Geometric Quantum Noise of Spin

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The presence of geometric phases is known to affect the dynamics of the systems involved. Here, we consider a quantum degree of freedom, moving in a dissipative environment, whose dynamics is described by a Langevin equation with quantum noise. We show that geometric phases enter the stochastic noise terms. Specifically, we consider small ferromagnetic particles (nanomagnets) or quantum dots close to Stoner instability, and investigate the dynamics of the total magnetization in the presence of tunneling coupling to the metallic leads. We generalize the Ambegaokar-Eckern-Schön effective action and the corresponding semiclassical equations of motion from the U(1) case of the charge degree of freedom to the SU(2) case of the magnetization. The Langevin forces (torques) in these equations are strongly influenced by the geometric phase. As a first but nontrivial application, we predict low temperature quantum diffusion of the magnetization on the Bloch sphere, which is governed by the geometric phase. We propose a protocol for experimental observation of this phenomenon.

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Introduction.—It is well-known that the kinetic part of the action of a free spin of length S, whose position is described in spherical coordinates by angles θ and ϕ reads $\mathcal{S}_{\text{spin}} = \int p dq$. Here, the generalized coordinate is $q \equiv \phi$ and the conjugate momentum is $p \equiv S(1 - \cos\theta)$. This action, a.k.a. geometric (Berry) phase action or Wess-Zumino-Novikov-Witten (WZNW) action, produces deterministic spin dynamics if accompanied by, e.g., a Zeeman term. If the spin is subject to dissipation its equations of motion are expected to contain deterministic friction terms, e.g., Gilbert damping, as well as stochastic Langevin terms. Here, we show that the geometric phase determines the form of these stochastic terms and analyze the consequence of this for observables. Specifically, we focus on the dynamics of the collective spin degree of freedom of either a nanomagnet or a paramagnetic quantum dot near the Stoner instability characterized by a large total spin [1–5]. The system is tunnel coupled to a normal lead, which gives rise to a dissipative behavior.

We find that in the quantum regime, i.e., when the precession frequency is higher than the temperature, the stochastic spin torques, represented through random Langevin terms, are substantially influenced by the Berry phase accumulated by the system in the course of precession. As an application of our theory, we calculate the diffusion rate for a large spin, which is artificially held on a high-energy precessing trajectory by a specific multiple echo (“bang-bang”) protocol [6].

Our approach can be viewed as a generalization of the Landau-Lifschitz-Gilbert (LLG)-Langevin equation [7,8], central to the field of spintronics [9], to a regime where quantum dynamics dominates. Stochastic LLG equations have been derived in numerous publications for both a localized spin in an electronic environment (a situation of the Caldeira-Leggett type) [10,11] and for a magnetization formed by itinerant electrons [12,13]. In all these papers, the precession frequency was assumed to be lower than the temperature or the voltage, thus justifying the semiclassical treatment of the problem. In this regime, the geometric phase did not influence the Langevin terms.

From a different perspective, the equation of motion presented here is derived from a new action which constitutes a generalization of the Ambegaokar-Eckern-Schön (AES) theory [14,15]. The latter was written to describe the dynamics of the charge degree of freedom [marked by an Abelian U(1) symmetry]. Our generalized AES action, which is the first main result of our analysis, is underscored by the non-Abelian SU(2) dynamics. As only two out of three SU(2) Euler angles are needed to describe the spin position, a gauge freedom emerges. A central element of our analysis is to employ this freedom and find a gauge, which allows for efficient calculation and

