Mesoscopic Stoner Instability in Open Quantum Dots: Suppression of Coleman-Weinberg Mechanism by Electron Tunneling

I. S. Burmistrov,[1,2] Y. Gefen,[3,4] D. S. Shapiro,[5,6,7] and A. Shnirman[8,4]

[1] *L. D. Landau Institute for Theoretical Physics, Akademika Semenova Avenue 1-a, 142432 Chernogolovka, Russia*
[2] *Laboratory for Condensed Matter Physics, National Research University Higher School of Economics, 101000 Moscow, Russia*
[3] *Department of Condensed Matter Physics, Weizmann Institute of Science, 76100 Rehovot, Israel*
[4] *Institut für Quantenmaterialien und Technologien, Karlsruhe Institute of Technology, 76021 Karlsruhe, Germany*
[5] *Department of Physics, National Research University Higher School of Economics, 101000 Moscow, Russia*
[6] *Dukhov Research Institute of Automatics (VNIIA), Moscow 127055, Russia*
[7] *V. A. Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Moscow 125009, Russia*
[8] *Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

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The mesoscopic Stoner instability is an intriguing manifestation of symmetry breaking in isolated metallic quantum dots, underlined by the competition between single-particle energy and Heisenberg exchange interaction. Here we study this phenomenon in the presence of tunnel coupling to a reservoir. We analyze the spin susceptibility of electrons on the quantum dot for different values of couplings and temperature. Our results indicate the existence of a "quantum phase transition" at a critical value of the tunneling coupling, which is determined by the Stoner-enhanced exchange interaction. This quantum phase transition is a manifestation of the suppression of the Coleman-Weinberg mechanism of symmetry breaking, induced by coupling to the reservoir.

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The physics of quantum dots (QDs) has been the focus of theoretical and experimental study for three decades [1–5]. A major breakthrough in this field was the introduction of the so-called universal Hamiltonian [6], rendering QDs as zero-dimensional objects. This is valid for metallic QDs, characterized by the Thouless energy being larger than the mean single-particle level spacing, $E_{\mathrm{Th}} \gg \delta$. The universal Hamiltonian comprises a charging energy term that leads to Coulomb blockade [7–11]. An additional term in the universal Hamiltonian is a ferromagnetic Heisenberg exchange term. Even relatively weak exchange interaction, $J \lesssim \delta/2$, seems to be important for a quantitative description of transport experiments in QDs at low temperatures, $T \lesssim \delta$ [12–15]. Moderate exchange, $\delta/2 \lesssim J < \delta$ [16], gives rise to "mesoscopic Stoner instability": the emergence of a finite (but nonextensive) value of the total electron spin S in the ground state of an isolated QD [6]. In the vicinity of the transition, $\delta - J \ll \delta$, the ground-state spin is estimated as $S = J_*/(2\delta) \gg 1$, where $J_* = J\delta/(\delta - J)$ denotes the Stoner-enhanced exchange interaction. At $J = \delta$ an extensive part of electron spins becomes polarized; i.e., a Stoner phase transition to a macroscopic ferromagnetic phase takes place. A nonzero value of S gives rise to a finite Curie spin susceptibility at low T [6,17–19]. Spin-charge coupling leads to signatures of the mesoscopic Stoner instability in electron transport through QDs [17,18,20,21].

The physics of the mesoscopic Stoner instability in an isolated QD is marked by total spin conservation. It is an example [19] of the Coleman-Weinberg mechanism for the emergence of spontaneous symmetry breaking [22]. Does the Coleman-Weinberg mechanism survive electron tunneling dynamics between the QD and the reservoir? Addressing this question is not straightforward, given the fact that spin conservation is then broken, resulting in a nontrivial dissipative dynamics of S [23,24]. Similar to the problems of a localized spin in an electronic environment [25–27] or that of an itinerant magnetization [28,29], the equation of motion for the total spin on the QD assumes the form of the Landau-Lifshitz-Gilbert-Langevin (LLGL) equation. We note in passing that in Refs. [23,24] the LLGL equation has been derived under the assumption that the tunneling between the QD and reservoir does not change the value of S.

