# Cross-phase modulation of surface magnetostatic spin waves

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(Submitted 14 July 1999)
Zh. Éksp. Teor. Fiz. **116**, 2058–2068 (December 1999)

This paper explains the modulation instability of two surface magnetostatic spin waves simultaneously propagating in a ferromagnetic film. Self-modulation of the spin waves appears when their power reaches a threshold, and this is a sign of cross-phase modulation. The parameters of the unstable process are calculated, and the gains of the perturbation amplitudes are determined. The results published earlier on the experimental detection of the cross-phase modulation of spin waves are explained. © *1999 American Institute of Physics.* [S1063-7761(99)01412-2]

## 1. INTRODUCTION

Magnetostatic spin waves (MSWs) propagating in magnetized films of yttrium iron garnet (YIG) are an extremely interesting object for research, since the nonlinear effects that appear when intense MSWs propagate begin to manifest themselves at relatively small powers.[1,2] Surface MSWs are especially interesting in this regard, since the energy of the wave is concentrated at the film surface in this case, and the losses are minimal when the signal is generated and detected. When a single surface MSW propagates, an increase in the power of the wave does not cause amplitude modulation of the envelope of the magnetostatic potential to appear spontaneously, and the wave is modulationally stable in this case.[3] However, recent experiments[4] with simultaneous propagation of two surface MSWs of different frequencies show that modulation instability appears under definite conditions. This effect is observed in the form of side frequencies on the peaks corresponding to the carrier frequencies on the output spectral characteristic. The cause of the instability is that the dispersion dependence for the MSWs changes, because the second wave propagates in a medium perturbed by the first wave. A phenomenon similar to that observed was theoretically explained for the first time in Ref. 5, which discussed the combined propagation of two waves of different polarizations in a plasma. Similar effects when signals propagated in optical waveguides were subsequently detected.[6] The instability causes the envelope of the MSWs to be modulated, since the ever-present noise serves as an initial perturbation.[7] This phenomenon has become known as modulation instability. The modulation instability that arises when two modulationally stable waves propagate simultaneously, due to cross-phase modulation,[5] is usually caused by induced modulation instability.[6] The derivation of the conditions for the appearance of modulation instability is one of the main problems in studying such processes. To do this, it is important to obtain the dispersion dependence of the amplitude perturbations of the magnetostatic potential of the carrier signal. In studying such processes, it is usual to neglect dissipative effects because they are small at the distances under consideration.[2] The output MSWs are attenuated if the dissipative terms are included, but the qualitative picture of the wave propagation does not change. This paper discusses a model of simultaneous nondissipative propagation of two surface MSWs in a ferromagnetic film. The dispersion dependences are derived in Sec. 2 in terms of this model. The equations for the evolution of the amplitudes of the coupled waves are obtained in Sec. 3. After this, an analysis of these equations is given in Sec. 4 in order to derive the conditions for the appearance of modulation instability. Finally, in Sec. 5, the results are used to explain the experimental results of Ref. 4.

## 2. DERIVATION OF THE DISPERSION DEPENDENCE

Let us consider the propagation of two surface MSWs in a thin ferromagnetic film of thickness $d$, placed in a saturating external magnetic field $\mathbf{H}$ (Fig. 1).

Under these conditions, it is possible to introduce into the discussion a magnetostatic potential that describes the propagating waves and has in our case the form[8]

$$\psi = A[\exp(k_1x)+\alpha\exp(-k_1x)]\exp(ik_1y) + B[\exp(k_2x)+\beta\exp(-k_2x)]\exp(ik_2y)+\text{c.c.}, \quad (1)$$

where $A$ and $B$ are the amplitudes of the two MSWs, $k_1$ and $k_2$ are the wave numbers, and $\alpha$ and $\beta$ are factors that depend on the film parameters and the external magnetic field and determine the propagation of the waves on some surface or the other.[9] The time dependence of the potential will be introduced later. It is easy to obtain the high-frequency components of magnetic field $\mathbf{h}=\nabla\psi$ from the given expression:

