(a) Band ferromagnetism

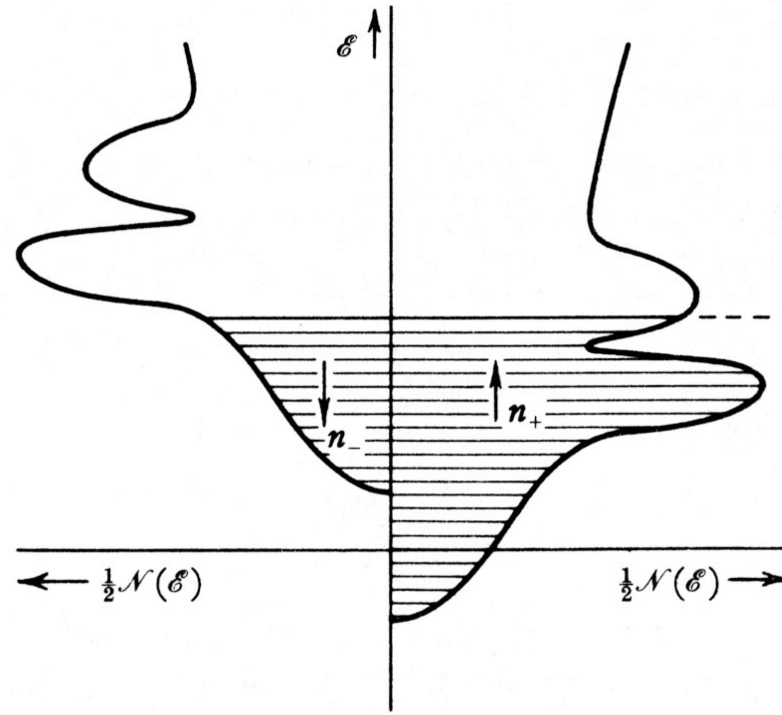
Stoner model:

$$H_{\text{int}} = \frac{U}{N}\sum_{\mathbf{k},\mathbf{k}'} n_{\mathbf{k}+} n_{\mathbf{k}'-}\,.$$

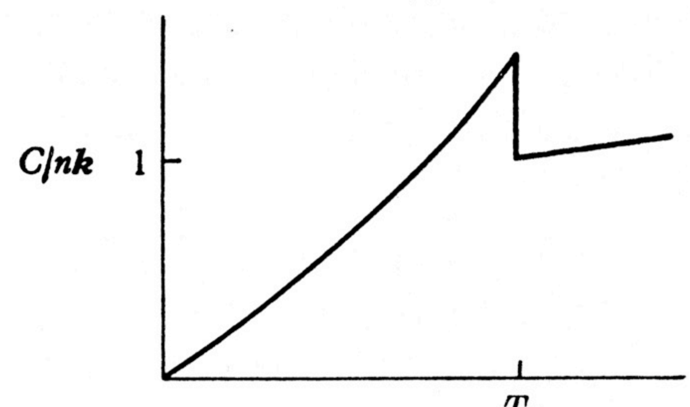
$$\chi_{H\to 0,\,T\to 0} = \frac{\mu_0^2 n\left(E_F\right)}{1-\frac{1}{2}Un\left(E_F\right)}.$$

Transition to FM: $\frac{1}{2}Un\left(E_F\right) > 1$

(b) Electronic specific heat of a ferromagnetic metal



(a)



(b)

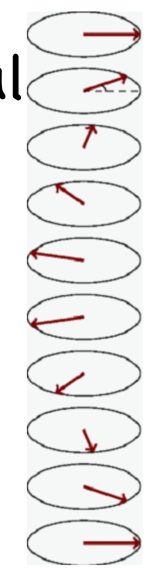
[from J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, 1979)]

Antiferromagnetic helical order (terbium and dysprosium)

Tb and Dy order ferromagnetically below 219 and 85 K, resp.

Above T_C these two elemental metals slip into a helical antiferromagnetic state, in which all the atomic moments in a basal plane layer are parallel, and oriented at a certain angle to the moments of adjacent layers.

In a transect along the hexagonal axis the moments would be observed to rotate around the transect line in a helical pattern. There is no net spontaneous magnetization.



Transitions from a helical antiferromagnetic state to a paramagnetic state occur at 230 and 179 K, respectively.

Helical or conical ferromagnetism

Erbium below 19.5 K also exhibits a helical magnetic order.

Each atom, however, has a component of magnetization parallel to the hexagonal axis, and, consequently, there is a net magnetic moment.

Between 19.5 and 80 K erbium displays modulated antiferromagnetism (hex. axis components vary continuously in magnitude in a sinusoidal pattern from layer to layer).

Sinusoidal AF ordering (thulium between 32 and 56 K).

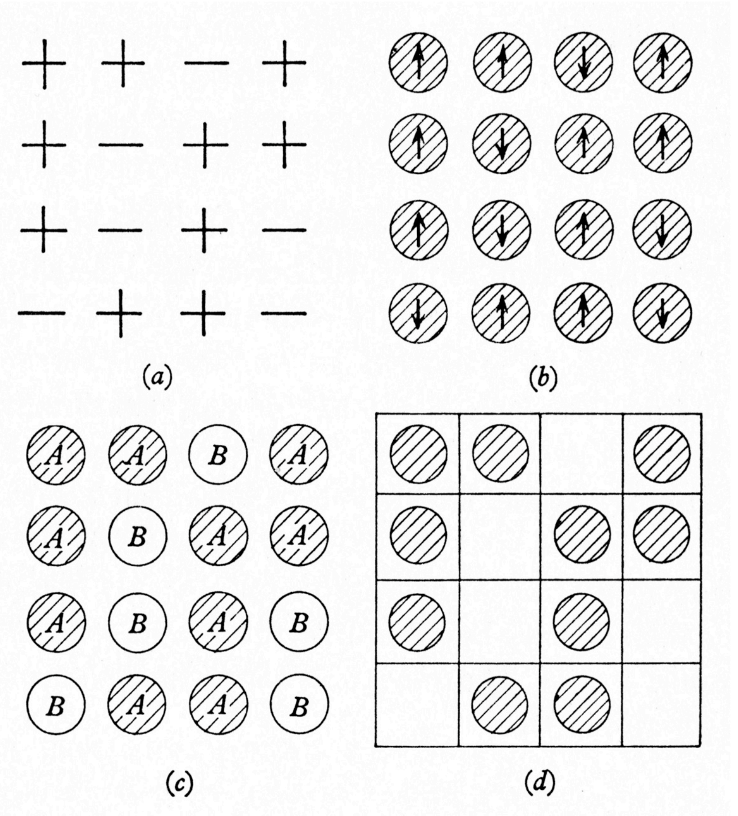
Modulated ferromagnetism (thulium below 32 K).

