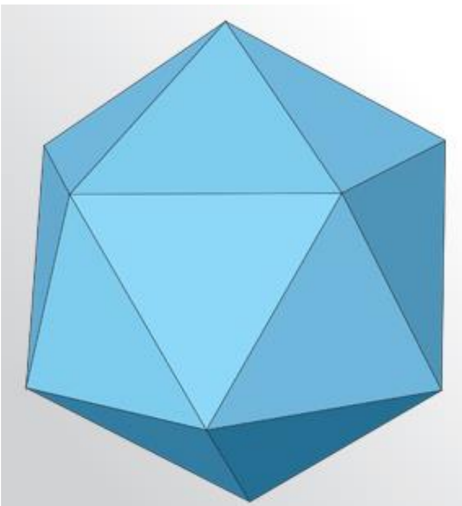
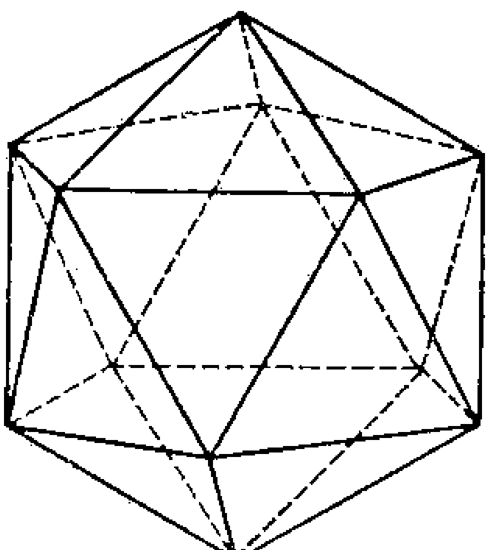
Icosahedral and other quasicrystals

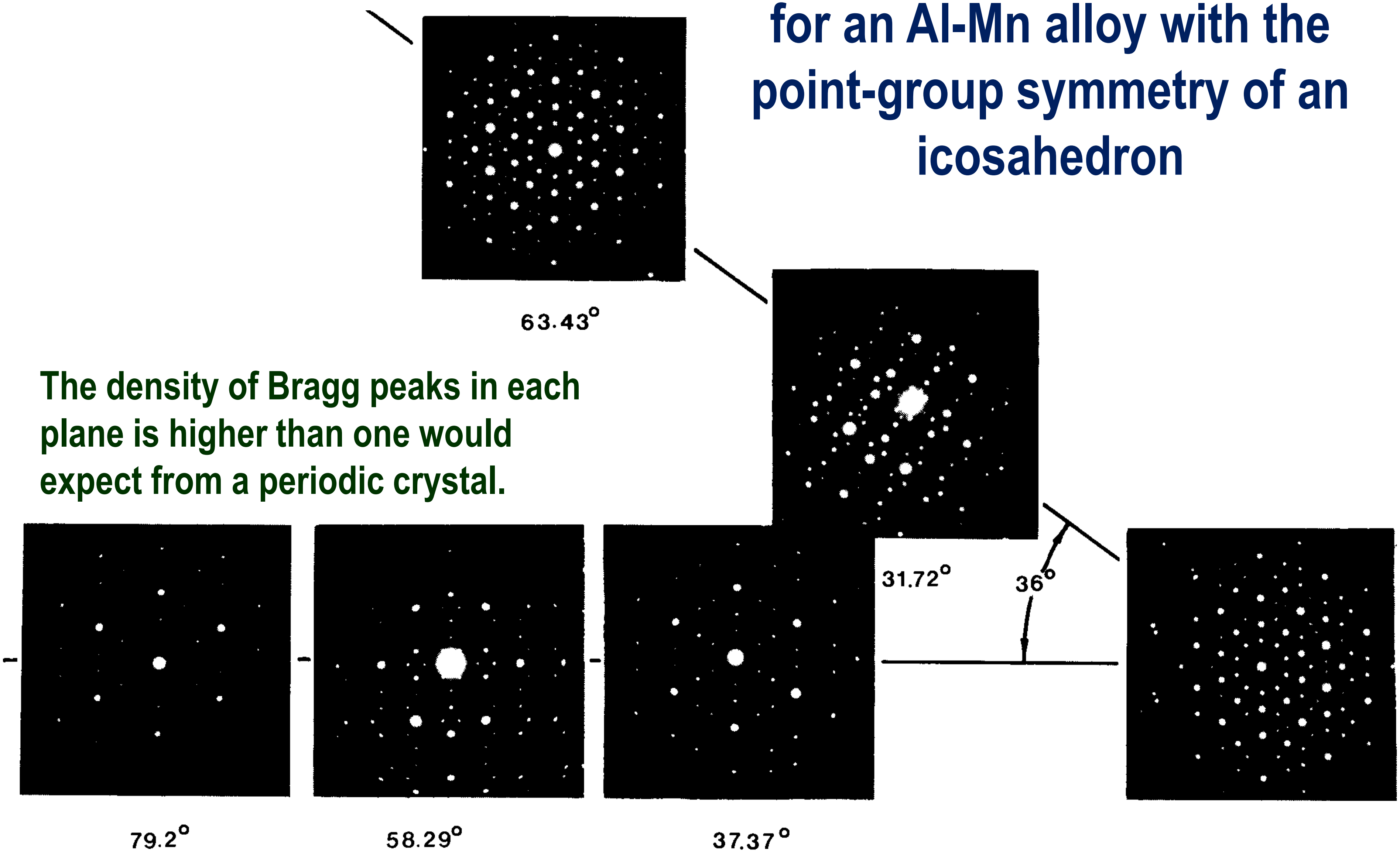
Nowadays, quasicrystals are commonly understood as hard metal alloys with long-range order whose diffraction peaks are located with noncrystallographic symmetry.