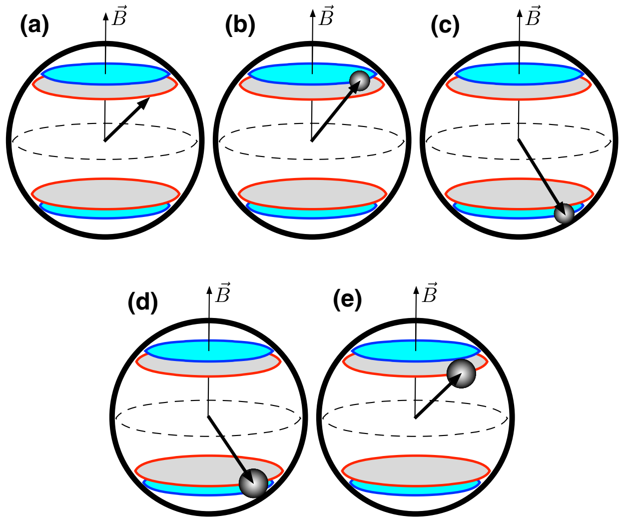

FIG. 1 (color online). "Bang-bang" protocol. Standard NMR techniques in the rotating frame are employed: rotations around, e.g., the x and the y axes of the rotating frame are achieved by applying resonant driving pulses, which are $\pi/2$ phase shifted with respect to each other. (a) First, a θ_0 pulse around the y axis drives the spin in the xz plane of the rotating frame to form angle θ_0 with the z axis; (b) During time $\Delta t \ll \tau_{\mathrm{rel}}$, the spin is left alone, and it relaxes to $\theta = \theta_0 - \delta\theta$, where $\delta\theta \approx \tilde{g}\tilde{B}\sin\theta_0\Delta t \ll \pi$; (c) A π pulse around x is performed. The spin is again in the xz plane but at $\theta = \pi/2 - (\theta_0 - \delta\theta)$; (d) The spin is left alone again for time Δt. The relaxation brings it to $\theta = \pi/2 - \theta_0$; (e) A π pulse around x is performed. The spin returns to $\theta = \theta_0$ in the xz plane. This cycle is repeated multiple times. At the end a $-\theta_0$ pulse around y axis would bring the spin back to the north pole, but with an accumulated uncertainty (gray cloud in all panels) due to the quantum geometric diffusion.

highlights the role of the Berry phase in the stochastic Langevin terms.

Effective action.—Our derivation here is technically close to that of Ref. [12]. However, in contrast to Ref. [12], we do not limit ourselves to small deviations of the spin from the instantaneous direction, but rather consider the action on global trajectories covering the whole Bloch sphere.

To demonstrate the emergence of an AES-like effective action, we consider a quantum dot with strong exchange interaction coupled to a normal lead. The Hamiltonian reads $H = H_{\mathrm{dot}} + H_{\mathrm{lead}} + H_{\mathrm{tun}}$. The quantum dot is described by the magnetic part [16] of the "universal" Hamiltonian [1]

$$H_{\mathrm{dot}} = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - J\boldsymbol{S}^2 + \boldsymbol{B}\boldsymbol{S}, \qquad (1)$$

where $\boldsymbol{S} \equiv (1/2)\sum_{\alpha,\sigma_1,\sigma_2} a^\dagger_{\alpha,\sigma_1} \boldsymbol{\sigma}_{\sigma_1,\sigma_2} a_{\alpha,\sigma_2}$ is the operator of the total spin on the quantum dot, $\boldsymbol{B}$ is the external magnetic field, and $J > 0$ is the corresponding "zero mode" ferromagnetic exchange constant. The Hamiltonian of the lead and that describing the tunneling between the dot and the lead are standard: $H_{\mathrm{lead}} = \sum_{\gamma,\sigma} \epsilon_\gamma c^\dagger_{\gamma,\sigma} c_{\gamma,\sigma}$ and $H_{\mathrm{tun}} = \sum_{\alpha,\gamma,\sigma} V_{\alpha,\gamma} a^\dagger_{\alpha,\sigma} c_{\gamma,\sigma} + \mathrm{H.c.}$ We assume here a nonmagnetic lead.

We consider the Keldysh generating functional $\mathcal{Z} = \int D\bar{\Psi} D\Psi \exp[i\mathcal{S}_\Psi]$, where the Keldysh action is given by $\mathcal{S}_\Psi = \oint_K dt (i\bar{\Psi}\partial_t \Psi - H)$ (plus the necessary source terms which are not explicitly written). Here, for brevity, Ψ denotes all fermionic fields and the time t runs along the Keldysh contour. After standard Hubbard-Stratonovich manipulations [3,4,17], decoupling the interaction term $-J\boldsymbol{S}^2$, we obtain $\mathcal{Z} = \int D\boldsymbol{\mathcal{M}} \exp[i\mathcal{S}_M]$ and the action for the bosonic vector $\boldsymbol{\mathcal{M}}(t)$ reads

$$i\mathcal{S}_M = \mathrm{tr}\ln\left[\begin{pmatrix} G^{-1}_{\mathrm{dot}} & -\hat{V} \\ -\hat{V}^\dagger & G^{-1}_{\mathrm{lead}} \end{pmatrix}\right] - i\oint_K dt \frac{|\boldsymbol{\mathcal{M}}|^2}{4J}. \qquad (2)$$

