

Metal-insulator transition in a interacting many-electron system



Igor Aleiner (Columbia)

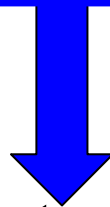
Collaborators: B.L. Altshuler (Columbia, NEC America)
D.M. Basko (Columbia)

Detailed paper: [Annals of Physics 321 \(2006\) 1126-1205](#)
Shorter version: [cond-mat/0602510](#); chapter in “Problems of CMP”

A.I. Larkin Conference, Chernogolovka, June 25th, 2007

Outline:

- Formulation and history of the problem
- Results
- Effective model
- Technique
- Stability of the metal
- Stability of the many-body insulator



- Metal insulator transition

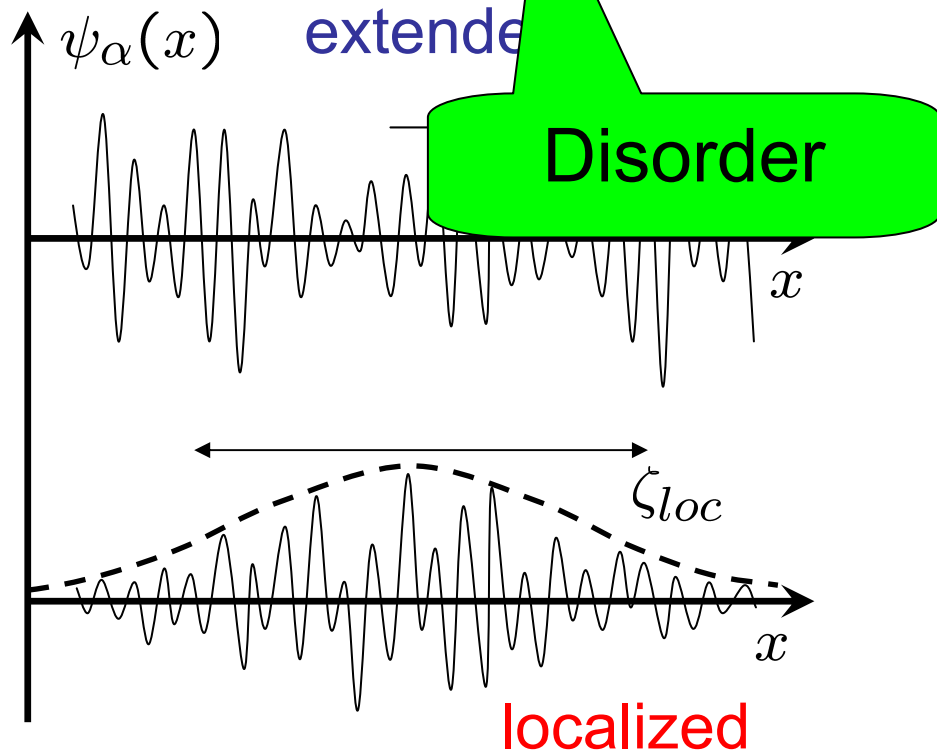
Problem: can e-e interaction alone
sustain hopping conduction
in a localized system?

- Given:**
1. All one-electron states are localized
 2. Electrons interact with each other
 3. The system is closed (no phonons)
 4. Temperature is low but finite

Find: DC conductivity $\sigma(T, \omega=0)$
(zero or finite?)

1. Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m} + U(\mathbf{r}) - \epsilon_F \right] \psi_\alpha(\mathbf{r}) = \xi_\alpha \psi_\alpha(\mathbf{r})$$



Most of the knowledge is based on extensions and Improvements of:

PHYSICAL REVIEW

VOLUME 109, NUMBER 5

MARCH 1, 1965

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON

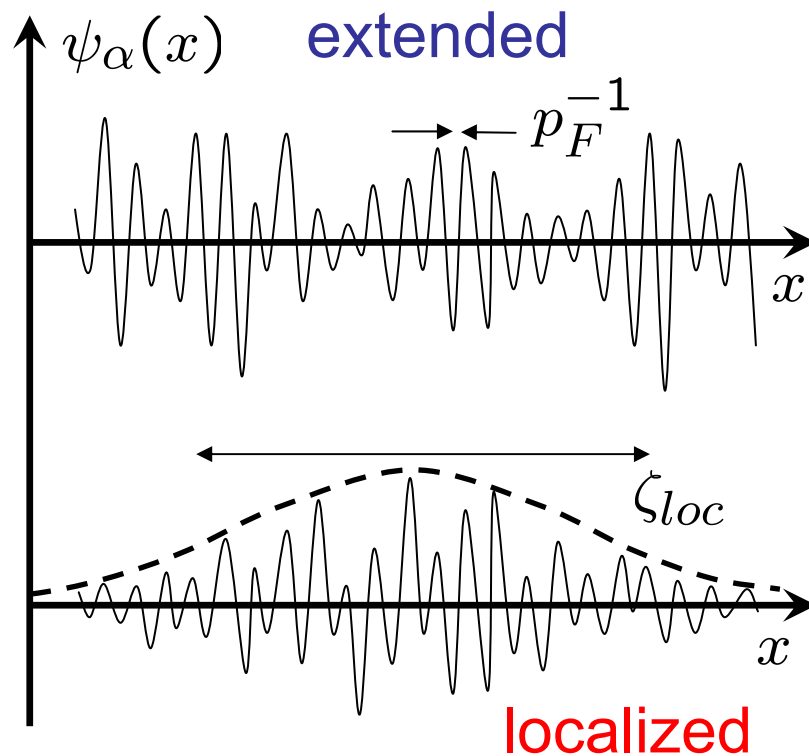
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

1. Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m} + U(\mathbf{r}) - \epsilon_F \right] \psi_\alpha(\mathbf{r}) = \xi_\alpha \psi_\alpha(\mathbf{r})$$



$d=1$; All states are localized

Exact solution for one channel:

M.E. Gertsenshtein, V.B. Vasil'ev, (1959)

“Conjecture” for one channel:

Sir N.F. Mott and W.D. Twose (1961)

Exact solution for $\sigma(\omega)$ for one channel:

V.L. Berezinskii, (1973)

Scaling argument for multi-channel :

D.J. Thouless, (1977)

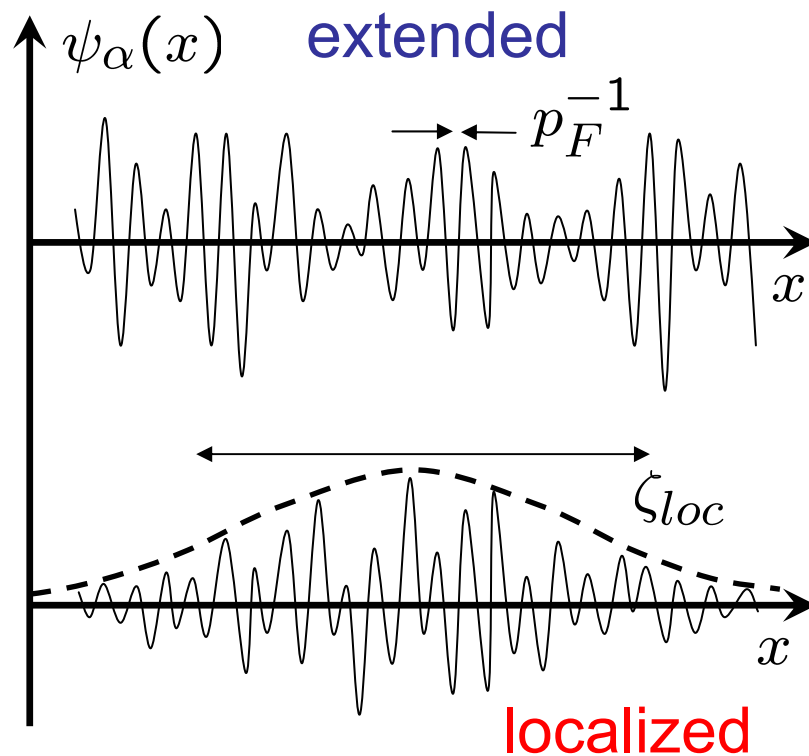
Exact solutions for multi-channel:

K.B.Efetov, A.I. Larkin (1983)

O.N. Dorokhov (1983)

1. Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m} + U(\mathbf{r}) - \epsilon_F \right] \psi_\alpha(\mathbf{r}) = \xi_\alpha \psi_\alpha(\mathbf{r})$$



$d=1$; All states are localized

$d=2$; All states are localized
If no spin-orbit interaction

Thouless scaling + ansatz:

E. Abrahams, P. W. Anderson, D. C. Licciardello, and T.V. Ramakrishnan, (1979)

Instability of metal with respect to quantum (weak localization) corrections:

L.P. Gorkov, A.I.Larkin, D.E. Khmel'nitskii, (1979)

First numerical evidence:

A Maccinon, B. Kramer, (1981)