The Ising model (simplification of the Heisenberg model)

$$H = -\frac{1}{2}\sum_{\mathbf{R},\mathbf{R}'} J(\mathbf{R}-\mathbf{R}')\mathbf{S}_z(\mathbf{R})\cdot\mathbf{S}_z(\mathbf{R}') - g\mu_B H \sum_{\mathbf{R}} \mathbf{S}_z(\mathbf{R}).$$

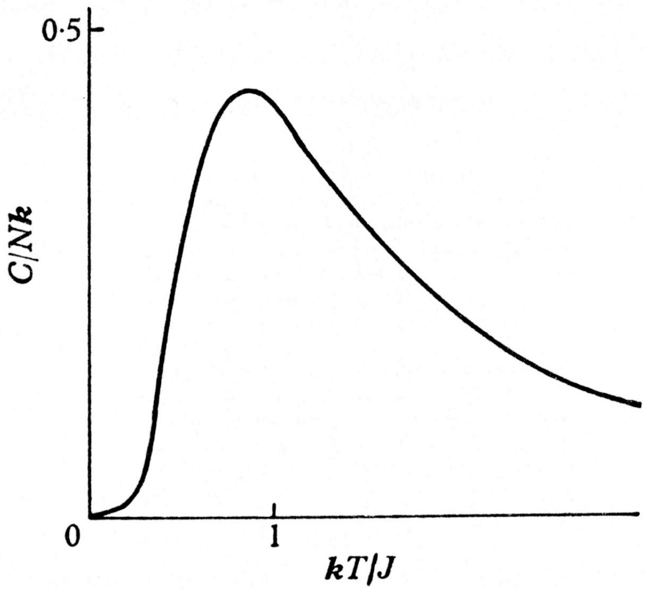
(a) A configuration of the Ising model; (b) an arrangement of spins; (c) an arrangement of atoms in a binary alloy; (d) a configuration of a "lattice gas".

[from J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, 1979)]

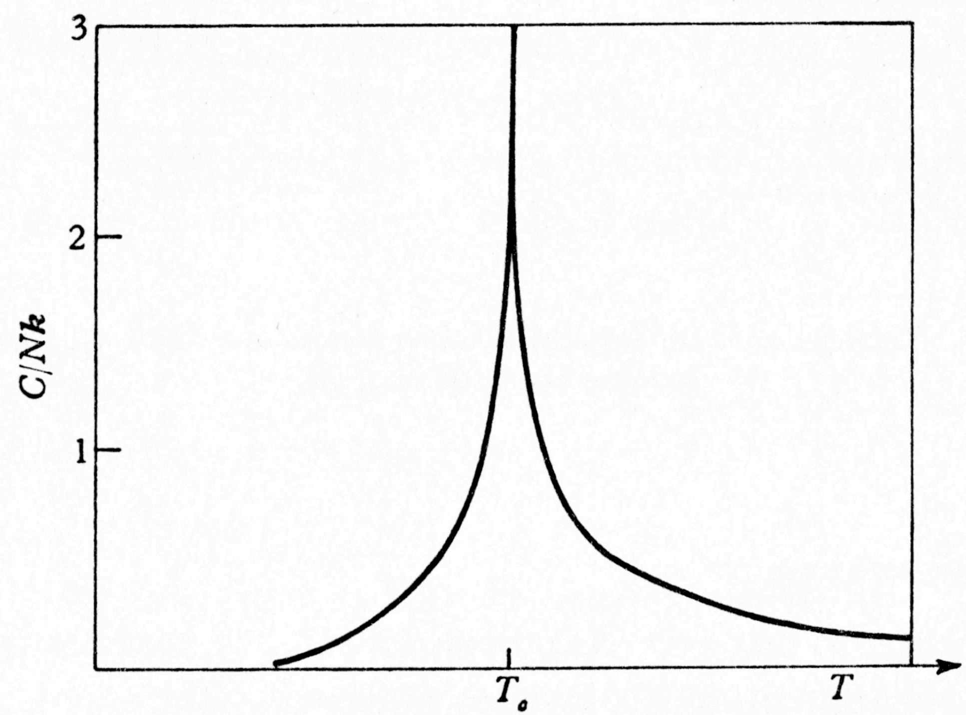


The Ising model: specific heat

one-dimensional lattice



2D square lattice



[from J. M. Ziman, *Principles of the Theory of Solids* (Cambridge University Press, 1979)]

Competing interactions

- Frustration
- Spin glasses
- Strongly geometrically frustrated magnets
- One-dimensional magnets, spin chains, the spin-Peierls transition
- Two-dimensional magnets
- Magnetism in heavy-electron metals
- Kondo semiconductors
- Quantum phase transitions

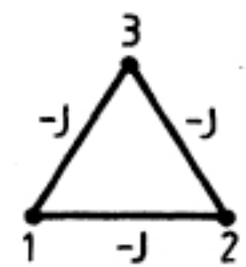
Frustration

The concept of "frustration" (Toulouse, 1977).

The random-bond model for Ising spins in zero external magnetic field.

Disorder due to the effect of "frustration", i.e., competition between interactions, which cannot be satisfied by any spin configuration.

Example is an equilateral triangle with all bonds negative $J_i = -1$. It is impossible to choose the orientations of the spins at the vertices without "frustrating" at least one bond.



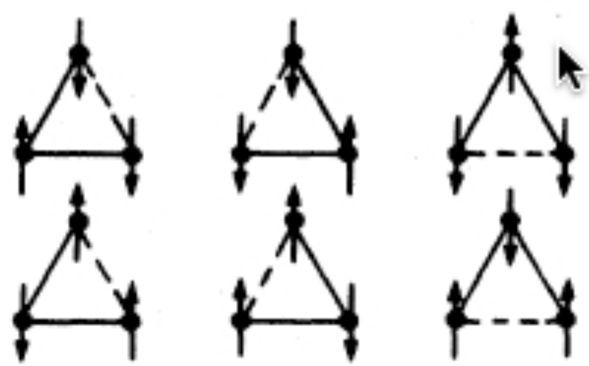
[from K. Binder and A. P. Young Rev. Mod. Phys. **58**, 801 (1986)]

The frustration effect can be expressed by the function

$$\phi_f = \frac{J_1 J_2 J_2}{\left|J\right|^3} \ ,$$

where $J_i = \pm 1$. Frustration is present when $\phi_f = -1$

Ground states of the frustrated triangle; broken lines are "unsatisfied" bonds.



[from K. Binder and A. P. Young Rev. Mod. Phys. **58**, 801 (1986)]

Frustration leads to the extra energy.

The frustration effect allows for a highly degenerate ground state and consequently creates numerous valleys in the phase space, which is a fundamental characteristic of a spin glass.

Spin frustration in high-symmetry magnetic nanomolecules.

Spin glasses

"Spin glasses" – the freezing behavior and low-temperature magnetic properties of various random magnetic systems.

Controversy over the question of a phase transition.

Time-dependent phenomena:

- Point to a gradual freezing over a wide temperature range
- Suggestive of a glasslike or thermally activated behavior.

Cooperative but time dependent "phase transition".