In 1984, an alloy of aluminum with manganese was discovered $Al_{0.86}Mn_{0.14}$ the sample of which scattered the electron beam in such a way that a clearly pronounced diffraction pattern with fifth-order symmetry in the arrangement of the diffraction maxima (icosahedral symmetry) was formed on the photo-plate. The presence of sharp diffraction maxima indicated the presence in the structure of a long-range order in the arrangement of atoms characteristic of crystals, since this means that the atoms in different parts of the sample equally reflect the electron beam. However, the symmetry of the observed diffraction pattern contradicted the fundamental concepts of classical crystallography: such symmetry is physically impossible for any crystalline substance. Further studies have shown that a new type of order is realized in the new material, non-crystalline and non-amorphous (amorphous matter is characterized by the presence of a short-range atomic order - crystalline order only within a few interatomic distances). Therefore, this substance was called a quasicrystal [1]. Later, other metal alloys with a long-range order and prohibited for crystals axes of symmetry 7, 8, 10, 12, etc., were found.

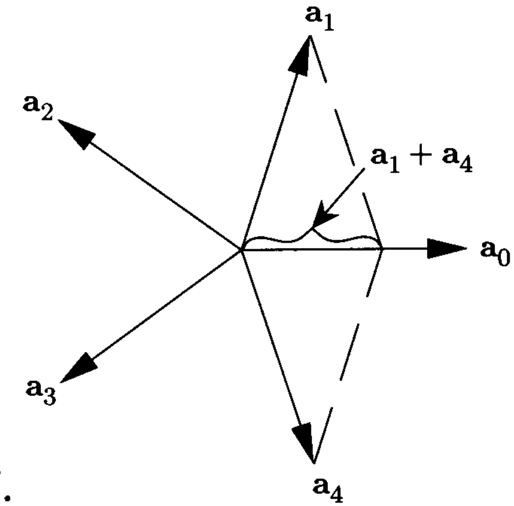
Electron diffraction pattern for an Al-Mn alloy with the point-group symmetry of an icosahedron



The density of Bragg peaks in each plane is higher than one would expect from a periodic crystal.

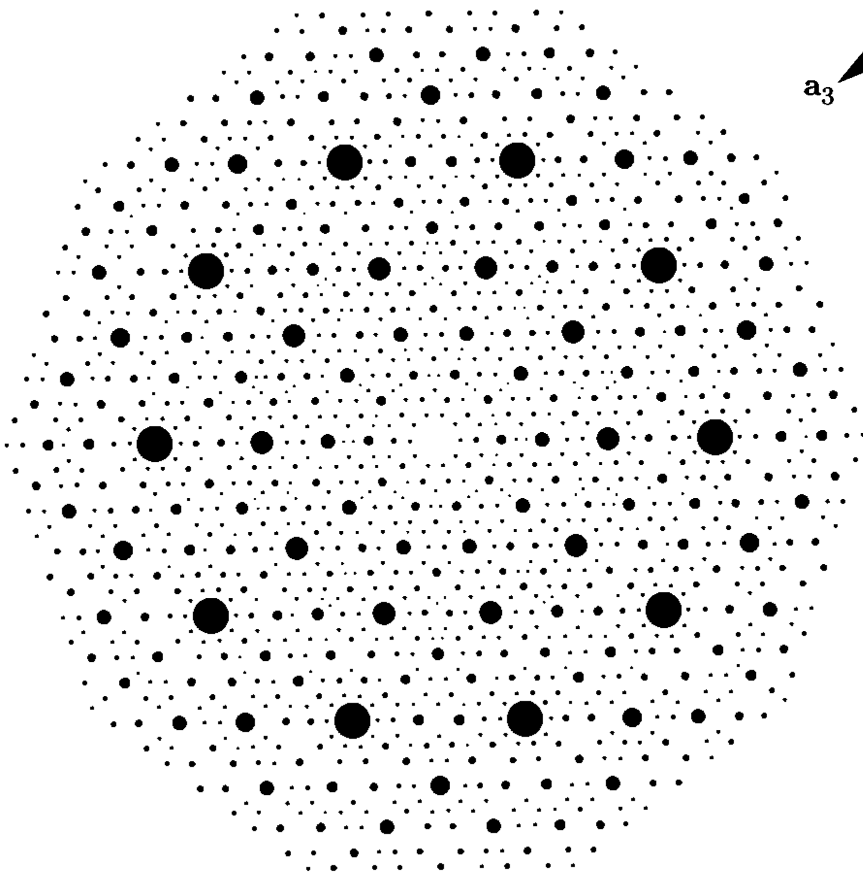
Fig. 2.10.2. Experimental diffraction from icosahedral $AlMn_6$ [D.S. Shechtman, I. Blech, D. Gratias, and J. W. Cahn, *Phys. Rev. Lett.* **53**, 1951 (1984)].

Pentagon symmetry in 2D quasicrystals



Though arbitrarily short vectors are permitted in a reciprocal lattice, atoms in real space cannot be arbitrarily close together.

How can there be a minimum distance in a quasicrystal?



Points G in a reciprocal lattice with ten-fold symmetry generated by

$\sum A_n \mathbf{a}_n$ with $A_n = 0, \pm 1, \pm 2$ with $|\mathbf{G}|/|\mathbf{a}_0| \le 4$.

The answer is provided by tiling of a 2D plane with five-fold symmetry invented by R. Penrose (1974).