Interaction-Induced Metallicity in a Two-Dimensional Disordered Non-Fermi Liquid

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The interplay of interactions and disorder in two-dimensional (2D) electron systems has actively been studied for decades. The paradigmatic approach involves starting with a clean Fermi liquid and perturbing the system with both disorder and interactions. Instead, we start with a clean non-Fermi liquid near a 2D ferromagnetic quantum critical point and consider the effects of disorder. In contrast with the disordered Fermi liquid, we find that our model does not suffer from runaway flows to strong coupling and the system has a marginally stable fixed point with perfect conduction.

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Despite enormous progress [1–11], the possible ground states of two dimensional (2D) interacting, disordered electron systems remain largely unexplored. In agreement with experimental observations [12–22], the theory of disordered Fermi liquids (FL) [23–28] suggests that, in some cases, interactions can stabilize 2D metallic behavior at low temperatures (T), while the noninteracting counterparts remain fully localized in 2D [29]. However, the ultimate understanding of experiments requires well-controlled theories of strong interactions and disorder, representing a fundamental challenge.

In the theory of 2D disordered FLs, metallic behavior occurs near a strong coupling fixed point, marking the onset of a magnetic instability [24,25]. This instability has been interpreted as indicating either the formation of local moments [25,30–32] or ferromagnetism [33–36]. At present, the strong coupling fixed point and the associated metallicity remain poorly understood. Experimental studies of 2D systems have revealed an enhancement of electron spin susceptibility [15,37–43] and the existence of spin droplets [44–47] in the metalliclike regime. Both theory and experiment call for an alternative approach in which magnetic fluctuations are treated beyond a mean-field approximation.

Close to the ferromagnetic ordering, FL breaks down via scattering of fermions off soft magnetic fluctuations, leading to a "non-Fermi liquid" (NFL) [48–50]. Within a phenomenological approach, the 2D NFL with vanishing density of states (DOS) at the chemical potential is stable to localization and remains a perfect conductor [51], in agreement with general scaling arguments [52,53]. Much less is known about the effect of disorder on 2D NFLs having nonzero DOS [60,61].

In this Letter, we study disorder effects near a metallic quantum critical point at which singular effects of interactions lead to a magnetic instability. Since the strong interactions require additional control parameters, we start with a recently studied tractable large N limit of a 2D NFL, which involves fermions coupled to quantum critical "magnetic" fluctuations [62]. Assuming that the characteristic energy scales of NFL behavior and diffusion are well separated, we incorporate the absence of quasiparticles already at the saddle-point level and study the combined effects of residual interactions and disorder by means of the renormalization group (RG) (see Fig. 1).

In contrast to the disordered FL, we find unconventional dynamical scaling of the diffusion propagator (diffuson) inherited from the NFL. Moreover, short-range interactions are irrelevant in the RG sense in our theory, and runaway flows to strong coupling disappear. The only remaining source of IR divergences is the small momentum scattering mediated by the diffusive Landau damped magnetic order parameter, which ultimately sets the new dynamical scaling $z_d = 4$ for diffusons below a certain energy scale. As a

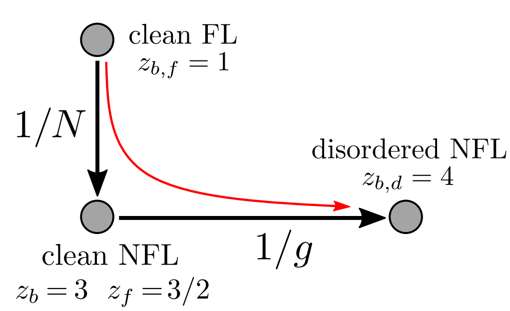

FIG. 1. Schematic RG flow for a disordered 2D fermionic system interacting with the magnetic order parameter fluctuations. The intermediate clean NFL fixed point is unstable to disorder and ultimately leads to a dirty fixed point with the dynamical scaling $z = 4$.

result, the coupling between diffusons and the magnetic fluctuations vanishes under the RG, giving way to a well-controlled fixed point with perfect conduction.