 

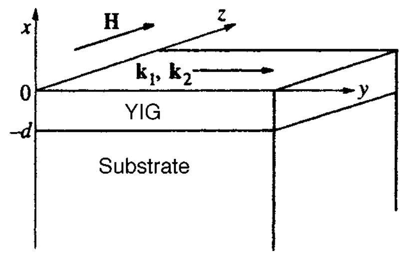


FIG. 1. Geometry of the problem. The MSWs propagate along the $y$ axis, and the external magnetic field **H** is directed along the $z$ axis.

$$h_x=\frac{\partial\psi}{\partial x}=Ak_1[\exp(k_1x)-\alpha\exp(-k_1x)]\exp(ik_1y)+Bk_2[\exp(k_2x)-\beta\exp(-k_2x)]\exp(ik_2y)+\text{c.c.},\tag{2}$$

$$h_y=\frac{\partial\psi}{\partial y}=iAk_1[\exp(k_1x)+\alpha\exp(-k_1x)]\exp(ik_1y)+iBk_2[\exp(k_2x)+\beta\exp(-k_2x)]\exp(ik_2y)+\text{c.c.}\tag{3}$$

The high-frequency part of the magnetization is connected with **h**,

$$\mathbf{m}=\hat{\chi}\mathbf{h},\tag{4}$$

by magnetic susceptibility tensor $\hat{\chi}$, which is written as

$$\hat{\chi}=\begin{pmatrix}\chi_1 & i\chi_2 & 0\\ -i\chi_2 & \chi_1 & 0\\ 0 & 0 & 1\end{pmatrix},\quad \mathbf{h}=\begin{pmatrix}h_x\\ h_y\\ h_z\end{pmatrix}.\tag{5}$$

As a result, we have

$$m_x=\chi_1\frac{\partial\psi}{\partial x}+i\chi_2\frac{\partial\psi}{\partial y},$$

$$m_y=-i\chi_2\frac{\partial\psi}{\partial x}+\chi_1\frac{\partial\psi}{\partial y}.\tag{6}$$

As will become clear later, we are interested only in the square of the modulus of the components of the magnetization. We consider $|\mathbf{m}|^2$ one component at a time:

$$|m_x|^2=\chi_1^2\left|\frac{\partial\psi}{\partial x}\right|^2+\chi_2^2\left|\frac{\partial\psi}{\partial y}\right|^2+i\chi_1\chi_2\left[\left(\frac{\partial\psi}{\partial y}\right)\left(\frac{\partial\psi}{\partial x}\right)^*-\left(\frac{\partial\psi}{\partial x}\right)\left(\frac{\partial\psi}{\partial y}\right)^*\right].\tag{7}$$

Recalling that Re $\psi=\psi$, we get

$$|m_x|^2=\chi_1^2\left|\frac{\partial\psi}{\partial x}\right|^2+\chi_2^2\left|\frac{\partial\psi}{\partial y}\right|^2.\tag{8}$$

Likewise, for $m_y$,

$$|m_y|^2=\chi_2^2\left|\frac{\partial\psi}{\partial x}\right|^2+\chi_1^2\left|\frac{\partial\psi}{\partial y}\right|^2.\tag{9}$$

Finally, we find

$$|\mathbf{m}|^2=|m_x|^2+|m_y|^2=(\chi_1^2+\chi_2^2)\left(\left|\frac{\partial\psi}{\partial x}\right|^2+\left|\frac{\partial\psi}{\partial y}\right|^2\right).\tag{10}$$

For the subsequent discussion, we need the quantity

$$\omega_M=4\pi\gamma\left[M_0-\frac{|m_x|^2+|m_y|^2}{2M_0}\right],\quad \omega_{M_0}=4\pi\gamma M_0,\tag{11}$$

which enters into the expression for the dispersion dependence[8] for a surface MSW:

$$\omega^2=\omega_H^2+\omega_H\omega_M+\frac{\omega_M^2}{4}[1-\exp(-2kd)].\tag{12}$$

