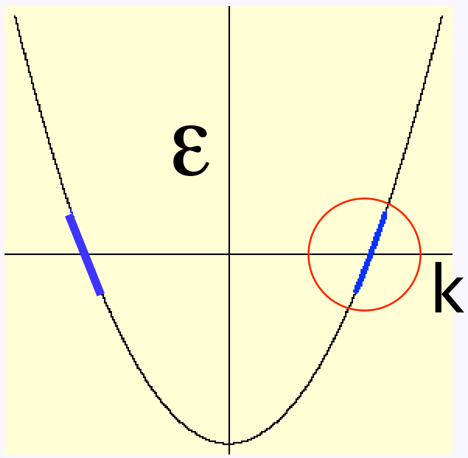
Dispersive shock waves in interacting one-dimensional systems and edge states of FQHE

Alexander Abanov
Stony Brook University

Collaboration: Paul B. Wiegmann, Eldad Bettelheim

Thanks: A. Guliani, M. Kulkarni, …
ITP, EPFL for hospitality

Gradient catastrophe

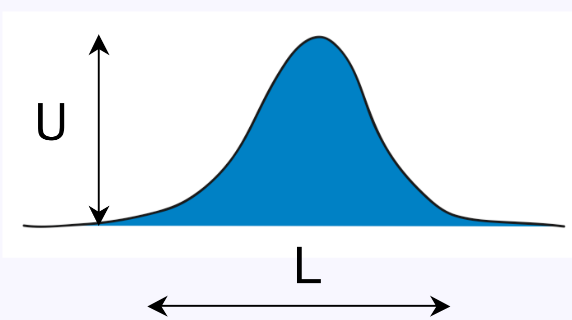


$$\psi_R \propto e^{ik_F x + i\phi_R}$$

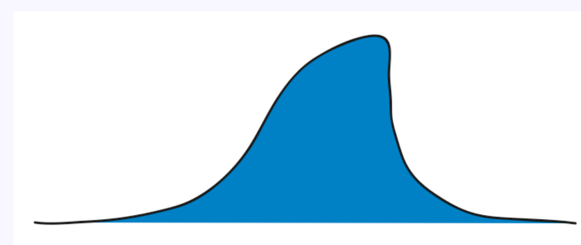
$$u \propto \partial_x \phi_R$$ current

$$u_t + c u_x = 0$$ chiral boson

$$u_t + c u_x + u u_x = 0$$ weak nonlinearity (curvature)



$$t_c \sim L/U$$



Gradient catastrophe in finite time!

Interactions -> Dispersion.

Outline

– Benjamin-Ono equation

– Calogero-Sutherland model

– Dispersive shock waves for CS and FQHE

Phys. Rev. Lett. 95, 076402 (2005)

J. Phys. A: Math.Theor. 40, F193 (2007)
Phys. Rev. Lett. 97, 246401 (2006)
Phys. Rev. Lett. 97, 246402 (2006)

Classical integrable equations

$$u_t + uu_x = 0$$

Riemann

$$u_t + u\,u_x + \epsilon\,u_{xxx} = 0$$

KdV

$$u_t + u\,u_x + \epsilon\,u_{xx} = 0$$

Burgers

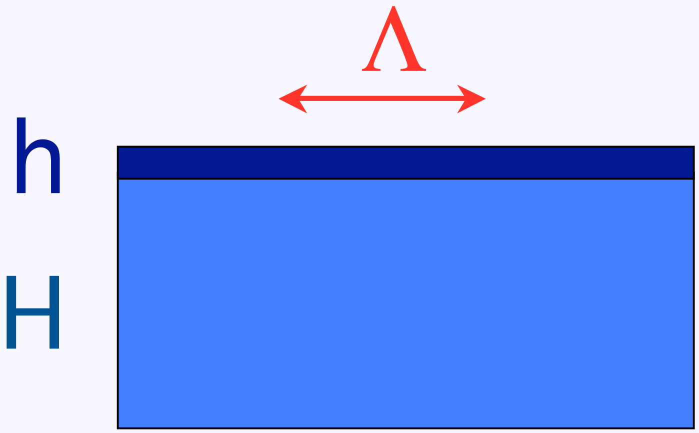
$$u_t + u\,u_x + \epsilon\,u_{xx}^H = 0$$

Benjamin-Ono

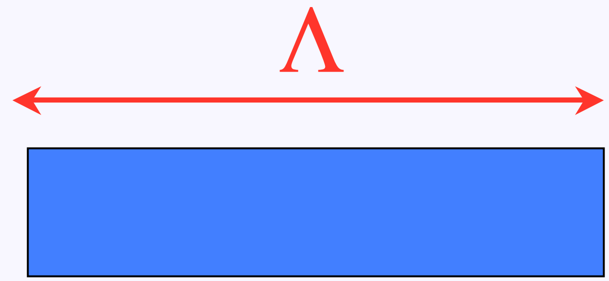
$$\omega = ck - \varepsilon k\,|\,k\,|$$

