Inverse versus Direct Cascades in Turbulent Advection

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A model of scalar turbulent advection in compressible flow is analytically investigated. It is shown that, depending on the dimensionality d of space and the degree of compressibility of the smooth advecting velocity field, the cascade of the scalar is direct or inverse. If $d > 4$ the cascade is always direct. For a small enough degree of compressibility, the cascade is direct again. Otherwise it is inverse; i.e., very large scales are excited. The dynamical hint for the direction of the cascade is the sign of the Lyapunov exponent for particles separation. Positive Lyapunov exponents are associated to direct cascade and Gaussianity at small scales. Negative Lyapunov exponents lead to inverse cascade, Gaussianity at large scales, and strong intermittency at small scales. [S0031-9007(97)04955-7]

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The keystone of the celebrated 1941 Kolmogorov-Obukhov [1,2] theory for 3D fully developed turbulence is the *direct* (downscales) energy cascade. Many other examples of direct cascades have later been found for turbulent systems (see [3,4] for a review). The presence of a direct cascade expresses the fact the average flux of an integral of motion, which holds for the system in the absence of forcing and dissipation (e.g., energy for 3D Navier-Stokes turbulence and many examples of wave turbulence, vorticity for 2D Navier-Stokes turbulence, etc.), is directed toward small scales and is constant along the scales. The variety of turbulent systems is not exhausted by direct cascades. When the pumping is supplied, the flux of an integral of motion can go toward large, and not small, scales. This is the case of 2D turbulence, where the dynamical constraints due to the presence of two integrals of motion lead to the remarkable phenomenon of the *inverse* energy cascade discovered by Kraichnan in [5]. Other examples of inverse cascade are known in wave turbulence [3].

Advection of a passive scalar $\theta(t;\mathbf{r})$ (it might be the concentration of a pollutant or temperature) by a turbulent *incompressible* flow belongs to the class of direct cascades [6]. The direct cascade survives if the Navier-Stokes turbulent velocity is replaced by a synthetic field with prescribed statistical properties [7,8] (this model, introduced by Kraichnan in [8], is attracting a great deal of attention for the anomalous scaling discovered there [9–11]). A one-dimensional compressible generalization of the Kraichnan model was recently introduced in [12]. We have considered the smooth limit of the model in [13] and shown that an inverse cascade takes place. This has led us to investigate the general relation between compressibility and the direction of the cascade. The aim of this Letter is to present and analyze a model where we can continuously move from inverse to direct cascade by varying two parameters: The dimensionality of space d and the degree of compressibility.

The dynamical equation is

$$[\partial_t + \mathbf{u}_\alpha(t;\mathbf{r})\nabla_\alpha - \kappa\triangle]\theta(t;\mathbf{r}) = \phi(t;\mathbf{r}), \tag{1}$$

for the Lagrangian tracer scalar field $\theta(t;\mathbf{r})$ (say temperature or entropy but generally not concentration of a pollutant), steadily supplied by the random Gaussian source ϕ and advected by the compressible d-dimensional Gaussian random velocity $u(t;\mathbf{r})$, having zero average. The correlation of the pumping is $\langle\phi(t_1;\mathbf{r}_1)\phi(t_2;\mathbf{r}_2)\rangle = \chi(\mathbf{r}_1 - \mathbf{r}_2)\delta(t_1 - t_2)$, with $\chi(\mathbf{r})$ regular at the origin and decaying fast at distances larger than the integral scale L. The molecular diffusivity κ is supposed small enough to be in the fully turbulent regime; i.e., L is much larger than the dissipative scale. The pair correlation function of the velocity fluctuations, $\langle\delta u_\alpha(t;\mathbf{r})\delta u_\beta(t';\mathbf{r})\rangle$, is

$$\frac{2(dC^2 - S^2)r_\alpha r_\beta - [2C^2 - (d+1)S^2]\delta_{\alpha\beta}r^2}{d(d-1)(d+2)}\,\delta(t-t'), \tag{2}$$

where $C^2 = \langle[\nabla_\alpha u_\alpha]^2\rangle$, $S^2 = \langle(\nabla_\alpha u_\beta)^2\rangle$, and $\delta u_\alpha(t;\mathbf{r}) = u_\alpha(t;\mathbf{r}) - u_\alpha(t;\mathbf{0})$.