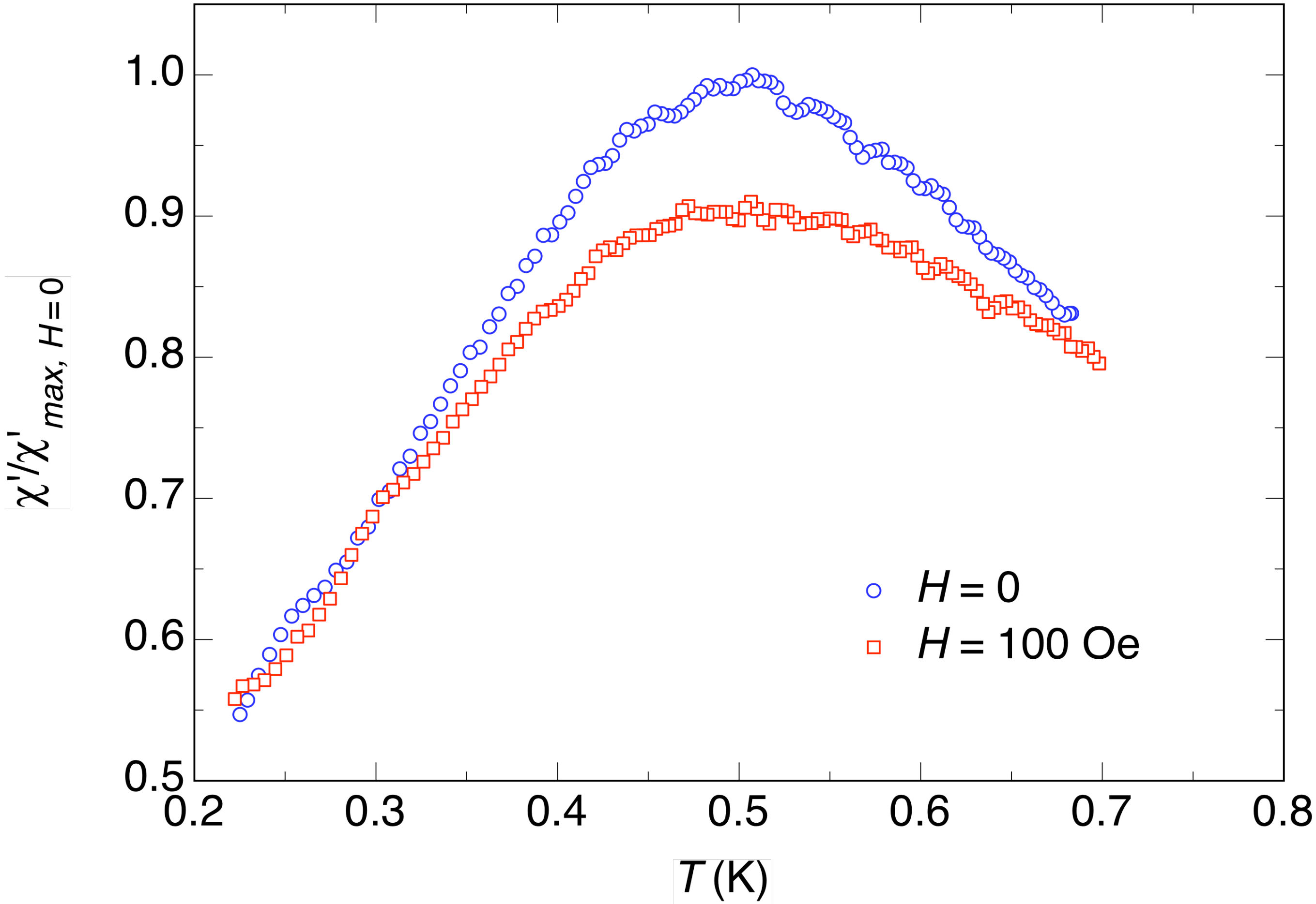
Spin-glasses are strongly affected by an external magnetic field in the temperature range around the freezing temperature T_f.

At T_f the cusp of the a.c. magnetic susceptibility $\chi'(T)$ becomes rounded in very low fields of order 100 Oe.

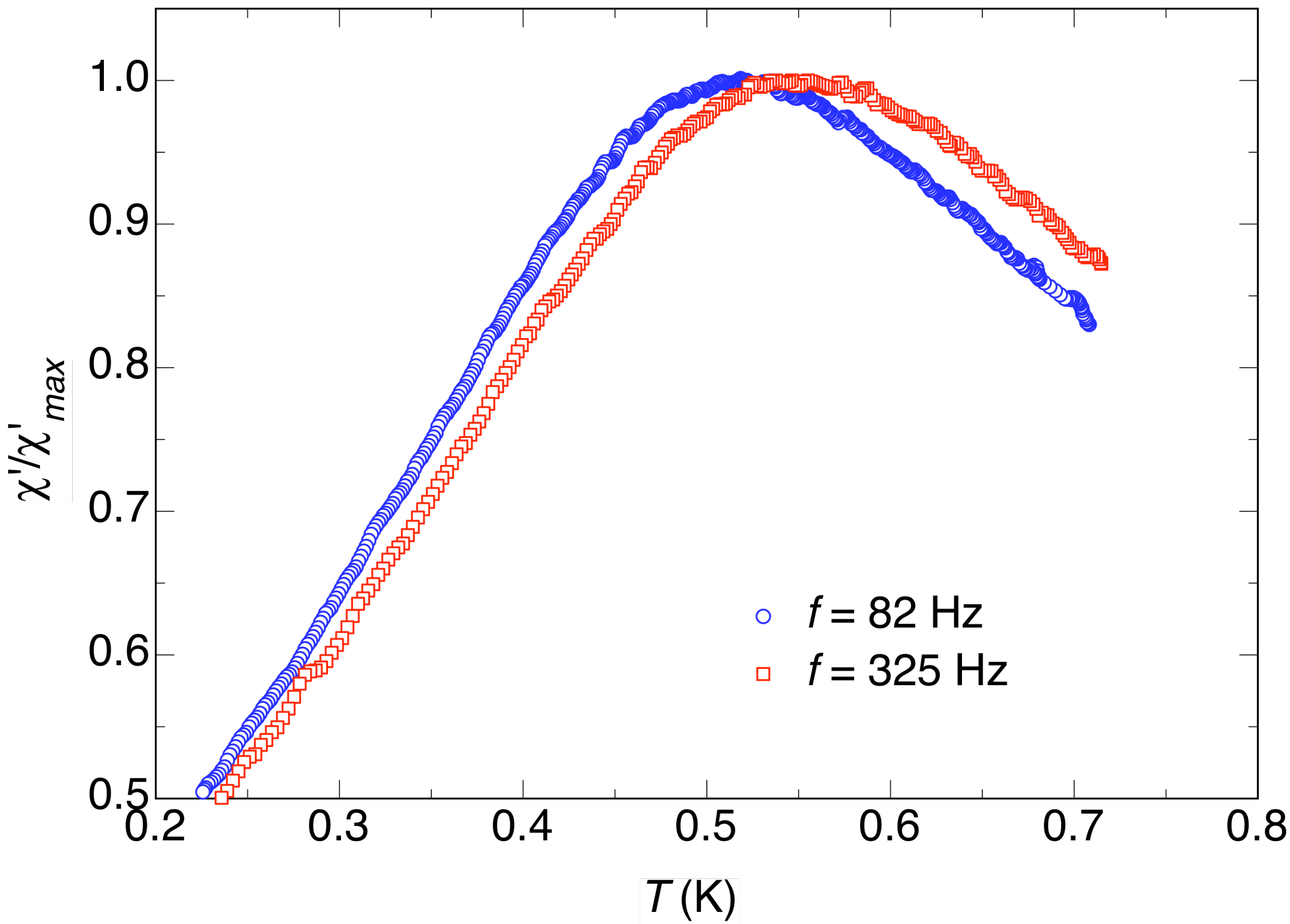
Complex response to an external magnetic field:

- Time-dependent and irreversible behavior of the magnetization M at $T < T_f$ in a wide range of magnetic fields.
- Even extremely high magnetic fields, however, cannot saturate $M(H)$.

