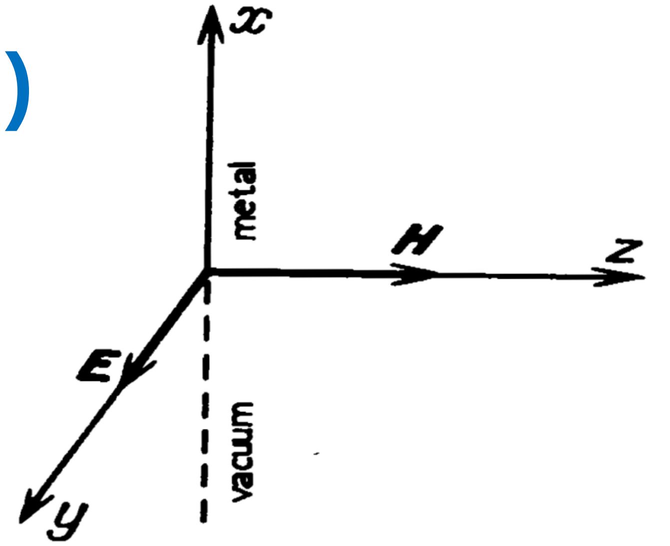
Metals in high-frequency electromagnetic field

- **Normal skin-effect ($\delta > l\tau$)**
- **Anomalous skin effect ($\delta < l\tau$). Inefficiency concept.**

Normal skin-effect ($\delta > l_\tau$)



Maxwell equations:

$$\text{rot}\,\boldsymbol{E} = -c^{-1}\frac{\partial \boldsymbol{H}}{\partial t}, \quad \text{rot}\,\boldsymbol{H} = \frac{4\pi}{c}j = \frac{4\pi}{c}\sigma E.$$

Substituting solution in the form of plane wave $exp(ikx-i\omega t)$ **we obtain**

$$\mathrm{i}kE_y = \frac{\mathrm{i}\omega}{c}H_z, \qquad -\mathrm{i}kH_z = \frac{4\pi}{c}\sigma E_y. \Rightarrow k^2 = \frac{4\pi \mathrm{i}\omega\sigma}{c^2}.$$

Hence, the wave number

$$k = \left(\frac{4\pi\mathrm{i}\omega\sigma}{c^2}\right)^{1/2} = \left(\frac{2\pi\omega\sigma}{c^2}\right)^{1/2}(1+\mathrm{i}) = k_1 + \mathrm{i}k_2.$$

Substituting this to $exp(ikx-i\omega t)$ **we see that electromagnetic wave decreases exponentially** ~ $exp(-k_2 x)$ **inside the metal.**

This phenomenon is call the *skin effect*.

The penetration depth is

$$\delta = k_2^{-1} = \left[\frac{c^2}{2\pi\omega\sigma}\right]^{1/2}.$$

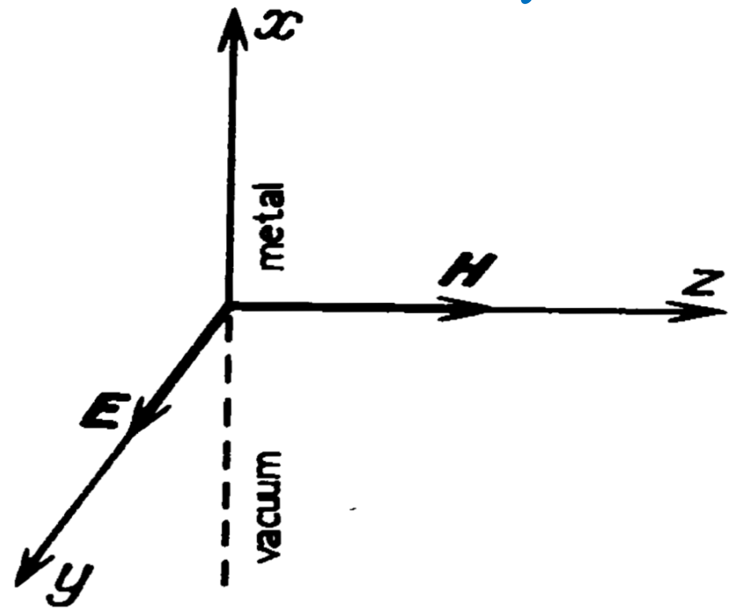
Surface impedance
in the normal skin-effect ($\delta > l_\tau$)

Electric field inside the metal decreases exponentially:

$E_y(x) = E_y(0)\exp(ikx)$.