We show for the model that *the cascade of the scalar is inverse if* $d < 4$ *and the degree of compressibility* $C^2/S^2 > d/4$; *otherwise it is direct*. The dimension $d = 4$ then turns out to be critical for the direction of the cascade. When the cascade is inverse and no infrared cutoffs are present, the smallest excited wave numbers become smaller and smaller with t. Moments of the scalar field are dominated by infrared contributions and will diverge in time (linearly, as shown in [13]). Moments of the scalar differences are, on the contrary, formed at finite wave numbers and will then reach stationary values. The probability density function (PDF) at the steady state

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of scalar differences is found explicitly in the convective interval, i.e., at scales much larger than the diffusive one, both for direct and inverse cascades. The Gaussianity of the scalar distribution, known to be present at small scales in the incompressible case [14], emerges at large scales when the cascade is inverse. Small scales are then shown to be strongly intermittent.

Since the scalar is a tracer, its statistics is very directly related to the one of the Lagrangian trajectories separation and, specifically for smooth velocities, of Lyapunov exponents, describing the rate of the exponential-in-time stretching (or contraction) of the separation. We shall derive below the following relation between the sign of the maximum Lyapunov exponent and the direction of the cascade: *$\overline{\lambda}$ is positive in the case of direct cascade and negative in the case of inverse cascade*. Before the systematic analysis, it is worth presenting first some simple intuitive arguments. Indeed, once the statistics of the scalar field is expressed in terms of Lagrangian paths properties, the crucial question becomes, given two particles initially separated by r, what is the probability that their distance will ever reach the typical scale of the pumping L? *If the Lyapunov exponent is positive*, separations will typically grow exponentially in time. Starting from a separation $r \ll L$, the scale of the pumping is almost certainly reached; vice versa, if $r \gg L$, there is practically no chance to reach L. This is the dynamical hint of the direct cascade. Consider now *a negative Lyapunov exponent*. The picture is completely reversed. Since typical trajectories are contracting, the probability to reach L is much higher at scales $r \gg L$ than $r \ll L$. The characteristic sign of the inverse cascade is emerging here. Furthermore, the described physics of the relation between the stretching-contraction interplay does have a certain generality. For incompressible flow, the strain tensor being traceless, no strong trapping phenomena are possible and the rate of typical separations growth is expected to be positive. Strong trapping appears for compressible flow, where it can lead to a substantial slowdown of transport (see [15]) and, possibly, to a negative rate of the Lagrangian separations stretching. Finally, the importance of traps for transport should definitely reduce when the dimensionality of space increases.

Let us now start the systematic analysis of the model by considering Lagrangian separations statistics. The stochastic differential equation governing the evolution of the separation $R_\alpha(t)$ between two Lagrangian particles, in the absence of molecular diffusion, is $\partial_t R_\alpha = \sigma_{\alpha\beta} R_\beta$. The statistics of the random strain $\sigma^{\alpha\beta}$ is Gaussian. Its irreducible correlation function $\langle[\sigma_{\gamma\alpha}(t)\sigma_{\delta\beta}(t')]\rangle$ is simply obtained operating with $\nabla_{\mathbf{r}}^\gamma \nabla_{\mathbf{r}}^\delta/2$ on (2). $\langle\sigma^{\alpha\beta}\rangle = -\delta^{\alpha\beta}C^2/[2d]$ as follows from the condition $d\langle R_\alpha(t)\rangle/dt = 0$, which is an immediate consequence of isotropy, and the symmetric temporal regularization (the choice of which is customary) corresponding to $\langle\sigma^{\alpha\beta}R^\beta\rangle = \langle\sigma^{\alpha\beta}\sigma^{\beta\gamma}\rangle\langle R^\gamma\rangle/2$. Note that the average of $\hat{\sigma}$ vanishes for incompressible flow. The $2n$th moment of $R(t)$ obeys the following differential equation:

$$\partial_t\langle R^{2n}\rangle = \frac{2n}{d}\langle R^{2n}\rangle \times \left[S^2\left(\frac{1}{2} + \frac{n-1}{d+2}\right) + \frac{2(n-1)}{d+2}C^2\right], \tag{3}$$

corresponding to the following Gaussian statistics of the exponential stretching rate $\lambda \equiv \ln[R(t)/R(0)]/t$:

$$\mathcal{P}(t;\lambda) = \sqrt{\frac{t}{2\pi\zeta^2}}\exp\left[-\frac{(\lambda-\overline{\lambda})^2 t}{2\zeta^2}\right], \qquad \overline{\lambda} = \frac{dS^2 - 4C^2}{2d(d+2)}, \qquad \zeta = \sqrt{\frac{S^2+2C^2}{d(d+2)}}. \tag{4}$$