Classical Benjamin-Ono equation

(internal waves in deep stratified fluid)



$$h \ll \Lambda \ll H$$



$$H \ll \Lambda$$

Benjamin-Ono

$$u_t + u u_x + u^H_{xx} = 0$$

KdV

$$u_t + u u_x + u_{xxx} = 0$$

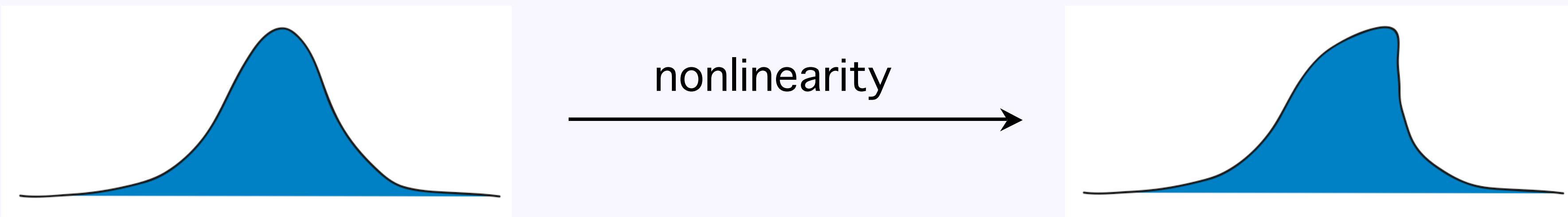
Benjamin 1967
Ono 1975

$$u^H(x) = \frac{1}{\pi} \int_{p.v.} dy \, \frac{u(y)}{y - x} = i(u^+ - u^-)$$

Nonlinearity vs. Dispersion

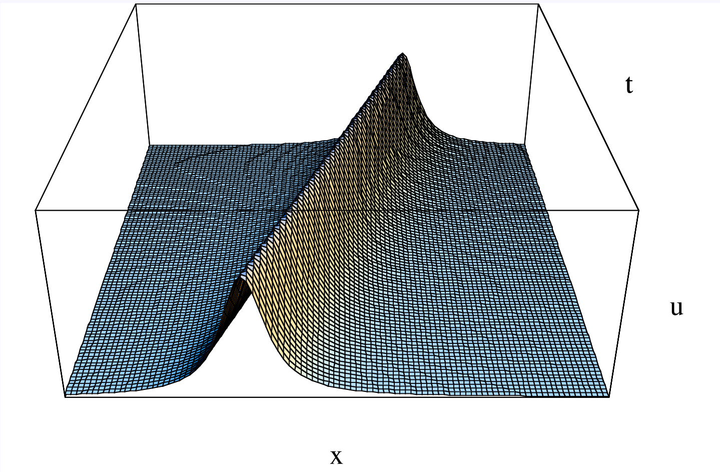
$$u_t + uu_x = 0$$

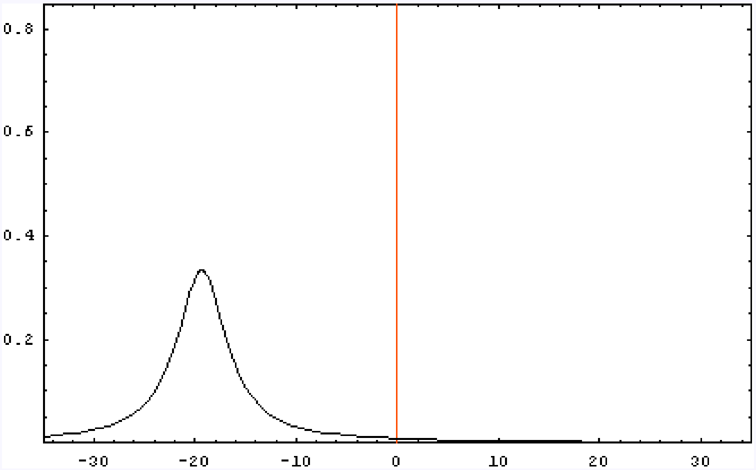
Gradient catastrophy



$$u_t + u_{xx}^H = 0$$

One-soliton solution of Benjamin-Ono equation



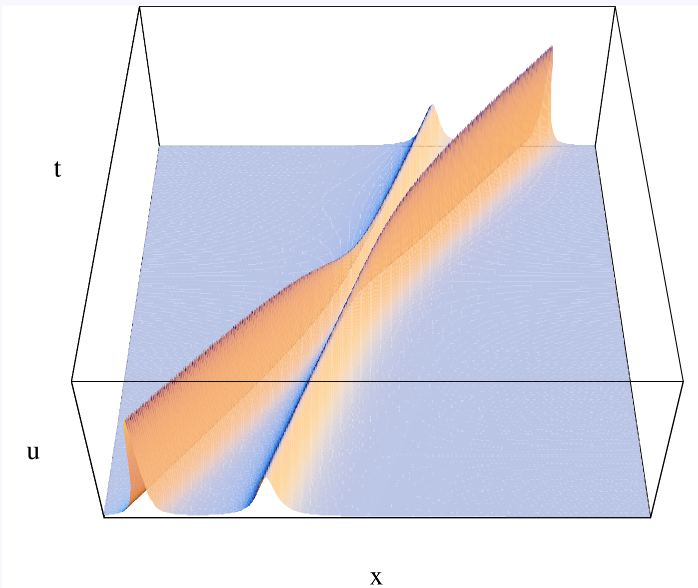


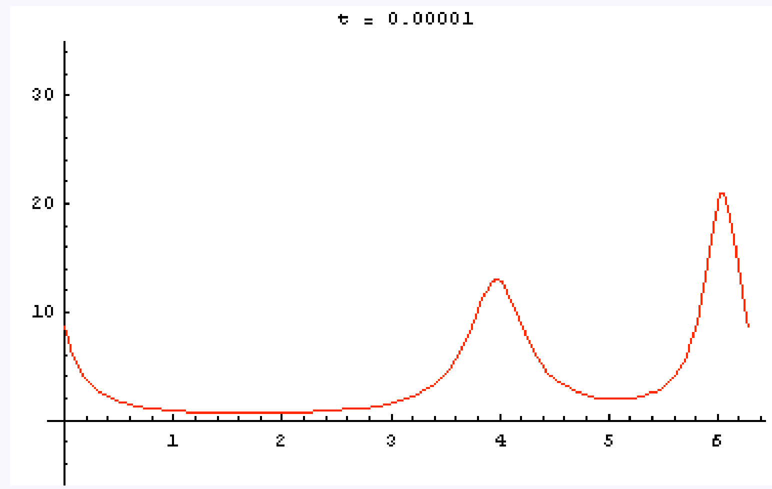
$$u_t + uu_x + u_{xx}^H = 0$$

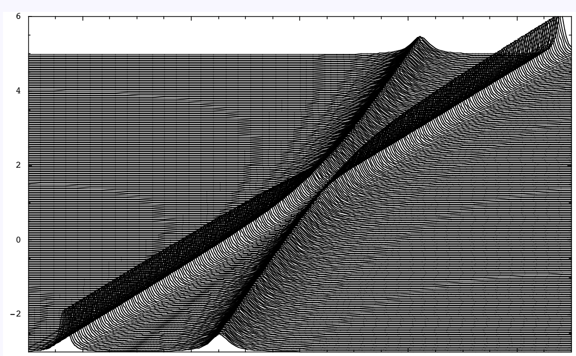
$$u(x,t) = \frac{2V}{V^2(x - x_0 - Vt)^2 + 1}$$

$$\int_{-\infty}^{+\infty} u(x,t)\,dx = 2\pi$$

Two-soliton solution of Benjamin-Ono equation (N=2)