We introduce into the discussion

$$\omega_{0i}^2=\omega_H^2+\omega_H\omega_{M_0}+\frac{\omega_{M_0}^2}{4}[1-\exp(-2k_id)],\tag{13}$$

where $i=\{1,2\}$. Finally,

$$\omega_i^2=\omega_{0i}^2-\frac{\pi\gamma}{M_0}\{2\omega_H+\omega_{M_0}[1-\exp(-2k_id)]\}\times(|m_x|^2+|m_y|^2).\tag{14}$$

Simple but tedious algebraic formations can be used to obtain an expression for $|\partial\psi/\partial x|^2+|\partial\psi/\partial y|^2$, which contains terms proportional to exp($iky$). In order to introduce the time dependence into the equations, it is necessary to make the substitution exp($iky$)→exp[$i(ky-\omega t)$]. This means that the terms containing $t$ and $y$ disappear after averaging over the period. These terms can play a role only if the waves are coherent. After this, we obtain

$$\left|\frac{\partial\psi}{\partial x}\right|^2+\left|\frac{\partial\psi}{\partial y}\right|^2=4\{|A|^2k_1^2[\exp(2k_1x)+|\alpha|^2\times\exp(-2k_1x)]+|B|^2k_2^2[\exp(2k_2x)+|\beta|^2\exp(-2k_2x)]\}.\tag{15}$$

We now have the following dispersion dependence:

$$\omega_i^2=\omega_{0i}^2-4\frac{\pi\gamma}{M_0}\{2\omega_H+\omega_{M_0}[1-\exp(-2k_id)]\}\times(\chi_1^2+\chi_2^2)\{|A|^2k_1^2[\exp(2k_1x)+|\alpha|^2\times\exp(-2k_1x)]+|B|^2k_2^2[\exp(2k_2x)+|\beta|^2\times\exp(-2k_2x)]\}.\tag{16}$$

The nonlinear dispersion Eqs. (16) for surface MSWs are derived in the limit of weak nonlinearity. Namely, nonlinear Eq. (11) describes the magnetization frequency, provided that the amplitude of the high-frequency magnetization is much less than the amplitude of the magnetization of the ferromagnetic film ($|m_x|,|m_y|\ll M_0$).

## 3. DERIVATION OF A SYSTEM OF EQUATIONS FOR THE EVOLUTION OF COUPLED WAVES

The dispersion dependence given by Eq. (16) can be written in general form as

$$G(k_1,k_2,\omega_1,\omega_2,|A|^2,|B|^2,|\alpha|^2,|\beta|^2)=0.\tag{17}$$

Let us introduce the dispersion dependence for an independently propagating wave:

$$G_{i0}=\omega_{0i}^2-\left\{\omega_H^2+\omega_H\omega_{M_0}+\frac{\omega_{M_0}^2}{4}[1-\exp(-2k_i d)]\right\}. \tag{18}$$

We expand Eq. (16) up to terms corresponding to second order in amplitude:

$$\begin{aligned}G_i=G_{i0}&+\frac{\partial G_{i0}}{\partial\omega_i}\Delta\omega_i+\frac{\partial G_{i0}}{\partial k_i}\Delta k_i+\frac{\partial^2 G_{i0}}{\partial k_i\partial\omega_i}\Delta\omega_i\Delta k_i\\&+\frac{1}{2}\frac{\partial^2 G_{i0}}{\partial\omega_i^2}(\Delta\omega_i)^2+\frac{1}{2}\frac{\partial^2 G_{i0}}{\partial k_i^2}(\Delta k_i)^2+\frac{\partial G_i}{\partial|A|^2}|A|^2\\&+\frac{\partial G_i}{\partial|B|^2}|B|^2+\frac{\partial G_i}{\partial|\alpha|^2}|\alpha|^2+\frac{\partial G_i}{\partial|\beta|^2}|\beta|^2=0.\end{aligned} \tag{19}$$