The focus of this Letter is the mesoscopic Stoner physics in open quantum dots. We study how tunneling to the reservoir (assigning a broadening γ to the single-particle levels) affects the mesoscopic Stoner instability. Addressing the vicinity of the transition to the macroscopic Stoner phase, $\delta - J \ll \delta$, our analysis indicates the existence of the quantum phase transition (QPT) at a critical broadening strength, $\gamma_c \simeq J_*$ (see Fig. 1). The quantum critical point (QCP) separates the ordered ($\gamma < \gamma_c$) and the disordered ($\gamma > \gamma_c$) phases. The QPT occurs since tunneling to the reservoir modifies the Coleman-Weinberg (CW) potential and suppresses the spontaneous symmetry breaking at $\gamma > \gamma_c$. Our analysis relies on the study of the spin susceptibility χ of the electrons on the QD.

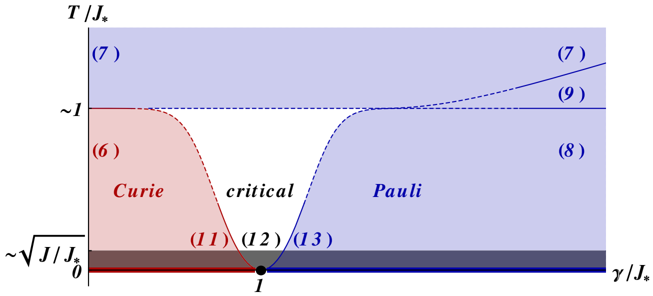

FIG. 1. A sketch of the phase diagram for the case $\delta - J \ll \delta$. The red color indicates the region with a Curie-type spin susceptibility above the zero-temperature ordered phase. The blue color indicates the regions with the Pauli-type spin susceptibility. The black dot indicates the position of the QCP. The thick, solid red (blue) line corresponds to the zero-temperature ordered (disordered) phase. The thin, solid curves correspond to the crossovers discussed in the text. The dashed lines are guides for the eye. A latin number indicates the equation for the corresponding region of the phase diagram. Black shaded region at the bottom marks the region above which our theory is applicable.

Model.—A metallic QD tunnel coupled to a reservoir is described by the following Hamiltonian: $H = H_d + H_r + H_t$. The effective Hamiltonian for a disordered metallic QD with large Thouless conductance is $H_d = H_0 + H_s$ [6], where $H_0 = \sum_{\alpha,\sigma} \epsilon_\alpha d^\dagger_{\alpha,\sigma} d_{\alpha,\sigma}$ is the free electron part and $H_s = -J\boldsymbol{S}^2$ takes into account the exchange interaction on the QD [30]. The free electrons in the reservoir are governed by the $H_r = \sum_{k,\sigma} \epsilon_k a^\dagger_{k,\sigma} a_{k,\sigma}$. The Hamiltonian $H_t = \sum_{k,\alpha,\sigma} t_{k\alpha} a^\dagger_{k,\sigma} d_{\alpha,\sigma} + \mathrm{H.c.}$ describes a multichannel tunneling junction between the QD and the reservoir with a small dimensionless (in units e^2/h) tunneling conductance of each channel. The total dimensionless tunneling conductance of the junction g is assumed large. This assumption allows us to neglect the Coulomb blockade effects associated with the charging energy term in the "universal" Hamiltonian [6]. Here ϵ_α, ϵ_k denote the energies of single-particle levels on the QD and in the reservoir, respectively, counted from the chemical potential. The operators $d^\dagger_{\alpha,\sigma}$, $a^\dagger_{k,\sigma}$ ($d_{\alpha,\sigma}$, $a_{k,\sigma}$) create (annihilate) an electron on the QD and the reservoir, respectively. $\boldsymbol{S} = \sum_{\alpha\sigma\sigma'} d^\dagger_{\alpha,\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} d_{\alpha,\sigma'}/2$ stands for the operator of the total electron spin in a QD. The vector $\boldsymbol{\sigma} = \{\sigma_x, \sigma_y, \sigma_z\}$ comprises the three Pauli matrices.