“All states are localized”

means

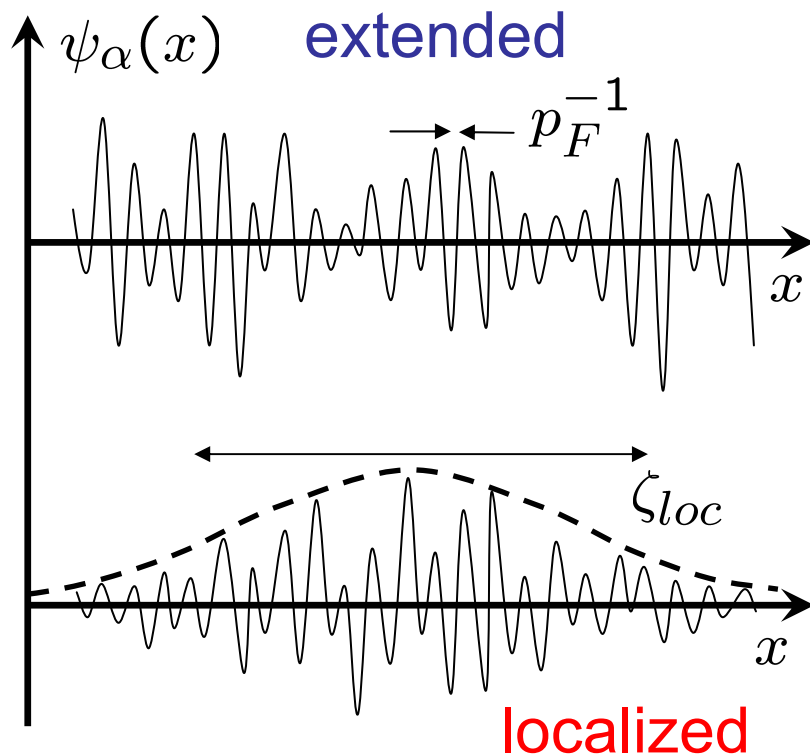
Probability to find an extended state:

$$\mathcal{P}_{ext} \propto \exp \left(-\# \frac{L}{\zeta_{loc}} \right)$$

System size

1. Localization of single-electron wave-functions:

$$\left[-\frac{\nabla^2}{2m} + U(\mathbf{r}) - \epsilon_F \right] \psi_\alpha(\mathbf{r}) = \xi_\alpha \psi_\alpha(\mathbf{r})$$

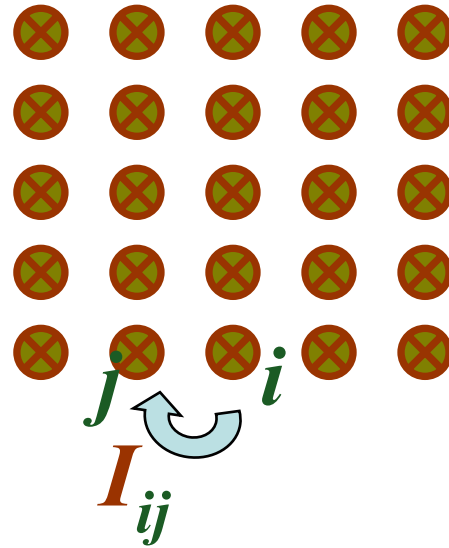


$d=1$; All states are localized

$d=2$; All states are localized

$d>2$; Anderson transition

Anderson Model



- *Lattice - tight binding model*
- *Onsite energies ε_i - **random***
- *Hopping matrix elements I_{ij}*

$$I_{ij} = \begin{cases} I & \textit{i and j are nearest neighbors} \\ 0 & \textit{otherwise} \end{cases}$$

Critical hopping:

$$-W < \varepsilon_i < W$$

uniformly distributed

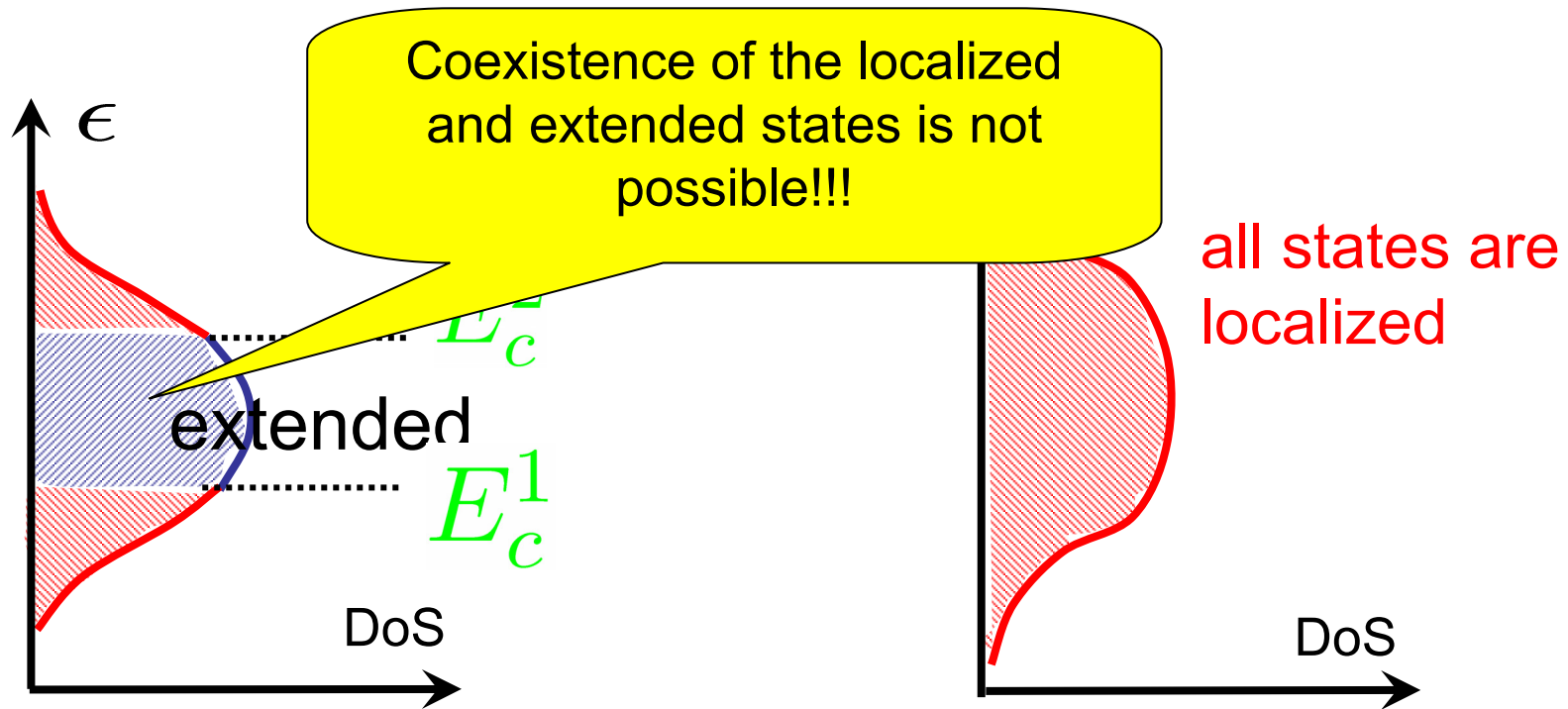
$$\frac{I_c}{W} \simeq \left(\frac{1}{2d} \right) \left(\frac{1}{\ln d} \right)$$

$$d \gtrsim 3 \gg 1$$

Anderson Transition

$$I > I_c$$

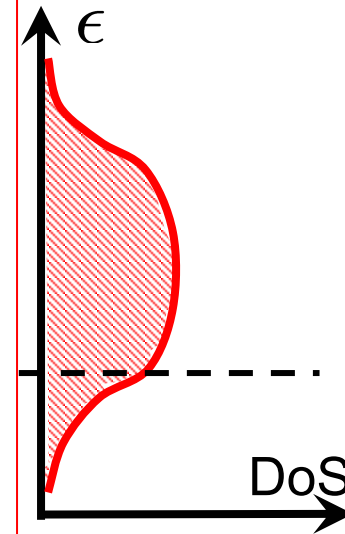
$$I < I_c$$



E_c - mobility edges (one particle)

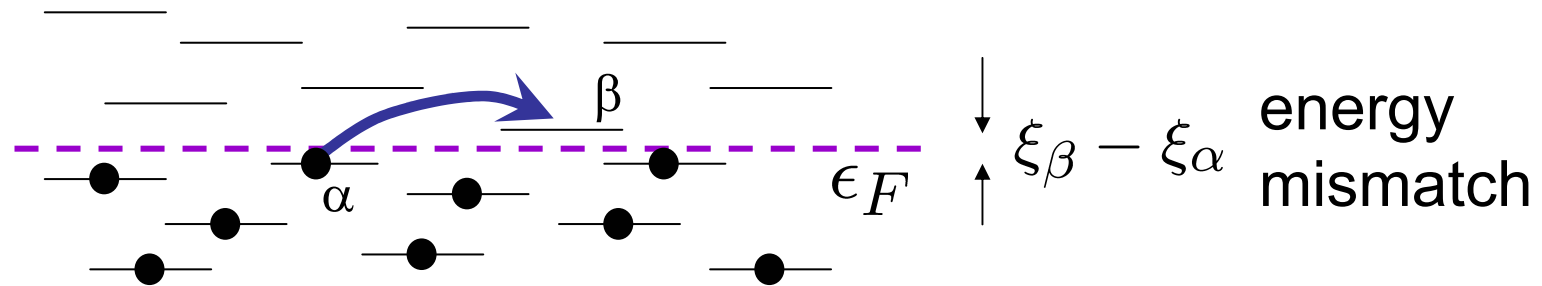
Temperature dependence of the conductivity (I)

