On Dissipation Function of Ocean Waves due to Whitecapping

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Abstract. The Hasselmann kinetic equation [1] provides a statistical description of waves ensemble. Several catastrophic events are beyond statistical model. In the case of gravity waves on the surface of the deep fluid may be the most frequent and important events of such kind are whitecapping and wave breaking. It was shown earlier that such effects leads to additional dissipation in the energy contaning region around waves spectral peak, which can be simulated by means of empiric dissipative term in kinetic equation. In order to find dependence of this term with respect to nonlinearity in the system (steepness of the surface) we preformed two numerical experiments: weakly nonlinear one in the framework of 3D hydrodynamics and fully nonlinear one for 2D hydrodynamic. In spite of significantly different models and initial conditions, both these experiments yielded close results. Obtained data can be used to define analytical formula for dependence of the dissipative term of dissipation coefficient with respect to mean steepness of the surface.

Keywords: Hasselmann kinetic equation, whitecapping, waves forecasting
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KINETIC EQUATION.

In frameworks of the weakly turbulent theory [2] the state of a wind-driven sea is described statistically by the spectrum of wave action $N(\vec{k},t)$, satisfying the Hasselmann kinetic equation [1]:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = S_{nl} + S_{in} + S_{diss}. \tag{1}$$

Here S_{nl} is responsible for four-wave resonant interaction, S_{in} — is an input of wave action from wind, S_{diss} — the dissipation function under discussion. In the heuristic models of white capping

$$S_{diss} = -\gamma_{diss} N(\vec{k},t). \tag{2}$$

In the model *WAM3* [3]

$$\gamma_{diss}^{(3)} = 2.58\tilde{\omega}\frac{k}{\tilde{k}}\mu_p^4. \tag{3}$$

In the model *WAM4* [3]

$$\gamma_{diss}^{(4)} = 0.98\tilde{\omega}\frac{k}{\tilde{k}}\left(1+\frac{k}{\tilde{k}}\right)\mu_p^4. \tag{4}$$

Here tilde means everaging, while μ_p is the steepness near the spectral peak

$$\mu_p^2 = \langle\eta^2\rangle\tilde{k}_p^2. \tag{5}$$

Here $\langle\eta^2\rangle$ is the mean squared surface elevation

$$\langle\eta^2\rangle = \int \omega_k N(\vec{k})\mathrm{d}\vec{k}. \tag{6}$$

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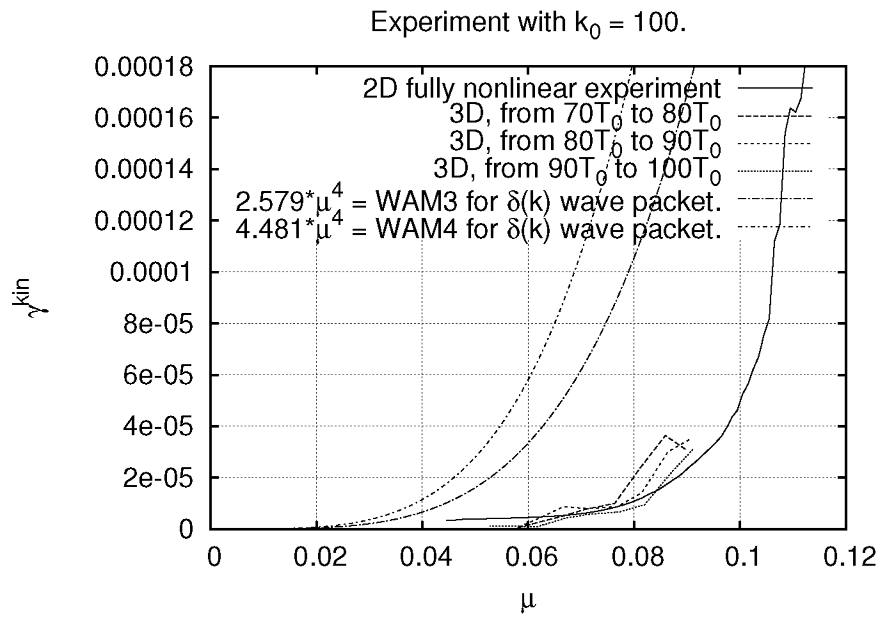

FIGURE 1. Comparison of the dissipation functions from numerical experiments and empirical formulae from *WAM3* and *WAM4*.

$$\gamma_k = C_{ds}\tilde{\omega}\frac{k}{\tilde{k}}\left((1-\delta)+\delta\frac{k}{\tilde{k}}\right)\left(\frac{\tilde{S}}{\tilde{S}_{pm}}\right)^p \tag{7}$$

where k and ω are the wave number and frequency, tilde denotes mean value; C_{ds}, δ and p are tunable coefficients; $S = \tilde{k}\sqrt{H}$ is the overall steepness; $\tilde{S}_{PM} = (3.02\times 10^{-3})^{1/2}$ is the value of $\tilde{S}$ for the Pierson-Moscowitz spectrum (note that the characteristic steepness is $\mu = \sqrt{2}S$). The values of the tunable coefficients for the *WAM3* case are:

$$C_{ds} = 2.35\times 10^{-5},\ \delta = 0,\ p = 4 \tag{8}$$

and for the *WAM4* case are:

$$C_{ds} = 4.09\times 10^{-5},\ \delta = 0.5,\ p = 4 \tag{9}$$

We must stress here that formulae (3) and (4) are purely heuristic. Neither theoretical nor experimental justification of these dissipation functions were offered.

RESULTS OF NUMERICAL SIMULATION OF DYNAMICAL EQUATIONS IN 3D AND 2D CASES.

In the 3D-hydrodynamics we have to use Hamiltonian expansion in orders of surface steepness (measure of nonlinearity in the system). Similar experiment was performed in [5]. Dissipation due to whitecapping in such weakly nonlinear model can be emulated by processes described and explored in [7]. For 2D-hydrodynamics situation is much better, because we can use fully nonlinear equations, conformal analog of Euler equations [6]. Results of comparison of results of numerical experiments and *WAM3* and *WAM4* dissipation terms are presented in Figure 1. One can see that we obtained threshold like dependence. Similar behavior was observed in field experiment [8]. We can state that empiric formulae have to be updated.

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