Letter

Minimal velocity of the traveling wave solutions in two coupled Fisher-Kolmogorov-Petrovsky-Piskunov equations with the global conservation law

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We investigate a system of two coupled one-dimensional Fisher-Kolmogorov-Petrovsky-Piskunov (FKPP) equations which possess a global conservation law. Such a system of equations has been recently derived for the quasiparticle densities in a two-band fermionic model with a particle-number conserving dissipative protocol. As the standard FKPP equation, the studied system has one unstable and one stable homogeneous solution, with a traveling wave switching between them. We demonstrate that the conservation law enforces the synchronization of traveling waves for both densities and determines their minimal possible velocity. Surprisingly, we find the existence of jumps in the minimal velocity as a function of control parameters. We obtain that the minimal velocity of propagating fronts in the coupled FKPP equations may significantly exceed the minimal velocity for a single FKPP equation in a wide range of control parameters.

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The Fisher-Kolmogorov-Petrovsky-Piskunov (FKPP) equation [1,2] is a well-known example belonging to the reaction-diffusion universality class [3,4]. This type of dynamics emerges in a large number of applications, e.g., the propagation of advantageous genes, combustion fronts, the dynamics of domain walls, fluid motion, chemical reactions, decoherence propagation, etc. [5–9].

Recently [10,11], a specific form of two coupled FKPP equations has been derived for the description of the quasiparticle density evolution in a two-band fermionic model [12,13] with a particle-number conserving dissipative protocol. Such dissipative evolution with particle conservation has generated interest [14] as a way to stabilize topologically nontrivial steady states. Particle conservation in the dissipative protocol makes the total number of particles an integral of motion of the corresponding FKPP equations.

Coupled reaction-diffusion equations emerge in many fields, e.g., in physics [15], in biology [16,17], genetics [18,19], ecology [20,21], as well as being studied from a purely mathematical interest [22–24]. However, we are not aware of any discussion regarding the role of a global conservation law (the integral of motion) on the behavior of their solutions.

In this Letter, we report an analysis of two coupled one-dimensional FKPP equations which possess a global conservation law. Similar to a standard FKPP equation, the studied system of equations [cf. Eq. (1)] has unstable and stable homogeneous solutions with traveling waves switching between them. We demonstrate that the conservation law enforces the synchronization of traveling waves for both densities and determines their minimal possible velocity. Surprisingly, we find the existence of jumps in the minimal velocity as a function of control parameters. We obtain that the minimal velocity of propagating fronts in the coupled FKPP equations may significantly exceed the minimal velocity for a single FKPP equation in a wide range of control parameters.

Formulation of the problem. We study the following two coupled one-dimensional (1D) FKPP equations:

$$\partial_t n_u - D_u \partial_x^2 n_u = F(n_u, n_d), \quad \partial_t n_d - D_d \partial_x^2 n_d = -F(n_u, n_d),$$
$$F(n_u, n_d) = n_u/\tau_u - n_d/\tau_d + \beta n_u n_d. \tag{1}$$

Such a system of equations has been derived in Ref. [11] for the dissipative two-band fermionic model. In that context $n_u(x,t) \geqslant 0$ describes the density of particles in the upper band, while $n_d(x,t) \leqslant 0$ is the density of holes in the lower band. Physically, $n_{u,d}$ are small and correspond to the deviations of the densities from the half-filled case (the upper band is empty while the lower band is fully occupied): $n_u = n_d = 0$. $D_{u,d} > 0$ denote the diffusion coefficients in each band. The rates $1/\tau_{u,d} > 0$ characterize the instability of the fully occupied lower band towards a process similar to impact ionization in semiconductors [25]. The quantity $\beta > 0$ describes the rate of recombination akin to nonradiative recombination in semiconductors [26].

Another realization of the system of equations (1) can be found in the biological problem of species invasion with two dispersal phenotypes. In this model, n_u and $-n_d$ represent the population densities of two morphs, β controls the competition between them, and $1/\tau_{u,d}$ are their respective growth and mutation rates [18].

Although the function in Eq. (1) seems generic and might describe two types of diffusive particles with recombination and interparticle transitions in many settings, the analysis

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below is, in fact, more general. It is applicable to a function $F(n_u, n_d)$ which has an unstable zero at $n_u = n_d = 0$ and a stable one at $n_u = -n_d = n_*$ on the line $n_d = -n_u$. In this case, the role of $1/\tau_{u,d}$ is played by the derivative $\pm\partial_{n_{u,d}} F(n_u, n_d)$ at $n_u = n_d = 0$, which is assumed positive. For the sake of concreteness, below we present the analytic and numerical results for the function F determined in Eq. (1) only.