Assume that all the states are localized



$$\underline{F} \quad \sigma(T) = 0$$

Inelastic processes \Rightarrow
transitions between localized states

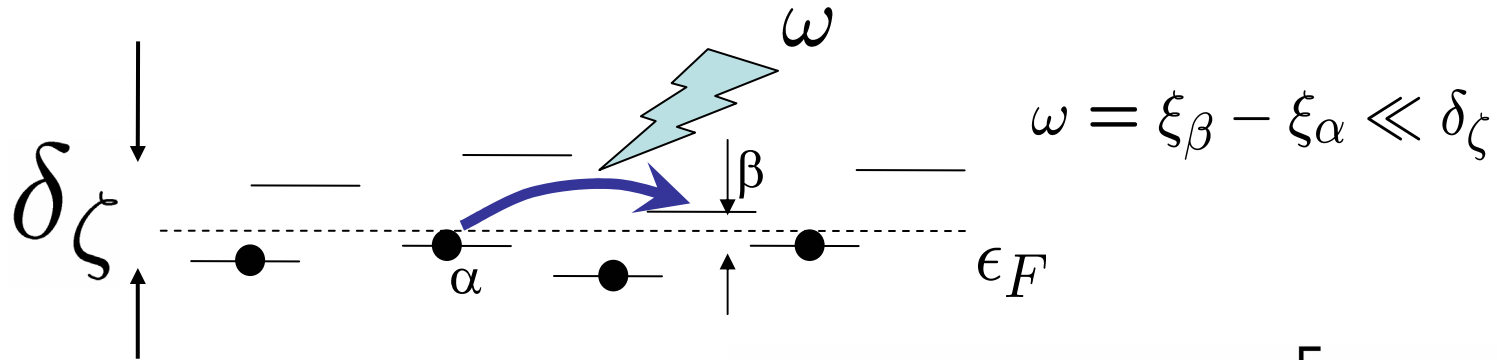


$$\sigma(T) \propto \Gamma_\alpha \text{ (inelastic lifetime)}^{-1}$$

$$T = 0 \quad \Rightarrow \quad \sigma = 0 \quad \text{(any mechanism)}$$

$$T > 0 \quad \Rightarrow \quad \sigma = ?$$

Phonon-induced hopping



Variable Range Hopping
 Sir N.F. Mott (1968)

$$\sigma(T) \propto T^\gamma \exp \left[- \left(\frac{\delta_\zeta}{T} \right)^{\frac{1}{d+1}} \right]$$

Mechanism-dependent prefactor

Without band gap
 A.L. Efros
 Mott (1975)

Optimized phase volume

Any bath with a continuous spectrum of delocalized excitations down to $\omega = 0$ will give the same exponential

Q: Can we replace phonons with e-h pairs and obtain phonon-less VRH?

A#1: Sure

Easy steps:

1) Recall phonon-less AC conductivity:

Sir N.F. Mott (1970)

$$\sigma(\omega) \simeq \frac{e^2 \zeta_{loc}^{d-2}}{\hbar} \left(\frac{\hbar\omega}{\delta\zeta} \right)^2 \ln^{d+1} \left| \frac{\delta\zeta}{\hbar\omega} \right|$$

2) Calculate the Nyquist noise.

3) Use the electric noise instead of phonons.

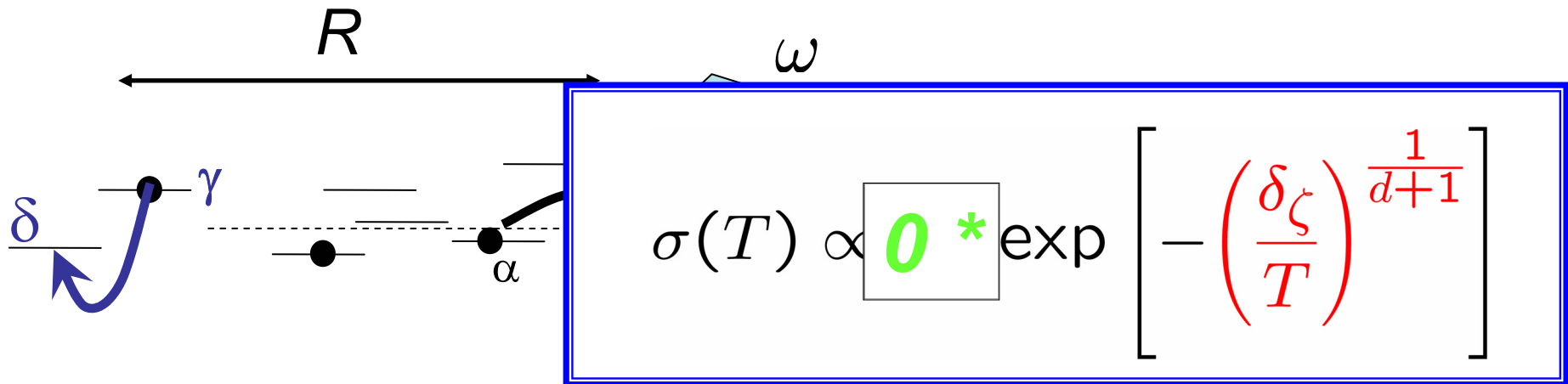
4) Do self-consistency (whatever it means).

Q: Can we replace phonons with e-h pairs and obtain phonon-less VRH?

A#1: Sure

A#2: No way [L. Fleishman, P.W. Anderson (1980)]
 (for Coulomb interaction in 2D — may be)

$R \rightarrow \infty$ Thus, the matrix element vanishes !!!



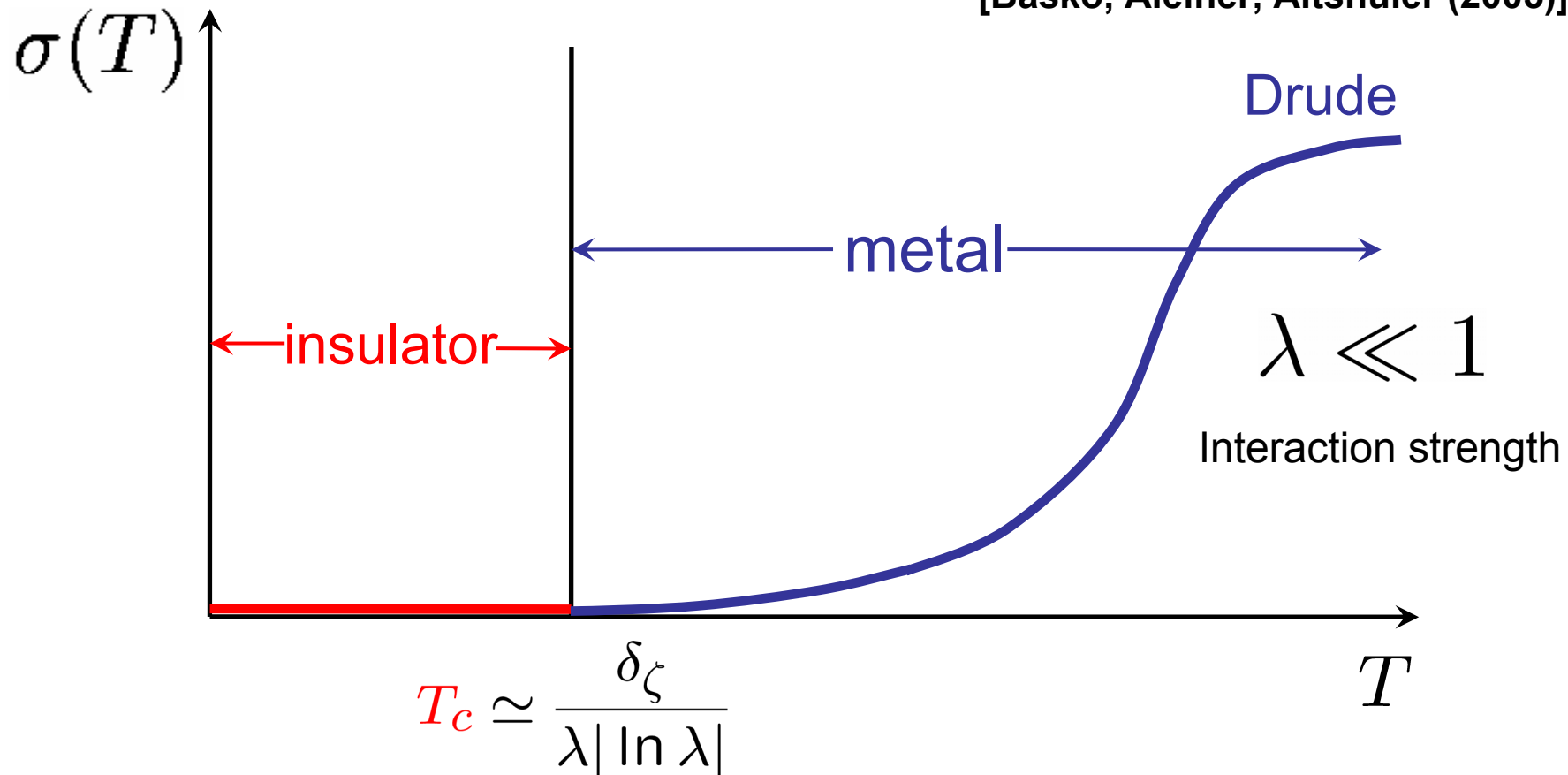
Q: Can we replace phonons with e-h pairs and obtain phonon-less VRH?