Here, $G^{-1}_{\mathrm{dot}} \equiv [i\partial_t - \epsilon_\alpha - (\boldsymbol{\mathcal{M}}(t) + \boldsymbol{B})\cdot\boldsymbol{\sigma}/2]$, while $G^{-1}_{\mathrm{lead}} \equiv i\partial_t - \epsilon_\gamma$. Both G^{-1}_{dot} and G^{-1}_{lead} are matrices with time, spin, and orbital indexes. We introduce $\boldsymbol{M}(t) \equiv \boldsymbol{\mathcal{M}}(t) + \boldsymbol{B}$. Expanding Eq. (2) in powers of the tunneling matrix $\hat{V}$ and resumming, we easily obtain

$$i\mathcal{S}_M = \mathrm{tr}\ln[G^{-1}_{\mathrm{lead}}] + \mathrm{tr}\ln[G^{-1}_{\mathrm{dot}} - \Sigma] - i\oint_K dt \frac{|\boldsymbol{M} - \boldsymbol{B}|^2}{4J}, \qquad (3)$$

where the self-energy reads $\Sigma \equiv \hat{V} G_{\mathrm{lead}} \hat{V}^\dagger$. The first term is trivial; i.e., it would never contain the source fields. Thus, it will be dropped in what follows.

Rotating frame.—We introduce a unit length vector $\boldsymbol{n}(t) = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ through $\boldsymbol{M}(t) = M(t)\boldsymbol{n}(t)$ and transform to a coordinate system in which $\boldsymbol{n}$ coincides with the z axis $\boldsymbol{n}(t)\cdot\boldsymbol{\sigma} = R(t)\sigma_z R^\dagger(t)$. This condition identifies the unitary rotation matrix R as an element of SU(2)/U(1). Indeed, if we employ the Euler angle representation $R = \exp[-(i\phi/2)\sigma_z]\exp[-(i\theta/2)\sigma_y]\times \exp[-(i\psi/2)\sigma_z]$, then the angles $\phi(t)$ and $\theta(t)$ determine the direction of $\boldsymbol{n}(t)$, while $\psi(t)$ is arbitrary; i.e., the condition $\boldsymbol{n}(t)\cdot\boldsymbol{\sigma} = R\sigma_z R^\dagger$ is achieved with any value of $\psi(t)$. Thus, ψ represents the gauge freedom of the problem. We introduce, first, a shifted gauge field $\chi(t) \equiv \phi(t) + \psi(t)$. This way a periodic boundary condition, e.g., in the Matsubara representation $R(\tau) = R(\tau + \beta)$, is satisfied for $\chi(\tau + \beta) = \chi(\tau) + 4\pi m$ (The fact that m is integer is intimately related to the spin quantization [18]). We can always assume trivial boundary conditions for χ; i.e., $m = 0$. We keep this representation of the rotation matrix R also for the Keldysh technique.

We perform a transition to the rotating frame and obtain $i\mathcal{S}_M = \mathrm{tr}\ln[R^\dagger(G^{-1}_{\mathrm{dot}} - \Sigma)R] - i\oint_K dt(M^2 - 2\boldsymbol{B}\boldsymbol{M})/(4J)$ (we omit the constant term $\propto |\boldsymbol{B}|^2$). For the Green's function of the dot, this gives $R^\dagger G^{-1}_{\mathrm{dot}} R = i\partial_t - \epsilon_\alpha - M(t)\sigma_z/2 - Q$, where we define the gauge (Berry) term as $Q \equiv R^\dagger(-i\partial_t)R = Q_\parallel + Q_\perp$. Here, $Q_\parallel \equiv [\dot{\phi}(1-\cos\theta) - \dot{\chi}]\sigma_z/2$ and $Q_\perp \equiv -\exp[i\chi\sigma_z][\dot{\theta}\sigma_y - \dot{\phi}\sin\theta\sigma_x]\exp[i\phi\sigma_z]/2$. Note, that Q depends on the choice of the gauge field χ. Finally, we obtain

$$iS_M = \mathrm{tr}\ln[G^{-1}_{\mathrm{dot},z} - Q - R^\dagger\Sigma R] - i\oint_K dt\left[\frac{M^2}{4J} - \frac{\boldsymbol{B}\boldsymbol{M}}{2J}\right], \quad (4)$$

where $G^{-1}_{\mathrm{dot},z} \equiv i\partial_t - \epsilon_\alpha - (1/2)M(t)\sigma_z$.