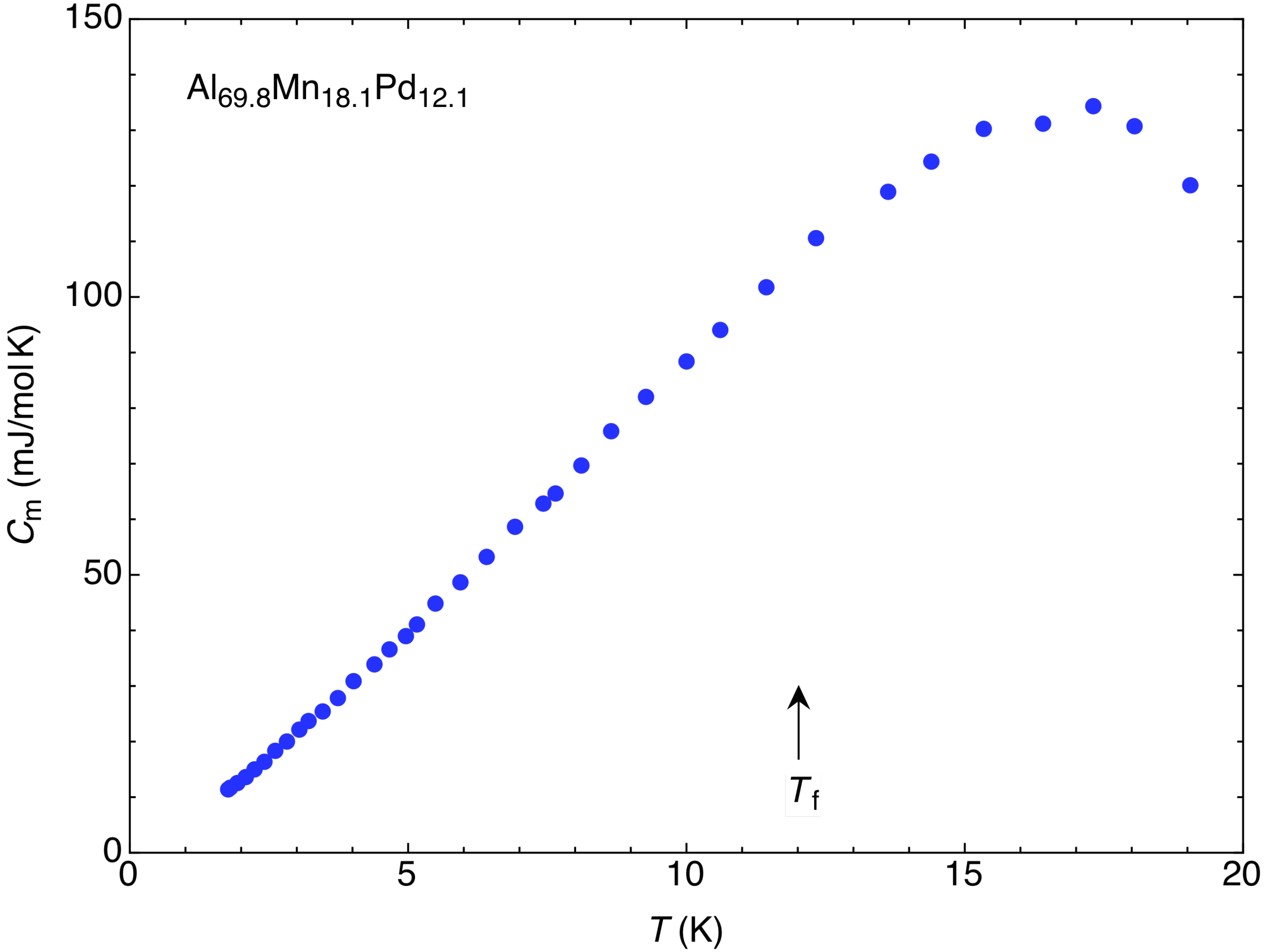
Low-field a.c. susceptibility of icosahedral Al-Mn-Pd as function of temperature. The amplitude of the excitation field is 0.1 Oe.



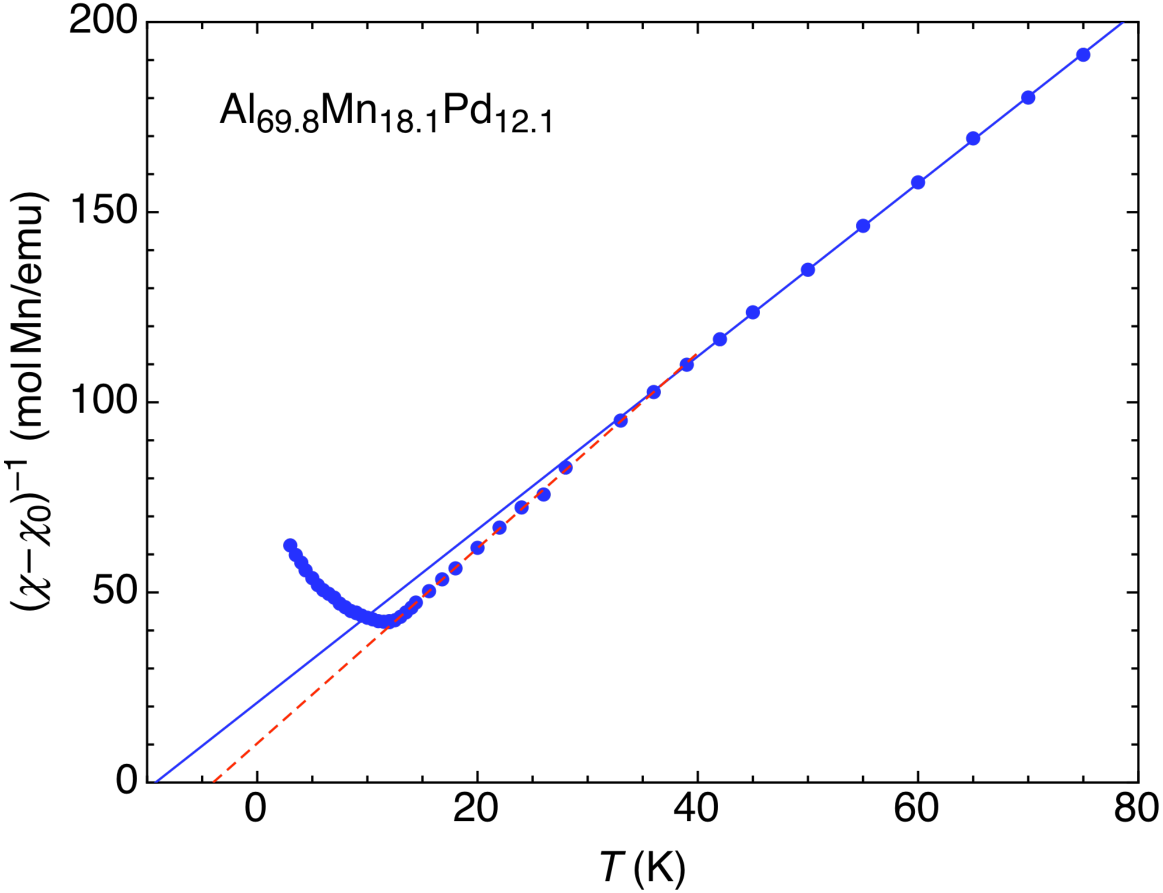
Low-field a.c. susceptibility of icosahedral Al-Mn-Pd as function of T for different frequencies. The amplitude of the excitation field is 0.1 Oe.