A#1: Sure [a person from the street (2005)]:

A#2: No way [L. Fleishman. P.W. Anderson (1980)]

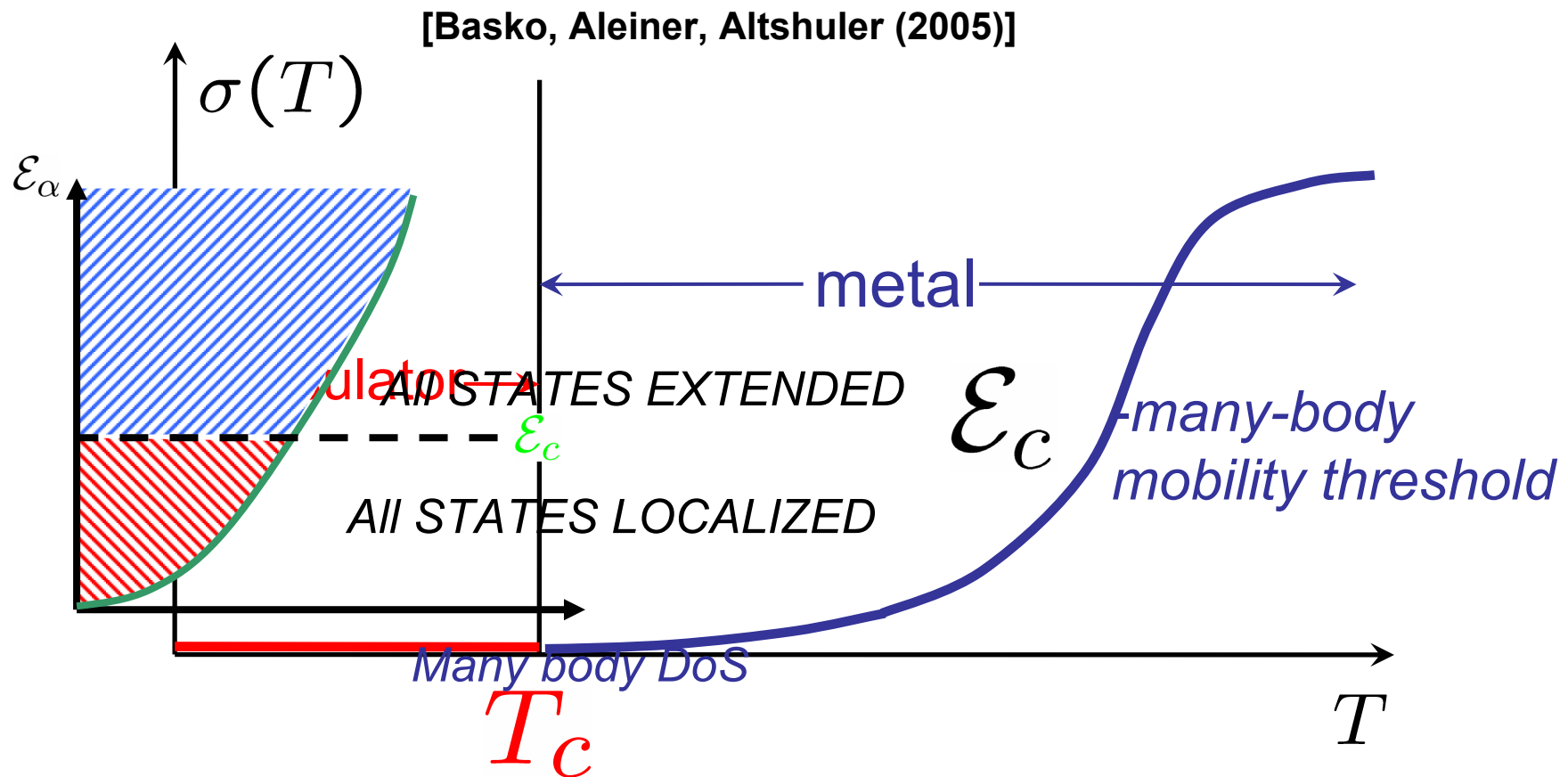
A#3: Finite T Metal-Insulator Transition

[Basko, Aleiner, Altshuler (2005)]



Many-body mobility threshold

$$\left[\hat{H}_1 + \hat{H}_{int} \right] \Psi_\alpha = \epsilon_\alpha \Psi_\alpha$$



“All states are localized”

means

Probability to find an extended state:

$$\mathcal{P}_{ext} \propto \exp \left(-\# \frac{\mathcal{V}}{\mathcal{V}_{loc}(\mathcal{E})} \right)$$

$$\lim_{\mathcal{E} \rightarrow \mathcal{E}_c - 0} \mathcal{V}_{loc}(\mathcal{E}) = \infty$$

System volume

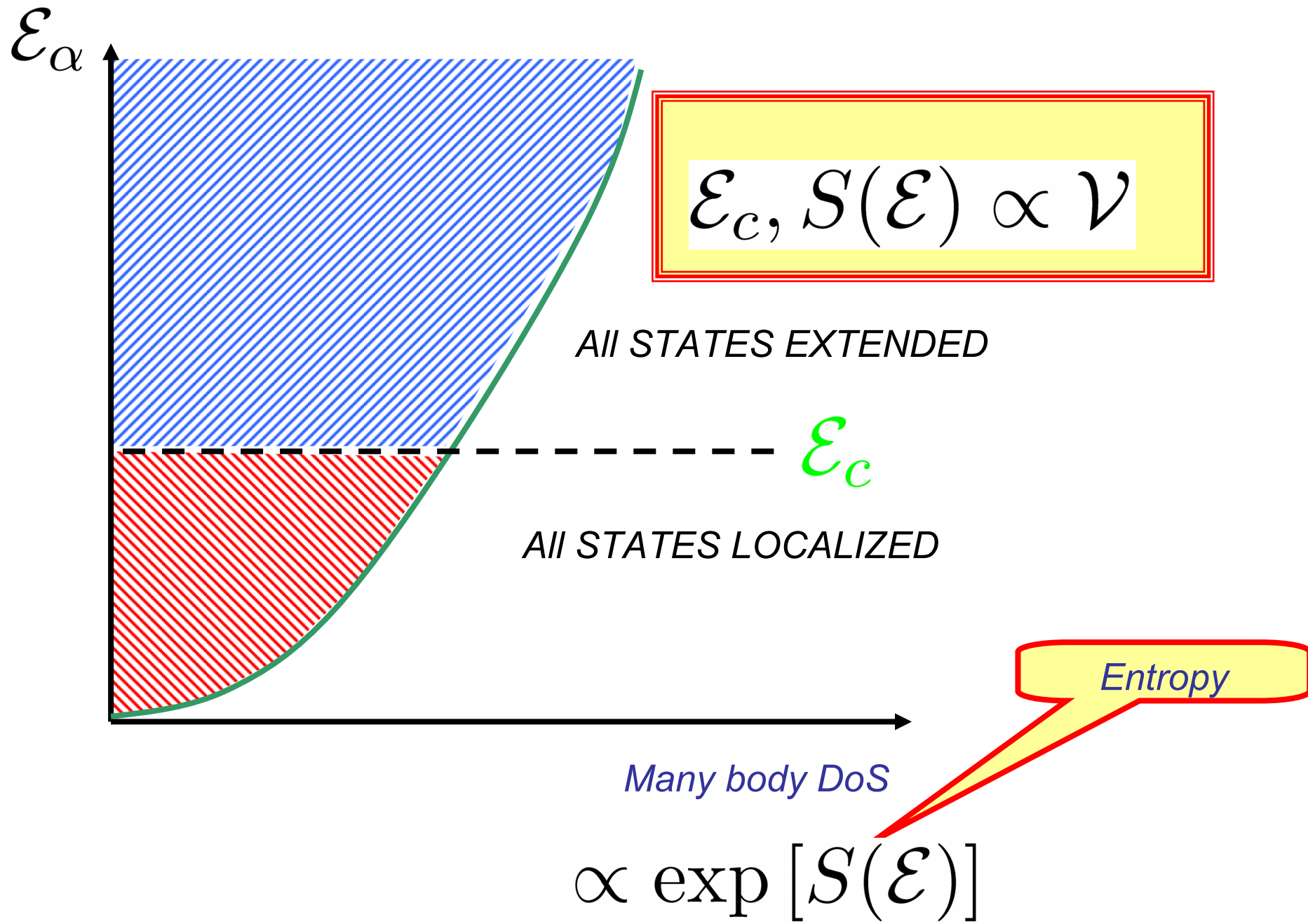
Localized one-body wave-function

Means, in particular:

$$\langle i | O(\mathbf{r}_1) | j \rangle \langle j | O(\mathbf{r}_2) | i \rangle \simeq \begin{cases} a \left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|}{L(\omega)} \right), & \omega = \xi_i - \xi_j \\ & \text{extended} \\ b \left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|}{\zeta_{loc}} \right), & \text{localized} \end{cases}$$

We define localized many-body wave-function as:

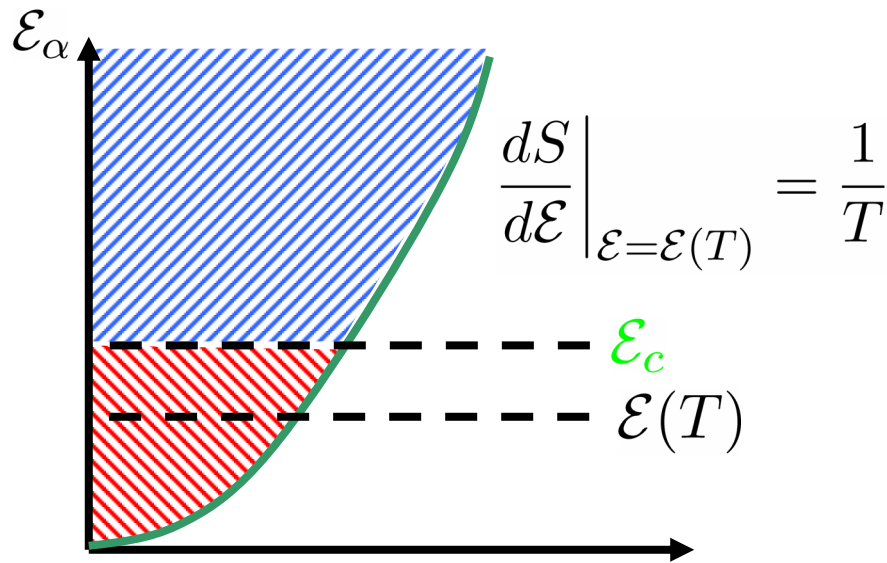
$$\langle \alpha | \hat{O}(\mathbf{r}_1) | \beta \rangle \langle \beta | \hat{O}(\mathbf{r}_2) | \alpha \rangle \simeq \begin{cases} \mathcal{A} \left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|}{L(\omega)} \right), & \omega = \mathcal{E}_\alpha - \mathcal{E}_\beta \\ & \text{extended} \\ \mathcal{B} \left(\frac{|\mathbf{r}_1 - \mathbf{r}_2|}{\zeta_{loc}} \right), & \text{localized} \end{cases}$$