To find the semiclassical trajectories of the magnetization, we need to consider paths $M(t)$, $\theta(t)$, $\phi(t)$ on the Keldysh contour such that the quantum components are small (in Supplemental Material C [19], we discuss the physical meaning of this approximation). The quantum (q) and classical (c) components of the fields are expressed in terms of the forward (u) and backward (d) components [17], e.g., $\phi_q(t) = \phi_u(t) - \phi_d(t)$ and $\phi_c(t) = (\phi_u(t) + \phi_d(t))/2$. Performing the standard rotation [17], we thus obtain

$$iS_M = \mathrm{tr}\ln[\tilde{G}^{-1}_{\mathrm{dot},z} - \tilde{Q} - \tilde{R}^\dagger\tilde{\Sigma}\tilde{R}] + i\int dt\frac{\boldsymbol{B}\boldsymbol{M}_q}{2J} - i\int dt\frac{M_c M_q}{2J}, \quad (5)$$

where $\tilde{G}^{-1}_{\mathrm{dot},z} \equiv \tau_x G^{-1}_{\mathrm{dot},z}$. The local in time matrix fields $Q(t)$ and $R(t)$ also acquire the 2×2 matrix structure in the Keldysh space, e.g., $\tilde{Q} = Q_c\tau_x + Q_q\tau_0/2$, where $\tau_{x,y,z,0}$ are the standard Pauli matrices.

Adiabatic limit.—Thus far, we have made no approximations. The action [Eq. (5)] governs both the dynamics of the magnetization amplitude $M(t)$ and of the magnetization direction $\boldsymbol{n}(t)$. Here, we focus on the case of a large amplitude M (more precisely, M fluctuates around a large average value M_0 (see also Supplemental Material F [19])). Such a situation arises either on the ferromagnetic side of the Stoner transition or on the paramagnetic side, but very close to the transition. In the latter case, as was shown in Refs. [3,4], it is the integration out of the fast angular motion of $\boldsymbol{n}$ which creates an effective potential for M, forcing it to acquire a finite average value. More precisely, the angular motion with frequencies $\omega \gg \max[T, B]$ (we adopt the units $\hbar = k_B = 1$) can be integrated out, renormalizing the effective potential for the slow part of $M(t)$. The very interesting question of the dissipative dynamics of slow longitudinal fluctuations of $M(t)$ in the mesoscopic Stoner regime will be addressed elsewhere. Here, we focus on the slow angular motion and substitute $M(t) = M_0$. Thus, the last term of Eq (5) can be dropped. We note that in the adiabatic limit, we may neglect $\tilde{Q}_\perp$ as it contributes only in the second order in $d\boldsymbol{n}/dt$ [4].

The idea now is to expand the action Eq. (5) in both $\tilde{Q}$ [which is small due to the slowness of $\boldsymbol{n}(t)$] and $\tilde{R}^\dagger\tilde{\Sigma}\tilde{R}$ (which is small due to the smallness of the tunneling amplitudes). A straightforward analysis reveals that a naive expansion to the lowest order in both violates the gauge invariance with respect to the choice of $\chi(t)$. One can show that the expansion in $\tilde{R}^\dagger\tilde{\Sigma}\tilde{R}$ is gauge invariant only if all orders of $\tilde{Q}$ are taken into account, that is if $(\tilde{G}^{-1}_{\mathrm{dot},z} - \tilde{Q})^{-1}$ is used as zeroth order Green's function in the expansion. This problem necessitates a clever choice of gauge, such that $(\tilde{G}^{-1}_{\mathrm{dot},z} - \tilde{Q})^{-1}$ is as close as possible to $\tilde{G}_{\mathrm{dot},z}$; i.e., the effect of $\tilde{Q}$ is "minimized".

Choice of gauge.—As the action [Eq. (5)] is gauge invariant, we are allowed to choose the most convenient form of $\chi(t)$. We make the following choice

$$\dot{\chi}_c(t) = \dot{\phi}_c(t)[1 - \cos\theta_c(t)],$$
$$\chi_q(t) = \phi_q(t)[1 - \cos\theta_c(t)], \quad (6)$$

which satisfies the necessary boundary conditions; i.e., $\chi_q(t = \pm\infty) = 0$.