Estimate of the magnetic contribution to the specific heat of the decagonal Al-Mn-Pd phase

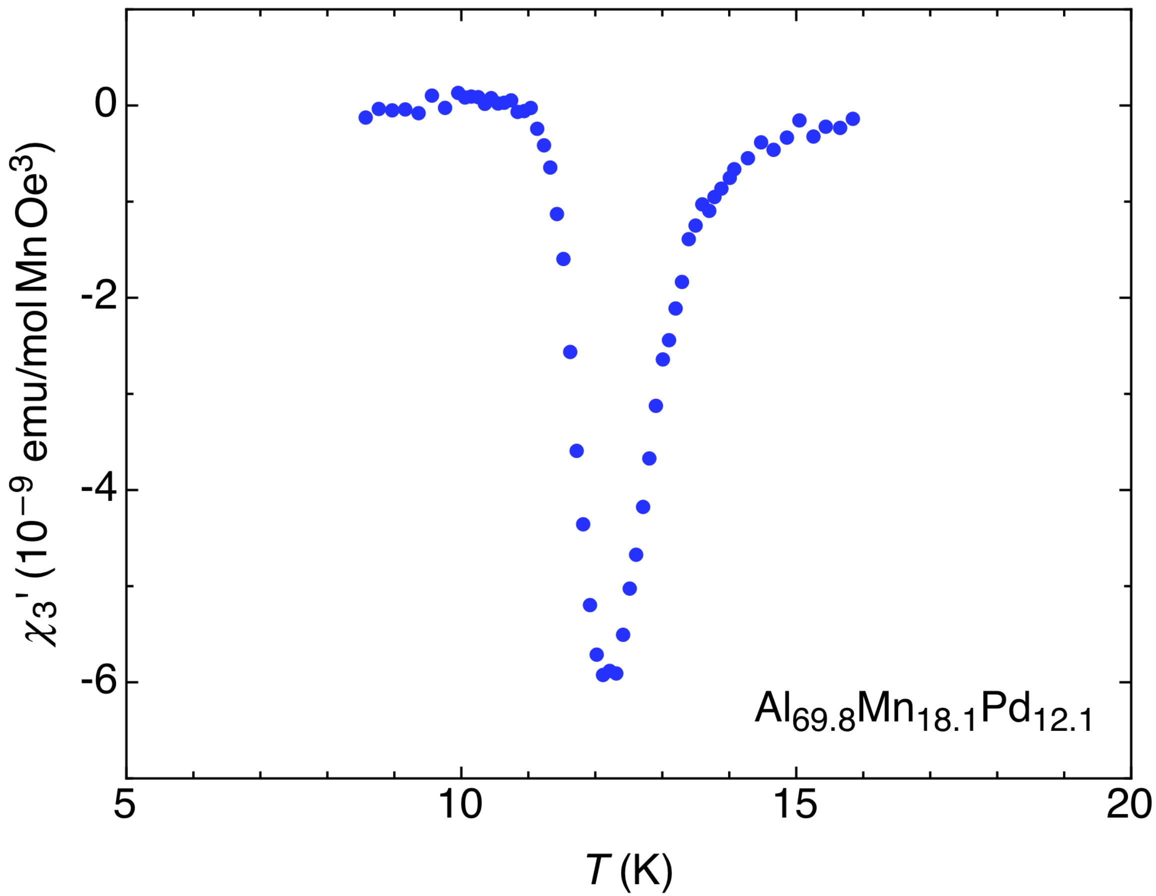


Magnetic susceptibility of decagonal Al-Mn-Pd plotted as $(\chi - \chi_0)^{-1}$ vs. T between 2 and 80 K

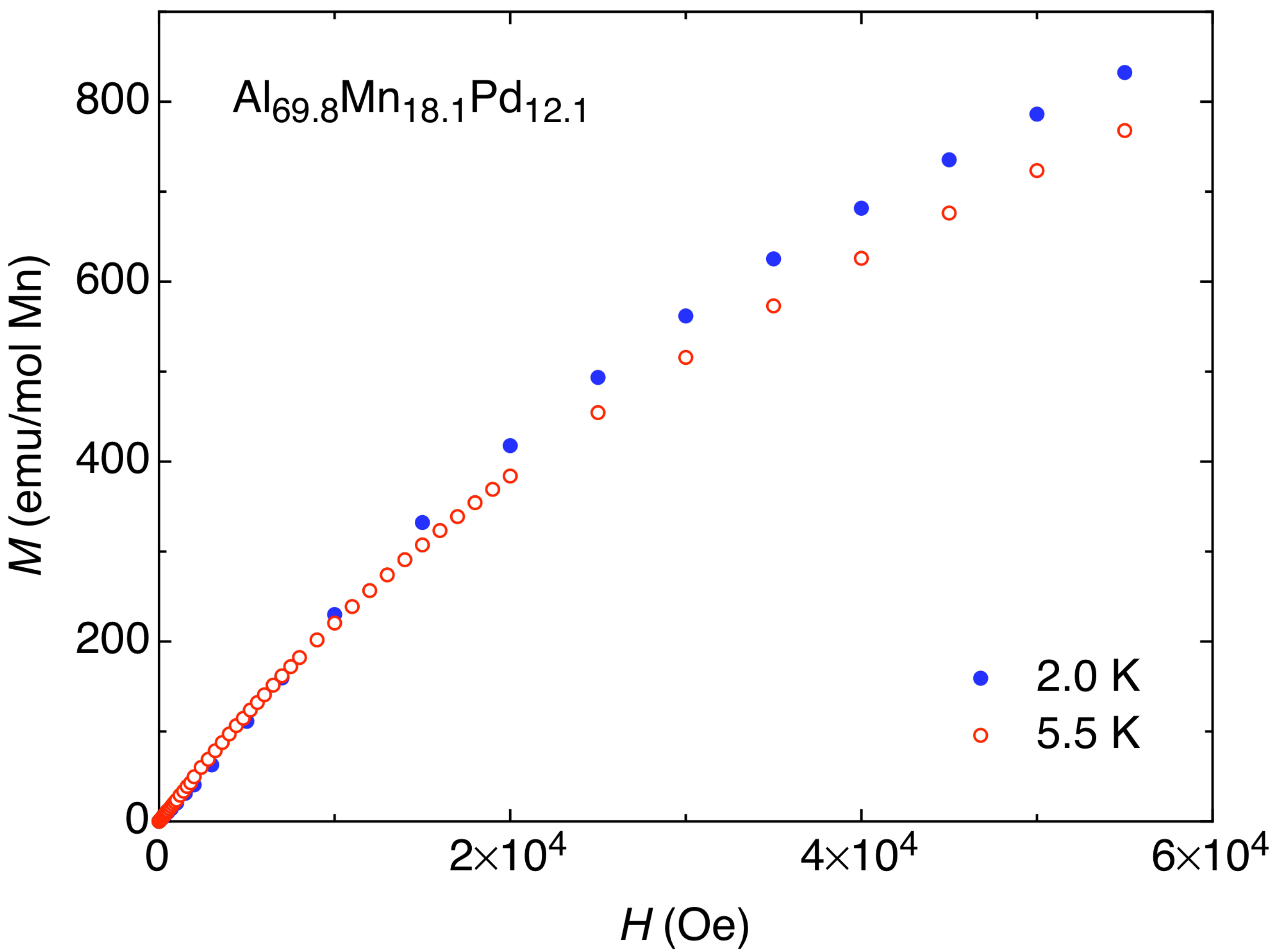


The solid line and the broken line are fits using the Curie-Weiss law in the temperature ranges between 60 and 300 K and between 15 and 30 K, respectively.

