Classical theory of elasticity. Deformation.

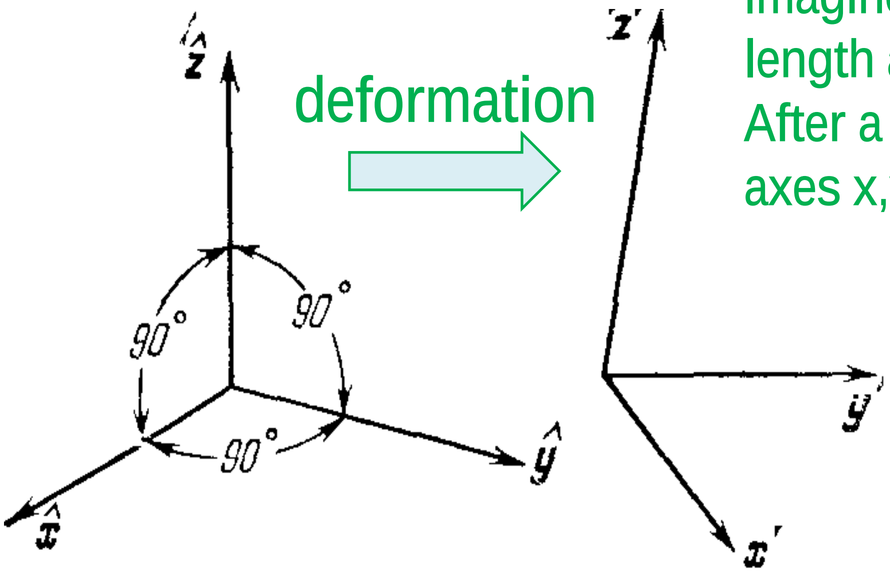


Fig.1. Coordinate axes (a) before a deformation and (b) after it.

Imagine that three orthogonal vectors x, y, z of unit length are embedded securely in the unstrained solid. After a small uniform deformation of the solid these axes x,y,z are distorted in orientation and in length.

In a uniform deformation each primitive cell of the crystal is deformed in the same way. The new axes x', y', z' may be written in terms of the old axes:

$$\mathbf{x}' = (1 + \epsilon_{xx})\hat{\mathbf{x}} + \epsilon_{xy}\hat{\mathbf{y}} + \epsilon_{xz}\hat{\mathbf{z}}$$

$$\mathbf{y}' = \epsilon_{yx}\hat{\mathbf{x}} + (1 + \epsilon_{yy})\hat{\mathbf{y}} + \epsilon_{yz}\hat{\mathbf{z}}$$

$$\mathbf{z}' = \epsilon_{zx}\hat{\mathbf{x}} + \epsilon_{zy}\hat{\mathbf{y}} + (1 + \epsilon_{zz})\hat{\mathbf{z}}$$

The coefficients $\varepsilon_{\alpha\beta}$, called *strain tensor*, define the deformation; they are dimensionless and << 1 (if the strain is small). The original axes were of unit length, but the new axes are not necessarily of unit length:

$$\mathbf{x}' \cdot \mathbf{x}' = 1 + 2\epsilon_{xx} + \epsilon_{xx}^2 + \epsilon_{xy}^2 + \epsilon_{xz}^2$$

$$x' \cong 1 + \epsilon_{xx} + \cdots .$$

The length of the unit vector changes

Change of length and volume during deformation

$$\mathbf{x}' = (1 + \epsilon_{xx})\hat{\mathbf{x}} + \epsilon_{xy}\hat{\mathbf{y}} + \epsilon_{xz}\hat{\mathbf{z}}$$
$$\mathbf{y}' = \epsilon_{yx}\hat{\mathbf{x}} + (1 + \epsilon_{yy})\hat{\mathbf{y}} + \epsilon_{yz}\hat{\mathbf{z}}$$
$$\mathbf{z}' = \epsilon_{zx}\hat{\mathbf{x}} + \epsilon_{zy}\hat{\mathbf{y}} + (1 + \epsilon_{zz})\hat{\mathbf{z}}$$

The displacement R due to the deformation is

$$\mathbf{R} \equiv \mathbf{r}' - \mathbf{r} = x(\mathbf{x}' - \hat{\mathbf{x}}) + y(\mathbf{y}' - \hat{\mathbf{y}}) + z(\mathbf{z}' - \hat{\mathbf{z}})$$

$$\mathbf{R}(\mathbf{r}) \equiv (x\epsilon_{xx} + y\epsilon_{yx} + z\epsilon_{zx})\hat{\mathbf{x}} + (x\epsilon_{xy} + y\epsilon_{yy} + z\epsilon_{zy})\hat{\mathbf{y}} + (x\epsilon_{xz} + y\epsilon_{yz} + z\epsilon_{zz})\hat{\mathbf{z}}$$
$$= u(\mathbf{r})\hat{\mathbf{x}} + v(\mathbf{r})\hat{\mathbf{y}} + w(\mathbf{r})\hat{\mathbf{z}}$$

Strain components $e_{xx} \equiv \epsilon_{xx} = \dfrac{\partial u}{\partial x}$; $e_{yy} \equiv \epsilon_{yy} = \dfrac{\partial v}{\partial y}$; $e_{zz} \equiv \epsilon_{zz} = \dfrac{\partial w}{\partial z}$

The fractional increase of volume associated with a deformation is called *dilation*. The dilation is negative for hydrostatic pressure.