$$u_t + uu_x + u_{xx}^H = 0$$

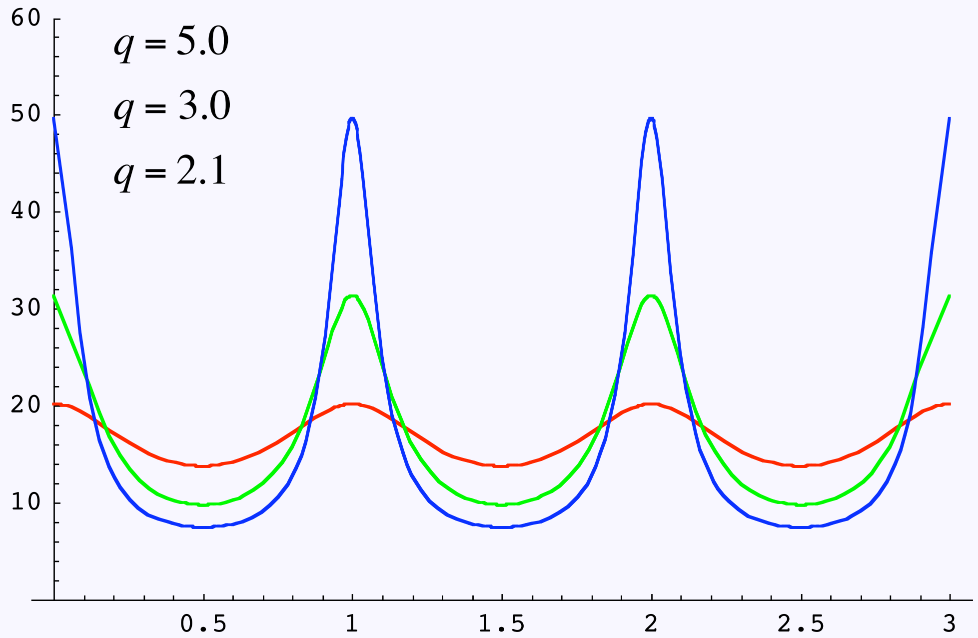
Periodic solutions of BO

$$u_t + uu_x + u_{xx}^H = 0$$

$$u = f(kx - \omega t)$$

$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

$$\theta = kx - \omega t = (p-q)x + \frac{1}{2}(p^2 - q^2)t$$



$$p > q > K$$

$$K = 2$$

$$p - q = 2\pi = k$$

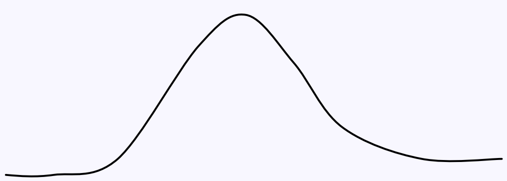
Applications of Classical BO

- Internal waves in deep stratified fluids
- Atmosphere waves

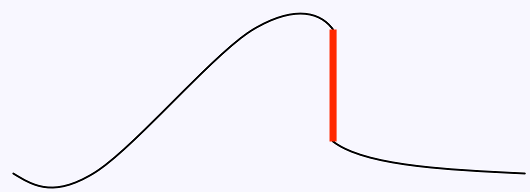
www.dropbears.com

Conventional shock waves

Solution of Riemann equation + discontinuity



Discontinuity



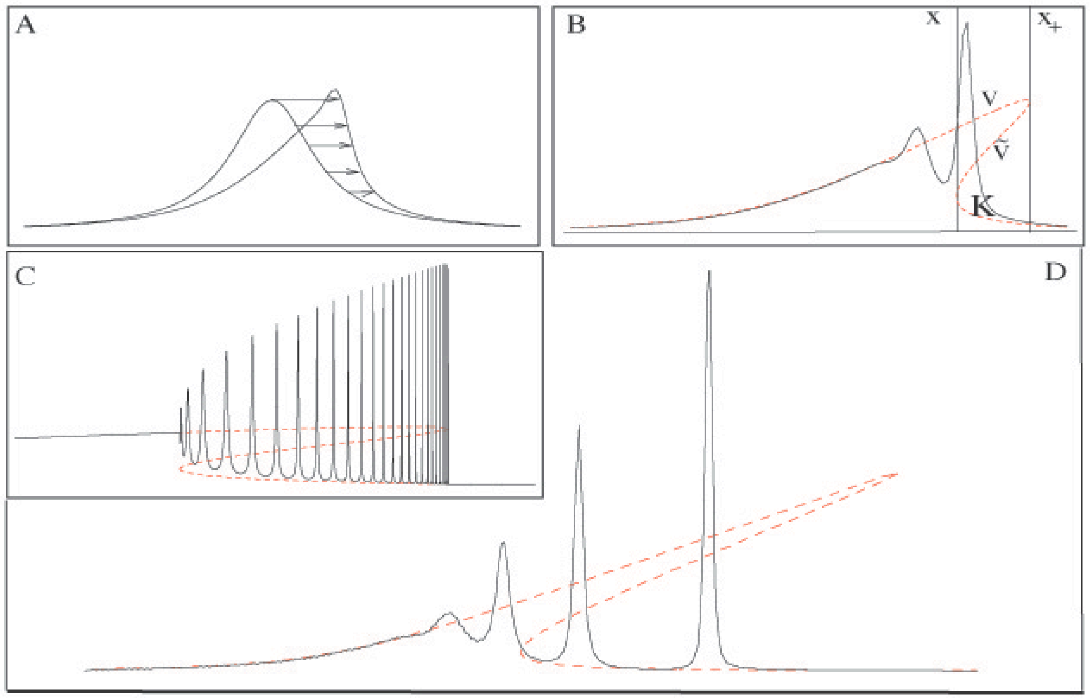
Dissipative region

Dissipation ⟶ Dispersion

Shock waves ⟶ ? Dispersive shock waves

Dispersive shock waves

Solution of Riemann equation + Modulated periodic solution of BO



Phys. Rev. Lett. 97, 246401 (2006)

Whitham theory

A. Gurevich, L. Pitaevsky (1973)