The system (1) has an unstable stationary homogeneous solution $n_u = n_d = 0$ and a stable stationary homogeneous solution $n_u = -n_d = n_* = \Gamma/\beta$, where $\Gamma = 1/\tau_u + 1/\tau_d$. The dynamics (1) has a globally conserved quantity, the total number of excess particles N:

$$N = \int dx[n_u(x,t) + n_d(x,t)] = 0. \tag{2}$$

The above constraints holds under the natural assumptions that $\lim_{x\to\infty} \partial_x n_{u,d}(x,t) = 0$. We emphasize that the conservation of the total number of particles imposes a strong constraint on the dynamics of the system (1). Indeed, it implies that the homogeneous solutions of Eq. (1) for n_u and n_d are not independent:

$$n_u(t) = -n_d(t) = \frac{n(0)n_* e^{\Gamma t}}{n_* + n(0)(e^{\Gamma t} - 1)}. \tag{3}$$

It is convenient to introduce the following dimensionless time τ and distance ρ: $t = (\tau_u\tau_d)^{1/2}\tau$ and $x = (D_u D_d \tau_u \tau_d)^{1/4}\rho$. Also we introduce dimensionless densities $U = n_u\beta\sqrt{\tau_u\tau_d}$ and $V = n_d\beta\sqrt{\tau_u\tau_d}$. Then Eq. (1) become

$$\partial_\tau U - \frac{1}{\sqrt{\alpha}}\partial_\rho^2 U - \frac{1}{\sqrt{\delta}}U + \sqrt{\delta}V - UV = 0,$$
$$\partial_\tau V - \sqrt{\alpha}\partial_\rho^2 V - \sqrt{\delta}V + \frac{1}{\sqrt{\delta}}U + UV = 0. \tag{4}$$

Here, we introduced two control parameters $\alpha = D_d/D_u$ and $\delta = \tau_u/\tau_d$. In dimensional units the unstable solution is $U = V = 0$ while the stationary one corresponds to $U = -V = U_* = \sqrt{\delta} + 1/\sqrt{\delta}$. Also we note that the dynamics of U and V is such that their magnitudes belong to the region $\sqrt{\delta}/U + 1/(|V|\sqrt{\delta}) \geqslant 1$.

Synchronization of the traveling wave solutions. As for the single FKPP equation, we are interested in the solutions of Eq. (4) that satisfy the following initial conditions,

$$\lim_{\rho\to-\infty} U(\rho,0) = -\lim_{\rho\to-\infty} V(\rho,0) = \sqrt{\delta} + 1/\sqrt{\delta},$$
$$\lim_{\rho\to+\infty} U(\rho,0) = \lim_{\rho\to+\infty} V(\rho,0) = 0. \tag{5}$$

We seek traveling wave solutions $U(\rho,\tau) = \mathcal{U}(\rho - v_1\tau)$ and $V(\rho,t) = \mathcal{V}(\rho - v_2\tau)$ with two independent velocities v_1 and v_2, in general. We note that the velocity v of the traveling wave in dimensionless Eq. (4) is related to the physical velocity c in dimensional Eq. (1) as $c = v[D_u D_d/(\tau_u\tau_d)]^{1/4}$. Integrating Eq. (4) over space, we find

$$\partial_t N \propto -\int d\rho[v_1\mathcal{U}'(\rho - v_1\tau) + v_2\mathcal{V}'(\rho - v_2\tau)] = 0, \tag{6}$$

provided $v_1 = v_2$. Therefore, the conservation law (2) forces synchronization of the traveling wave solutions, making their velocities identical, $v_1 = v_2 = v$. Below we assume that $v > 0$. The results for $v < 0$ can be obtained by means of trivial redefinitions.