The largest Lyapunov exponent (generally there are d of them) has been denoted by $\overline{\lambda}$ and ζ is the variance of λ. For incompressible velocity fields, $C^2 = 0$ and the Lyapunov exponent $\overline{\lambda}$ is always positive. The opposite limit is the one of gradient-type velocity fields $\mathbf{u} = \nabla\psi$, where the equality $C^2 = S^2 = \langle(\Delta\psi)^2\rangle$ holds. The interesting conclusion arising from (4) is that $d = 4$ is actually a critical dimension for gradient-type fields. For generic smooth flow, the largest Lyapunov exponent is always positive for $d > 4$ and its precise behavior (including the value of the possible critical dimension $d_c = 4C^2/S^2$) depends on the specific value of the degree of compressibility C^2/S^2. It follows from (4) that, if the Lyapunov exponent is negative, the low-order moments of the Lagrangian separation decay in time, but their high-order moments (e.g., integer moments) grow exponentially. As first highlighted in [16], this means that the dynamics of Lagrangian separations is dominated by rare events and this is the origin of the strong intermittency of the scalar at small scales which will be evidenced below.

We proceed now with the derivation of the scalar statistics. The relation between Lagrangian trajectories and the passive scalar field in (1) is very direct. The solution of the equation can indeed be presented as $\theta(t;r) = \int_{-\infty}^{t} dt'\,\phi[t';\boldsymbol{\rho}(t')]$, where the Lagrangian trajectory $\boldsymbol{\rho}(t)$ satisfies $\dot{\boldsymbol{\rho}}(t') = \mathbf{u}(t';\boldsymbol{\rho}(t'))$ and $\boldsymbol{\rho}(t) = \mathbf{r}$. For the sake of simplicity of the presentation, we have not taken into account molecular diffusion (see, for example, [13] for the detailed path integral formulation including diffusion). Its smearing effects on Lagrangian trajectories can be neglected in the analysis presented in the sequel. The simultaneous $2n$th order scalar correlation function can be rewritten in terms of the average over $2n$ Lagrangian trajectories as

$$\langle\theta_1\cdots\theta_{2n}\rangle = \left\langle\sum_{\text{permut}}\prod_{k=1}^{n}\int_0^T dt\,\chi\left[\frac{\mathbf{R}_{i_k j_k}(t)}{L}\right]\right\rangle, \tag{5}$$

where the sum is over all the possible permutations $\{i_1, \dots, i_n, j_1, \dots, j_n\}$ of the indices $\{1, \dots, 2n\}$. The Lagrangian separation $\mathbf{R}_{ij}(t)$ satisfies the Langevin equation associated with (3) and the two Lagrangian particles are initially located at $\mathbf{r}_i$ and $\mathbf{r}_j$, respectively. T stands for the time of evolution and should tend to infinity for the system to attain the stationary state. The statistics of the whole set of d Lyapunov exponents is required to reconstruct the Lagrangian dynamics of a general d dimensional structure. However, to analyze the structure functions of the scalar (or multipoints correlation functions where all the points are elongated on a straight line), we do not need to go into the heavy details of subleading Lyapunov hierarchy. The point is that, modulo the dissipation that will be discussed later on, the collinear geometry is preserved by the dynamics [11,17]. Here, we shall therefore consider only collinear geometry $\mathbf{r}_i = \mathbf{n} x_i$, with all the distances large enough for dissipative effects to be negligible. The $2n$-order structure function $S_{2n}(x)$ can be easily reconstructed from the $2n$th order correlation function $\langle \omega(T, \mathbf{r}_1) \cdots \omega(T, \mathbf{r}_{2n}) \rangle$ of the scalar gradient $\omega(t; \mathbf{r}_i) \equiv \partial_{x_i} \theta(t; \mathbf{r}_i)$, in the collinear geometry. This is simply done integrating all the coordinates x_i from 0 to x and the final result for $S_{2n} = \langle [\theta(x) - \theta(0)]^{2n} \rangle$ is

$$S_{2n} = a_n \left\langle \left[\int_0^T dt [\chi(e^{\lambda t} r_d / L) - \chi(e^{\lambda t} r / L)] \right]^n \right\rangle, \tag{6}$$