S. Dobrokhotov, I. Krichever (1991)

Y. Matsuno (1998)

The Whitham theory for BO

$$u_t + uu_x + u_{xx}^H = 0$$

$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

p, q, K - moduli

$$p > q > K$$

$$p(x,t), q(x,t), K(x,t)$$

$$\theta_x = p - q$$

$$\theta_t = \frac{1}{2}(p^2 - q^2)$$

Whitham's equations

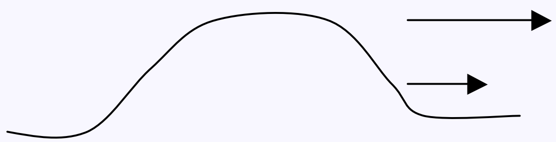
$$\begin{aligned} p_t + pp_x &= 0 \\ q_t + qq_x &= 0 \\ K_t + KK_x &= 0 \end{aligned}$$

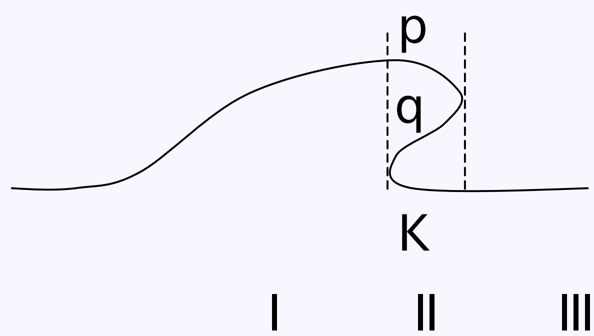
Gurevich-Pitaevskii approach

$$u_t + uu_x + u_{xx}^H = 0$$

$$u_t + uu_x = 0$$

$$u = u_0(x - ut)$$





$$u = \frac{(p-q)^2}{\frac{p+q}{2} - K - \sqrt{(p-K)(q-K)}\cos\theta} + 2K$$

$$\int \frac{dx}{2\pi} u >> 1$$

$$p_t + pp_x = 0$$

$$q_t + qq_x = 0$$

$$K_t + KK_x = 0$$

$$\theta_x = p - q$$

$$\theta_t = \frac{1}{2}(p^2 - q^2)$$

Calogero-Sutherland model

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i<j}\frac{\lambda(\lambda-1)}{(x_i - x_j)^2}$$