In order to address H_s we employ the Hubbard-Stratonovich transformation, introducing the bosonic vector field $\boldsymbol{\Phi}$. Integrating out fermions, we obtain an effective action in the imaginary time,

$$S = \frac{1}{4J}\int_0^\beta d\tau \boldsymbol{\Phi}^2 - \mathrm{Tr}\,\ln\left(-\partial_\tau - \hat{\epsilon} + \frac{1}{2}\boldsymbol{\sigma\Phi} - \hat{\Sigma}\right). \quad (1)$$

Here $\beta = 1/T$, $\hat{\epsilon}_{\alpha\alpha'} = \epsilon_\alpha \delta_{\alpha\alpha'}$, and $\hat{\Sigma}_{\alpha\alpha'} = \sum_k t^*_{\alpha k}(-\partial_\tau - \epsilon_k)^{-1} t_{k\alpha'}$ is the self-energy induced by the tunneling to the reservoir. In what follows, we neglect the mesoscopic fluctuations in the tunneling amplitudes $t_{k\alpha}$ and approximate the self-energy as $\hat{\Sigma}_{\alpha\alpha'}(i\varepsilon_n) = -i(\gamma/\pi)\mathrm{sgn}\varepsilon_n \delta_{\alpha\alpha'}$. Here $\varepsilon_n = \pi T(2n+1)$, and $\pi^2 \sum_k |t_{k\alpha}|^2 \delta(\epsilon_k) \to \gamma$ characterizes the uniform broadening of a single-particle level on the QD [31]. It is related to the tunneling conductance of the junction through $g = 4\gamma/\delta$. The spin susceptibility of electrons on the QD can be computed as [19]

$$\chi = \frac{T}{12J^2}\left\langle \left| \int_0^\beta d\tau \boldsymbol{\Phi} \right|^2 \right\rangle - \frac{1}{2J}, \quad (2)$$

where the averaging is carried out with respect to the action (1).

Wei-Norman-Kolokolov trick.—In order to proceed further, one needs to be able to compute the Tr ln in the action (1). A solution of this complicated problem requires the knowledge of the matrix $U(\tau) = \mathcal{T}_\tau \exp[\int_0^\tau d\tau' \boldsymbol{\sigma\Phi}(\tau')/2]$, where $\mathcal{T}_\tau$ denotes the time ordering along the imaginary time contour. For an arbitrary trajectory $\boldsymbol{\Phi}(\tau)$, direct evaluation of $U(\tau)$ is impossible. It is possible, though, to perform a transformation in the functional integral from the variables $\boldsymbol{\Phi}$ to new variables ρ, κ, and $\tilde{\kappa}$ [33–38]: $\Phi_z = \rho - 2\kappa\tilde{\kappa}$, $\Phi_- = \tilde{\kappa}$, and $\Phi_+ = \partial_\tau \kappa + \kappa\rho - \kappa^2\tilde{\kappa}$, where $\Phi_\pm = (\Phi_x \pm i\Phi_y)/2$. While ρ is the degree of freedom related to the length of $\boldsymbol{\Phi}$, κ and $\tilde{\kappa}$ describe small rotations of $\boldsymbol{\Phi}$. The Jacobian of this transformation is equal to $\exp(\beta h)$, where $h = T\int_0^\beta d\tau \rho(\tau)/2$ is a half of the zeroth Matsubara harmonics of $\rho(\tau)$ [38]. This transformation is supplemented by the initial condition $\kappa(0) = 0$, which guarantees $U(0) = 1$. The 2×2 matrix $U(\tau)$ can be written explicitly in terms of new variables ρ, κ, and $\tilde{\kappa}$ [39].