The unit cube of edges x, y, z after deformation has a volume

$$\mathbf{x}' \cdot \mathbf{y}' \times \mathbf{z}' = \begin{vmatrix} 1 + \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & 1 + \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & 1 + \epsilon_{zz} \end{vmatrix} \cong 1 + e_{xx} + e_{yy} + e_{zz}$$

Relative volume change during deformation $\delta \equiv \dfrac{V' - V}{V} \cong e_{xx} + e_{yy} + e_{zz}$

Tensors. Definition as multidimensional arrays.

An **n**-th-rank tensor in **m**-dimensional space is a mathematical object that has **n** indices and m^n components and obeys certain transformation rules (its components transform as a products of the components of n vectors). Each index of a tensor ranges over the number of dimensions of space. Tensors are generalizations of scalars (0-rank tensor that have no indices), vectors (that have one index => tensor of rank 1), and matrices (that have two indices => 2nd-rank tensor) to an arbitrary number of indices.

Tensors may have upper (contravariant) and lower (covariant) indices: A^i or A_i .
They are related by the metric tensor g_{ik} : $A_i = g_{ik} A^k$. Usually, $g_{ik} = \delta_{ik}$, => $A^i = A_i$

The tensor product takes two tensors, *S* and *T*, and produces a new tensor *S* ⊗ *T*, whose order is the sum of the orders of the original tensors: $A_{ik} \otimes B_{lm} = C_{iklm}$ **.**

Tensor contraction is an operation that reduces a type (*n*, *m*) tensor to a type (*n*−1,*m*−1) tensor. It thereby reduces the total order of a tensor by two. The operation is achieved by summing components for which one contravariant index is the same as one covariant index to produce a new component. Components for which those two indices are different are discarded. (This is like taking the trace of a matrix.) For example, a (1,1) tensor $A^i{}_k$ can be contracted to a scalar: $A^i{}_i = \Sigma_i a^i{}_i$. **Or** $A_{ik} B^{ik} = \Sigma_{ik} a_{ik} b^{ik} = C$**.**

Deformation in tensor notations.

The displacement vector $\boldsymbol{r}' - \boldsymbol{r}$ has the components $u_i = x'_i - x_i$.

The distance between two points is $\mathrm{d}l = \sqrt{(\mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2)}$

It changes due to deformation: $\mathrm{d}l^2 = \mathrm{d}x_i^2$, $\mathrm{d}l'^2 = \mathrm{d}x'_i{}^2 = (\mathrm{d}x_i + \mathrm{d}u_i)^2$

Substituting $\mathrm{d}u_i = (\partial u_i/\partial x_k)\mathrm{d}x_k$ we rewrite dl'^2 as (one may interchange the suffixes i and l due to summation)

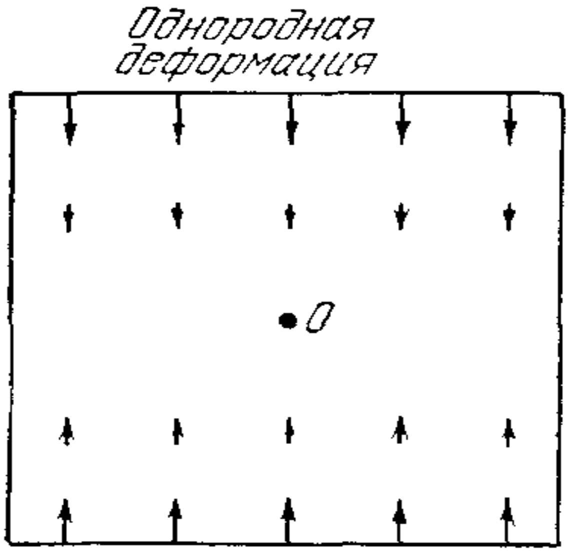
$$\boxed{\mathrm{d}l'^2 = \mathrm{d}l^2 + 2\frac{\partial u_i}{\partial x_k}\mathrm{d}x_i\,\mathrm{d}x_k + \frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_l}\mathrm{d}x_k\,\mathrm{d}x_l}$$