Calogero 1969
Sutherland 1971

$$\Psi_{ground} \sim \prod_{i<j}(x_i - x_j)^{\lambda}$$

λ=0 - free bosons
λ=1 - free fermions

$$k_F = \pi\rho_0$$

$$v_s = \lambda\pi\rho_0$$

$$\epsilon(\rho) = \frac{1}{6}(\lambda\pi\rho)^2$$

Exactly solvable
by Bethe Ansatz

Dispersion of quasiparticles

$$\varepsilon_p(k) = \lambda k_F\,|k|\left(1 + |k|/2\lambda k_F\right)$$

$$\varepsilon_h(k) = \lambda k_F\,|k|\left(1 + |k|/2k_F\right)$$

Hydrodynamics of CS model

$$H = \int dx \left(\frac{\rho v^2}{2} + \rho \varepsilon(\rho) \right) \qquad [\rho(x), v(y)] = -i\delta'(x-y)$$

$$\varepsilon(\rho) = \frac{1}{6}(\pi\lambda\rho)^2 + \frac{1}{2}\pi\lambda(\lambda-1)\rho_x^H + \frac{(\lambda-1)^2}{8}(\partial_x \log\rho)^2$$

$$\sum_{j<k} \frac{\lambda(\lambda-1)}{(x_j - x_k)^2} \rightarrow \int dx \rho(x) \int dy \frac{\lambda(\lambda-1)\rho(y)}{(y-x)^2} \rightarrow \pi\lambda(\lambda-1)\int dx \rho\rho_x^H$$

$$\rho_t + (\rho v)_x = 0$$

$$v_t + \left(\frac{v^2}{2} + w[\rho] \right)_x = 0$$

Specific enthalpy $w[\rho] = \dfrac{\delta(\rho\varepsilon)}{\delta\rho}$

Free fermions $\lambda = 1$

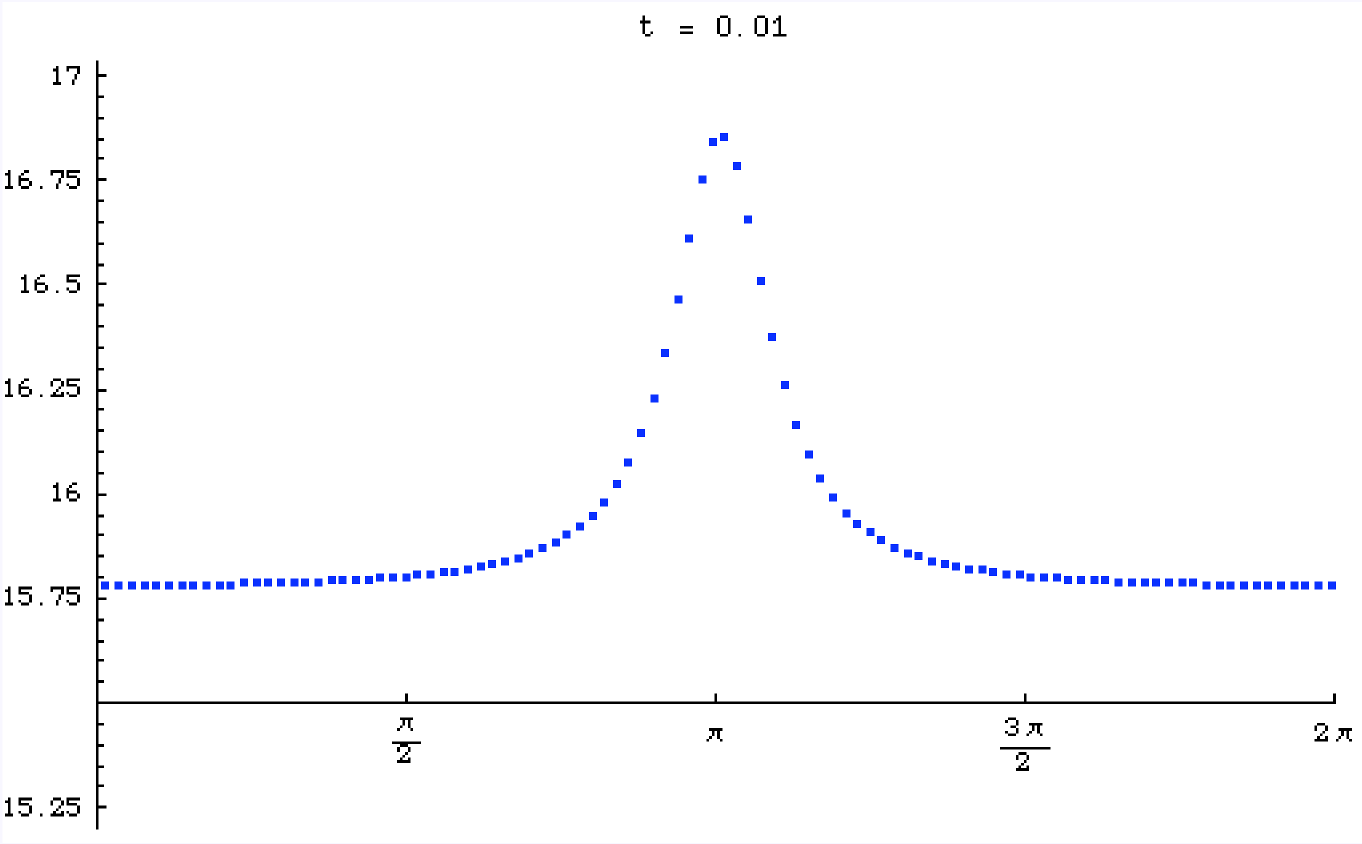
Classical Calogero $\lambda - 1 \to \lambda$

