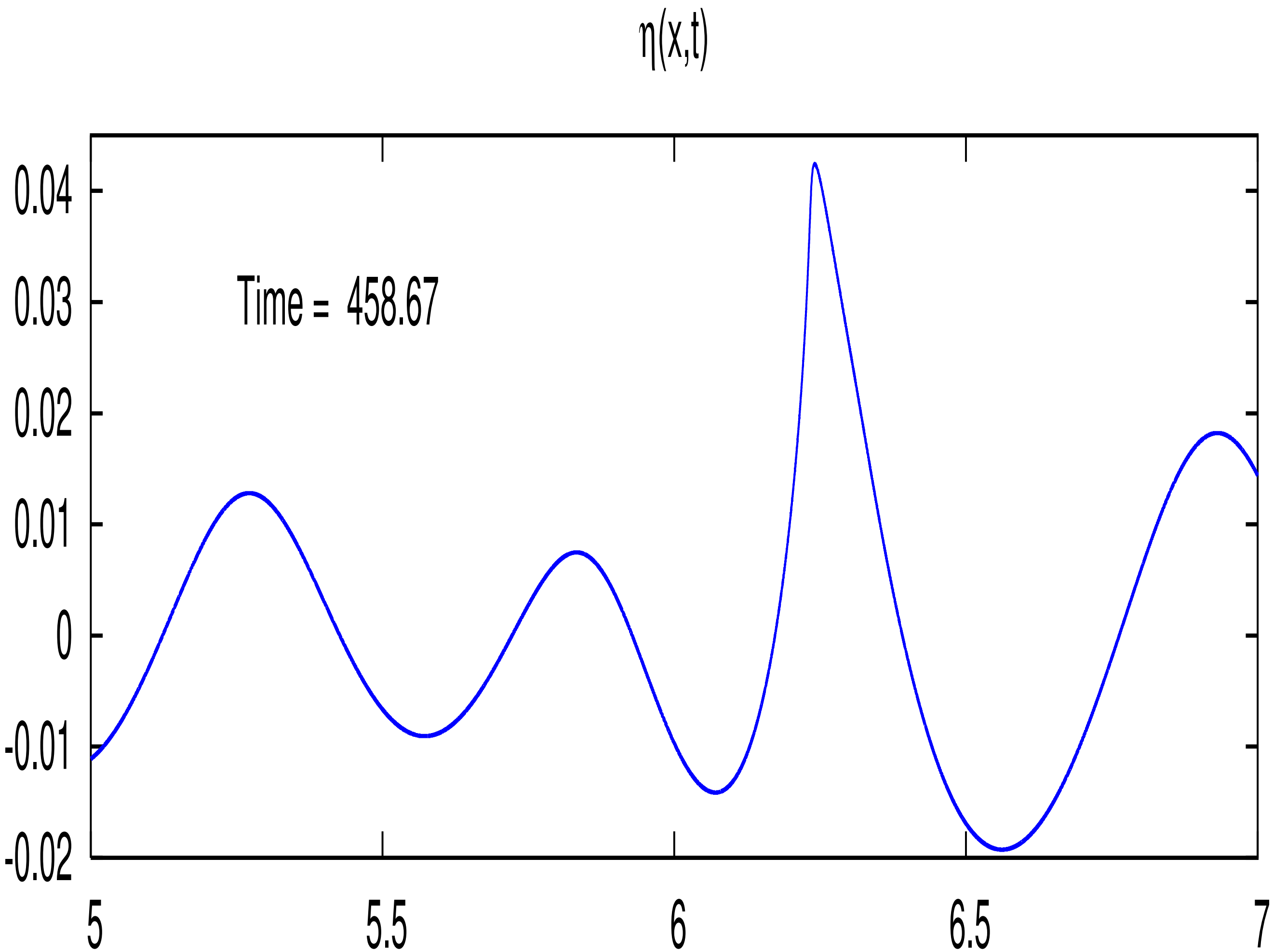


Figure 1: Close view