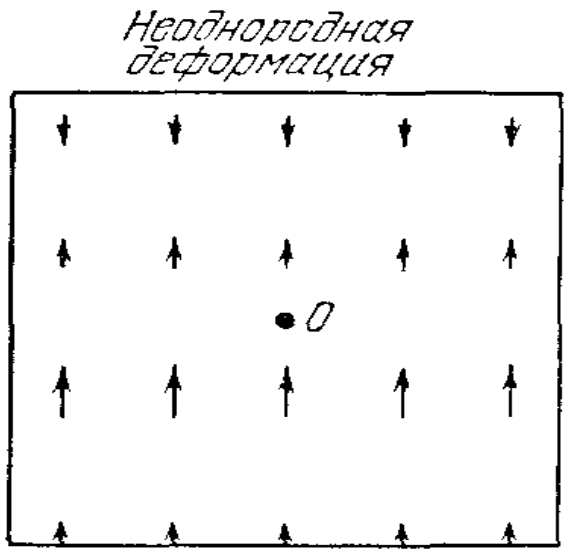
Then after introducing the ***strain tensor***

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}\right)$$

dl'^2 takes the final form ($u_{ik} = u_{ki}$):

$$\mathrm{d}l'^2 = \mathrm{d}l^2 + 2u_{ik}\,\mathrm{d}x_i\,\mathrm{d}x_k,$$





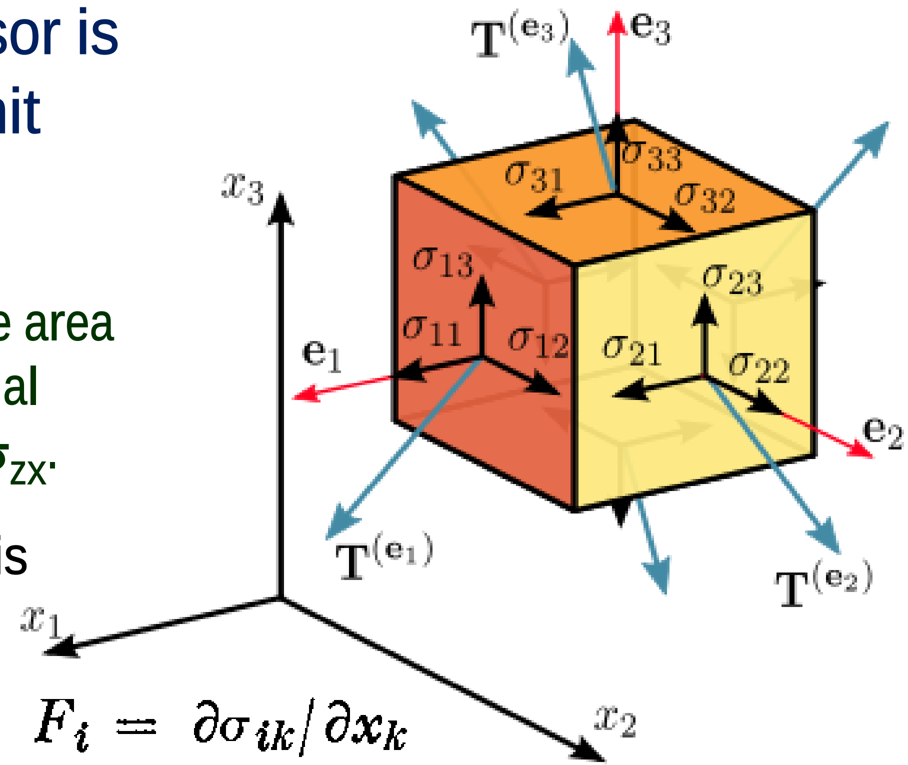
In almost all cases occurring in practice, the strain is small.

The stress tensor and Hooke's law.

Stress tensor $\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \equiv \begin{bmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{bmatrix}$

The component $\boldsymbol{\sigma}_{ik}$ of the stress tensor is the i-th component of the force on unit area perpendicular to the x_k-axis.

For instance, the force acting on unit area perpendicular to the x-axis and normal to the area (i.e. along the x-axis) is $\boldsymbol{\sigma}_{xx}$, and the tangential forces (along the y and z axes) are $\boldsymbol{\sigma}_{yx}$ and $\boldsymbol{\sigma}_{zx}$.



The total force acting on a solid of volume V is

$$\int F_i \, \mathrm{d}V = \int \frac{\partial \sigma_{ik}}{\partial x_k} \mathrm{d}V = \oint \sigma_{ik} \, \mathrm{d}f_k,$$

$$F_i = \partial \sigma_{ik} / \partial x_k$$

The moment of the forces on a portion of the body is

$$M_{ik} = \int (F_i x_k - F_k x_i) \, \mathrm{d}V = \oint (\sigma_{il} x_k - \sigma_{kl} x_i) \, \mathrm{d}f_l.$$

For hydrostatic compression $\sigma_{ik} = -p\delta_{ik}$ and the moment of forces is $M_{ik}=0$.

The energy of deformations and Hooke's law.