I. Andric, V. Bardek, 1988

Collective motion in Sutherland model

$$\rho = \rho_0\left(1 + \frac{a}{(\pi\rho_0(x - Vt))^2 + a^2}\right)$$

N=100



Chiral (right) sector

$$\rho_t + (\rho v)_x = 0$$

$$v_t + \left(\frac{v^2}{2} + w[\rho] \right)_x = 0$$

Chiral constraint

$$v = \pi\lambda\rho + \frac{\lambda - 1}{2} \partial_x (\log \rho)^H$$

$$\rho_t + \pi\lambda\partial_x \left[\rho^2 + \frac{\lambda - 1}{2\pi\lambda} \rho \partial_x (\log \rho)^H \right] = 0$$

$$\rho = \rho_0 + \delta\rho$$

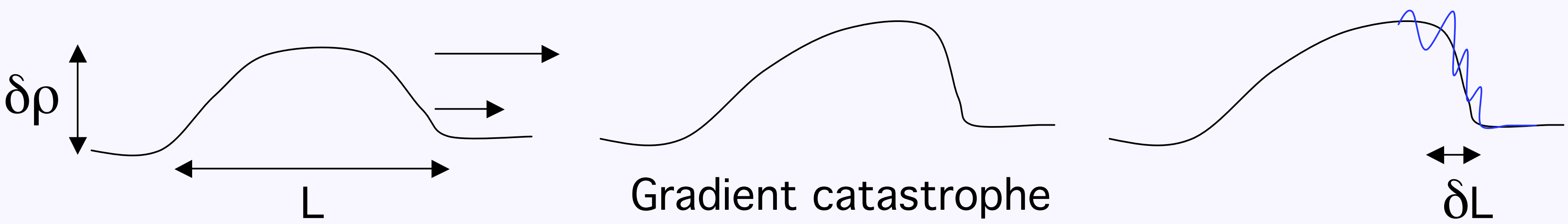
$$u = 2\pi\lambda\delta\rho$$

$$u_t + uu_x + \frac{\lambda - 1}{2} u^H_{xx} = 0$$

Shock waves in Sutherland model

$$u_t + uu_x + \frac{\lambda - 1}{2} u_{xx}^H = 0$$

$$\rho = \rho_0 + \delta\rho$$
$$u = 2\pi\lambda\delta\rho$$



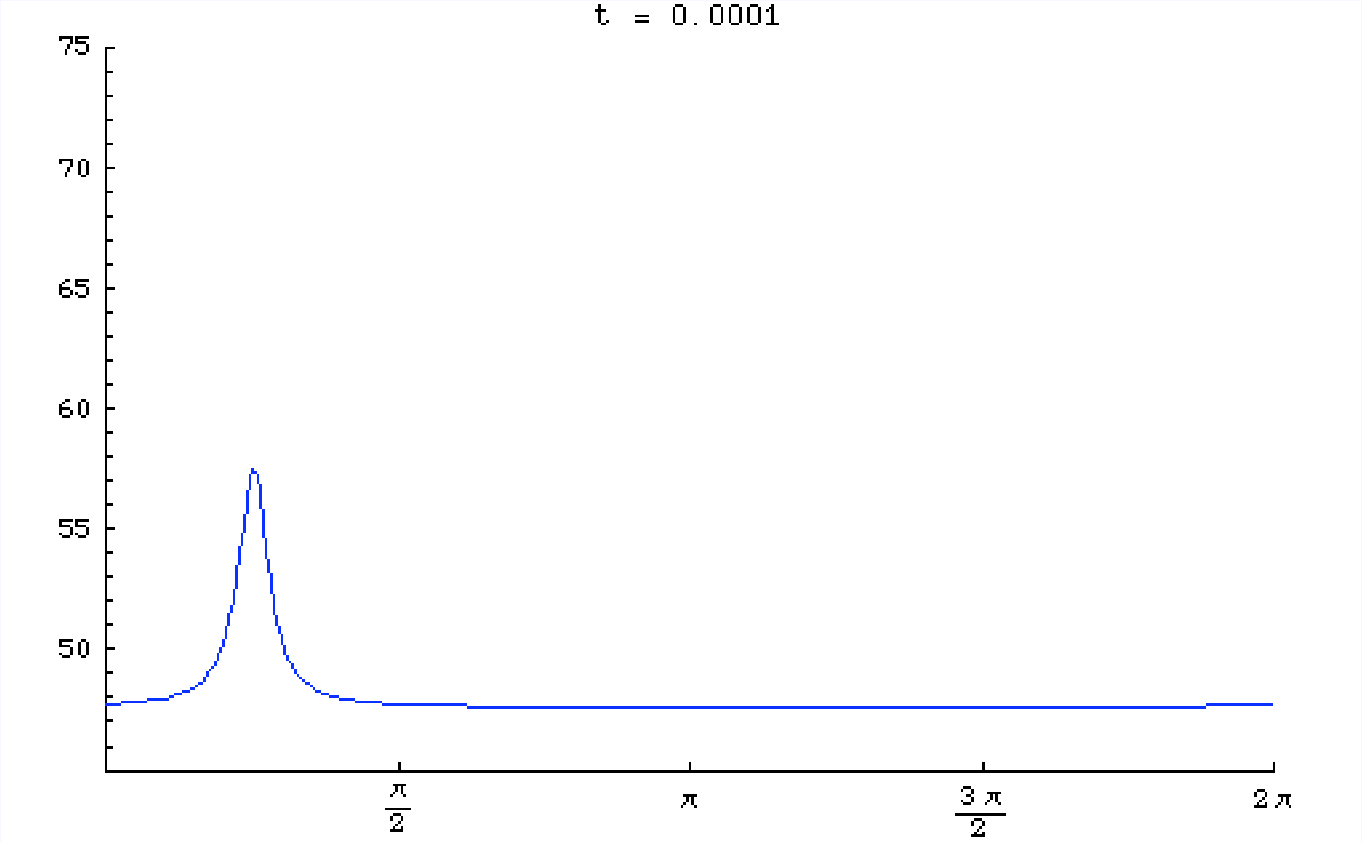
$$t_c \propto \frac{L}{\delta\rho}$$

$$\delta L \propto \frac{\lambda - 1}{\delta\rho}$$

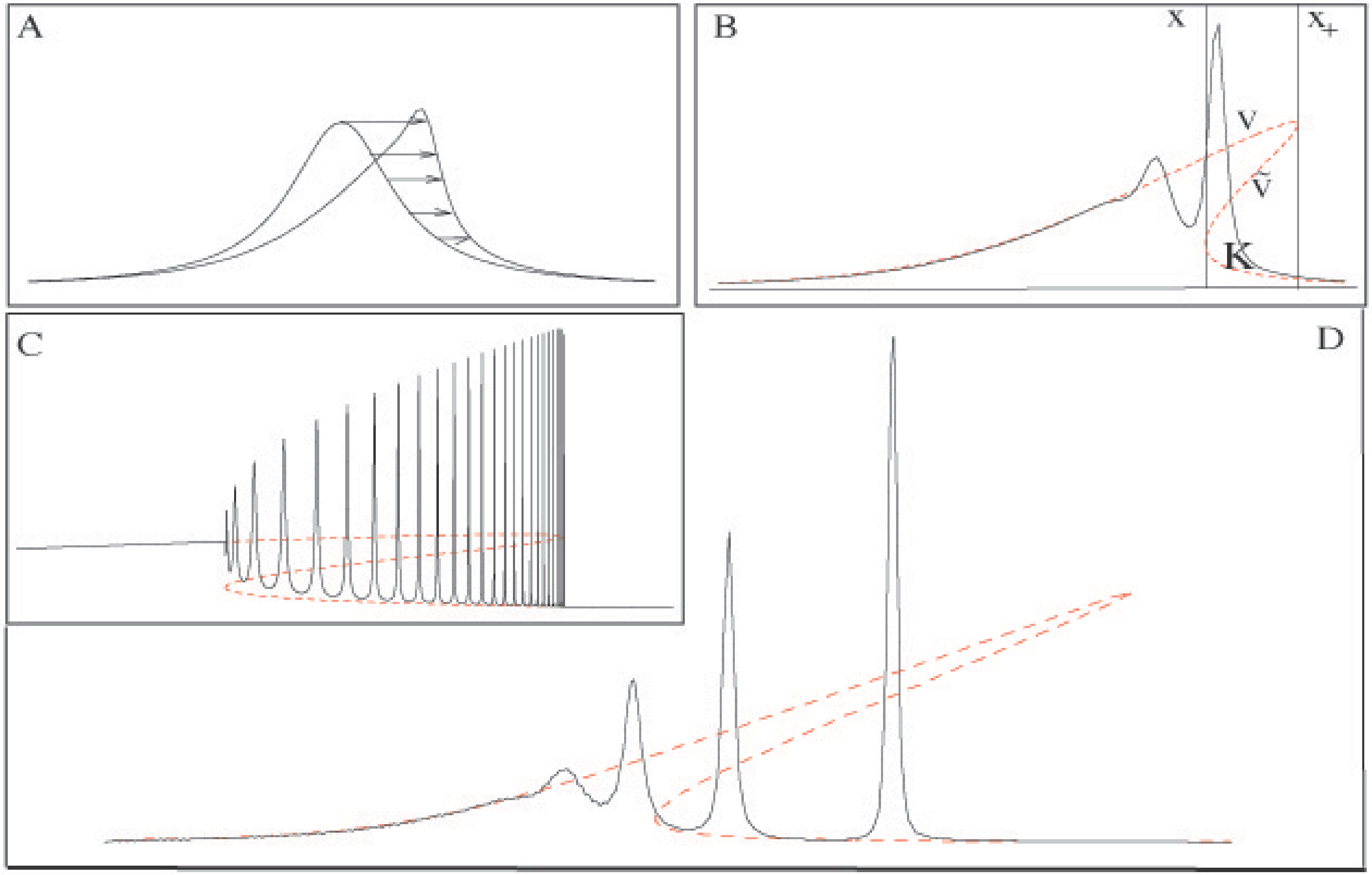
$$\rho_0 \delta L \propto (\lambda - 1) \frac{\rho_0}{\delta\rho} >> 1$$