Asymptotic analysis near the unstable solution. The standard approach to derive the minimal velocity of a traveling wave for the FKPP equation consists of an asymptotic analysis near the unstable homogeneous solution. The minimal velocity is determined from the condition that $U(\rho,\tau) \geqslant 0$ and $-V(\rho,\tau) \geqslant 0$ tend monotonously to zero as $\tau \to -\infty$. Substituting the traveling wave ansatz $U(\rho,\tau) = \mathcal{U}(z)$ and $V(\rho,\tau) = \mathcal{V}(z)$, where $z = \rho - v\tau$, into Eq. (4) and taking the limit $z \to +\infty$ (for which $\mathcal{U}, \mathcal{V} \to 0$), we obtain the linearized system of equations

$$\begin{pmatrix} v\partial_z + \frac{1}{\sqrt{\alpha}}\partial_z^2 + \frac{1}{\sqrt{\delta}} & -\sqrt{\delta} \\ -\frac{1}{\sqrt{\delta}} & v\partial_z + \sqrt{\alpha}\partial_z^2 + \sqrt{\delta} \end{pmatrix} \begin{pmatrix} \mathcal{U} \\ \mathcal{V} \end{pmatrix} = 0. \tag{7}$$

Seeking the solution in the standard form $\mathcal{U}, \mathcal{V} \propto \exp(\lambda z)$, we obtain the characteristic equation

$$\lambda^4 + 2v\lambda^3\cosh a + [v^2 + 2\cosh(a-b)]\lambda^2 + 2v\lambda\cosh b = 0. \tag{8}$$

Here, we parametrize α and δ as $\alpha = \exp(2a)$ and $\delta = \exp(2b)$. We are interested in real negative solutions of Eq. (8) in order to preserve the non-negativity of $\mathcal{U}$ and $-\mathcal{V}$, as well as to guarantee that $\mathcal{U}, \mathcal{V} \to 0$ as $z \to +\infty$. Due to the inequality $2v\cosh b > 0$, Eq. (8) has at least one real negative root.

Let us first consider the case $\alpha = 1$ ($a = 0$) that corresponds to equal diffusion constants, $D_u = D_d$. Then we obtain the following four roots of Eq. (8):

$$\lambda_1 = 0, \quad \lambda_2 = -v, \quad \lambda_{3,4} = -\frac{v \pm \sqrt{v^2 - 8\cosh b}}{2}. \tag{9}$$

The roots $\lambda_{1,2}$ control the asymptotic behavior of the total density $\mathcal{U} + \mathcal{V}$. The existence of a zero root, $\lambda_1 = 0$, is a consequence of the conservation law (2) for the total number of particles. The other two roots, $\lambda_{3,4}$, correspond to the eigenvector $\{1, -1\}^T$ and describe the asymptotic behavior of the imbalance $\mathcal{U} - \mathcal{V}$. For $v \to \infty$ the root λ_4 tends to the magnitude $2v\cosh b$ that corresponds to the homogeneous solution (3).

For the roots $\lambda_{3,4}$ to be real, the velocity has to be larger than the minimal one: $v^2 \geqslant v_{\min}^2 = 8\cosh b$. Returning to the dimensional units for the velocity, we find that for the case of equal diffusion coefficients $D_u = D_d = D$ ($\alpha = 1$) the traveling wave velocity satisfies the following inequality:

$$c^2 \geqslant c_{\min}^2 = v_{\min}^2[D^2/(\tau_u\tau_d)]^{1/2} = 4D\Gamma. \tag{10}$$

The above result can be understood as follows. For $D_u = D_d$ Eq. (1) have the solution $n_u = -n_d = n$ that satisfies the following single FKPP equation: $\partial_t n - D\partial_x^2 n - \Gamma n + \beta n^2 = 0$. In what follows, we will compare the minimal velocity obtained in the general case $D_u \neq D_d$ with the result (10) that corresponds to a single FKPP equation. We note that we can write $c_{\min}^2 = c_{\min,u}^2 + c_{\min,d}^2$ where $c_{\min,u/d} = 2\sqrt{D/\tau_{u/d}}$ is the minimal velocity for independent FKPP equations $\partial_t n_{u/d} - D\partial_x^2 n_{u/d} - n_{u/d}/\tau_{u/d} + \beta n_{u/d}^2 = 0$.