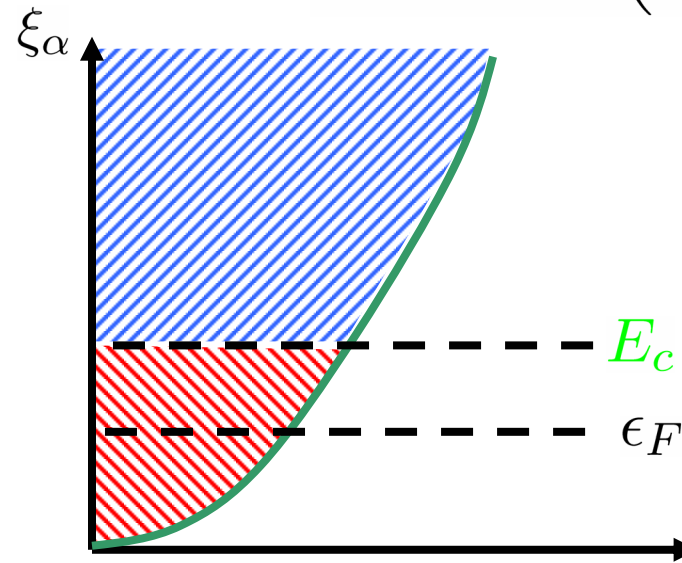
Is it similar to Anderson transition?

Why no activation?

$$\sigma(T) \propto \exp\left(-\frac{E_c - \epsilon_F}{T}\right)$$



Many body DoS



One-body DoS

$$\sigma(T) = \frac{\int_{\epsilon_c}^{\infty} d\mathcal{E} e^{S(\mathcal{E}) - \mathcal{E}/T} \sigma(\mathcal{E})}{\int_0^{\infty} d\mathcal{E} e^{S(\mathcal{E}) - \mathcal{E}/T}} \simeq \exp\left[-\frac{1}{T} \int_{\mathcal{E}(T)}^{\epsilon_c} \mathcal{E} d\mathcal{E} \frac{d^2 S}{d^2 \mathcal{E}}\right] \xrightarrow{\mathcal{V} \rightarrow \infty} 0$$

$\propto \mathcal{V}$

$$\sigma = 0$$

Physics: Many-body excitations turn out to be localized in the Fock space

VOLUME 78, NUMBER 14

PHYSICAL REVIEW LETTERS

7 APRIL 1997

Quasiparticle Lifetime in a Finite System: A Nonperturbative Approach

Boris L. Altshuler,¹ Yuval Gefen,² Alex Kamenev,² and Leonid S. Levitov³

¹*NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540*

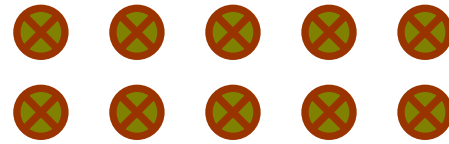
²*Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot, 76100, Israel*

³*Massachusetts Institute of Technology, 12-112, Cambridge, Massachusetts 02139*

(Received 30 August 1996)

The problem of electron-electron lifetime in a quantum dot is studied beyond perturbation theory by mapping onto the problem of localization in the Fock space. Localized and delocalized regimes are identified, corresponding to quasiparticle spectral peaks of zero and finite width, respectively. In the localized regime, quasiparticle states are single-particle-like. In the delocalized regime, each eigenstate is a superposition of states with very different quasiparticle content. The transition energy is $\epsilon_c \approx \Delta(g/\ln g)^{1/2}$, where Δ is mean level spacing, and g is the dimensionless conductance. Near ϵ_c there is a broad critical region not described by the golden rule. [S0031-9007(97)02895-0]

Anderson Model



• *Lattice - tight binding model*

In fact, i, j can be states in any space (not necessarily coordinate)

Critical hopping: I_{ij}

Interpretation:

$$\frac{I_c}{W} \simeq \left(\frac{1}{2d} \right)$$
$$d \gtrsim 3 \gg 1$$

W – maximal energy mismatch;
 $2d$ – number of coupled neighbors;
(connectivity)

At $I > I_c$ there will be always level mismatched from given by $|\varepsilon_i - \varepsilon_j| < I$

and the resonance transport will occur

Fock space localization in quantum dots (AGKL, 1997)

**1-particle
excitation**

**3-particle
excitation**

**5-particle
excitation**

$$\xi_\alpha \xrightarrow{\lambda\delta_1} \xi_\gamma + \xi_\delta - \xi_\beta \xrightarrow{\lambda\delta_1} \xi_1 + \xi_2 + \xi_3 - \xi_4 - \xi_5 \dots \xrightarrow{\lambda\delta_1}$$

$$(2d) \frac{I_c}{W} \simeq 1$$

↓

$$\left(\frac{T_c}{\delta_1}\right)^2 \lambda \simeq 1$$

$$I \rightarrow \lambda\delta_1$$

$$W \rightarrow \delta_1$$

$$2d \rightarrow \left(\frac{T}{\delta_1}\right)^2$$

δ_1 - one-particle level spacing;

Metal-Insulator “Transition” in zero dimensions

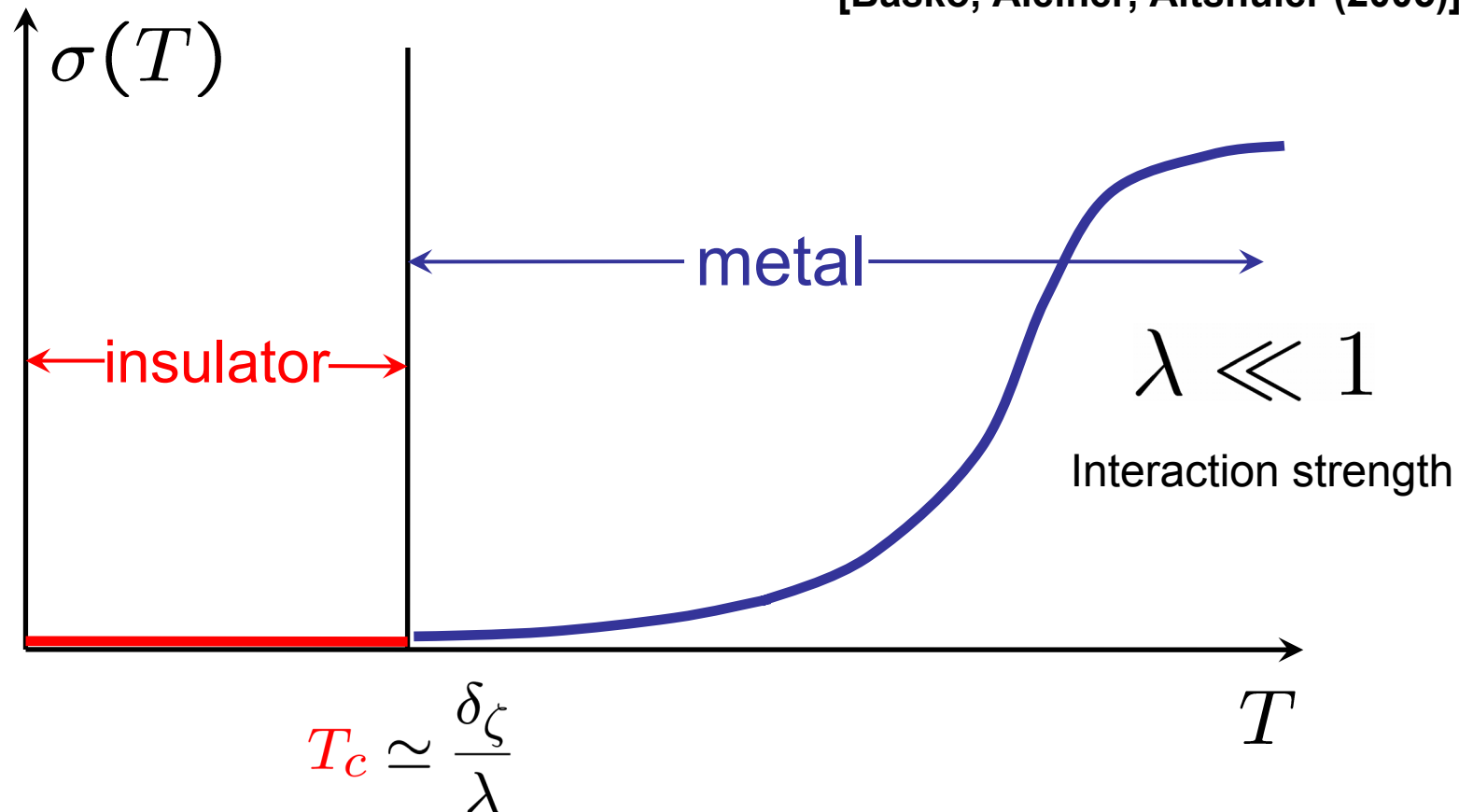
$$\left(\frac{T_c}{\delta_1}\right)^2 \simeq \frac{1}{\lambda}$$

[Altshuler, Gefen, Kamenev, Levitov (1997)]

In the paper: $\left(\frac{\epsilon_c}{\delta_1}\right)^2 \simeq \frac{1}{\lambda} \ln \frac{1}{\lambda}$

Vs. finite T Metal-Insulator Transition in the bulk systems

[Basko, Aleiner, Altshuler (2005)]



Metal-Insulator “Transition” in zero dimensions

$$\left(\frac{T_c}{\delta_1}\right)^2 \simeq \frac{1}{\lambda}$$

[Altshuler, Gefen, Kamenev, Levitov (1997)]

δ_1 - one-particle level spacing;

Vs. finite T Metal-Insulator Transition in the bulk systems

[Basko, Aleiner, Altshuler (2005)]

$$T_c \simeq \frac{\delta_\zeta}{\lambda}$$

δ_ζ 1-particle level spacing in localization volume;

$$\delta_1 \longrightarrow \delta_\zeta$$

1) Localization in Fock space

= Localization in the coordinate space.