Collective description is still valid
for interacting system

Shock waves in Sutherland model

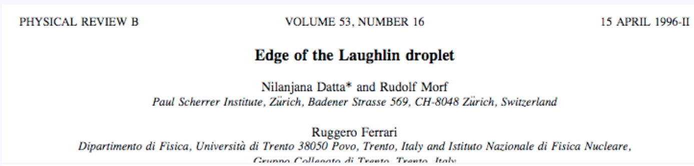


Dynamics of 300 particles on a circle. Initial profile: Lorentzian with area 3.



Bettelheim, AGA, Wiegmann, Phys. Rev. Lett. 97, 246401 (2006)

BO equation for the edge of FQHE


PHYSICAL REVIEW B VOLUME 53, NUMBER 16 15 APRIL 1996-II

Edge of the Laughlin droplet

Nilanjana Datta* and Rudolf Morf
Paul Scherrer Institute, Zürich, Badener Strasse 569, CH-8048 Zürich, Switzerland

Ruggero Ferrari
Dipartimento di Fisica, Università di Trento 38050 Povo, Trento, Italy and Istituto Nazionale di Fisica Nucleare,

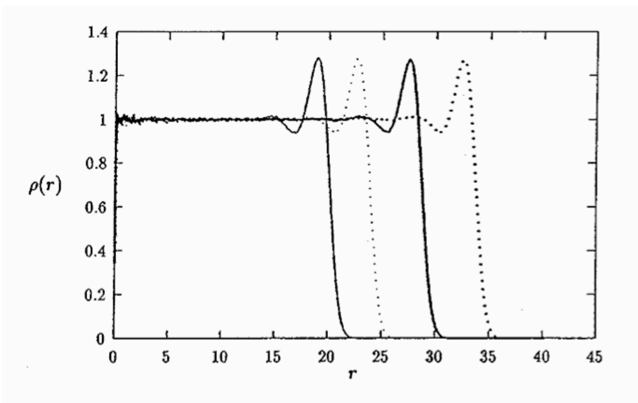
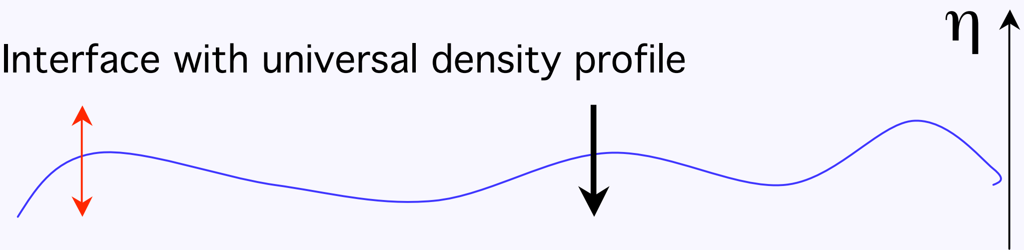


FIG. 3. Plot of $\rho(r)$ vs r for $m=3$ and $N=72$, 100, 144, and 196 electrons.



$$\frac{1}{c}\eta_t + \eta_x + \frac{\beta}{l_0}\eta\eta_x + \frac{\alpha}{2} l_0 \eta_{xx}^H = 0$$

soliton charge $= \dfrac{\alpha}{\beta}\dfrac{1}{m}$ m=3

$$t_c \propto \tau_F (\rho_0 L) \frac{\rho_0}{\delta\rho} \approx 1ns$$

Gradient catastrophe time

$$\alpha = \beta$$

Consequence of the universal density profile!

Conclusions

- Dispersion and nonlinearity in interacting systems -> solitons, dispersive shock waves etc.
- Collective field theory for Calogero-Sutherland -- Benjamin-Ono equation.
- Benjamin-Ono equation - solitons with "quantized" charge.
- Useful for nonlinear phenomena
  - Shock waves in electronic systems and systems of cold atoms,
  - Quantum Hall effect edge states,
  - Tunneling into coherent states,
  - Instantons (rare fluctuations),
  - Spin-Charge coupling in systems with dispersion,...