Summary of results.—We consider a Euclidean Lagrangian density $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm dis}$ which involves a 2D system of fermions ψ at a finite density interacting with a critical magnetic collective mode ϕ

$$\mathcal{L}_0 = tr[(\partial_\tau\phi)^2 + (\nabla\phi)^2] + \bar{\psi}_j[\partial_\tau + \xi(i\nabla)]\psi^j,$$
$$\mathcal{L}_{\rm int} = \frac{\lambda}{\sqrt{N}}\phi^j_k\bar{\psi}_j\psi^k, \qquad \mathcal{L}_{\rm dis} = \frac{1}{\sqrt{N}}V(x)\bar{\psi}_j\psi^j, \tag{1}$$

where $\xi(p) = p^2/2m - \mu$. Here, m denotes the fermion mass and μ is the chemical potential. This model has a global $\mathrm{SU}(N)$ flavor symmetry, with ψ_i and ϕ^i_j transforming in the fundamental and adjoint representations, respectively. A random potential $V(x)$ coupled to the fermionic density has a Gaussian distribution with the zero mean and a variance $\langle V(x)V(x')\rangle = (2\pi\nu\tau)^{-1}\delta(x - x')$. Here, ν is the DOS per flavor and τ is the mean free time. We assume that $1/N$ and $1/g$ (where g is the dimensionless Drude conductivity per flavor measured in units e^2/h) are the only expansion parameters of the model. In order to distinguish the interaction-induced effects from localization corrections, we follow [63] and consider a situation in which the Cooper channel is suppressed by a small time-reversal and parity breaking field.

The low-temperature behavior of theory (1) is governed by the RG equations derived at the leading order in $1/N$ and $1/g$, with no restrictions on the strength of the Yukawa coupling λ, cf. Eq. (8). They exhibit an IR marginally stable fixed point at $1/g = \lambda = 0$ with the following features: (1) The large-N low-energy quantum dynamics is set by $\sim N^2$ multiplet diffusons with dynamical scaling $z_d = 4$. (2) The average fermionic DOS $\nu(E)$ diverges very weakly at sufficiently small energies, $|E| \ll \Lambda_4$

$$\nu(E) \simeq \nu_0 \exp\{\alpha \ln^2[\ln(\Lambda_4/|E|)]\}, \tag{2}$$

where $\alpha \approx 0.104$ is the universal exponent and ν_0 is the density of states per one flavor at the emergent energy scale Λ_4 below which the system flows to the fixed point. (3) The conductivity diverges as $T \to 0$, in a slow logarithmic manner with the universal exponent $s \approx 0.704$

$$g(T) \simeq g_0^{1-s}\zeta_0^s \ln^s(\Lambda_4/T), \qquad T \ll \Lambda_4. \tag{3}$$

Here, g_0 and ζ_0 are the conductance and dimensionless interaction strength (see below) at the energy scale Λ_4. (4) N^2 bosonic modes with dynamical scaling $z_b = 4$ are responsible for the anomalous temperature dependence of the specific heat at $T \ll \Lambda_4$, $c_v \sim T^{2/z}$ with $z = 4$.

We now turn to the detailed description of these results.