2) Interaction is local;

Metal-Insulator “Transition” in zero dimensions

$$\left(\frac{T_c}{\delta_1}\right)^2 \simeq \frac{1}{\lambda}$$

[Altshuler, Gefen, Kamenev, Levitov (1997)]

δ_1 - one-particle level spacing;

Vs. finite T Metal-Insulator Transition in the bulk systems

[Basko, Aleiner, Altshuler (2005)]

$$T_c \simeq \frac{\delta_\zeta}{\lambda}$$

δ_ζ 1-particle level spacing in localization volume;

1,2) Locality:

$$\delta_1 \longrightarrow \delta_\zeta$$

3) Interaction matrix elements

$$\left(\frac{T}{\delta_\zeta}\right)^2 \longrightarrow \left(\frac{T}{\delta_\zeta}\right) \times \left(\frac{\omega}{\delta_\zeta}\right) \longrightarrow \left(\frac{T}{\delta_\zeta}\right) \times 1$$

Effective Hamiltonian for MIT.

We would like to describe the low-temperature regime only.

Spatial scales of interest \gg



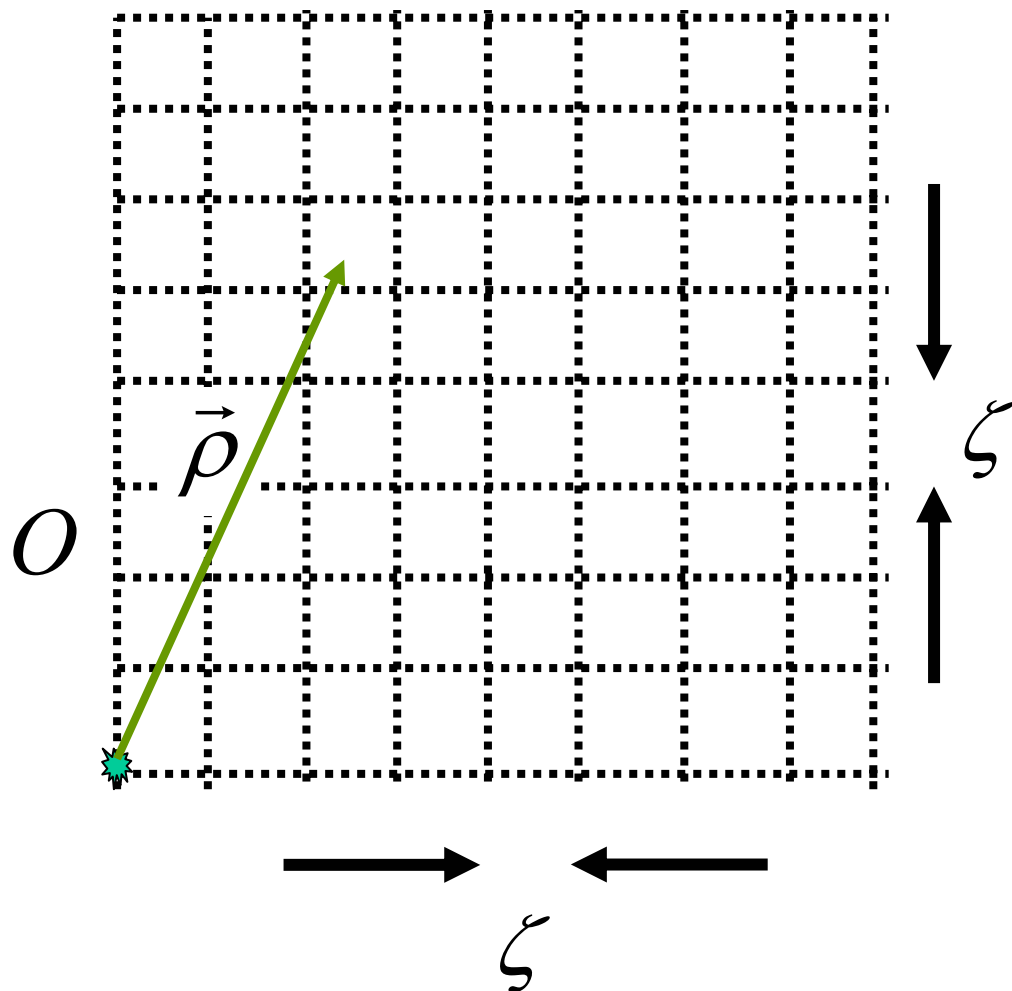
ζ_{loc}

1-particle
localization length

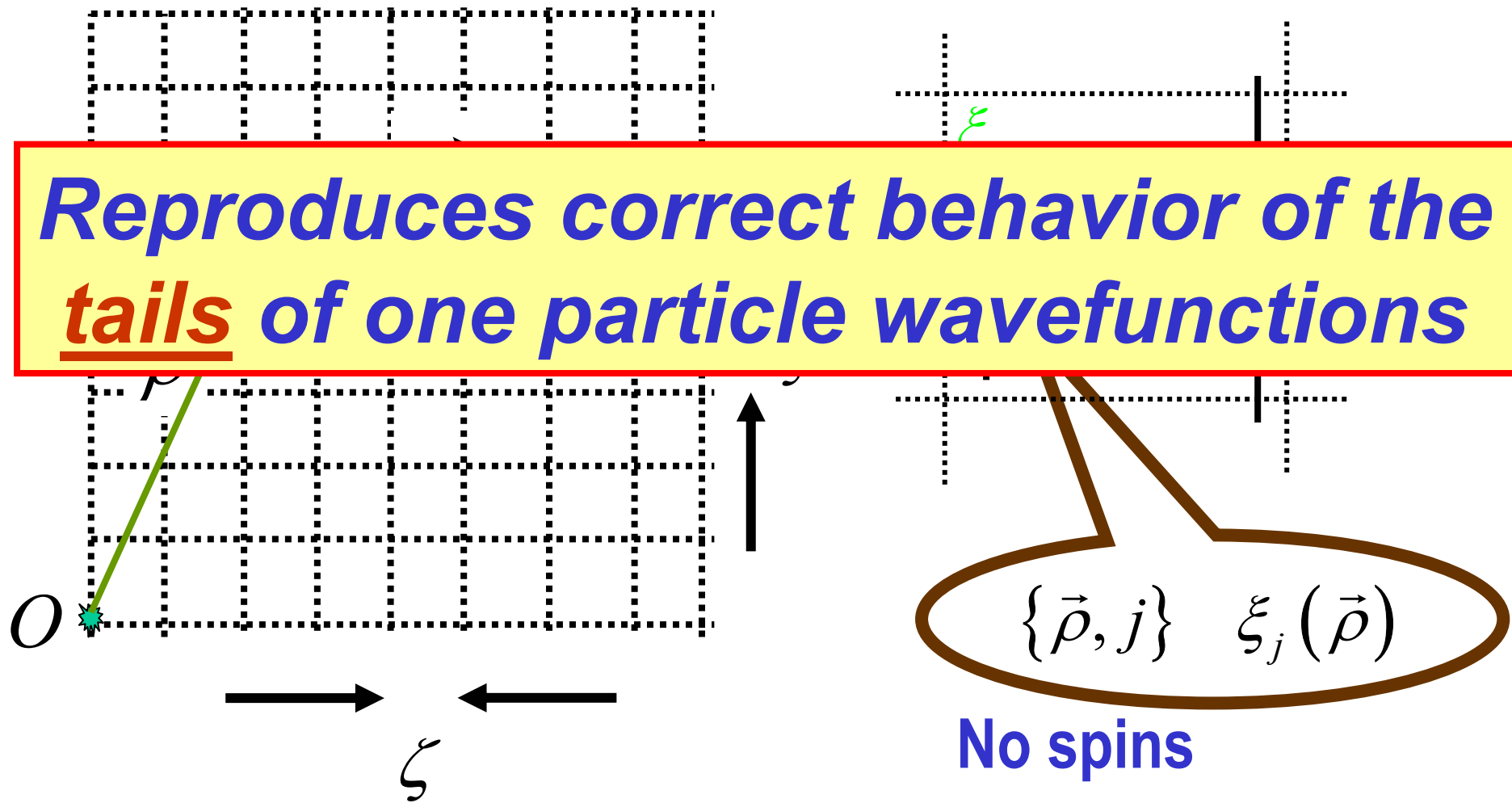
Otherwise, conventional perturbation theory for disordered metals works.