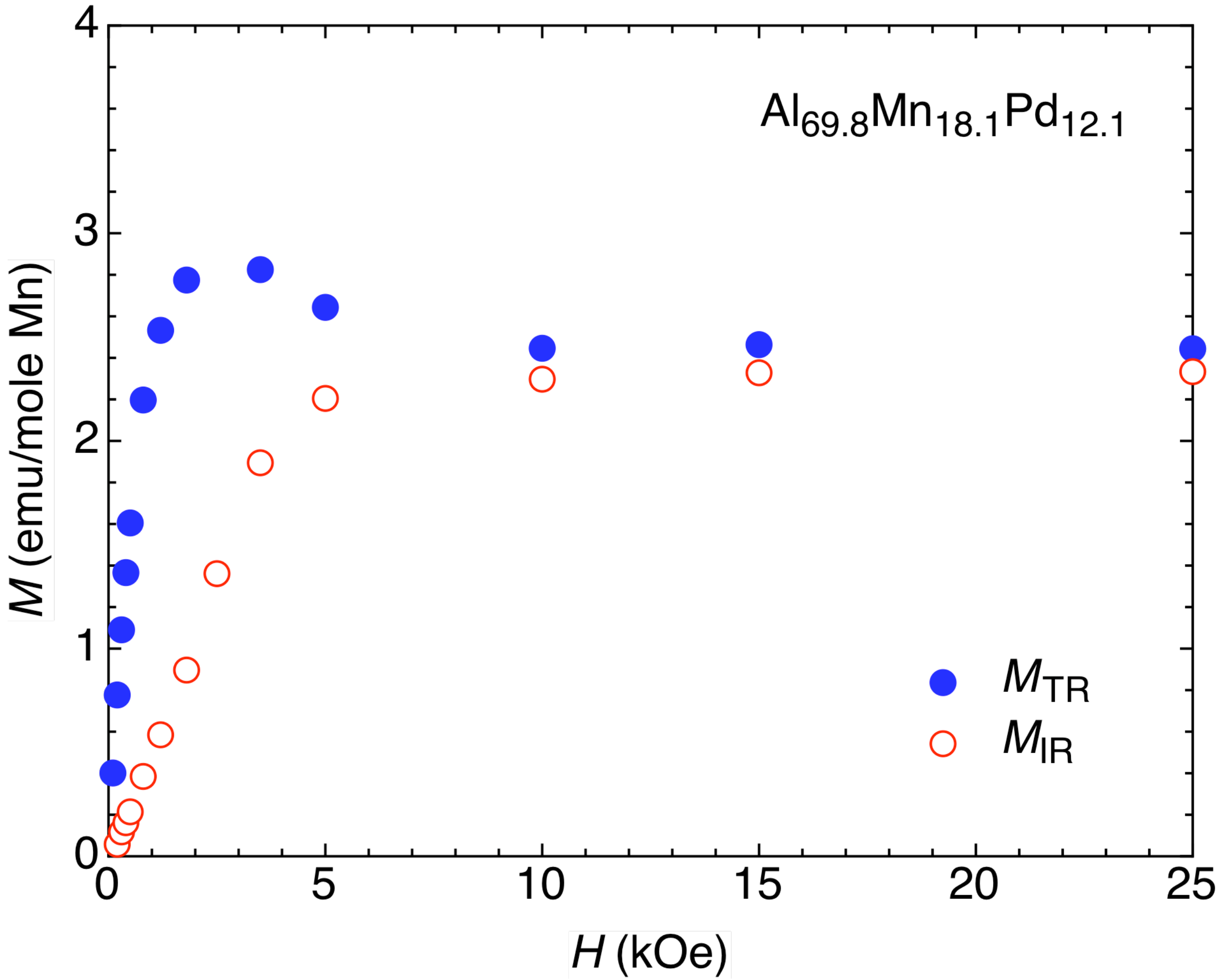
The real part of the complex a.c. third-order magnetic susceptibility of decagonal Al-Mn-Pd as a function of temperature



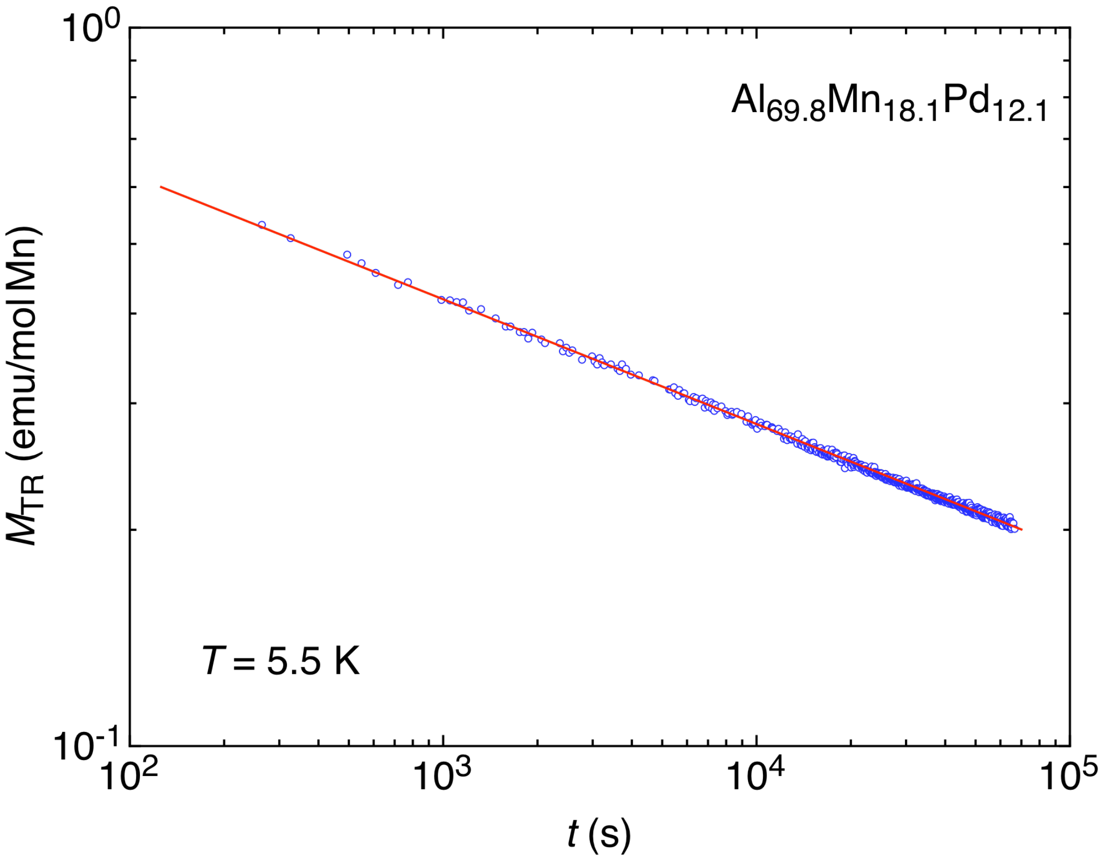
Magnetization M of decagonal Al-Mn-Pd as a function of magnetic field H at 2 and at 5 K.