The work δR in terms of the change in the strain tensor is $\delta R = -\sigma_{ik}\delta u_{ik}$

The total work on a body is $\int \delta R \, \mathrm{d}V = \int (\partial \sigma_{ik}/\partial x_k)\delta u_i \, \mathrm{d}V$

Change in the internal energy $\mathrm{d}\mathscr{E} = T\mathrm{d}S + \sigma_{ik}\,\mathrm{d}u_{ik}$

Change in the free energy $\mathrm{d}F = -S\mathrm{d}T + \sigma_{ik}\,\mathrm{d}u_{ik}$

Hence, the stress tensor is given by the derivative: $\sigma_{ik} = (\partial \mathscr{E}/\partial u_{ik})_S = (\partial F/\partial u_{ik})_T.$

Hooke's law in a general form (for crystals).

In equilibrium the Taylor series $F(u_{ik})$ does not contain terms linear in u_{ik}, and the free energy of a deformed crystal is

$$F = \tfrac{1}{2}\lambda_{iklm}u_{ik}u_{lm},$$

where λ_{iklm} is a tensor of rank four, called the *elastic modulus tensor*.

Hence, the stress tensor σ_{ik} is linear in strain tensor u_{ik} (Hooke's law): $\sigma_{ik} = \partial F/\partial u_{ik} = \lambda_{iklm}u_{lm}$

If the crystal possesses symmetry, relations exist between the various components of the tensor λ_{ikim}, so that the number of independent components is less than 21.

The least number of non-zero moduli that is possible by suitable choice of the co-ordinate axes is the same for all the classes in each system :

Triclinic 18
Monoclinic 12
Orthorhombic 9
Tetragonal 6
Rhombohedral 6
Hexagonal 5
Cubic 3

Hooke's law in isotropic media

The free energy can be expanded in a Taylor series for small strain tensor u_{ik}:

$$F = F_0 + \tfrac{1}{2}\lambda u_{ii}^2 + \mu u_{ik}^2$$

λ and μ are called *Lamé coefficients*.

In equilibrium this Taylor series does not contain terms linear in u_{ik}, => the stress tensor is linear in strain tensor u_{ik}: $\sigma_{ik} = (\partial\mathscr{E}/\partial u_{ik})_S = (\partial F/\partial u_{ik})_T$

Any deformation can be represented as the sum of a pure shear and a hydrostatic compression:

$$u_{ik} = \left(u_{ik} - \tfrac{1}{3}\delta_{ik}u_{ll}\right) + \tfrac{1}{3}\delta_{ik}u_{ll}.$$

Then the free energy becomes $F = \mu\left(u_{ik} - \tfrac{1}{3}\delta_{ik}u_{ll}\right)^2 + \tfrac{1}{2}K u_{ll}^2$

K is called the bulk modulus of hydrostatic compression (or simply the modulus of compression) and μ is shear modulus or modulus of rigidity.

The stress tensor $\sigma_{ik} = \partial F/\partial u_{ik}$ is linear in the strain tensor (Hooke's law):

$$\sigma_{ik} = K_{\mathrm{ad}} u_{ll}\delta_{ik} + 2\mu\left(u_{ik} - \tfrac{1}{3}\delta_{ik}u_{ll}\right)$$

In crystals the Hooke's law is $\sigma_{ik} = \lambda_{iklm}u_{lm}$

Elastic waves in crystals (from Landau & Lifshitz, Vol. 7)

The general equations of motion (second Newton's law)

$\rho \ddot{u}_i = F_i = \partial\sigma_{ik}/\partial x_k$ **where** $\sigma_{ik} = \lambda_{iklm} u_{lm}$

Substituting also the strain tensor (expressed via displacement derivatives): $u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} + \cancel{\frac{\partial u_l}{\partial x_i}\frac{\partial u_l}{\partial x_k}}\right)$

One obtains $$\rho \ddot{u}_i = \lambda_{iklm}\frac{\partial u_{lm}}{\partial x_k} = \frac{\lambda_{iklm}}{2}\frac{\partial}{\partial x_k}\left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l}\right) = \lambda_{iklm}\frac{\partial^2 u_m}{\partial x_k \partial x_l}$$

Since the tensor λ_{iklm} is symmetric with respect to the indices l and m, we can inter-change these in the first term, which then becomes identical with the second term.

Thus the equations of motion are $$\rho \ddot{u}_i = \lambda_{iklm}\frac{\partial^2 u_m}{\partial x_k \partial x_l} \quad (1)$$

Let us consider a monochromatic elastic wave in a crystal.
We seek a solution of the equations of motion in the form: $u_i = u_{0i} e^{i(\mathbf{kr}-\omega t)}$

Substituting this to (1) we obtain $\rho\omega^2 u_i = \lambda_{iklm} k_k k_l u_m$

This is a set of three homogeneous equations of the first degree for 3 unknowns u_x, u_y, u_z. Such equations have non-zero solutions only if the determinant of coefficients is zero.