*Altshuler, Aronov, Lee (1979); Finkelshtein (1983) – T-dependent SC potential
Altshuler, Aronov, Khmelnitskii (1982) – inelastic processes*

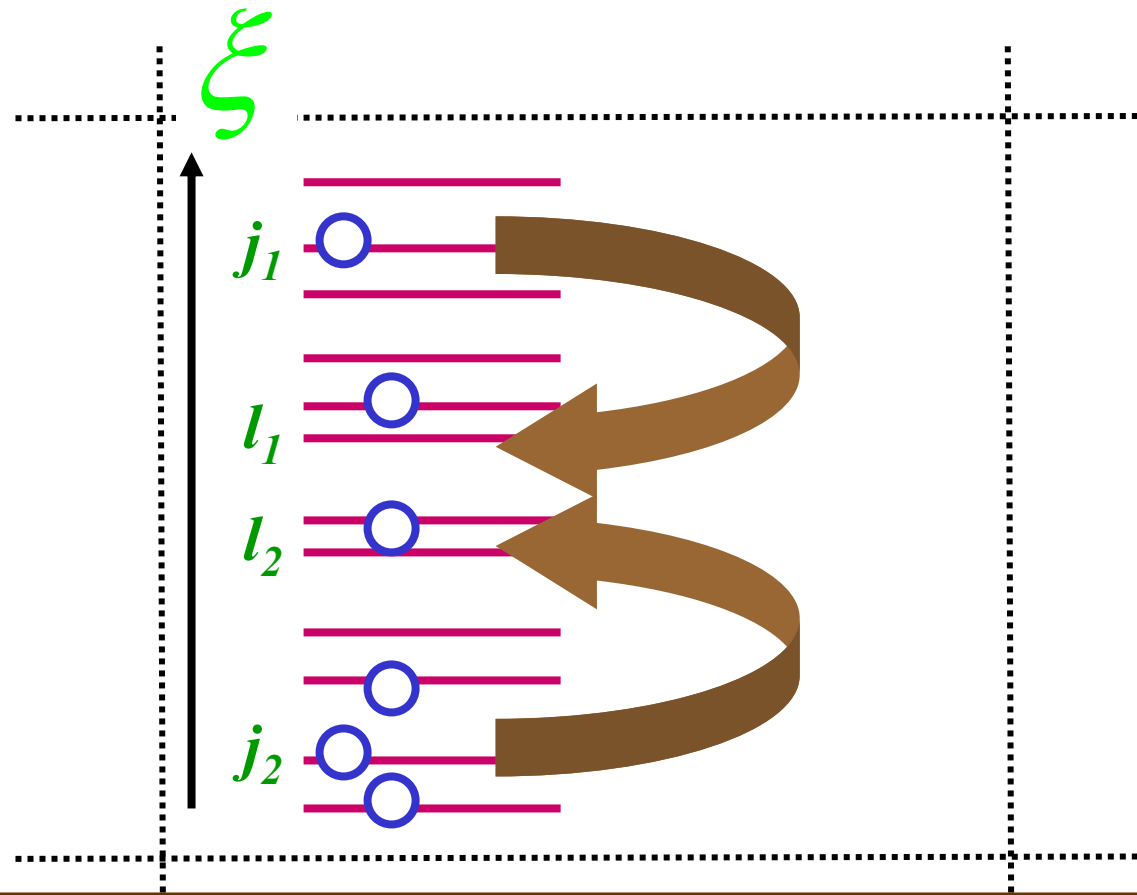
Coarse-graining of the non-interacting Hamiltonian



Reproduces correct behavior of the tails of one particle wavefunctions



$$\hat{H}_0 = \sum_{\rho, l} \hat{c}_l^\dagger(\rho) \left[\xi_l(\rho) \hat{c}_l(\rho) + I \delta_\xi \sum_{\mathbf{a}, m} \hat{c}_m(\rho + \mathbf{a}) \right]$$



$$\hat{V}_{int} = \frac{1}{2} \sum_{l_1 l_2 j_1 j_2; \rho} V_{l_1 l_2}^{j_1 j_2}(\rho) \hat{c}_{l_1}^\dagger(\rho) \hat{c}_{l_2}^\dagger(\rho) \hat{c}_{j_2}(\rho) \hat{c}_{j_1}(\rho)$$

Interaction only within the same cell;

$$\hat{H}_0 = \sum_{\rho, l} \hat{c}_l^\dagger(\rho) \left[\xi_l(\rho) \hat{c}_l(\rho) + I \delta_\xi \sum_{\mathbf{a}, m} \hat{c}_m(\rho + \mathbf{a}) \right]$$

$$\hat{V}_{int} = \frac{1}{2} \sum_{l_1 l_2 j_1 j_2; \rho} V_{l_1 l_2}^{j_1 j_2}(\rho) \hat{c}_{l_1}^\dagger(\rho) \hat{c}_{l_2}^\dagger(\rho) \hat{c}_{j_2}(\rho) \hat{c}_{j_1}(\rho)$$

Statistics of matrix elements?

Energy transfer $\omega \gg \delta_\zeta$

corresponds to the special scale $L_\omega = \sqrt{D/\omega} \ll \zeta$.

$$\hat{H}_0 = \sum_{\rho, l} \hat{c}_l^\dagger(\rho) \left[\xi_l(\rho) \hat{c}_l(\rho) + \underline{I} \delta_\xi \sum_{\mathbf{a}, m} \hat{c}_m(\rho + \mathbf{a}) \right]$$

$$\hat{V}_{int} = \frac{1}{2} \sum_{l_1 l_2 j_1 j_2; \rho} V_{l_1 l_2}^{j_1 j_2}(\rho) \hat{c}_{l_1}^\dagger(\rho) \hat{c}_{l_2}^\dagger(\rho) \hat{c}_{j_2}(\rho) \hat{c}_{j_1}(\rho)$$

$$V_{l_1 l_2}^{j_1 j_2} = \frac{\lambda \delta_\zeta \sigma_{l_1}^{j_1} \sigma_{l_2}^{j_2}}{2} \Upsilon \left(\frac{\xi_{j_1} - \xi_{l_1}}{\delta_\zeta} \right) \Upsilon \left(\frac{\xi_{j_2} - \xi_{l_2}}{\delta_\zeta} \right) - (l_1 \leftrightarrow l_2)$$

$$\Upsilon(x) = \theta \left(\frac{\underline{M}}{2} - |x| \right); \quad 1 \ll M \lesssim \frac{1}{\sqrt{\lambda}}$$

Parameters:

$$\lambda, I, M^{-1} \ll 1$$

σ_l^j

random
signs

$$\hat{H}_0 = \sum_{\rho, l} \hat{c}_l^\dagger(\rho) \left[\xi_l(\rho) \hat{c}_l(\rho) + I \delta_\zeta \sum_{\mathbf{a}, m} \hat{c}_m(\rho + \mathbf{a}) \right]$$

$$\hat{V}_{int} = \frac{1}{2} \sum_{l_1 l_2 j_1 j_2; \rho} V_{l_1 l_2}^{j_1 j_2}(\rho) \hat{c}_{l_1}^\dagger(\rho) \hat{c}_{l_2}^\dagger(\rho) \hat{c}_{j_2}(\rho) \hat{c}_{j_1}(\rho)$$

$$V_{l_1 l_2}^{j_1 j_2} = \frac{\lambda \delta_\zeta \sigma_{l_1}^{j_1} \sigma_{l_2}^{j_2}}{2} \Upsilon \left(\frac{\xi_{j_1} - \xi_{l_1}}{\delta_\zeta} \right) \Upsilon \left(\frac{\xi_{j_2} - \xi_{l_2}}{\delta_\zeta} \right) - (l_1 \leftrightarrow l_2)$$

$$\Upsilon(x) = \theta \left(\frac{M}{2} - |x| \right); \quad 1 \ll M \lesssim \frac{1}{\sqrt{\lambda}}$$

Parameters:

$$\lambda, I, M^{-1} \ll 1$$

σ_l^j

random
signs

Ensemble averaging over: $\xi_l(\rho); \sigma_i^j = \pm 1$

Level repulsion: Only within one cell.

Probability to find n levels in the energy interval of the width E :

$$P(n, E) = \frac{e^{-E/\delta_\zeta}}{n!} \left(\frac{E}{\delta_\zeta} \right)^n \exp \left[-F \left(\frac{n\delta_\zeta}{E} \right) \right]$$

$$\lim_{x \rightarrow \infty} \frac{F(x)}{x} = \infty$$

What to calculate?

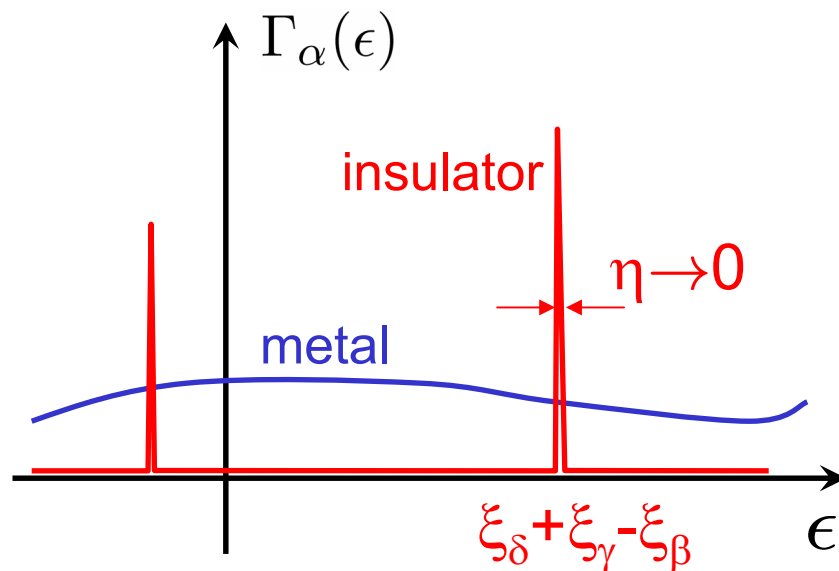
Idea for one particle localization Anderson, (1958);

MIT for Cayley tree: Abou-Chakra, Anderson, Thouless (1973);