where $a_n \equiv (2n - 1)!!\, 2^n$ and the averaging with respect to λ is fixed by (4). The regularity of the pumping $\chi(x)$ at the origin has been exploited in (6) and the integrations over the x_i's have been cut from below by the diffusive scale $r_d = \sqrt{\kappa}$. We will indeed see that in those cases when the cutoff is important, the resulting dependence on r_d is logarithmic, thus justifying the used regularization. Equation (6) works generically for all the r from the convective interval and all the n with one exception. Equation (6) is not applicable for the highest moments, $n \gg \ln[L/r]$, in the case of the direct cascade and high enough dimensionality, $d > 2$ [18]. The exceptional limit requiring an account for off-collinear trajectories will be discussed elsewhere. The generating function $\mathcal{Z}(r/L, q)$ (Fourier transform of the respective PDF) of scalar differences is simply restored from (6) in the form of a matrix element over an auxiliary quantum mechanics (see [13] for more details) with the Hamiltonian $\hat{H} = -\frac{\zeta^2}{2} \partial_\eta^2 + q^2 [\chi(e^\eta / \mathrm{Pe}) - \chi(e^\eta r / L)]$. Indeed, $Z = [e^{-T \bar{\lambda}^2 / (2\zeta^4)} \Psi(T; \eta)]_{\eta = 0}$, where the wave function Ψ satisfies the Schrödinger equation $[\partial_T + \hat{H}] \Psi = 0$, with the initial condition $\Psi(0; \eta) = e^{\bar{\lambda} \eta / \zeta^2}$. The asymmetry between negative and positive $\bar{\lambda}$ clearly emerges in the different asymptotic behaviors of Ψ. For negative $\bar{\lambda}$, the nonvanishing at $\eta \to -\infty$ initial condition $\exp[\bar{\lambda} \eta / \zeta^2]$ will survive in the stationary ($T \to \infty$) limit, since the potential part of the Hamiltonian ($\sim q^2$) is also vanishing at $\eta \to -\infty$. Analogous consideration of the positive λ case fixes the same boundary condition for Ψ, but in the opposite limit $\eta \to +\infty$ (it is worth mentioning that a finite diffusivity is needed for this). We thus arrive at the Fokker-Planck equation

$$\left[y^2 \partial_y^2 + \left(1 + \frac{2\bar{\lambda}}{\zeta^2} \right) y \partial_y - \frac{2q^2}{\zeta^2} [\chi(0) - \chi(y)] \right] \mathcal{Z} = /, \tag{7}$$

with the following boundary conditions on $\mathcal{Z}(y, q)$:

$$\text{at } \bar{\lambda} < 0 \quad \mathcal{Z}|_{y \to 0} \to 1, [\mathcal{Z} y^{\bar{\lambda}/\zeta^2}]|_{y \to 1/\xi \to \infty} \to 0, \tag{8}$$

$$\text{at } \bar{\lambda} > 0 \quad \mathcal{Z}|_{y \to 1/\xi \to \infty} \to 1, [\mathcal{Z} y^{\bar{\lambda}/\zeta^2}]|_{y \to 0} \to 0. \tag{9}$$

Here, ξ should be considered as a y-independent parameter to be replaced by r_d/r after integration of (7).

Note that the Fokker-Planck equation (7) is the same as the one found directly from (1), taking two replicas, averaging with the appropriate weight and discarding the contribution of the molecular diffusivity term. If the latter term is kept, the so-called anomaly term appears (following the field-theoretical terminology introduced by Polyakov in [19]) and the equation is not closed. In turbulent systems, it is generally expected that the anomaly term (for a discussion of the Polyakov theory in the context of the present passive scalar model, see [20]) does not vanish in the zero diffusivity limit. The absence of the anomaly term is therefore neither a trivial nor a general fact. To derive (7) rigorously we started from the multipoint object given by a compact average of a $\hat{\sigma}(t)$ dependent functional (5). Fussion of the points along a collinear path allowed to reduce (5) to (6) which was shown to be equivalent to the anomaly-free Eq. (7) conditioned by (8) or (9), respectively. No doubt in the general case of a nonsmooth incompressible velocity an analogous procedure would give a nonzero anomaly [9–11]. The respective question addressed to the compressible yet nonsmooth case is not yet resolved.