Recalling that

$$\Delta\omega_i\leftrightarrow i\frac{\partial}{\partial t},\quad \Delta k_i\leftrightarrow -i\frac{\partial}{\partial y},\quad \frac{\partial^2}{\partial t^2}\leftrightarrow v_{gi}\frac{\partial^2}{\partial y^2},$$

we get

$$\begin{aligned}&i\frac{\partial A}{\partial t}+iv_{g1}\frac{\partial A}{\partial y}+\frac{1}{2}\beta_1\frac{\partial^2 A}{\partial y^2}=f_1(a^2|A|^2+b^2|B|^2)A,\\&i\frac{\partial B}{\partial t}+iv_{g2}\frac{\partial B}{\partial y}+\frac{1}{2}\beta_2\frac{\partial^2 B}{\partial y^2}=f_2(a^2|A|^2+b^2|B|^2)B,\end{aligned} \tag{20}$$

with

$$v_{gi}=2d\frac{\omega_{M0}^2}{4\omega_i}\exp(-2k_i d), \tag{21}$$

$$\beta_i=-\frac{v_{gi}}{\omega_i}[v_{gi}+\omega_i d], \tag{22}$$

$$\begin{aligned}f_i=&-\frac{\omega_{M_0}}{2M_0^2\omega_i}\{2\omega_H+\omega_{M_0}[1-\exp(-2k_i d)]\}\\&\times(\chi_{1i}^2+\chi_{2i}^2),\end{aligned} \tag{23}$$

$$\begin{aligned}&a^2=k_1^2[\exp(2k_1x)+2|\alpha|^2\exp(-2k_1x)],\\&b^2=k_2^2[\exp(2k_2x)+2|\beta|^2\exp(-2k_2x)].\end{aligned} \tag{24}$$

Parameters $a$ and $b$ characterize the amplitude attenuation of the wave with distance from the surface. Equations (20) are in essence a system of equations of the type of the nonlinear Shrödinger equation and describe the evolution of the amplitudes of the coupled surface MSWs.

## 4. INVESTIGATION OF MODULATION INSTABILITY

Let us introduce the energy of the wave at depth $x$ from the surface:

$$P_1=a^2|A|^2,\quad P_2=b^2|B|^2. \tag{25}$$

Then the steady-state equations have the form

$$\begin{aligned}&iv_{g1}\frac{\partial A}{\partial y}+\frac{1}{2}\beta_1\frac{\partial^2 A}{\partial y^2}=f_1(P_1+P_2)A,\\&iv_{g2}\frac{\partial B}{\partial y}+\frac{1}{2}\beta_2\frac{\partial^2 B}{\partial y^2}=f_2(P_1+P_2)B.\end{aligned} \tag{26}$$

Let us consider, for example, the first equation. We seek a solution in the form

$$A=C\exp(-i\alpha y). \tag{27}$$

After substitution, we find

$$\alpha_{1,2}=\frac{v_{g1}}{\beta_1}\left[1\mp\sqrt{1-\frac{2\beta_1 f_1(P_1+P_2)}{v_{g1}^2}}\right]. \tag{28}$$

We have the following solution:

$$A=C_1\exp(-i\alpha_1 y)+C_2\exp(-i\alpha_2 y). \tag{29}$$

The amplitude close to the antenna is constant, and consequently the envelope is constant:

$$\begin{aligned}\frac{\partial A}{\partial y}=&C_1(-i\alpha_1)\exp(-i\alpha_1 y)\\&+C_2(-i\alpha_2)\exp(-i\alpha_2 y)=0.\end{aligned} \tag{30}$$