Thus we obtain the cubic equation on ω: $|\lambda_{iklm} k_k k_l - \rho\omega^2 \delta_{im}| = 0$
It has 3 solutions (branches of sound).

This equation gives the relation ω(k) between frequency and wave vector of waves, called the dispersion relation.

Dispersion relation and group velocity of sound waves

Equation $(\rho\omega^2\delta_{im} - \lambda_{iklm}k_k k_l)\,u_m = 0$ determines eigenvectors u_i (polarization) and eigenvalues $\omega(k)$ (dispersion) of the sound waves.

The velocity of propagation of the wave (the group velocity) is given by the derivative of the frequency with respect to wave vector: $\mathbf{U} = \partial\omega/\partial\mathbf{k}$

In an isotropic body, the frequency is proportional to the magnitude of k, and the direction of the velocity is the same as that of k. Then the sound velocity c is constant (independent of k) and directed along k.

In crystals the direction of wave velocity is, generally, not the same as of its wave vector

Example: Dispersion relation for elastic waves propagating in hexagonal crystal.

Hexagonal crystal has 5 independent elastic modules:

$$\lambda_{xxxx} = \lambda_{yyyy} = a, \quad \lambda_{xyxy} = b, \quad \lambda_{xxyy} = a - 2b,$$
$$\lambda_{xxzz} = \lambda_{yyzz} = c, \quad \lambda_{xzxz} = \lambda_{yzyz} = d, \quad \lambda_{zzzz} = f.$$

Take finite angle θ between k and z: $k_x = k\sin\theta,\ k_y = 0,\ k_z = k\cos\theta,$

Solving equation $|\lambda_{iklm}k_k k_l - \rho\omega^2\,\delta_{im}| = 0$ we find $\rho\omega_1^2 = k^2\,(b\sin^2\theta + d\cos^2\theta),$

and $\rho\omega_{2,3}^2 = {}^1/_2 k^2\,\{a\sin^2\theta + f\cos^2\theta + d \pm$

$$\pm\,[((a-d)\sin^2\theta + (d-f)\cos^2\theta)^2 + 4\,(c+d)^2\sin^2\theta\cos^2\theta]^{1/2}\}.$$

Equations of motion and relation between strain and stress tensors in isotropic media

In an isotropic body, the free energy

$$F = \mu\left(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}\right)^2 + \frac{K}{2}u_{ll}^2 = \frac{E}{2(1+\sigma)}\left(u_{ik}^2 + \frac{\sigma}{1-2\sigma}u_{ll}^2\right)$$

K is the modulus of hydrostatic compression and μ is shear modulus,

the modulus of extension (*Young's modulus*:) $E = 9K\mu/(3K+\mu)$

The ratio of transverse compression to longitudinal extension, called ***Poisson's coefficient***, is

$$\sigma = \frac{1}{2}\frac{3K-2\mu}{3K+\mu}$$

The stress tensor $\sigma_{ik} = \partial F/\partial u_{ik}$ is then given in terms of strain tensor by

$$\sigma_{ik} = \frac{E}{1+\sigma}\left(u_{ik} + \frac{\sigma}{1-2\sigma}u_{ll}\delta_{ik}\right) \quad \text{or} \quad u_{ik} = \frac{1}{E}[(1+\sigma)\sigma_{ik} - \sigma\sigma_{ll}\delta_{ik}]$$

Then $$\frac{\partial\sigma_{ik}}{\partial x_k} = \frac{E\sigma}{(1+\sigma)(1-2\sigma)}\frac{\partial u_{ll}}{\partial x_i} + \frac{E}{1+\sigma}\frac{\partial u_{ik}}{\partial x_k}$$

Substituting this to the equation of motion $\rho\ddot{u}_i = F_i = \partial\sigma_{ik}/\partial x_k$

we obtain $$\rho\ddot{\mathbf{u}} = \frac{E}{2(1+\sigma)}\Delta\mathbf{u} + \frac{E}{2(1+\sigma)(1-2\sigma)}\operatorname{grad}\operatorname{div}\mathbf{u}.$$

Dispersion relation and group velocity of sound waves in isotropic media

Equation of motion

$$\rho\ddot{\mathbf{u}} = \frac{E}{2(1+\sigma)}\Delta\mathbf{u} + \frac{E}{2(1+\sigma)(1-2\sigma)}\,\mathrm{grad\,div}\,\mathbf{u}. \qquad (1)$$

Consider a plane elastic wave in an infinite isotropic medium, i.e. a wave in which the deformation u is a function only of one coordinate (x) and of the time. All derivatives with respect to y and z in equation (1) are then zero, and we obtain

$$\frac{\partial^2 u_x}{\partial x^2} - \frac{1}{c_l^2}\frac{\partial^2 u_x}{\partial t^2} = 0, \quad \text{and} \quad \frac{\partial^2 u_y}{\partial x^2} - \frac{1}{c_t^2}\frac{\partial^2 u_y}{\partial t^2} = 0 \qquad (2)$$

longitudinal wave   transverse wave

where the sound velocities are

$$c_l = \sqrt{\frac{E(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}}, \qquad c_t = \sqrt{\frac{E}{2\rho(1+\sigma)}}.$$

Equations (2) are ordinary wave equations in one dimension, and the quantities c_l and c_t which appear in them are the velocities of propagation of the wave. We see that the velocity of propagation for the component u_x is different from that for u_y and u_z.