We next motivate the choice of Eq. (6). Ideally, we should have chosen a gauge that would lead to $Q_\parallel = 0$. However, any gauge has to satisfy the boundary condition $\chi_q(t = \pm\infty) = 0$. This condition is violated by the naive gauge, in which on both forward and backward Keldysh contours $\dot{\chi} = \dot{\phi}(1 - \cos\theta)$, and, thus, $Q_\parallel$ vanishes identically. The gauge [Eq. (6)] satisfies the boundary conditions and leads to the desired cancellation $Q_{\parallel,c} = 0$, whereas the quantum component of $Q_\parallel$ remains nonzero:

$$Q_{\parallel,q} = \frac{1}{2}\sigma_z\sin\theta_c[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q]. \quad (7)$$

At the same time, this choice allows for the expansion of the Keldysh action in the small ϕ_q and θ_q as there are no $\dot{\phi}_q$ terms in Eq. (7) (see Supplemental Material A [19]).

Berry phase (WZNW action).—Expanding the zeroth order in $\tilde{\Sigma}$ term of the action [Eq. (5)] to first order in $\tilde{Q}$, we obtain the well known in spin physics (see, e.g., Refs. [18,23]) Berry phase (WZNW) action $iS_{\mathrm{WZNW}} = -\frac{1}{2}\int dt\,\mathrm{tr}[G^K_{\mathrm{dot},z}(t,t)Q_{\parallel,q}(t)]$, which after a straightforward calculation reads

$$iS_{\mathrm{WZNW}} = iS\int dt\sin\theta_c[\dot{\phi}_c\theta_q - \dot{\theta}_c\phi_q], \quad (8)$$

where $S \equiv N(M_0)/2$ is the (dimensionless) spin of the dot. Here, $N(M_0)$ is the number of orbital levels of the dot in the energy interval M_0 around the Fermi energy. Roughly, $S = M_0\bar{\rho}_{\mathrm{dot}}/2$, where $\bar{\rho}_{\mathrm{dot}}$ is the density of states averaged over the energy interval M_0. The effects of mesoscopic fluctuations of the density of states were considered in Ref. [5].

AES action.—The central result of the current Letter is the AES-like [14,15] effective action, which we obtain by expanding [Eq. (5)] to the first order in $\tilde{R}^\dagger\tilde{\Sigma}\tilde{R}$: $iS_{AES} = -\mathrm{tr}[\tilde{G}_{\mathrm{dot},z}\tilde{R}^\dagger\tilde{\Sigma}\tilde{R}]$. This gives

$$iS_{AES} = -g \int dt_1 dt_2 \times \mathrm{tr}\left[\begin{pmatrix} R_c^\dagger(t_1) & \frac{R_q^\dagger(t_1)}{2} \end{pmatrix} \begin{pmatrix} 0 & \alpha_A \\ \alpha_R & \alpha_K \end{pmatrix}_{(t_1-t_2)} \begin{pmatrix} R_c(t_2) \\ \frac{R_q(t_2)}{2} \end{pmatrix}\right], \quad (9)$$

where $g = \frac{1}{2}(\pi|V|^2\rho_{\mathrm{dot}}^{\uparrow}\rho_{\mathrm{lead}} + \pi|V|^2\rho_{\mathrm{dot}}^{\downarrow}\rho_{\mathrm{lead}})$ is the (spin-independent) conductance per spin direction. Here, $\rho_{\mathrm{dot}}^{\uparrow/\downarrow}$ are the densities of states at the respective $\uparrow$ and $\downarrow$ Fermi levels, whereas the density of states in the lead, ρ_{lead}, is spin independent. The standard [15] Ohmic kernel functions are given by $\alpha_R(\omega) - \alpha_A(\omega) = 2\omega$ and $\alpha_K(\omega) = 2\omega\coth(\omega/2T)$. The action Eq. (9) strongly resembles the AES action [15], with U(1) exponents $\exp[i\varphi/2]$ replaced by the SU(2) matrices R. Fixing the gauge of R is an essential part of our procedure.