From this, we obtain the relationship between the amplitudes:

$$C_1=-\frac{\alpha_2}{\alpha_1}C_2. \tag{31}$$

Let us evaluate this relationship for a thin film ($k_i d\ll 1$). In this case, the ratio under the radical in Eq. (28) equals

$$\epsilon_i=\frac{2\omega_{M_0}\omega_H}{M_0^2\omega_i^2}(\chi_{1i}^2+\chi_{2i}^2)(P_1+P_2). \tag{32}$$

We estimate the factors in Eq. (32) by the following approximations:

$$(\chi_{1i}^2+\chi_{2i}^2)\approx 3,\quad \frac{2\omega_{M_0}\omega_H}{\omega_i^2}\approx\frac{1}{2},$$

and consequently we get

$$\epsilon\approx 3(P_1+P_2)/M_0^2. \tag{33}$$

However, this ratio is small in the approximation considered here (in the experiment considered below, it equals 1/11). Consequently, $\epsilon\ll 1$. It follows from this that $C_1\gg C_2$. We shall neglect the quantity $C_2$ in the subsequent calculations. Then we can write

$$\alpha=\alpha_1=\frac{v_{g1}}{\beta_1}\left[1-\sqrt{1-\frac{2\beta_1 f_1(P_1+P_2)}{v_{g1}^2}}\right]. \tag{34}$$

Finally,

$$A=\sqrt{P_1}e^{-i\alpha y}. \tag{35}$$

We now impose a perturbation on this solution:

$$A=[\sqrt{P_1}+\tilde{a}(y,t)]e^{-i\alpha y},\quad \tilde{a}\ll\sqrt{P_1}. \tag{36}$$

After substituting this expression into the equations, we carry out a similar procedure for $B$, and we linearize the resulting equations in terms of the perturbations:

$$i\frac{\partial\tilde{a}}{\partial t}+iv_{g1}\phi_1\frac{\partial\tilde{a}}{\partial y}+\frac{1}{2}\beta_1\frac{\partial^2\tilde{a}}{\partial y^2}=f_1[P_1(\tilde{a}+\tilde{a}^*)+\sqrt{P_1P_2}(\tilde{b}+\tilde{b}^*)],$$

$$i\frac{\partial\tilde{b}}{\partial t}+iv_{g2}\phi_2\frac{\partial\tilde{b}}{\partial y}+\frac{1}{2}\beta_2\frac{\partial^2\tilde{b}}{\partial y^2}=f_2[P_2(\tilde{b}+\tilde{b}^*)+\sqrt{P_1P_2}(\tilde{a}+\tilde{a}^*)], \quad (37)$$

where

$$\phi_i=v_{gi}\sqrt{1-\frac{2\beta_i f_i(P_1+P_2)}{v_{gi}^2}}. \quad (38)$$

We seek $\tilde{a}$ and $\tilde{b}$ in the following form:

$$\tilde{a}=u_1\cos[K_1(y-\phi_1 t)-\Omega t]+iv_1\times\sin[K_1(y-\phi_1 t)-\Omega t],$$

$$\tilde{b}=u_2\cos[K_2(y-\phi_2 t)-\Omega t]+iv_2\times\sin[K_2(y-\phi_2 t)-\Omega t], \quad (39)$$

where $K_1$ and $K_2$ are the wave numbers of the amplitude perturbations, while $\Omega$ is their frequency. After substitution, we must set the real and imaginary parts equal to zero. We obtain a system of linear equations with the matrix

$$M=\begin{pmatrix} m_1 & q_1 & l_1 & 0 \\ n_1 & p_1 & 0 & 0 \\ l_2 & 0 & m_2 & q_2 \\ 0 & 0 & n_2 & p_2 \end{pmatrix}, \quad (40)$$

where

$$m_i=-\frac{1}{2}\beta_i K_i^2\cos[K_i(y-\phi_i t)-\Omega t]-2f_iP_i\cos[K_i(y-\phi_i t)-\Omega t],$$

$$q_i=(K_i\phi_i+\Omega)\cos[K_i(y-\phi_i t)-\Omega t]-\phi_i K_i\cos[K_i(y-\phi_i t)-\Omega t],$$