$$k = \left(\frac{4\pi i\omega\sigma}{c^2}\right)^{1/2} = \left(\frac{2\pi\omega\sigma}{c^2}\right)^{1/2}(1+i)$$

Penetration depth $\delta = k_2^{-1} = \left[\frac{c^2}{2\pi\omega\sigma}\right]^{1/2}$



The impedance, or surface resistance is

$$Z = \frac{E_y(0)}{\int_0^\infty j_y(x)\,dx}$$

Using the Maxwell equations,

$$\text{rot}\,H = \frac{4\pi}{c} j \quad \text{and} \quad \text{rot}\,E = -c^{-1}\frac{\partial H}{\partial t},$$

we obtain

$$Z = R - iX = \frac{E_y(0)}{-(c/4\pi)H_z|_0^\infty} = \frac{4\pi}{c}\frac{E_y(0)}{H_z(0)} = \frac{4\pi}{c^2}\frac{\omega}{k}.$$

Results:

Surface impedance (2)
in the normal skin-effect ($\delta > l_\tau$)

The impedance Z is a complex quantity and is written in the form $R - \mathrm{i}X$, where R is the resistance and X is the reactance. The quantities R and X can be determined from the change in the amplitude and phase of the wave reflected from the metal surface. The resistance R determines the energy loss of the electromagnetic wave upon reflection and can be found from the production of heat in the metal when it is place in a high-frequency field.

$$Z = R - \mathrm{i}X = \frac{4\pi}{c^2}\frac{\omega}{k} = \left[\frac{4\pi\omega}{\mathrm{i}\sigma c^2}\right]^{1/2} = \left[\frac{2\pi\omega}{\sigma c^2}\right]^{1/2}(1-\mathrm{i})$$

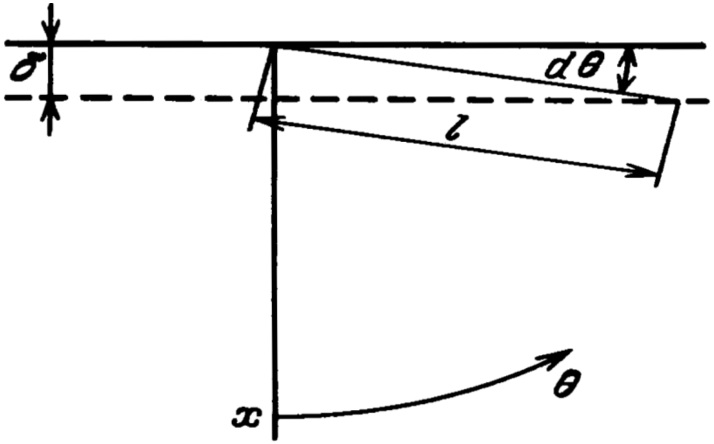
valid for any relation between j and E (first equality)

valid in the case of Ohm's law $j=\sigma E$ (second equality)

$$R = X = \left[\frac{2\pi\omega}{\sigma c^2}\right]^{1/2}.$$

The penetration depth is $$\delta = k_2^{-1} = \left[\frac{c^2}{2\pi\omega\sigma}\right]^{1/2}.$$

Anomalous skin-effect ($\delta < l_\tau \Rightarrow j \neq \sigma E$)



The electrons moving at a large angle to the surface spend little time in electric field and therefore almost do not interact with it. This is called ***inefficiency concept.***

In the isotropic case the number of electrons with momenta directed within a certain solid angle is proportional to that angle. Essential electrons are those which are moving within the skin layer throughout their entire mean free path. For these electrons (Fig. 28) $d\theta \sim \delta/l$ and, hence,

$$d\Omega \sim 2\pi \sin\theta \, d\theta \sim 2\pi \, d\theta \sim \frac{2\pi\delta}{l} \qquad (\theta \approx \tfrac{1}{2}\pi).$$

Thus, the effective electron density is $n_{\text{eff}} \sim n_e \dfrac{d\Omega}{4\pi} \sim \dfrac{n_e\delta}{l}$.