Semiclassical equations of motion.—From the effective action [Eq. (9)], we derive the following semiclassical equation of motion (see [24] and Supplemental Material B [19] for details)

$$\dot{\theta}_c + \tilde{g}\sin\theta_c\dot{\phi}_c = \eta_\theta,$$
$$\sin\theta_c(\dot{\phi}_c - \gamma B) - \tilde{g}\dot{\theta}_c = \eta_\phi, \quad (10)$$

where $\tilde{g} \equiv (g/2S)$ and $\gamma = (J\bar{\rho}_{\mathrm{dot}})^{-1}$ is the "giromagnetic" constant of order unity. The Langevin forces (torques) are given by

$$\eta_\theta = \frac{1}{2S}\cos\frac{\theta_c}{2}\left[\xi_x\cos\left(\phi_c - \frac{\chi_c}{2}\right) + \xi_y\sin\left(\phi_c - \frac{\chi_c}{2}\right)\right] - \frac{1}{2S}\sin\frac{\theta_c}{2}\left[\xi_z\cos\frac{\chi_c}{2} + \xi_0\sin\frac{\chi_c}{2}\right],$$
$$\eta_\phi = -\frac{1}{2S}\cos\frac{\theta_c}{2}\left[\xi_x\sin\left(\phi_c - \frac{\chi_c}{2}\right) - \xi_y\cos\left(\phi_c - \frac{\chi_c}{2}\right)\right] - \frac{1}{2S}\sin\frac{\theta_c}{2}\left[\xi_z\sin\frac{\chi_c}{2} - \xi_0\cos\frac{\chi_c}{2}\right]. \quad (11)$$

The lhs of Eqs. (10) represent the standard Landau-Lifshitz-Gilbert (LLG) equations [7] (without a random torque). The rhs represent the random Langevin torque. The latter is expressed in terms of four independent stochastic variables ξ_j $(j = 0, x, y, z)$, which satisfy $\langle\xi_j(t_1)\xi_k(t_2)\rangle = \delta_{jk}g\alpha_K(t_1 - t_2)$ and $\langle\xi_j\rangle = 0$. On the Gaussian level, i.e., if fluctuations of θ_c and ϕ_c are neglected in Eqs. (11), the Langevin forces η_θ and η_ϕ are independent of each other and have the same autocorrelation functions: $\langle\eta_\theta(t_1)\eta_\phi(t_2)\rangle = 0$ and $\langle\eta_\theta(t_1)\eta_\theta(t_2)\rangle = \langle\eta_\phi(t_1)\eta_\phi(t_2)\rangle$. We emphasize that, in general, the noise depends on the angles θ_c and ϕ_c leading to complicated dynamics within Eqs. (10). In the classical domain, i.e., for frequencies much lower than T, we can approximate $\langle\xi_j(t_1)\xi_k(t_2)\rangle = 4gT\delta(t_1 - t_2)\delta_{jk}$. Then $\langle\eta_\phi(t_1)\eta_\phi(t_2)\rangle = \langle\eta_\theta(t_1)\eta_\theta(t_2)\rangle = (gT/S^2)\delta(t_1 - t_2)$. Thus, the situation is simple, and we reproduce Ref. [8].

In the quantum high-frequency domain, the situation is different. We cannot interpret the four independent fields ξ_n as representing the components of a fluctuating magnetic field. Solving Eqs. (10) for $\dot{\theta}$ and $\dot{\phi}$, we obtain (see Refs. [8,12])

$$\dot{\phi}_c - \tilde{B} = \frac{1}{\sin\theta_c}\xi_\phi,$$
$$\dot{\theta}_c + \sin\theta_c\tilde{g}\tilde{B} = \xi_\theta, \quad (12)$$

where $\xi_\phi \equiv [(\eta_\phi + \tilde{g}\eta_\theta)/(1 + \tilde{g}^2)]$ and $\xi_\theta \equiv [(\eta_\theta - \tilde{g}\eta_\phi)/(1 + \tilde{g}^2)]$ and $\tilde{B} \equiv [(\gamma B)/(1 + \tilde{g}^2)]$. A close inspection of these equations shows that in the regime of weak dissipation, $S \gg 1$ and $\tilde{g} \ll 1$, the spin can precess with frequency $\tilde{B}$ at an almost constant θ for a long time of order (shorter than) $(\tilde{g}\tilde{B})^{-1}$. For such times, we can approximate $\phi_c = \tilde{B}t$ and $\chi_c = (1 - \cos\theta_c)\phi_c = (1 - \cos\theta_c)\tilde{B}t$. Thus, the Langevin fields ξ_n in Eq. (11) are multiplied by fast oscillating cosines and sines with frequencies $\omega_c \equiv \tilde{B}\cos^2(\theta_c/2)$ and $\omega_s \equiv \tilde{B}\sin^2(\theta_c/2)$. Thus, [25]

$$\langle\eta_{\phi,\theta}(t_1)\eta_{\phi,\theta}(t_2)\rangle_{\omega=0} = \frac{g}{4S^2}[\cos^2(\theta_c/2)\alpha_K(\omega_c) + \sin^2(\theta_c/2)\alpha_K(\omega_s)]. \quad (13)$$