$$l_i=-2f_i\sqrt{P_1P_2}\cos[K_{2-i}(y-\phi_{2-i}t)-\Omega t],$$

$$n_i=-(K_i\phi_i+\Omega)\sin(K_i(y-\phi_i t)-\Omega t)+\phi_i K_i\sin[K_i(y-\phi_i t)-\Omega t],$$

$$p_i=\frac{1}{2}\beta_i K_i^2\sin[K_i(y-\phi_i t)-\Omega t]. \quad (41)$$

For a nontrivial solution to exist in the system, it is necessary that the determinant of this matrix equal zero:

$$\det M=0, \quad (42)$$

or

$$(n_1q_1-p_1m_1)(n_2q_2-p_2m_2)-p_1p_2l_1l_2=0.$$

We obtain the following equation for $\Omega$:

$$(\Omega^2-c_1^2)(\Omega^2-c_2^2)=\zeta, \quad (43)$$

where

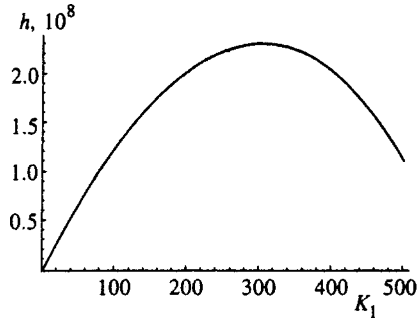


FIG. 2. Dependence of gain $h$ on $K_1$ for fixed $K_2$.

$$c_i=\frac{1}{2}\beta_i K_i^2, \quad \zeta=\beta_1\beta_2 f_1 f_2 P_1 P_2 K_1^2 K_2^2. \quad (44)$$

It is easy to express $\Omega^2$ as

$$\Omega^2=\frac{c_1^2+c_2^2\pm\sqrt{(c_1^2+c_2^2)^2-4(c_1^2c_2^2-\zeta)}}{2}. \quad (45)$$

We find the condition for which $\Omega^2<0$:

$$\zeta>c_1^2c_2^2. \quad (46)$$

This is the condition for modulation instability to appear. In this case, it can be seen from Eqs. (44) and (46) that both waves are modulationally unstable regardless of the signs of the nonlinearity coefficients given in Eq. (23). These results also agree with the results published earlier in Refs. 10 and 11 concerning the nonlinear interaction between spin and acoustic waves and between spin and electromagnetic waves.

## 5. ANALYSIS OF EXPERIMENT

In Ref. 4, two MSWs with frequencies $\omega_1=6.55$ GHz and $\omega_2=6.75$ GHz, which correspond to wave numbers $k_1=52.97\ \mathrm{cm}^{-1}$ and $k_2=379.6\ \mathrm{cm}^{-1}$, were generated in a film of yttrium iron garnet of thickness $d=1.15\times10^{-3}$ cm. The film in this experiment had a saturation magnetization of $M_0=135.6$ G and was located in an external magnetic field of $H_0=1627$ Oe, with $\gamma=2.8$ MHz/Oe. The signal from the film was fed to a spectrum analyzer. The mistuning between the nearest side band and the carrier peak equalled 1.4 MHz, but the side band is caused by the interaction with another wave, and it is consequently necessary to consider the spacing not between the nearest peaks but between the farthest. This makes it possible to explain the presence of the asym-

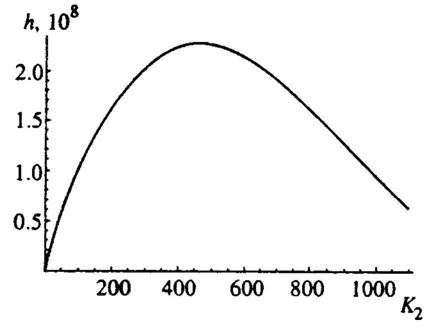


FIG. 3. Dependence of gain $h$ on $K_2$ for fixed $K_1$.