Coleman-Weinberg potential.—As is known from studies of the mesoscopic Stoner phase in an isolated QD [18,19], the zeroth Matsubara harmonics of $\rho(\tau)$ plays the role of an order parameter. Therefore, our strategy is to derive the effective free energy for h by integrating out the fluctuations with nonzero Matsubara frequency components in the action (1). We thus split the field ρ as $\rho(\tau) = 2h + \delta\rho(\tau)$ and integrate over $\delta\rho$, κ, and $\tilde{\kappa}$ within the Gaussian approximation. We then obtain the following free energy (CW potential) (see Supplemental Material for details [40])

$$F(h) = \frac{h^2}{J_*} - h + 2T\,\mathrm{Re}\,\ln\frac{\Gamma(1 + \frac{ih}{\pi T} + \frac{\gamma}{\pi^2 T})}{\Gamma(1 + \frac{ih}{\pi T})\Gamma(1 + \frac{\gamma}{\pi^2 T})}. \quad (3)$$

Here $\Gamma(z)$ is the Gamma function. The origin of different terms in the expression for $F(h)$ is the following. The first term on the rhs of Eq. (3) is the sum of two contributions, h^2/J and $-h^2/\delta$. The former comes from the first term on the rhs of Eq. (1), whereas the latter is a paramagnetic part of the thermodynamic potential of free electrons in the presence of a constant magnetic field $2h$. The second term on the rhs of Eq. (3) is an equivalent of the CW potential, as can be concluded from [19]. In the calculation based on the Wei-Norman-Kolokolov transformation, it appears from the Jacobian of that transformation. The third term of Eq. (3) is the result of integration over dynamical fluctuations of κ and $\tilde{\kappa}$, which are coupled to h in the presence of nonzero tunneling. We thus observe that tunneling to the reservoir indeed modifies the form of the CW potential.

The Gaussian approximation for integration over dynamical fluctuations is justified under the conditions [40]

$$|h|, T \gg \max\{J, \min\{J_*, \sqrt{J\gamma}\}\}. \qquad (4)$$

Instead of working with the full action (1), we can now use $F(h)$ for the purpose of analyzing the spin susceptibility. Under conditions (4), expression (2) can be simplified to

$$\chi = \frac{1}{3TJ^2}\int_{-\infty}^{\infty} dh h^2 e^{-\beta F(h)} \Big/ \int_{-\infty}^{\infty} dh e^{-\beta F(h)}. \qquad (5)$$

An isolated QD.—Before turning to the analysis of an open system, it is instructive to recover the CW potential (3) for the case of an isolated QD. For $\gamma = 0$, $F(h)$ possesses a minimum at $h = J_*/2$. At low temperatures, $T \ll J_*$, this minimum is narrow and Eqs. (3) and (5) yield the Curie law for the spin susceptibility: $\chi = J_*^2/(12TJ^2)$ [6,17,18]. At high temperatures, $T \gg J_*$, the minimum at $h = J_*/2$ becomes shallower. The thermal fluctuations then determine the typical value of $h \sim \sqrt{TJ_*}$. Equations (3) and (5) reproduce correctly the Pauli-type spin susceptibility, known from the exact solution [6,17,18]. We find from the CW potential (3) that $\chi = cJ_*/J^2$ with $c = 1/6$. The exact solution, however, yields the value of $c = 1/2$. Such a discrepancy in the prefactor arises since the free energy (3) reproduces the Gibbs weight $\exp[-\beta F(h)]$ up to a multiplicative prefactor proportional to h, which is irrelevant for the subsequent analysis.