Now we return to the general case. The discriminant of the cubic equation corresponding to Eq. (8) acquires the following form:

$$Q_\lambda = -v^6 \sinh^2 a + 2v^4 \sinh a[4\cosh(a+b)\sinh a + \sinh(2a-b)] + v^2\{4[1-2\cosh(2a)]\cosh^2(a-b) + 3[\cosh(2a-b) - 2\cosh b]^2\} + 8\cosh^3(a-b). \quad (11)$$

We note that Q_λ is unchanged after a simultaneous change of the signs of a and b: $a, b \to -a, -b$. For $Q_\lambda < 0$ there exists a single real negative root of Eq. (8) and two complex conjugated roots. Such a situation does not allow us to construct an asymptotic solution for $\mathcal{U} \geqslant 0$ and $\mathcal{V} \leqslant 0$. In the opposite case we have three real roots with at least one negative one. With three such roots we can build the proper asymptotic behavior. The minimal velocity required to have three real roots is determined from the equation $Q_\lambda = 0$. The corresponding cubic equation for v^2 has at least one positive real root. Again, to characterize the number of real roots for v^2 we compute the discriminant of the corresponding cubic equation:

$$Q_v = \cosh b\{[12 - 14\cosh(2a) + \cosh(4a)]\cosh b - 8\cosh^3 a \sinh a \sinh b\}^3. \quad (12)$$

For $Q_v < 0$ there is only a single real positive root for v^2. In the opposite case of $Q_v > 0$, there are three real roots. Thus the minimal positive root determines the minimal velocity square in this case. The boundary between the two regimes is determined by the equation $Q_v = 0$. It can be resolved explicitly in terms of the dependence of b on a:

$$b = a + \frac{3}{2}\ln\frac{3\tanh a - 1}{3\tanh a + 1}. \quad (13)$$

We note that for fixed b there are two solutions of the above equation: $a_b^\pm$ with $a_b^- < 0 < a_b^+$. For $a < a_b^-$ we have three real solutions for v^2, $Q_v > 0$. For $a_b^- < a < a_b^+$ there is only a single real solution. Finally, for $a_b^+ < a$ we have three real solutions again. We note that $|a_b^\pm| > (1/2)\ln 2$ (see Fig. 1).

Instead of writing long analytic expressions for the minimal velocity v, we present the following asymptotic results,

$$v_{\min}^2 \simeq 4\begin{cases} 2\cosh b + 2a\sinh b + a^2\frac{25\cosh(2b)-19}{2\cosh b}, & a\to 0,\\ e^{-|a+b|} - e^{-3|a+b|}, & a\to\infty, \end{cases} \quad (14)$$

and numerical dependence of $v_{\min}^2$ on a for a fixed b (see Fig. 2). It is worthwhile to compare the minimal velocity for the general case $D_u \neq D_d$ $(a \neq 0)$ with the one in the symmetric case $D_u = D_d$ $(a = 0)$ in which Eq. (1) can be reduced to a single equation [see Eq. (10)]. As one can see from Fig. 2, in the interval $a_b^- < a < a_b^+$ the minimal velocity can significantly exceed its value, $2\sqrt{2\cosh b}$, at $a = 0$.

Therefore, the asymptotic analysis near the unstable homogeneous solution reveals the dependence of the minimal velocity of the propagating front of the synchronized traveling waves on the dimensionless parameters of the model α and δ.

Asymptotic analysis near the stable solution. Now let us consider the vicinity of the stable homogeneous solution $U = -V = 2\cosh b$. Since this solution is expected to be realized at $z \to -\infty$, we are interested in the positive eigenvalues $\bar\lambda$, where we write $\mathcal{U} - 2\cosh b \propto \exp(\bar\lambda z)$ and $\mathcal{V} + 2\cosh b \propto \exp(\bar\lambda z)$. Then the characteristic equation for the corresponding eigenvalues becomes

$$\bar\lambda^4 + 2v\bar\lambda^3\cosh a + [v^2 - 2\cosh(a-b)]\bar\lambda^2 - 2v\bar\lambda\cosh b = 0. \quad (15)$$

Since the last term on the left-hand side, $-2v\cosh b < 0$, Eq. (15) has at least one real positive root. As one can check, the discriminant of Eq. (15) coincides with Q_v. Therefore, the analysis of the vicinity of the stable stationary solution does not impose additional conditions for $v_{\min}$.