An intermediate clean fixed point.—We begin with the UV limit, where both $\mathcal{L}_{\rm dis}$ and $\mathcal{L}_{\rm int}$ are relevant perturbations, so we are free to take into account, first, the cubic Yukawa interaction before introducing any effects of disorder. The large–N solution in the clean limit stems from the coupled set of Schwinger-Dyson equations. The vertex corrections can be neglected at large N. Under these conditions, the clean large-N solution immediately leads to two crucial effects [62]. At first, the boson self-energy is dominated by the Landau damping, $\Pi(\omega_n, q) = \gamma|\omega_n|/(Nq)$. Here, q is the momentum, $\omega_n = 2n\pi T$ is the bosonic Matsubara frequency, and $\gamma = \nu\lambda^2/2v$ with $v = \sqrt{2m\mu}$. Second, fermions become dressed into a NFL, with a self-energy correction $\Sigma_f(\varepsilon_n) = i\beta N^{1/3}|\varepsilon_n|^{2/3}\mathrm{sgn}\varepsilon_n$ where $\beta \propto \lambda^{4/3}\mu^{-1/3}$ and $\varepsilon_n = (2n+1)\pi T$ stands for the fermionic Matsubara frequency. $\Sigma_f(\varepsilon_n)$ is parametrically larger than the bare $i\varepsilon_n$ term at low energies. As a result, the clean interacting fixed point is described by the effective Lagrangian density $\mathcal{L}_{\rm eff} = \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{\rm int}$ where

$$\mathcal{L}_f = \bar{\psi}_{j,\varepsilon_n}[i\beta N^{1/3}|\varepsilon_n|^{2/3}\mathrm{sgn}\varepsilon_n - \xi(i\nabla)]\psi^j_{\varepsilon_n},$$
$$\mathcal{L}_b = tr\left[\phi_{\omega_n}(q)\left(q^2 + \frac{\gamma}{N}\frac{|\omega_n|}{q}\right)\phi_{-\omega_n}(-q)\right]. \tag{4}$$

The dynamical exponents are $z_b = 3$ for the boson, and $z_f = 3/2$ for the fermion (where z_f is defined with respect to the momentum component perpendicular to the Fermi surface). In addition, various symmetry-allowed interactions, such as ϕ^4 and four-Fermi forward scattering, become irrelevant, and only the Yukawa coupling λ remains marginal. Further analysis beyond the planar limit reveals that there are no leading $1/N$ logarithmic corrections to (4) which can potentially destabilize the fixed point at large but finite N [64].

Disorder at large N.—Our next step is to reintroduce disorder $\mathcal{L}_{\rm dis}$ as a relevant perturbation at the one-loop Lagrangian (4). Diagrammatically, first, we dress the fermion propagator by noncrossing impurity lines within the self-consistent Born approximation, $G(\varepsilon_n, p) = \{i[\beta N^{1/3}|\varepsilon_n|^{2/3} + (2N\tau)^{-1}]\mathrm{sgn}\varepsilon_n - \xi(p)\}^{-1}$. One might expect the disorder-induced lifetime to dominate at low energies, rendering the NFL frequency dependence insignificant. However, the actual low-energy gapless degrees of freedom of the disordered system are particle-hole excitations dressed with impurity ladders (diffusons), see Fig. 2(a). In our case, they acquire anomalous dynamical scaling $z_d = 3$ set by incoherent fermionic dynamics at low momenta, $q \ll (Nv\tau)^{-1}$, and frequencies, $|\varepsilon_n, \varepsilon'_n| \ll (N^{4/3}\beta\tau)^{-3/2}$: $\mathcal{D}_{\varepsilon_n,\varepsilon'_n}(q) = [NDq^2 + N^{1/3}\beta(|\varepsilon_n|^{2/3} + |\varepsilon'_n|^{2/3})]^{-1}$, where $D = v^2\tau/2$ is the diffusion constant. As a result, the diffusion pole in a particle-hole propagator (if analytically continued to real frequencies) is replaced by an effective IR-divergent energy relaxation time, which is, in some