Weak tunneling regime, $\gamma \ll J_*$.—We next analyze the spin susceptibility in the regime of weak tunneling, $\gamma \ll J_*$. Then the situation is similar to the case of an isolated QD. The free energy $F(h)$ has its minimum at $h = J_*[1 - 4\gamma/(\pi^2 J_*)]/2$ (see Fig. 2). At $T \ll J_*$ this minimum is narrow and Eq. (5) yields the Curie law

$$\chi \sim (J_*/J)^2[1 - 8\gamma/(\pi^2 J_*)]/T. \qquad (6)$$

At $T \gg J_*$ the CW potential $F(h)$ has the shallow minimum at $h = J_*[1 + \psi''(1)\gamma J_*/(\pi^4 T^2)]/2$. Here $\psi(z)$ denotes the digamma function. Then the typical value of h is dominated by the thermal fluctuations, which are of the order of $\sqrt{TJ_*[1 + \psi''(1)\gamma J_*/(\pi^4 T^2)]}$. Hence, at $T \gg J_*$ we find the Pauli-type spin susceptibility

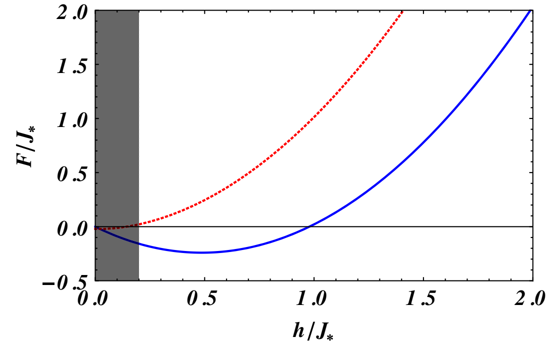

FIG. 2. The approximate free energy as given by Eq. (3) for $\gamma \ll J_*$ (blue solid curve) and $\gamma \gg J_*$ (red dotted curve). We note that Eq. (3) may not reflect correctly the behavior of the free energy for small h indicated by the black shaded region [see Eq. (4)].

$$\chi \sim J_*[1 + \psi''(1)\gamma J_*/(\pi^4 T^2)]/J^2. \qquad (7)$$

Therefore, in the weak tunneling regime, $\gamma \ll J_*$, the dependence of the spin susceptibility on temperature is qualitatively the same as in the case of an isolated QD.

Strong tunneling regime, $\gamma \gg J_*$.—For a strong tunneling, $\gamma \gg J_*$, the CW potential (3) has the minimum whose position depends on temperature. At low temperatures, $T \ll J_*$, the minimum of $F(h)$ is at $h = 0$ (see Fig. 2). The spin susceptibility then is determined by the thermal fluctuations of h, which are of the order of $\sqrt{TJ_*(1 + J_*/\gamma)}$. Thus, for $T \ll J_*$, we find

$$\chi \sim J_*(1 + J_*/\gamma)/J^2. \qquad (8)$$

For intermediate temperatures, $J_* \ll T \ll \sqrt{J_*\gamma}$, the free energy (3) has a shallow minimum at $h = J_*[1 - J_*/(6T)]$. h is typically of the order of $\sqrt{T[1 - J_*/(6T)]}$. Then the spin susceptibility is given by

$$\chi \sim J_*[1 - J_*/(6T)]/J^2. \qquad (9)$$

Finally, at $T \gg \sqrt{J_*\gamma}$ the behavior of the CW potential is similar to the one for weak tunneling and high temperatures, $T \gg J_*$. It follows that the spin susceptibility at $T \gg \sqrt{J_*\gamma}$ is given by Eq. (7).

Quantum phase transition.—The above analysis demonstrates that at low temperatures, $T \ll J_*$, the minimum of $F(h)$ at nonzero value of h survives at weak tunneling, $\gamma \ll J_*$, but disappears at strong tunneling, $\gamma \gg J_*$. This suggests the existence of the QPT at $\gamma = \gamma_c \sim J_*$. At $\gamma < \gamma_c$, there is a broken symmetry phase with a nonzero order parameter $\Delta = \lim_{T\to 0} T\chi$. For $\gamma > \gamma_c$, the symmetry is restored such that $\Delta = 0$.