The conductivity is proportional to the number of electrons. Therefore, the effective conductivity compared to the ordinary conductivity contains a factor of order δ/l.

$$\sigma_{\text{eff}} = \frac{ia\sigma}{kl}, \; k = \left(\frac{4\pi i\omega\sigma_e}{c^2}\right)^{1/2} = \left[\frac{4\pi\omega a\sigma}{c^2 l}\right]^{1/3} e^{i\pi/3}, \; \delta = \left(\frac{c^2 l}{4\pi\omega a\sigma}\right)^{1/3} \frac{1}{\sin\frac{1}{3}\pi}.$$

Surface impedance
in the anomalous skin-effect ($\delta < l_\tau$)

The surface impedance is

$$Z = \frac{E_y(0)}{\int_0^\infty j_y(x)\,\mathrm{d}x}$$

Using the Maxwell equations,

$$\mathrm{rot}\,\boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j} \quad \text{and} \quad \mathrm{rot}\,\boldsymbol{E} = -c^{-1}\frac{\partial \boldsymbol{H}}{\partial t},$$

we obtain

$$Z = R - \mathrm{i}X = \frac{E_y(0)}{-(c/4\pi)H_z|_0^\infty} = \frac{4\pi}{c}\frac{E_y(0)}{H_z(0)} = \frac{4\pi}{c^2}\frac{\omega}{k}.$$

Substituting

$$k = \left(\frac{4\pi\mathrm{i}\omega\sigma_e}{c^2}\right)^{1/2} = \left[\frac{4\pi\omega a\sigma}{c^2 l}\right]^{1/3} \mathrm{e}^{\mathrm{i}\pi/3}$$

we obtain the impedance:

$$Z = \frac{4\pi\omega}{c^2 k} = \left(\frac{4\pi\omega}{c^2}\right)^{2/3}\left(\frac{l}{a\sigma}\right)^{1/3} \mathrm{e}^{-\mathrm{i}\pi/3} = \left(\frac{2}{a}\right)^{1/3}\left(\frac{\pi\omega}{c^2}\right)^{2/3}\left(\frac{l}{\sigma}\right)^{1/3}(1 - \mathrm{i}\sqrt{3}).$$

Results:

Anomalous skin-effect ($\delta < l_\tau$)

The penetration depth:

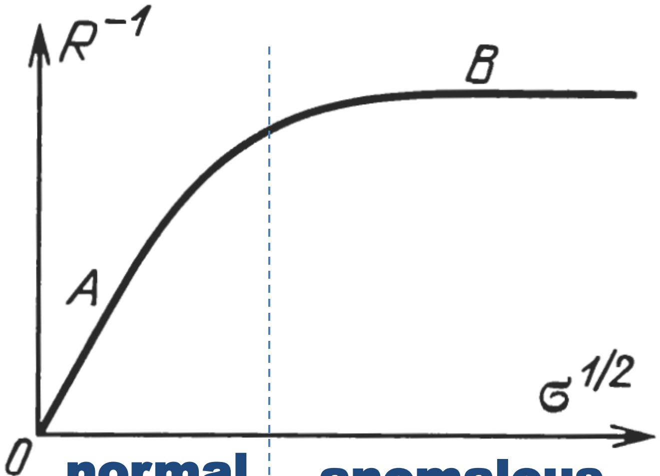
$$\delta = \left(\frac{c^2 l}{4\pi\omega a\sigma}\right)^{1/3} \frac{1}{\sin\frac{1}{3}\pi}.$$

The impedance:

$$Z = \left(\frac{2}{a}\right)^{1/3} \left(\frac{\pi\omega}{c^2}\right)^{2/3} \left(\frac{l}{\sigma}\right)^{1/3} (1 - \mathrm{i}\sqrt{3}).$$

Thus, we have obtained the following results: (a) Z is proportional to $\omega^{2/3}$; (b) $X = \sqrt{3}R$; (c) the conductivity enters into eqs. (7.11) and (7.12) only in the combination σ/l; but since $\sigma \sim n_e e^2 \tau/m \sim n_e e^2 l/p_0$, it follows that $\sigma/l \sim n_e e^2/p_0$ (this relation is independent of the temperature and is determined by the electronic spectrum alone).

Regions of normal and anomalous skin effect



normal skin effect, low conductivity

anomalous skin effect, (high conductivity)

The anomalous skin effect is observed when $\delta < l_\tau$

Substituting $\delta = k_2^{-1} = \left[\frac{c^2}{2\pi\omega\sigma}\right]^{1/2}$

and $\sigma = n_e e^2 l / p_0$

we obtain the condition for anomalous skin effect:

$$\omega > \frac{c^2 p_0}{2\pi n_e e^2 l^3}$$

Assuming that $n_e \sim 10^{22}\,\text{cm}^{-3}$ and $p_0 = \hbar(3\pi^2 n_e)^{1/3} \sim 10^{-19}\,\text{g}\,.\,\text{cm/s}$

we obtain $\omega > 10^{-2}(l[\text{cm}])^{-3}\,\text{s}^{-1}$

If $l \sim 10^{-3}$ cm (this is the residual resistance region), then $\omega > 10^7\,\text{s}^{-1}$.