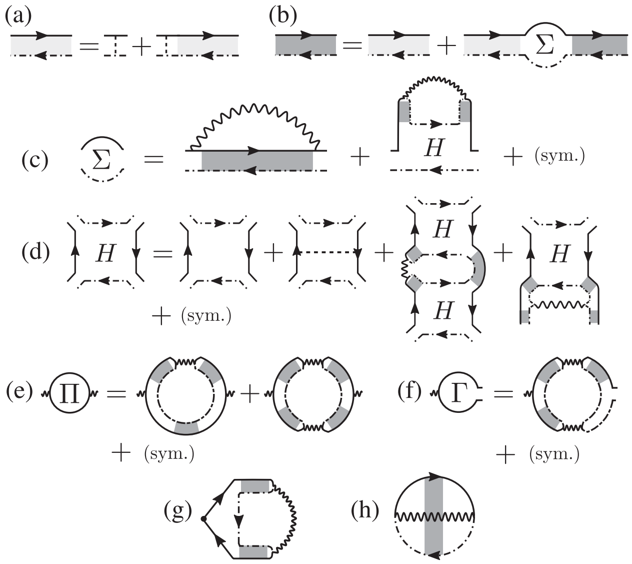

FIG. 2. The shaded rectangular stands for a bare diffuson (a). The impurity line is denoted by dashed line, and the retarded (advanced) single-particle fermionic propagator is denoted by solid (dash-dotted) line. Dark grey rectangular area represents the diffuson (b) with self-consistent self-energy due to interaction shown in (c). Wavy solid line represents the dynamically screened bosonic propagator. White rectangular area with symbol H stands for the Hikami box which acquires self-consistent corrections (d). Leading in $1/N$ loop corrections to the bosonic self-energy are shown in (e) and the corrected boson-diffuson vertex is presented in (f). Representative diagrams for the tunneling density of states and the free energy are depicted in (g) and (h), respectively.

sense, very similar to the spin-flip time for triplet diffusons in the presence of dynamical magnetic impurities [65–72].

The fate of residual interactions can now be naturally formulated in terms of anomalous diffusons. The appropriate field-theoretical description is given by the analog of the Finkel'stein nonlinear sigma model (FNLSM) which incorporates the nontrivial dynamical scaling already at the saddle point level. The details of this FNLSM approach are given in the Supplemental Material [73]. Below, we derive the same results within the standard fermionic perturbation theory.

There are three major possible interaction effects, controlled by two small parameters, $1/N$ and $1/g$. At first, the diffusons (which, by definition, include only processes with both particle and hole energies independently conserved) can be dressed by the self-energy effects via the Dyson equation [Fig. 2(b)]. Selection rules for both small parameters single out only diagrams of Fig. 2(c). Second, the bosonic propagator can also acquire corrections via the fermionic polarization operator. The simplest RPA-type resummation results in the form $[D_n^\phi(q)]^{-1} = q^2 + (2\beta)^{-1}\nu g^2|\omega_n|^{1/3}\mathcal{F}_{2/3}(Dq^2/\beta|\omega_n|^{2/3})$ with the scaling function $\mathcal{F}_\kappa(x) = \int_0^1 dt[x + t^\kappa + (1-t)^\kappa]^{-1}$. At large N, the one-loop corrections to $D_n^\phi(q)$ are limited to two diagrams of Fig. 2(e) only. Finally, various vertex functions [74] can also be renormalized by interactions. The only surviving diagram for quadratic boson-diffuson coupling is shown in Fig. 2(f), and the leading corrections to "self-interactions" between diffusons are depicted in Fig. 2(d).

Self-consistent solution.—The problem at hand very closely resembles the clean case, with diffusons playing a role similar to that of the fermions in the clean limit. We see this explicitly when we compute the one-loop diffuson self-energy diagrams [Fig. 2(c)] in a self-consistent way at the leading order in $1/(Ng)$, while ignoring higher order corrections (which turn out to be logarithmically divergent, so we will come back to them later). It is also worth noting that singular Hartree-type diagrams are absent because ϕ is traceless, and other processes involving large momentum transfer are either $1/N$ suppressed or irrelevant due to the anomalous dynamical scaling.