The velocity of longitudinal waves is greater than of transverse waves: $c_l > (4/3)^{1/2} c_t$.

The velocities c_l and c_t are often called the *longitudinal* and *transverse* velocities of sound.

Surface waves (Rayleigh waves).

Equation of motion (where u is any component of the vectors $\boldsymbol{u}_l$ or $\boldsymbol{u}_t$, and $\boldsymbol{c}$ is $\boldsymbol{c}_l$ or $\boldsymbol{c}_t$):

$$\frac{\partial^2 u}{\partial t^2} - c^2 \triangle u = 0, \quad (1)$$

has a solution $u = \mathrm{const}\, e^{i(kx-\omega t)} e^{\varkappa z}$ where $\varkappa = (k^2 - \omega^2/c^2)^{1/2}$

For $k^2 - \omega^2/c^2 < 0$ this gives usual plain wave.

However, for $k^2 - \omega^2/c^2 > 0$ this solution gives a surface wave.

The amplitude of surface wave is exponentially damped toward media.

Boundary condition at the surface $\sigma_{ik} n_k = 0$ gives the relation between various components of the displacement vector $\boldsymbol{u}$ of the wave.

Together with equation of motion this gives the dispersion ω(k):

$$\left(2k^2 - \frac{\omega^2}{c_t^2}\right)^4 = 16k^4 \left(k^2 - \frac{\omega^2}{c_t^2}\right)\left(k^2 - \frac{\omega^2}{c_l^2}\right)$$

<=>

$$\boxed{\omega = c_t k \xi}$$

where the number $\xi \approx 0.9$ satisfies the equation

$$\xi^6 - 8\xi^4 + 8\xi^2 \left(3 - 2\frac{c_t^2}{c_l^2}\right) - 16\left(1 - \frac{c_t^2}{c_l^2}\right) = 0$$

The ratio of longitudinal and transverse velocity depends only on the Poisson's coefficient, being material constant:

$$\frac{c_t^2}{c_l^2} = \frac{1 - 2\sigma}{2(1 - \sigma)}$$

Notations in the Kittel's book (1)

Stress Components. The force acting on a unit area in the solid is defined as the stress. There are nine stress components: $X_x, X_y, X_z, Y_x, Y_y, Y_z, Z_x, Z_y, Z_z$. The capital letter indicates the direction of the force, and the subscript indicates the normal to the plane to which the force is applied. The stress component X_x represents a force applied in the x direction to a unit area of a plane whose normal lies in the x direction; the stress component X_y represents a force applied in the X direction to a unit area **of a plane whose normal lies in the y** direction(Fig.1). The number of independent stress components is reduced from nine to six by applying to an elementary cube (as in Fig. 2) the condition that the angular acceleration vanish, => the total torque must be zero.

In tensor notations

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

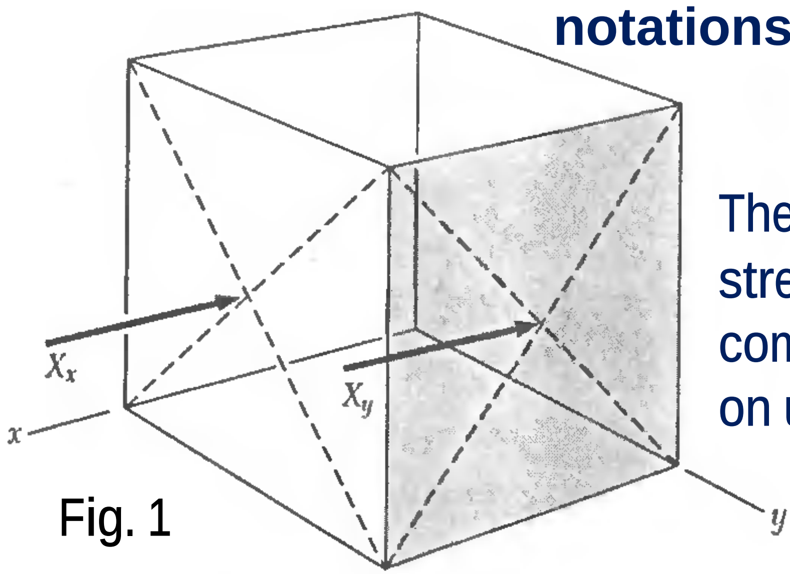


Fig. 1

The component σ_{ik} of the stress tensor is the i-th component of the force on unit area $\perp$ to x_k-axis.