In order to further substantiate the existence of a QPT we now consider the low-temperature regime, $T \ll \gamma \sim J_*$. One can show that, pushing toward the vicinity of the

QCP, the relevant values of h lie within the range $T \ll h \ll \gamma$. Taking the limit $h, \gamma \gg T$ in Eq. (3) and then expanding in h/γ to the fourth order, we obtain

$$F(h) \simeq (1/J_* - 1/\gamma)h^2 + \pi^2 h^4/(6\gamma^3). \qquad (10)$$

Taking this expression for $F(h)$ literally at $T = 0$ may suggest that there is indeed a QCP at $\gamma_c = J_*$. We recall, though, that setting the temperature to zero is not allowed in view of the inequality (4). Our strategy to detect the presence of the QCP will be to sweep γ near $\gamma_c \simeq J_*$ at the lowest possible temperature, $T \simeq \sqrt{JJ_*}$. We note that Eq. (10) resembles the standard form of the Landau free energy with h playing the role of the order parameter. We stress, though, that unlike the Landau free energy, which is valid only for small values of the order parameter, here Eq. (10) is valid for the entire interval $T \simeq \sqrt{JJ_*} \ll h \ll J_*$.

The form (10) of the CW potential implies a scaling form of the spin susceptibility $\chi = \sqrt{J_*^3/T} f(T_X/T)/J^2$, with a characteristic temperature scale $T_X = J_* \alpha^2$, and $\alpha = \gamma_c/\gamma - 1$. Notwithstanding the fact that we cannot determine the precise form of the scaling function $f(X)$, as we know $\exp[-\beta F(h)]$ only with exponential accuracy, Eq. (10) suffices for the evaluation of the asymptotic behavior of $f(X)$.

For $\gamma < \gamma_c$, the free energy (10) has its minimum at $h = J_* \sqrt{3\alpha}/\pi$. Then, at sufficiently low temperatures and away from the QCP, $T \ll T_X$, we can treat the thermal fluctuations around the minimum as being weak. We then find

$$\chi \sim J_*^2 \alpha/(TJ^2), \qquad T \ll T_X. \qquad (11)$$

At high temperatures, $J_* \gg T \gg T_X$, the typical value of h due to the thermal fluctuations is dictated by the quartic term in Eq. (10): $h \sim (TJ_*^3)^{1/4}$. Since this value of h is within the range $T \ll h \ll J_*$, the use of Eq. (10) is justified. Using Eq. (5), we obtain

$$\chi \sim J_*^{3/2}/(J^2 T^{1/2}), \qquad T_X \ll T \ll J_*. \qquad (12)$$

For $\gamma > \gamma_c$, the free energy (10) has a minimum at $h = 0$. Then, at low enough temperatures, $T \ll T_X$, and away from the quantum critical point, the quadratic term dominates over the fourth-order term in Eq. (10). Thus, the typical value of h due to the thermal fluctuations is given by $h \sim \sqrt{TJ_*/\alpha}$. Hence, the spin susceptibility reads

$$\chi \sim J_*/(3J^2|\alpha|), \qquad T \ll T_X. \qquad (13)$$

At higher temperatures, $J_* \gg T \gg T_X$, the spin susceptibility is given by Eq. (12).

For $\gamma < \gamma_c$, cf. Eq. (11), the spin susceptibility exhibits the Curie-type behavior at $T \ll T_X$, with the effective spin $\propto J_* \sqrt{\alpha}/J$. The latter decreases as the QCP is approached. For $\gamma > \gamma_c$, cf. Eq. (13), the spin susceptibility at $T \ll T_X$ has the Pauli form with the effective exchange $\propto J_*/|\alpha|$ diverging at the QCP. At high temperatures $T \gg T_X$, cf. Eq. (12), the spin susceptibility has a critical behavior, $\chi \propto 1/\sqrt{T}$, which is neither Curie- nor Pauli-like. Thus, the overall behavior of the spin susceptibility at low temperature is typical for the vicinity of a QCP (see Fig. 1).