As in the clean case [75], the possibility of obtaining a controllable self-consistent solution dramatically depends on how the large-N and low-energy limits are simultaneously taken. We follow the procedure introduced in [62] and rescale the bosonic and fermionic fields, momenta, and temperature as $\{\phi, \psi, \bar{\psi}, q, T\} \to \{N^2\phi, N^{3/4}\psi, N^{3/4}\bar{\psi}, q/N, T/N^2\}$ with new $q, T \sim \mathcal{O}(N^0)$. Then, the rescaled diffuson and bosonic propagator are free from any factors of N, and all N–dependence appears only in the vertices. We show [73] that, within this rescaling, none of the irrelevant operators are enhanced by a positive power of N. Physically, the rescaling procedure implies such a hierarchy of energy scales, when NFL effects (associated with γ/N) take place at energies higher than the onset of the diffusive regime [controlled by $1/(N\tau)$].

After the N–rescaling, the self-consistent solution for the diffuson propagator, see Fig. 2(b), takes the form

$$[\mathcal{D}_{\varepsilon_n,\varepsilon_n'}(q)]^{-1} = Dq^2 + \beta_4(|\varepsilon_n|^{1/2} + |\varepsilon_n'|^{1/2}), \qquad (5)$$

where $\beta_4 = \sqrt{(v\gamma/D)}/(4\pi\nu)$. Here, the frequency dependence $\sim|\varepsilon_n|^{1/2}$ comes from the self-energy correction to diffusion due to Yukawa interaction. This frequency dependence overshadows the bare frequency dependence of the diffuson $\sim|\varepsilon_n|^{2/3}$ for energies below the emergent scale $\Lambda_4 = (v\gamma/\beta^2 D)^3$. In contrast, we did not find any nonanalytic corrections to the bare momentum dependence $\sim q^2$ at the same order $\mathcal{O}(1/\sqrt{g})$, and thus, the IR dynamical scaling of the diffuson becomes $z_d = 4$.

The scaling (5) is "self-consistent" in the following sense: if one feeds this new diffuson back to the boson via quadratic vertices, then the dynamical scaling of the boson remains the same $z_b = 4$,

$$[D_n^\phi(q)]^{-1} = c^2q^2 + \frac{v\gamma|\omega_n|^{1/2}}{\beta_4}\mathcal{F}_{1/2}\left(\frac{Dq^2}{\beta_4|\omega_n|^{1/2}}\right), \qquad (6)$$

where $\mathcal{F}_{1/2}(x) = 2 - (\pi x/2) + (2(x^2-1)/\sqrt{x^2-2}) \times \mathrm{arccot}(2 + x/\sqrt{x^2-2})$ and $c = 1$. Although the exact form of the frequency-dependent term is modified, we find that, asymptotically, for $q \gg \sqrt{\beta_4/D}|\omega_n|^{1/4}$, the standard Landau damping, $|\omega_n|/Dq^2$, restores. Then, if we use the renormalized boson propogator, Eq. (6), to reevaluate the same self-energy diagram, Fig. 2(b), for the diffuson, (that gave us $z_d = 4$ in the first place), we obtain essentially the same result (up to small in $1/g$ corrections). Physically, the Landau damping controls the dynamical scaling of appropriate low energy modes associated with fermions even in the dirty case (in the clean scaling, $z_f = 3/2$ is also dictated by the ballistic form of the Landau damping).

The crucial observation here is that the coupling between the critical boson ϕ and the diffuson at the fixed point $z_{b,d} = 4$ is marginal. Thus, the dirty and clean limits share the common strategy of finding a self-consistent solution that results from resumming the effects of interactions. Contrary to the clean case, logarithmic divergences appear at $\mathcal{O}(1/g)$ and, thus, require extra caution.

Logarithmic corrections and RG.—The scale-dependent corrections to the self-consistent solution should be derived by reevaluating the diagrams in Fig. 2 with the modified propagators (5) and (6) near the fixed point $z_{b,d} = 4$. There are no double-counting issues because the self-consistent propagators include only the leading correction $\sim\mathcal{O}(1/\sqrt{g})$, while logarithmic divergences $\sim\mathcal{O}(1/g)$ are subleading. The renormalization of the diffusion coefficient, the density of states, and β_4 can be extracted from diagrams in Figs. 2(b)–2(d). The divergent part of the bosonic self-energy diagrams depicted in Fig. 2(e) renormalizes the coefficient c^2 in Eq. (6). The vertex correction of Fig. 2(f) results in the renormalization of λ.