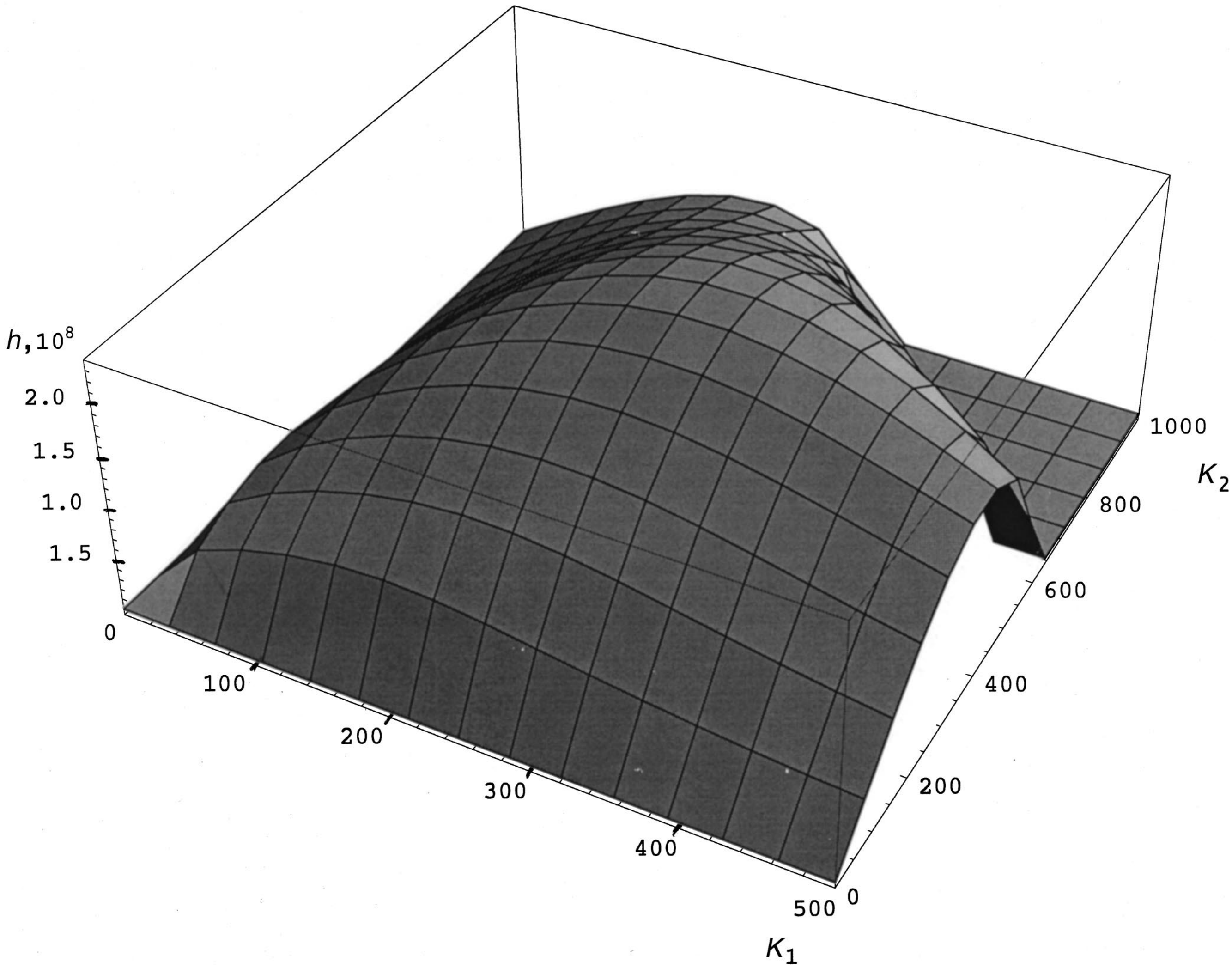


FIG. 4. Dependence of gain $h$ on $K_1$ and $K_2$.