In the quantum regime $T \ll \tilde{B}$, these correlation functions differ substantially from the classical ones, $\langle\eta_\phi(t)\eta_\phi(t')\rangle_{\omega=0} = \langle\eta_\theta(t)\eta_\theta(t')\rangle_{\omega=0} = gT/S^2$. Thus, if the spin could be held on a constant θ trajectory for a long time, the diffusion would be determined by the quantum noise at frequencies ω_c and ω_s, which are governed by the geometric phase.

We are now ready to discuss the physical meaning of the semiclassical approximation, i.e., the expansion of the action [Eq. (9)] up to the second order in θ_q and ϕ_q (see also Supplemental Material C [19]). The nonexpanded action is periodic in both θ_q and ϕ_q. The periodicity in ϕ_q corresponds to the quantization of the z spin component $S_z = S\cos\theta_c$. By expanding, we restrict ourselves to the long time limit, in which S_z has already "jumped" many times by $\Delta S_z = 1$ in the course of spin diffusion. We neglect, thus, higher than the second cumulants of spin noise (see, e.g., Ref. [26] for similar discussion of charge noise). We obtain, however, a correct second cumulant with down-converted quantum noise (similar to shot noise in the charge sector). This is due to the "multiplicative noise" character of our Keldysh action [Eq. (9)] similar to the original AES case [15] (see also [27]).

Measurement protocol.—The simplest idea on how to observe the Langevin terms influenced by the Berry phase, would be to perform a Ramsey protocol [28] to measure dephasing. Unfortunately, this is not a viable option, as for $T \ll \tilde{B}$, the deterministic relaxation time $\tau_{\mathrm{rel}} \sim (\tilde{g}\tilde{B})^{-1} \sim S(g\tilde{B})^{-1}$ is much shorter than the characteristic diffusion time $\tau_{\rm diff} \sim S^2(g\tilde{B})^{-1}$. Thus, at the time at which substantial dephasing takes place, the spin is long at the north pole ($\theta = 0$). To circumvent this hurdle, we propose to implement a "bang-bang" protocol [6] as shown in Fig. 1 (See also Supplemental Material D [19]). In our protocol, we keep the spin at $\theta_c \approx \pm\theta_0$ for a long time. Thus, the diffusion will be determined by the noise [Eq. (13)] at $\theta_c = \theta_0$. More precisely, the spread of θ_c and ϕ_c (in the rotating frame) will be given by $(\Delta\theta)^2 = \sin^2\theta_0(\Delta\varphi)^2 = Dt$, where

$$D = (g/S^2)T_{\rm eff}, \tag{14}$$

and the effective temperature is calculated from Eq. (13) to be [29]

$$T_{\rm eff} = \frac{\tilde{B}}{2}\cos^4\left(\frac{\theta_0}{2}\right)\coth\left[\frac{\tilde{B}}{2T}\cos^2\left(\frac{\theta_0}{2}\right)\right] + \frac{\tilde{B}}{2}\sin^4\left(\frac{\theta_0}{2}\right)\coth\left[\frac{\tilde{B}}{2T}\sin^2\left(\frac{\theta_0}{2}\right)\right]. \tag{15}$$

At $T \gg \tilde{B}$, we obtain $T_{\rm eff} \approx T$, and the geometric effects are completely washed out. We are thus back to the classical regime of [8]. In the quantum regime, $T \ll \tilde{B}$, the effective temperature has a characteristic θ_0 dependence $T_{\rm eff} = (1/2)\tilde{B}[\cos^4(\theta_0/2) + \sin^4(\theta_0/2)]$ which, if measured, would provide a direct evidence in favor of the geometric noise derived in this Letter.

Summary and conclusions.—We have derived an SU(2) generalization of the AES effective action for a large spin. The latter gives rise to semiclassical LLG-Langevin equations with Langevin torques being influenced by geometric phases. We have proposed here a driving protocol that would allow us to observe geometric spin diffusion in the quantum regime. We envision our formalism being applied to a broad range of other problems, e.g., easy-axis spin switching, the line width associated with persistent precession in magnetic tunnel junctions [12], or transport in arrays of quantum dots [30].

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