To extend the perturbative results into the RG form, we implement the following scaling procedure. We assign engineering dimensions as $[q] = 1$, $[\bar{\psi}\psi] = -2 + \eta_w$, $[\phi] = -2 + \eta_\phi$, $[T] = 4 + \eta_T$, where η_w and η_ϕ are anomalous field dimensions, and we also choose to scale temperature with some exponent η_T. This exponent is determined from the condition that the coefficient β_4 in the diffusion propagator (5) does not run under the RG.

As a result, similar to the case of the disordered FL [24,63,76], the scaling of our theory is given by two-parameter RG flow for the dimensionless resistance $t = 2/(\pi g)$ and the effective interaction $\zeta = (2\pi\beta_4)^{-2}\lambda^2/t$

$$\frac{dt}{d\ln y} = -t^2\zeta f_t(\zeta), \qquad \frac{d\zeta}{d\ln y} = -t\zeta^2 f_\zeta(\zeta). \tag{7}$$

Here, y is the running RG energy scale. The functions $f_{t,\zeta}(\zeta)$ are both positive, see Fig. 3(a), with the following asymptotic behavior: $f_t \approx (\ln\zeta)/(2\zeta)$, $f_\zeta \approx 0.048$ at $\zeta \gg 1$ and $f_t \approx 1/2 + \zeta\ln\zeta$, $f_\zeta \approx \Delta - 1/2$ at $\zeta \ll 1$ where $\Delta \approx 0.71$ [73]. We emphasize that we did not make any

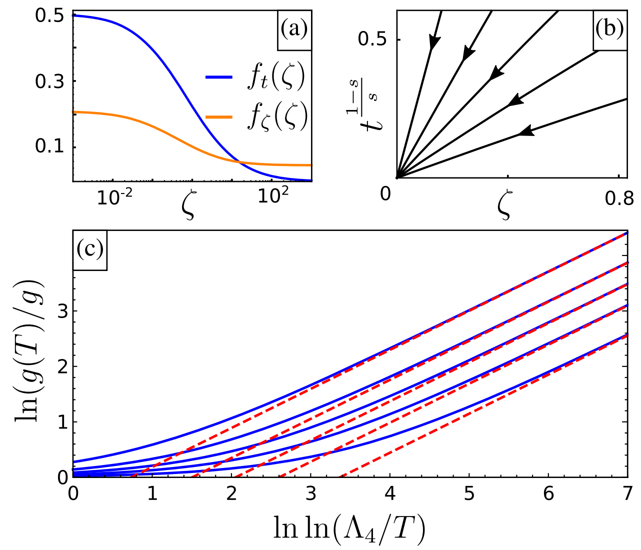

FIG. 3. (a) The RG flow governed by Eqs. (7). The arrows mark direction of RG flow towards the infrared. (b) The functions $f_t(\zeta)$ and $f_\zeta(\zeta)$ used in (7). (c) Temperature dependence of the logarithm of conductivity $\ln[g(T)/g]$ as a function of $\ln\ln(\Lambda_4/T)$ (blue curves). The red dashed lines represent an asymptotic behavior (3). The used bare parameters are $t_0 = 0.3$ and $\zeta_0 = 0.16$, 0.35, 0.60, 1.04, 2.25 from bottom to top.