The "isothermal remanent" magnetization M_{IR} and the "thermoremanent" magnetization M_{TR} measured at 5.5 K



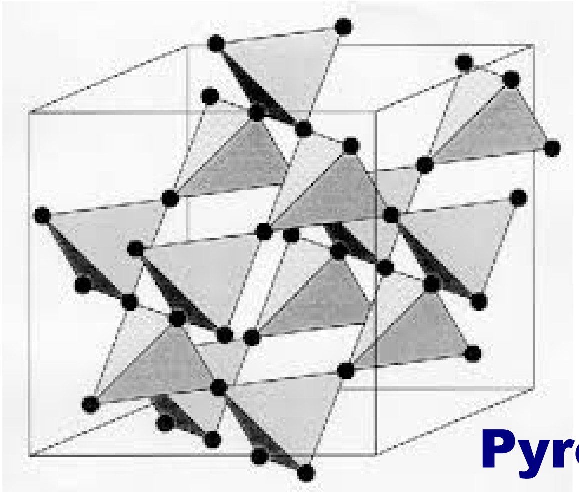
The saturation value of M_{TR} at 5.5 K, plotted as function of time t on logarithmic scales. The $M_{\mathrm{TR}}(t)$ data can be described by a power law $M_{\mathrm{TR}}(t) = mt^{-a}$, implying that the relaxation involves a distribution of activation energies.



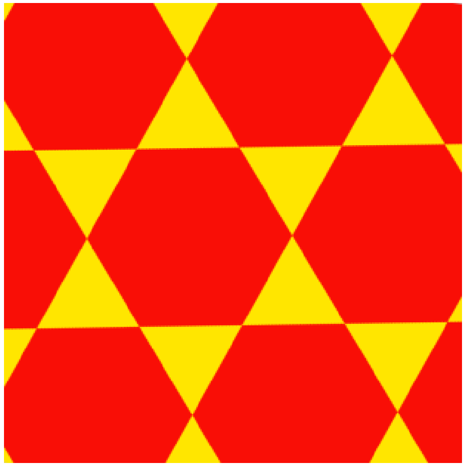
Strongly geometrically frustrated magnets

Geometrically frustrated lattices:

1) triangular,
2) Kagomé,
3) Pyrochlore,
4) fcc



Pyrochlore lattice



Kagomé

A. P. Ramirez, Ann. Rev. Mater. Sci. **24**, 453 (1994).

Quasi-one-dimensional magnets

Examples: $A_{1-x}CuO_2$ (A=Sr, Ca, Ba)

Significance of quantum fluctuations (tend towards spontaneous spin flips)

Structure: square planar CuO_2 units form edge-sharing infinite chains alternating with Sr, Ca, or Ba atoms.

The chains of copper spins are weakly coupled magnetically.

Spins dimerisation; long-range magnetic order below 10 K.

A. Shengelaya *et al.*, Phys. Rev. Lett. **80**, 3626 (1998)

2D magnets

Example: $Cu_2(OH)_3(C_mH_{2m+1}COO)$, m = 7, 9, 11.

Hybrid organic/inorganic triangular quantum Heisenberg antiferromagnets with additional Dzyaloshinskii-Moriya interaction.

No ordered noncollinear Néel ground state.

An unusual state with both canted antiferromagnetic and spin glass-like features.

M. A. Girtu et al., Phys. Rev. B **57**, 11058 (1998)

Heavy electron metals

Heavy electron physics ($CeAl_3$; Andres, Graebner, Ott, 1975) LT properties: a Fermi liquid, in which the quasiparticles have effective masses up to 1000 free electron masses.

Compounds in which such properties are evident: UPt_3, URu_2Si_2, U_2Zn_{17}, UBe_{13}, $CeAl_2$, $CeAl_3$, CeB_6, $CeCu_6$, $CeCu_2Si_2$, $CeRu_2Si_2$ and YbBiPt.

They are all intermetallic compounds of lanthanides or actinides with non-magnetic elements in which, at LT, a correlated fluid of electron quasiparticles forms as a result of weak hybridization between the almost localized 4f or 5f electrons and delocalized s, p and d electrons.

Transition from HT local moment behaviour to the LT state in which coherence between the moments develops and leads to a heavy electron ground state.

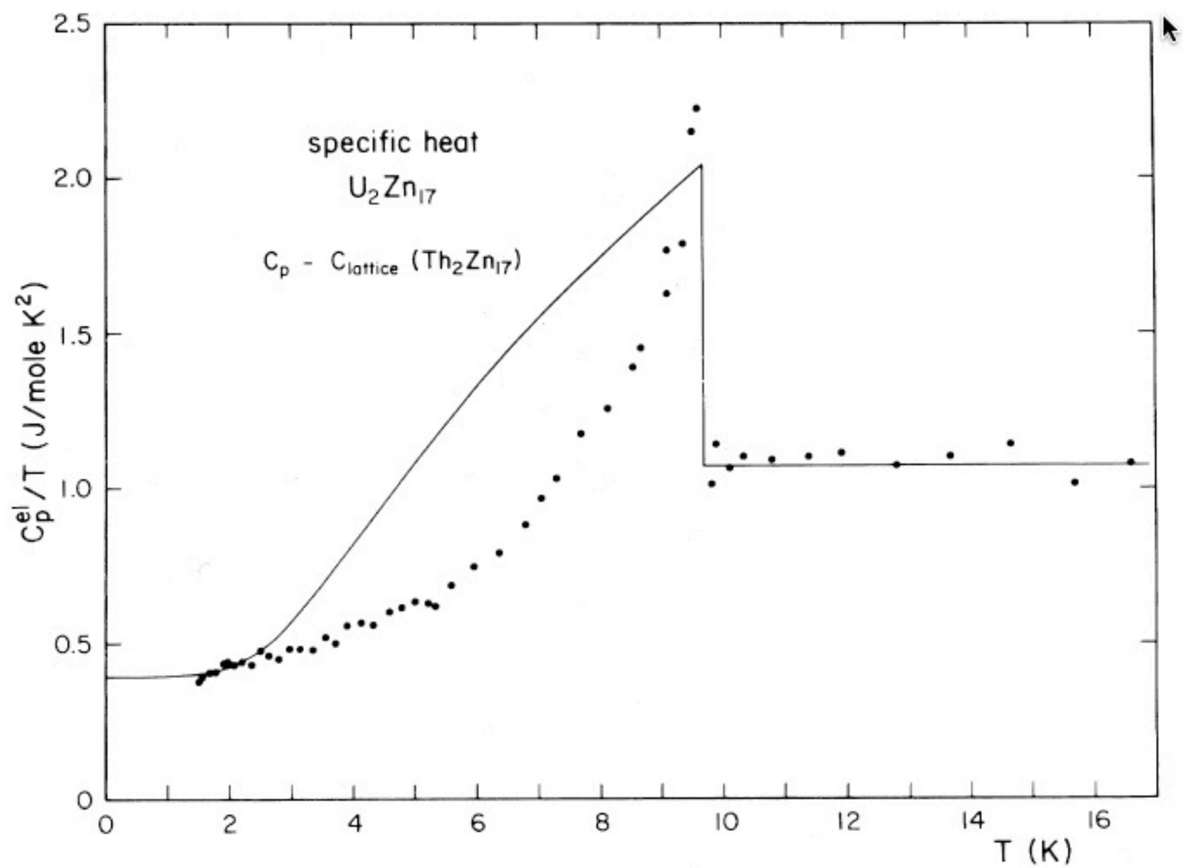
Insulating behaviour, Fermi liquid behaviour, AF ordering with very low moments, AF correlations, metamagnetic behaviour and superconductivity.