Since the range of validity of our analysis is limited from below by the temperature $T \simeq \sqrt{JJ_*}$, we can determine the position of the QCP only with a limited accuracy: $\gamma_c = J_*\{1 + O[(J/J_*)^{1/4}]\}$. This indicates that our theory becomes asymptotically exact as the system is approaching the bulk Stoner transition at $J = \delta$.

Discussion.—In Ref. [23], it has been demonstrated that electron tunneling between the QD and the reservoir in the mesoscopic Stoner regime induces a Gilbert damping term $g/(4\pi S)$ in the LLGL equation. Our present results imply that the LLGL equation of Ref. [23] applies to not-too-large values of the conductance, $g \lesssim g_c = 8S$. We note that the QCP corresponds to a value of the Gilbert damping of the order unity.

Recalling the mesoscopic Stoner phase for an isolated QD, it is marked by a nonzero value of the total spin in the ground state. This is the case for a finite interval of $J < \delta$. A state with a given value of the total spin S is separated by QPTs [at $J = \delta(2S \pm 1)/(2S + 1 \pm 1)$] from states with spin $S \pm 1$. One important implication of our analysis is that the presence of a very weak tunneling, $\gamma \ll \delta$, does not destroy these transitions. We expect that the lines of these QPTs in the J/δ, γ/δ parameter space terminate at $\gamma \sim \delta$ [40].

The universal Hamiltonian involves also a term with a Cooper channel interaction. This term represents superconducting correlations in the QDs [41–47]. Throughout our analysis we have assumed the absence of bare attraction, hence we have disregarded this Cooper channel interaction. Moreover, we have also neglected the effect of fluctuations in the matrix elements of the interaction [48,49]. These corrections are typically small in the regime $\delta/E_{\mathrm{Th}} \ll 1$, but may still be responsible for interesting physics beyond the universal Hamiltonian paradigm [5].

Another effect we have not considered here is the fluctuations of single-particle levels on the QD. Such fluctuations are particularly important in the case of Ising exchange interaction. For the latter, assuming equidistant quasiparticle spectrum, the phenomenon of the mesoscopic Stoner instability is completely absent [6]. The universal Hamiltonian with an Ising exchange is realizable in the limit of a strong spin-orbit coupling [50–54]. Considering an Ising exchange and an equidistant single-particle spectrum, the electron spin susceptibility is Pauli-like for all temperatures [20,55]. Accounting for single-particle level fluctuations (e.g., due to the presence of static disorder in the QD), a mesoscopic Stoner phase does exist for an isolated dot, with an averaged spin susceptibility yielding a Curie-type behavior at low temperatures [6,56,57]. In this case, one might expect the emergence of a QPT at a certain value of level broadening (tunnel coupling to external reservoirs), similar to the case of Heisenberg exchange studied here.

Finally, our results are amenable to experimental verification, employing a single electron box based on nanoparticles made up of materials with parameters close to the Stoner instability. There is a host of such nearly ferromagnetic materials [58–65]. Promising candidates are the compounds YFe_2Zn_{20} ($J = 0.88\delta$) and $LuFe_2Zn_{20}$ ($J = 0.89\delta$) [66,67].

Summary.—We have studied here the mesoscopic Stoner instability in open QDs, coupled to external fermionic reservoirs. We have developed a detailed theory for the regime close to the macroscopic Stoner instability, $0 < \delta - J \ll \delta$. The resulting temperature dependence of χ suggests the existence of a QPT at a critical value of the tunneling broadening, $\gamma_c = J_*$. This transition as function of the tunnel coupling strength is between the symmetry broken phase with nonzero value of the total spin in the ground state and spin-symmetric phase. The smoking gun evidence for the QPT is the electron spin susceptibility, switching between Curie and Pauli behaviors. This QPT (and the onset of the symmetry-conserved phase) marks the suppression of the Coleman-Weinberg mechanism of symmetry breaking by tunnel coupling to the reservoir.

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