assumptions regarding the magnitude of Yukawa coupling λ, so that our RG equations are formally valid to all orders in λ. The RG flow governed by Eqs. (7) is depicted in Fig. 3(b). Both the resistance t and the normalized interaction ζ scale to zero values, improving the validity of our RG equations. There is only a single stable IR fixed point at $t = \zeta = 0$. The asymptotic form of the RG equations near this fixed point is given by

$$\frac{1}{\Delta}\frac{dt}{d\ln y} = -st^2\zeta, \qquad \frac{1}{\Delta}\frac{d\zeta}{d\ln y} = -(1-s)t\zeta^2, \tag{8}$$

where $s = 1/(2\Delta) \approx 0.704$. Equations (8) indicate that both t and ζ (and thus, also λ) are marginally irrelevant. One can immediately notice that all RG trajectories have the form $\zeta t^{(s-1)/s} = \mathrm{const}$. By solving Eqs. (8), we find the scale dependence of the conductivity $g \sim 1/t$ depicted in Fig. 3(c) and with the asymptotic form given by Eq. (3). From $\lambda^2 \sim t\zeta$, one can easily obtain the RG equation for the Yukawa coupling, $d\lambda^2/d\ln y \sim -\lambda^4$. It yields $\lambda^2(E) \sim 1/\ln(\Lambda_4/E)$ in the infrared. We also note that all nonuniversal corrections to the anomalous dimensions vanish at this fixed point, $\eta_w, \eta_\phi, \eta_T \to 0$, implying that $z_d = 4$ is a true dynamical scaling of the problem at low energies.

The correction to the tunneling DOS is shown in Fig. 2(g). As explained above, this perturbative correction can be cast in the form of RG flow. Near the fixed point, the corresponding RG equation becomes [73], $d\ln\nu/d\ln y = t\zeta\ln(1/\zeta)/2$. In the low-energy limit, $|E| \ll \Lambda_4$, this RG equation yields the asymptotic form given in Eq. (2).

The behavior of the specific heat with T can be deduced from interaction corrections to the free energy [77]. Since the diagram of Fig. 2(h) is IR finite, we find $c_v \sim T^{2/z}$ with $z = 4$ [73]. This implies that, contrary to the case of disordered FL, the anomalous dimension η_T does not influence the T dependence of the specific heat.

Conclusions.—We have developed the RG theory describing the interplay between diffusive modes and soft magnetic fluctuations in a disordered 2D fermionic system. To keep the analysis parametrically under control, we employed expansion in two small parameters: $1/N$ (where N corresponds to a global SU(N) symmetry group) and dimensionless resistance t. By starting with an intermediate clean NFL fixed point, we found a self-consistent anomalous dynamical scaling $z_d = 4$ of the multiplet diffusive particle-hole excitations (see Fig. 1). The residual infrared logarithmic corrections to this self-consistent solution were summed up by the RG equations, see Eq. (7). The RG flow exhibits an IR stable fixed point with both the resistivity and the residual interaction approaching zero in spite of weak divergence of the local density of states. By contrast, in the large N extension of RG theory for the disordered FL [28], the magnetic instability still persists in the metallic regime.

While our theory predicts a phase with perfect conduction when $t \ll 1$, we cannot exclude the existence of a metal–insulator transition expected on general grounds for sufficiently large disorder $t \sim 1$. Even in the noninteracting unitary case (without Cooperons), certain higher order corrections due to diffusons favor suppression of conductivity [78] and can, in principle, compete with the interaction-induced effects [28].

Although it remains unknown whether our predictions hold in realistic systems with $N = 2$, we may speculate on experimental implications of our results. We have provided a model example where the resistance looks essentially finite for any realistically low accessible temperatures, but ultimately is zero at $T = 0$. One can speculate that this behavior is similar to a seemingly saturating resistance that has been observed in several experiments [12] at low temperatures. In addition, recent experiments [44–47] indicate the formation of spin droplets near apparent metal-insulator transition, which makes the model studied in this Letter extremely relevant for describing disordered 2D electron systems.

In the future, we wish to extend our results to the case when time-reversal symmetry is restored, leading to a competition [79] between weak-localization corrections and superconducting fluctuations. We expect both effects to be significantly modified because an anomalous dynamical scaling sets an unconventional temperature dependence of the phase-breaking time, and generally enhances the BCS instability [80].

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