Competing many-body interactions:

- the RKKY interaction in which there is a tendency towards moment ordering, mediated by the itinerary electrons,
- the Kondo effect in which a local spin forms a spin singlet with the surrounding itinerary electron cloud.

Magnetic order in the presence of heavy electrons

Example – U_2Zn_{17}, $C_{\mathrm{el}}/T \approx 550\,\mathrm{mJ/mol\,K^2}$ above T_N.



[from Ott H.R., Rudigier H., Delsing P. and Fisk Z., Phys. Rev. Lett. **52**, 1551 (1984)]

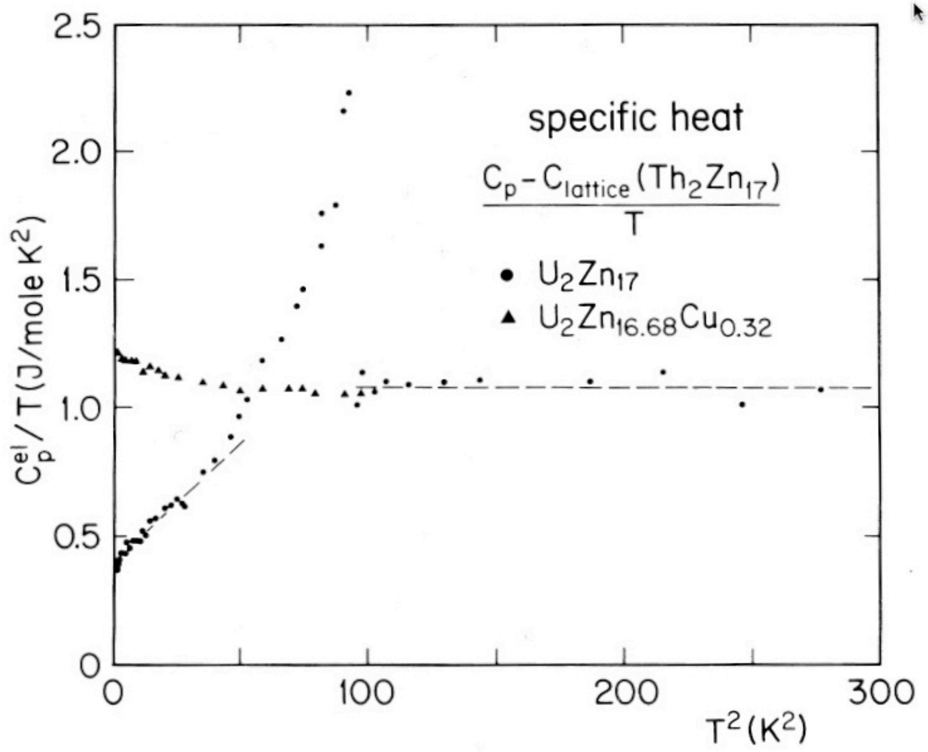
$\chi(T)$: transition to an antiferromagnetically ordered state.

Neutron diffraction: the saturated ordered moment in zero magnetic field is 0.8 μ_B/U.

This is distinctly smaller than expected for either free U^{3+} or U^{4+} ions.

The entropy loss due to the transition at T_N is very small compared with the entropy $\Delta S = 2R \ln 2 = 11.52\,\mathrm{J/mol\,K}$ that would be released by lifting the degeneracy of a doubly degenerate ground state of the uranium ions.

The AF ordering in U_2Zn_{17} is very sensitive to impurities replacing Zn, much more sensitive than what is typical for a conventional antiferromagnet:



[from Barth S., Ott H.R., Gygax F.N., Hitti B., Lippelt E., Schenck A. and Baines C., Phys. Rev. B **39**, 11695 (1989)]

Kondo insulators

Strongly correlated semiconductors (direct gap) and semi-metals (indirect gap)

Interaction of localized electrons from unfilled 3d- or 4f-shells with itinerary electrons opens a narrow gap at the Fermi energy.

Contrary to conventional semiconductors, the magnitude of the gap is strongly temperature dependent (metallic behaviour at high temperatures).

Examples are $Ce_3Bi_4Pt_3$, SmB_6, YB_{12}, CeNiSn.
Also FeSi. Co and Al doped FeSi

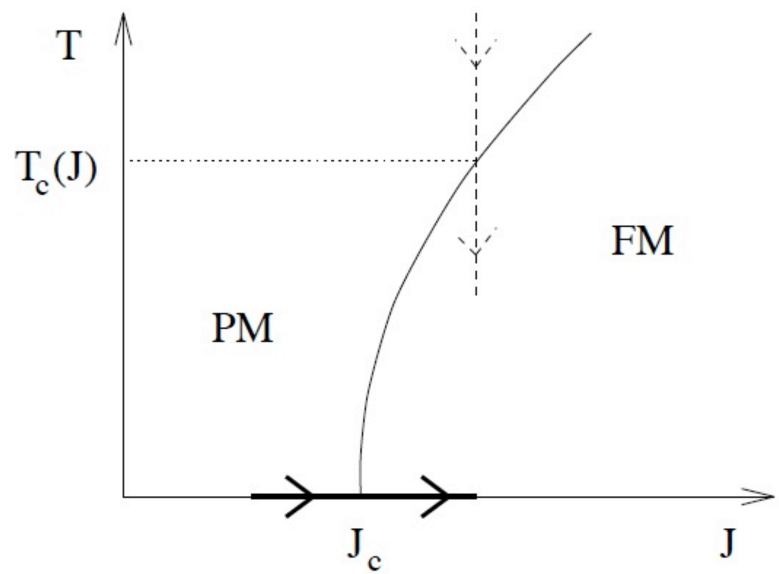
G. Aeppli, Z. Fisk, Comments Cond. Matt. Phys. **16**, 155 (1992).

Phase transitions between different quantum phases at $T = 0$ (quantum phase transitions)

Many-body systems: an abrupt change in the ground state due to quantum fluctuations.

Quantum vs. classical phase transitions:

Varying a physical parameter, e.g., pressure or magnetic field, at zero temperature.



[from D. Belitz and T. R. Kirkpatrick, arXiv:cond-mat/9811058v1]