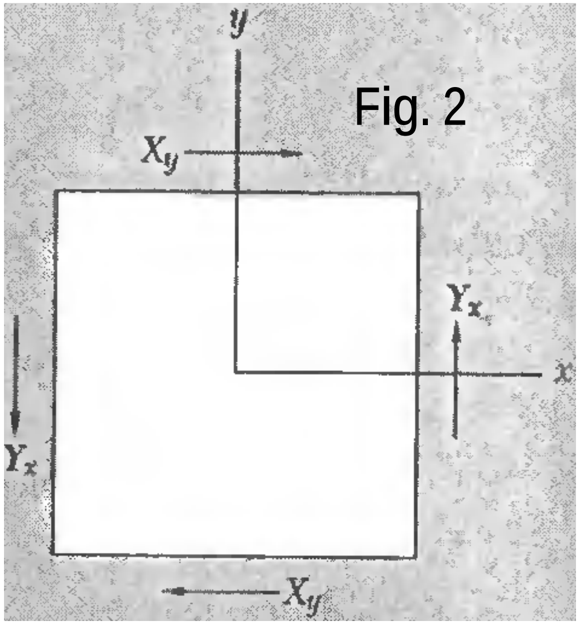


Fig. 2

Notations in the Kittel's book (2)

The Hooke's law $\sigma_{ik} = \partial F/\partial u_{ik} = \lambda_{iklm}u_{lm}$ in Kittel's notations is

$$X_x = C_{11}e_{xx} + C_{12}e_{yy} + C_{13}e_{zz} + C_{14}e_{yz} + C_{15}e_{zx} + C_{16}e_{xy}$$
$$Y_y = C_{21}e_{xx} + C_{22}e_{yy} + C_{23}e_{zz} + C_{24}e_{yz} + C_{25}e_{zx} + C_{26}e_{xy}$$
$$Z_z = C_{31}e_{xx} + C_{32}e_{yy} + C_{33}e_{zz} + C_{34}e_{yz} + C_{35}e_{zx} + C_{36}e_{xy}$$
$$Y_z = C_{41}e_{xx} + C_{42}e_{yy} + C_{43}e_{zz} + C_{44}e_{yz} + C_{45}e_{zx} + C_{46}e_{xy}$$
$$Z_x = C_{51}e_{xx} + C_{52}e_{yy} + C_{53}e_{zz} + C_{54}e_{yz} + C_{55}e_{zx} + C_{56}e_{xy}$$
$$X_y = C_{61}e_{xx} + C_{62}e_{yy} + C_{63}e_{zz} + C_{64}e_{yz} + C_{65}e_{zx} + C_{66}e_{xy}$$

Bulk Modulus and Compressibility

Объемный модуль упругости, 10^{12} дин/см^2 (или 10^{11} Н/м^2)

Сжимаемость, 10^{-12} см^2/дин (или 10^{-11} м^2/Н)

H (d) 0.002 500																	He (d) 0.00 1168
Li 0.116 8.62	Be 1.003 0.997											B 1.78 0.562	C (d) 5.45 0.183	N (e) 0.012 80	O	F	Ne (d) 0.010 100
Na 0.068 14.7	Mg 0.354 2.82											Al 0.722 1.385	Si 0.988 1.012	P (b) 0.304 3.29	S (r) 0.178 5.62	Cl	Ar (a) 0.016 93.8
K 0.032 31.	Ca 0.152 6.58	Sc 0.435 2.30	Ti 1.051 0.951	V 1.619 0.618	Cr 1.901 0.526	Mn 0.596 1.68	Fe 1.683 0.594	Co 1.914 0.522	Ni 1.86 0.538	Cu 1.37 0.73	Zn 0.598 1.67	Ga (b) 0.569 1.76	Ge 0.772 1.29	As 0.394 2.54	Se 0.091 11.0	Br	Kr (a) 0.018 56
Rb 0.031 32.	Sr 0.116 8.62	Y 0.366 2.73	Zr 0.833 1.20	Nb 1.702 0.587	Mo 2.725 0.366	Tc (2.97) (0.34)	Ru 3.208 0.311	Rh 2.704 0.369	Pd 1.808 0.553	Ag 1.007 0.993	Cd 0.467 2.14	In 0.411 2.43	Sn (g) 1.11 0.901	Sb 0.383 2.61	Te 0.230 4.35	I	Xe
Cs 0.020 50.	Ba 0.103 9.97	La 0.243 4.12	Hf 1.09 0.92	Ta 2.00 0.50	W 3.232 0.309	Re 3.72 0.269	Os (4.18) (0.24)	Ir 3.55 0.282	Pt 2.783 0.359	Au 1.732 0.577	Hg (c) 0.382 2.60	Tl 0.359 2.79	Pb 0.430 2.33	Bi 0.315 3.17	Po (0.26) (3.8)	At	Rn
Fr (0.020) (50.)	Ra (0.132) (7.6)	Ac (0.25) (4.)															

