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##### The Nonlinear Schrödinger Equation and Canonical Transformation

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Consider 1D NLSE for the field

(1)

in the periodic domain of . Making Fourier transformation of (1) one can get:

(2)

Here

Wave numbers are integers in the periodic domain of . The Hamiltonian of the equation is the following:

Some canonical transformation is applied to (2). This transformation has the following form of infinite series:

(3)

It converges only for small values of :

This transformation removes all nonresonant terms in the Hamiltonian and (what is the most important thing) simplifies the fourth order term. So, that

(4)

Six order term in the Hamiltonian vanishes also. For small values of one can consider a reduced Hamiltonian in which only the terms of the fourth order are kept. Then the equation for is also very simple:

(5)

The solution of the equation (5) is very simple:

(6)

Here

does not depend on and is defined by initial conditions only.

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