metry of the side bands as the result of the interaction of already modulated waves. Then the modulation frequency $\Omega=\Delta\omega$ is 201.4 MHz. Keeping in mind that the gain must have a maximum at this frequency, we get a system of equations for $K_1$ and $K_2$. Solving it, we find

$$K_1=283.3 \ \text{cm}^{-1}, \quad K_2=444.4 \ \text{cm}^{-1}. \tag{47}$$

The power of the wave can be obtained from[3]

$$W\kappa\simeq\frac{1}{16\pi}Ld^2\omega P, \tag{48}$$

where $L=0.3\,\text{cm}$ is the length of the antenna, while $\kappa=0.25$ is a factor that characterizes the part of the supplied power $W$ that goes into the generation of MSWs. From this we get

$$k=\frac{1}{16\pi}Ld^2=7.89\times10^{-9} \ \text{cm}^3,$$

$$P=\frac{W\kappa}{k\omega}, \quad P_1=134 \ \text{Oe}^2, \quad P_2=45 \ \text{Oe}^2. \tag{49}$$

Let us check whether the conditions $\zeta>c_1^2c_2^2$ for modulation instability are satisfied:

$$4f_1f_2P_1P_2>\frac{1}{2}\beta_1K_1^2\cdot\frac{1}{2}\beta_2K_2^2,$$

$$\frac{16f_1f_2}{\beta_1\beta_2}P_1P_2>K_1^2K_2^2,$$

$$\kappa^2 1.38\times10^{12} \ \text{cm}^{-4}>K_1^2K_2^2,$$

$$8.63>1.59. \tag{50}$$

As can be seen, the condition for cross-phase modulation is satisfied. In order to understand the spectral content of the received signal, it is necessary to know how the gain of the modulation perturbations depends on $K_1$ and $K_2$. The gain of the perturbation amplitudes equals

$$h(K_1,K_2)=2\,\text{Im}(\Omega)$$
$$=\sqrt{2[\sqrt{(c_1^2+c_2^2)^2+4(\zeta-c_1^2c_2^2)}-(c_1^2+c_2^2)]}. \tag{51}$$

Figure 2 shows a graph of the $h(K_1,K_2)$ dependence with $K_2$ fixed. Figure 3 shows the $h(K_1,K_2)$ dependence with $K_1$ fixed. The $h(K_1,K_2)$ surface is shown in Fig. 4.

It can be seen from the curves in the figures that the gain has a maximum at definite values of $K_1$ and $K_2$, corresponding to the frequencies $\Omega_1$ and $\Omega_2$ of the modulation instability observed in experiment.

## 6. CONCLUSION

Instability with respect to the amplitude of intense travelling surface MSWs in ferromagnetic films has been theoretically treated. Surface MSWs in the case of propagation of only one wave are modulationally stable even when the power of the wave varies within wide limits. When two waves simultaneously propagate in a ferromagnetic film, they become modulationally unstable when they reach a certain threshold power. Such instability cannot be explained in terms of a model of parametric instability. A model of the phase modulation of two intense waves is proposed here that explains the appearance of modulation instability of the waves. Calculations are carried out for the parameters of the waves, in particular, the threshold powers required for the phenomena discussed here. An explanation is given for the experimental results of Ref. 4, in which cross-phase modulation of surface MSWs was observed for the first time. Qualitative agreement is obtained between the theory developed here and the experimental data. Since this paper uses a model of nondissipative propagation of surface MSWs, while quenching of the waves plays a substantial role in the example studied, which involved a thick film, it is problematical to obtain quantitative agreement with experiment. Further development of the theory, taking into account wave dissipation, and additional experimental work should clarify the details of the process of cross-phase modulation of magnetostatic spin waves.

This work was carried out with the partial support of the Russian Fund for Fundamental Research (Project 99-02-17660) and the Ministry of Science and Education of the Russian Federation (the Atomic Surface Structures Program).

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Translated by W. J. Manthey