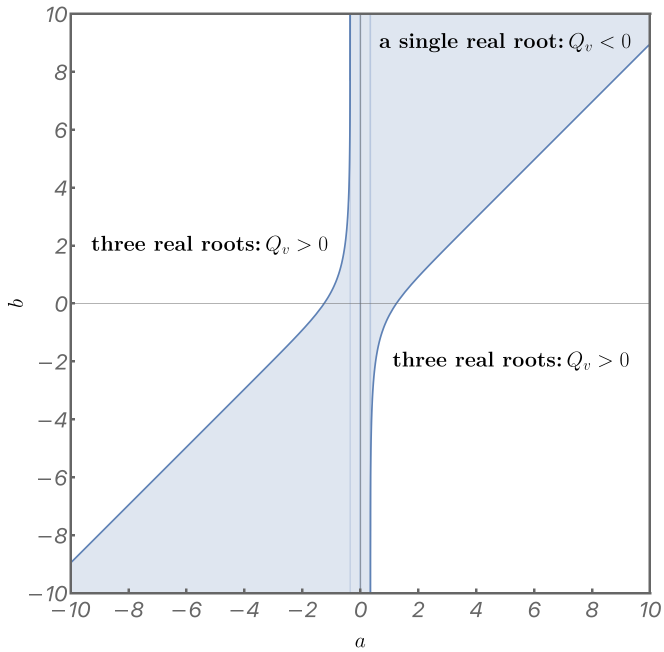

FIG. 1. The regions of positive and negative Q_v on the b-a plane [see Eq. (13)].

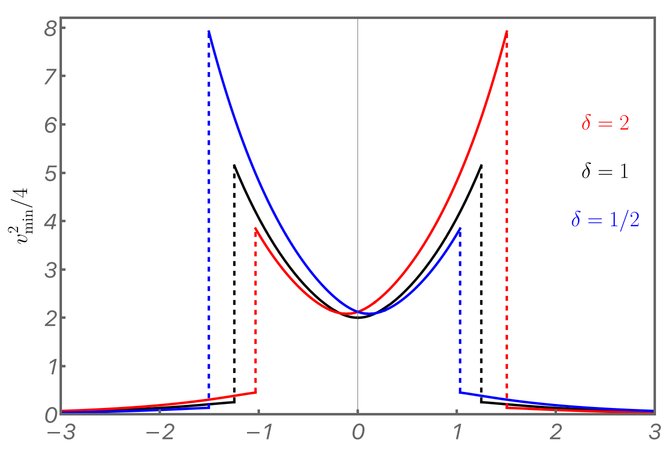

FIG. 2. Dependence of $v_{\min}^2$ on $a = (1/2)\ln\alpha$ for $\delta = 1/2, 1, 2$. The jumps occur at the points $a_b^\pm$ which are the solutions of Eq. (13). We mention that in the interval $a_b^- < a < a_b^+$ the minimal velocity can be larger than its value, $2\sqrt{2\cosh b}$, at $a = 0$. The latter corresponds to the minimal velocity of the single FKPP equation [see Eq. (10)] (see text).