Ce (γ) 0.239 4.18	Pr 0.306 3.27	Nd 0.327 3.06	Pm (0.35) (2.85)	Sm 0.294 3.40	Eu 0.147 6.80	Gd 0.383 2.61	Tb 0.399 2.51	Dy 0.384 2.60	Ho 0.397 2.52	Er 0.411 2.43	Tm 0.397 2.52	Yb 0.133 7.52	Lu 0.411 2.43

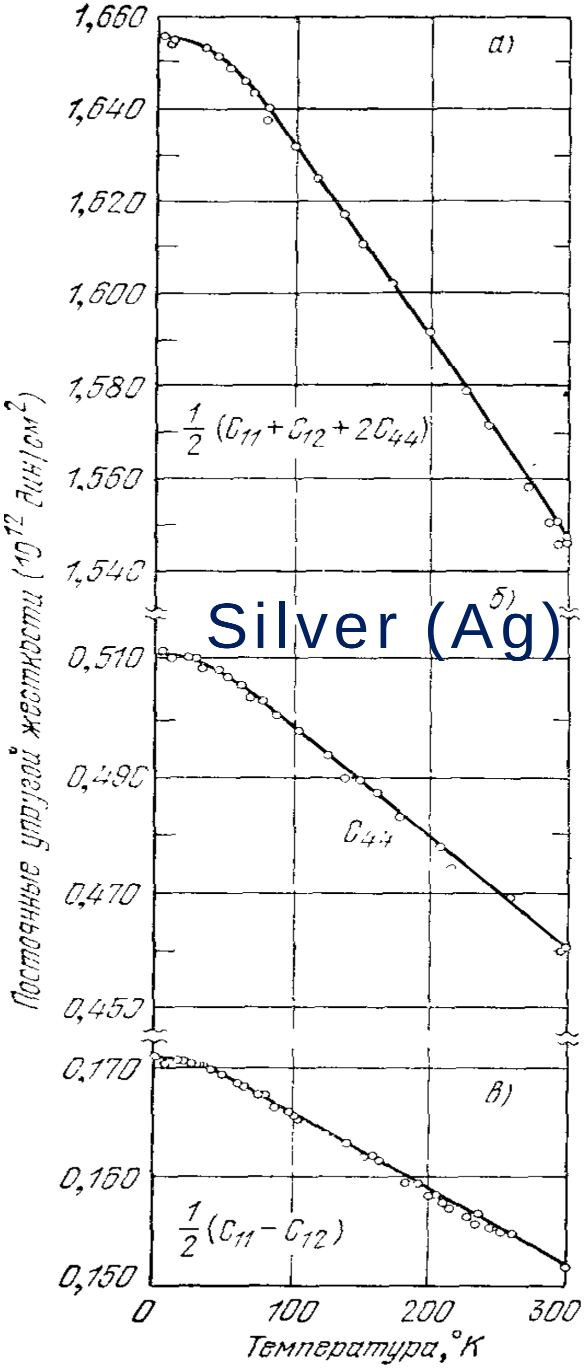
Stiffness constants of cubic crystals

Adiabatic elastic stiffness constants of several cubic crystals at room temperature or 300 K.

	Stiffness constants, in 10^{12} dyne/cm^2 or 10^{11} N/m^2		
	C_{11}	C_{12}	C_{44}
Diamond	10.76	1.25	5.76
Na	0.073	0.062	0.042
Li	0.135	0.114	0.088
Ge	1.285	0.483	0.680
Si	1.66	0.639	0.796
GaSb	0.885	0.404	0.433
InSb	0.672	0.367	0.302
MgO	2.86	0.87	1.48
NaCl	0.487	0.124	0.126

	Stiffness constants, in 10^{12} dyne/cm^2 (10^{11} N/m^2)			
Crystal	C_{11}	C_{12}	C_{44}	Temperature, K
W	5.326	2.049	1.631	0
	5.233	2.045	1.607	300
Ta	2.663	1.582	0.874	0
	2.609	1.574	0.818	300
Cu	1.762	1.249	0.818	0
	1.684	1.214	0.754	300
Ag	1.315	0.973	0.511	0
	1.240	0.937	0.461	300
Au	2.016	1.697	0.454	0
	1.923	1.631	0.420	300
Al	1.143	0.619	0.316	0
	1.068	0.607	0.282	300
K	0.0416	0.0341	0.0286	4
	0.0370	0.0314	0.0188	295
Pb	0.555	0.454	0.194	0
	0.495	0.423	0.149	300
Ni	2.612	1.508	1.317	0
	2.508	1.500	1.235	300
Pd	2.341	1.761	0.712	0
	2.271	1.761	0.717	300

Temperature dependence of stiffness constants



Stiffness constants decrease with increasing temperature