Let us finally show how to get from (7) and (8) or (9) the scalar differences PDF, with a special accent on the issue of universality. The PDF is indeed not globally universal, in the sense that it will generally depend on the explicit form of the pumping function $\chi(x)$. A relevant question to ask is then: What is universal in the PDF? There are two different universality issues which may be discussed in this respect. A kind of universality, typical of the case of positive Lyapunov exponents both at large and small scales, suggests that the only relevant parameter is the flux of θ^2 [$\chi(0)$] pumped into the system at the integral scale [14]. Indeed, at $\bar{\lambda} > 0$ the asymptotic solution of (7) can be found replacing $\chi(0) - \chi(x/L)$ by $\chi(0)$. Using (9) and the fact that ξ is the smallest value in the problem, one gets an algebraic form for the generating function $\mathcal{Z} = (r_d / \min\{r, L\})^a$, with the exponent $a = \sqrt{\bar{\lambda}^2/\zeta^4 + 2q^2 \chi[0]/\zeta^2} - \bar{\lambda}/\zeta^2$. Calculating the Fourier integral for the PDF in the saddle-point manner (the large parameter is r/r_d), one gets the Gaussian behavior at $|\delta\theta_r| \ll \ln[x^*]$

$$\mathcal{P}(r/L, \delta\theta_r) = \frac{\exp[-\delta\theta_r^2\overline{\lambda}/(4\chi[0]\ln[x^*])]}{\sqrt{\overline{\lambda}/\pi\chi[0]\ln[x^*]}}, \quad (10)$$

where $x^* \equiv \min\{r, L\}/r_d$. For very large values $|\delta\theta_r| \gg \ln[x^*]$, the Gaussian behavior transforms into an exponential tail [14,21]. The detailed form of the tail requires however the knowledge of the whole hierarchy of Lyapunov exponents and cannot be calculated within the collinear approach used in the present Letter at $d > 2$ [18]. The unknown exponential tail does not affect the behavior of the structure functions of order $n \ll \ln[x^*]$, that are equal to $(2n - 1)!!\, n!\, 2^n \ln^n[x^*]\chi^n[0]/\overline{\lambda}^n$. This kind of universality (and the corresponding restrictions on collinear considerations) can be easily extended to the case of negative Lyapunov exponents for $r \gg L$. To this aim, we replace again $\chi(0) - \chi(x/L)$ by $\chi(0)$ in (7), impose the boundary condition (8), and perform the saddle-point integral, as discussed above. We arrive finally at the expressions for the PDF and the structure functions (valid for $r \gg L$ and negative $\overline{\lambda}$), which are identical to (10) with the replacements of $\min\{r, L\}/r_d$ by r/L and $\overline{\lambda}$ by $|\overline{\lambda}|$.

For negative $\overline{\lambda}$, another kind of universal behavior is found at small scales $r \ll L$. In the vicinity of the origin, the PDF depends only on the second derivative of the pumping $\chi''(0)$. This part of the PDF is indeed formed at the largest $q \gg 1$, where one can keep only the first term of the expansion of $\chi(x/L)$ in the "potential" of (7). We arrive then at the following expression for the PDF at $|\delta\theta_r| \ll 1$ and $r \ll L$:

$$\mathcal{P} = c\left(\frac{\chi''[0]r^2}{\zeta^2L^2}\right)^{-\overline{\lambda}/\zeta^2}\left[\delta\theta_r^2 + \frac{\chi''[0]r^2}{\zeta^2L^2}\right]^{\overline{\lambda}/\zeta^2-1/2}, \quad (11)$$

$c \equiv \Gamma[1/2 - \overline{\lambda}/\zeta^2]/(\Gamma[-\overline{\lambda}/\zeta^2]\sqrt{\pi})$. The expression (11) shows that all the structure functions $S_n(r)$ of low orders $-1 < n < -2\overline{\lambda}/\zeta^2$ are controlled by (11) and scale normally $\sim r^n$. The behavior of the PDF at values larger than those where (11) holds is not universal; i.e., it depends on the whole form of $\chi(x/L)$. For very large values $\delta\theta_r \gg 1$ and at small scales $r \ll L$, it is however possible to show, from the expression of (7) and (8) at $q \lesssim 1$ and $y \ll 1$, that the far tail is exponential, $\mathcal{P} \sim [r/L]^{-2\overline{\lambda}/\zeta^2}\exp(-\sqrt{\chi}\,\zeta|\delta\theta_r|/\overline{\lambda})$. The nonuniversality emerges via the dimensional constant χ, which depends on the precise shape of the source function (for detailed explanations on the asymptotics, see [13], where the tail was fully determined in the one-dimensional case for a specific form of the pumping). The form of the tail, along with (11), shows that the structure functions of order larger than $-2\overline{\lambda}/\zeta^2$ all scale in the same way $S_{2n}(r) \sim (r/L)^{-2\overline{\lambda}/\zeta^2}$ at $r \ll L$. This collapse of the high-order structure functions scaling exponents should be contrasted with the Gaussian normal scaling observed for the direct cascade. These different scalings are an illustration of the profoundly different behaviors that have been found for the model below and above its threshold of compressibility where the direction of the cascade is reversed.

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