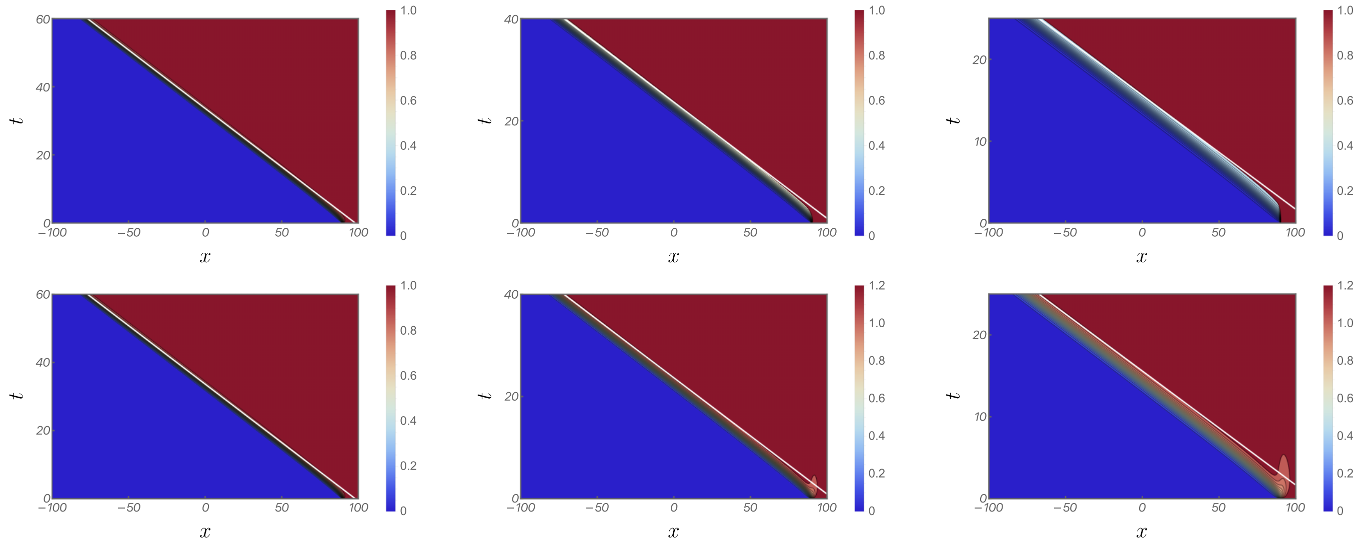

FIG. 3. The color-density plot of $U(t,x)/U_*$ (upper row) and $-V(t,x)/U_*$ (lower row) for $\delta = 2$ and $a = 0$ (left column), $a = 1$ (middle column), and $a = 2$ (right column). We solve Eq. (4) on the interval $x \in [-100, 100]$ with the following initial and boundary conditions: $U(0,x) = -V(0,x) = U_* \theta(x - 90)$, $U(t,-100) = V(t,-100) = U_*$, $U(t,100) = V(t,100) = U_*$. The straight white line is a guide for the eye and corresponds to the velocity of the traveling wave, $v \simeq 2.91, 4.39, 7.18$, for left, middle, and right columns, respectively. We note that for $a = 0$ and $a = 1$ the observed velocities correspond exactly to the minimal velocities, whereas the velocity for $a = 2$ is approximately 13 times larger than the minimal one (see text).

Numerical results. To illustrate our analytical results we solve Eq. (4) numerically using *Mathematica* (see Supplemental Material [27]). We focus on sharp initial conditions $U(0,x) = -V(0,x) = U_* \theta(x - x_0)$, where $\theta(x)$ denotes the Heaviside step function. The numerical results for $\delta = 2$ and for three values $a = 0, 1, 2$ are shown in Fig. 3. We demonstrate that the velocities of traveling waves for U and V are indeed synchronized. Interestingly, for such sharp initial conditions the velocity of the traveling waves in the case of $a = 0$ and $a = 1$ coincides exactly with the minimal velocity for a given value of a (which is smaller than a_b^+). In contrast, the velocity for $a = 2$ is much larger than the minimal velocity, which is equal to 0.61 (see Fig. 2). In fact, the observed velocity for $a = 2$ coincides with the maximal real root of the equation $Q_v = 0$ [see Eq. (12)]. We note some structure in the bottom right-hand corner of each panel in Fig. 3. It is a result of diffusive spreading of concentrations at the initial time stage. In the case of coupled FKPP equations it occurs in the same way as for a standard FKPP equation.

Conclusions. In our study we investigated the behavior of a system of two coupled one-dimensional FKPP equations for densities of fermions in the upper and lower bands. The crucial difference of the considered system of equations from the ones studied before is the existence of a global conservation law. The phenomenology for the behavior of the studied system of equations is similar to the standard FKPP equation: the existence of unstable and stable stationary homogeneous solutions as well as traveling waves switching the system between them. However, the conservation law enforces the synchronization of traveling waves for both densities and determines their minimal possible velocity. The conservation law results in the existence of a zero root of the characteristic polynomial for equations linearized near the unstable and stable solutions. We performed the linear analysis and found analytically the minimal velocity at which the traveling waves can spread out. We obtained that the minimal velocity is a nontrivial function of the two control parameters of the model. Moreover, we demonstrated the existence of jumps of the minimal velocity as a function of these control parameters. The existence of such jumps is a direct consequence of the conservation law and synchronization of traveling waves. Interestingly, we found that the minimal velocity of propagating fronts in the coupled FKPP equations may significantly exceed the minimal velocity for a single FKPP equation in a wide range of parameters. In particular, it implies that by varying the ratio of diffusion coefficients one may enhance the speed of the front propagation in such coupled FKPP equations.

As a possible direction for future research we mention a detailed study of the dependence of the wave front and its velocity on the initial conditions. Also it would be interesting to study the possibility of existence of nontrivial spatial structures at the front of the traveling wave in the case of two and three dimensions (see, e.g., Ref. [15]). We note that for the ordinary one-dimensional FKPP equation the exact solution for the traveling wave with a particular velocity is known [28]. It would be interesting to find similar exact solutions for the coupled FKPP equations, perhaps, using the approach of Ref. [29]. In addition, it would be worthwhile to extend our analysis to a more general form of equations of FKPP type which possesses a global conservation law.

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Data availability. The data that support the findings of this article are openly available [27].

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