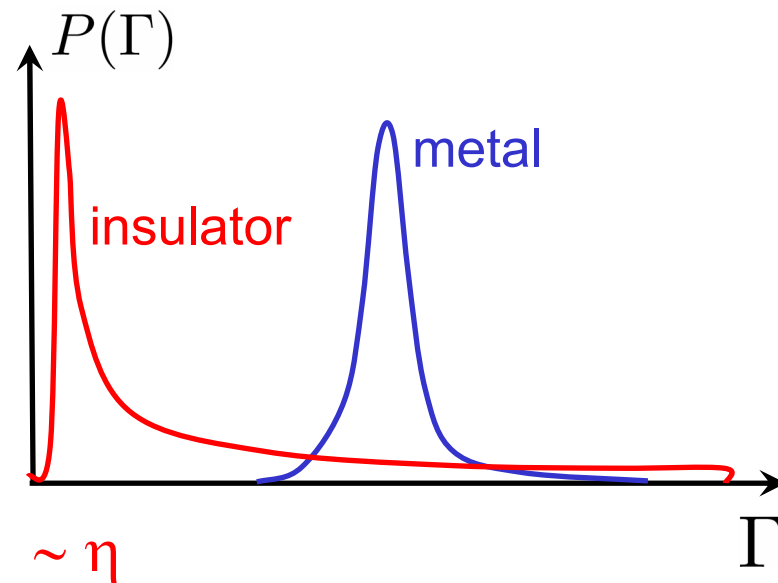
Critical behavior: Efetov (1987)

$$\Gamma_\alpha(\epsilon) = \text{Im} \Sigma_\alpha^A(\epsilon) - \text{random quantity}$$

No interaction: $\Gamma_\alpha(\epsilon) = \eta \rightarrow +0$

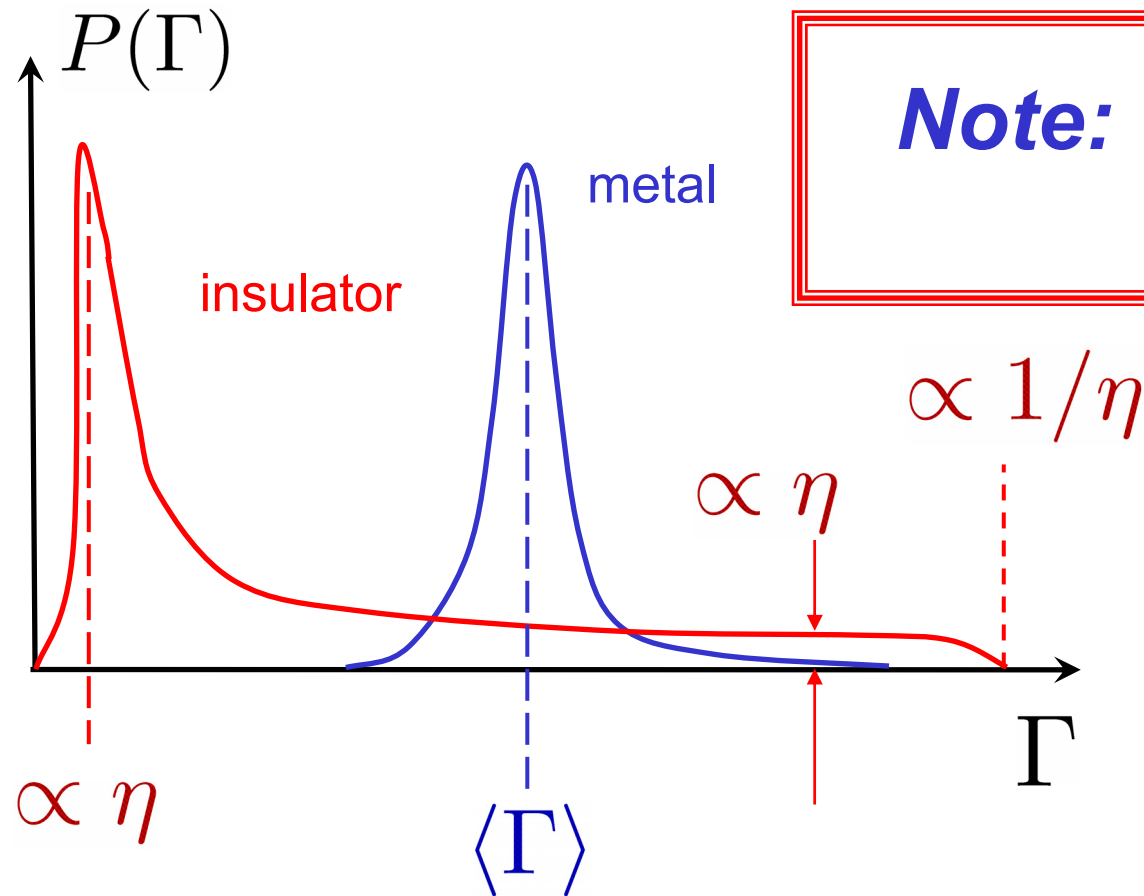


behavior for a
given realization



probability distribution
for a fixed energy

Probability Distribution



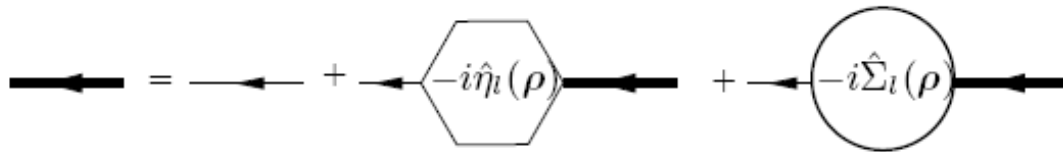
Note: $\langle \Gamma \rangle = \langle \Gamma \rangle$

Look for:

$$\lim_{\eta \rightarrow +0} \lim_{\nu \rightarrow \infty} P(\Gamma > 0) = \begin{cases} > 0; & \text{metal} \\ 0; & \text{insulator} \end{cases}$$

How to calculate?

non-equilibrium (arbitrary occupations) → Keldysh

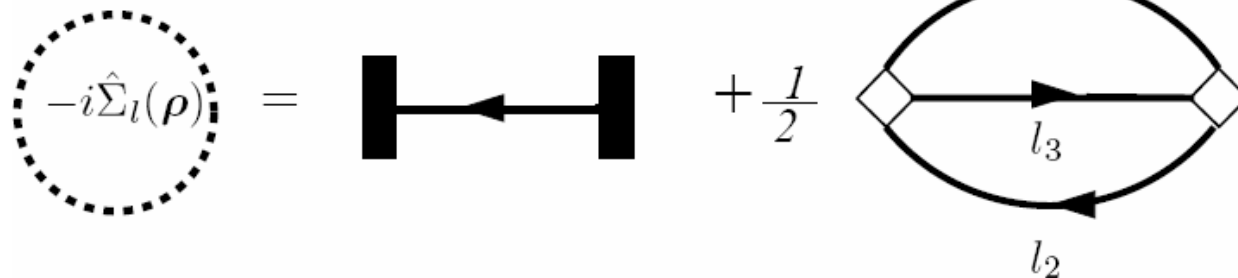


Parameters:

allow to select the most relevant series

$$\lambda, I, M^{-1} \ll 1$$

(a)



SCBA

Nonlinear integral equation with **random** coefficients

after standard simple tricks:

Decay due to tunneling

$$\Gamma_l(\epsilon) = \Gamma_l^{(el)}(\epsilon) + \Gamma_l^{(in)}(\epsilon) + n_l$$

$$\Gamma_l^{(el)}(\epsilon, \rho) = \pi I^2 \delta_\zeta^2 \sum_{l_1, \mathbf{a}} A_{l_1}(\epsilon, \rho + \mathbf{a})$$

Decay due to e-h pair creation

$$\Gamma_l^{(in)}(\epsilon) = \pi \lambda^2 \delta_\zeta^2 \sum_{l_1, l_2, l_3} Y_{l_1, l_2}^{l_3, l} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 A_{l_1}(\epsilon_1) A_{l_2}(\epsilon_2) A_{l_3}(\epsilon_3) \delta(\epsilon - \epsilon_1 - \epsilon_2 + \epsilon_3) F_{l_1, l_2; l_3}^{\Rightarrow}(\epsilon_1, \epsilon_2; \epsilon_3);$$

$$A_l(\epsilon) = \frac{\pi^{-1} \Gamma_l(\epsilon)}{[\epsilon - \xi_l]^2 + [\Gamma_l(\epsilon)]^2}$$

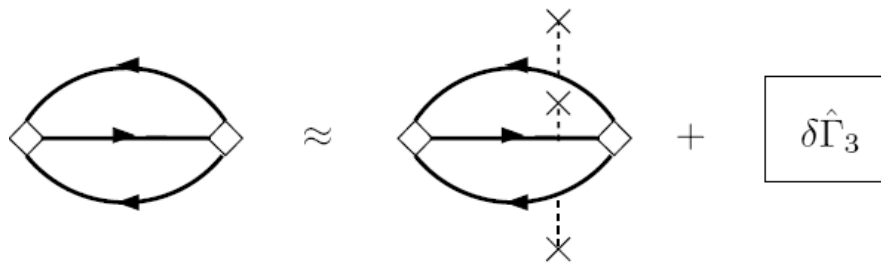
$$Y_{l_1, l_2}^{l_3, l} \equiv \frac{1}{2} \left[\Upsilon\left(\frac{\xi_{l_2} - \xi_l}{\delta_\zeta}\right) \Upsilon\left(\frac{\xi_{l_1} - \xi_{l_3}}{\delta_\zeta}\right) - \Upsilon\left(\frac{\xi_{l_1} - \xi_l}{\delta_\zeta}\right) \Upsilon\left(\frac{\xi_{l_2} - \xi_{l_3}}{\delta_\zeta}\right) \right]^2$$

$$F_{l_1, l_2; l_3}^{\Rightarrow}(\epsilon_1, \epsilon_2; \epsilon_3) = \frac{1}{4} \left\{ 1 + n_{l_1}(\epsilon_1) n_{l_2}(\epsilon_2) - n_{l_3}(\epsilon_3) [n_{l_1}(\epsilon_1) + n_{l_2}(\epsilon_2)] \right\};$$

+ kinetic equation for occupation function $n_l(\epsilon)$

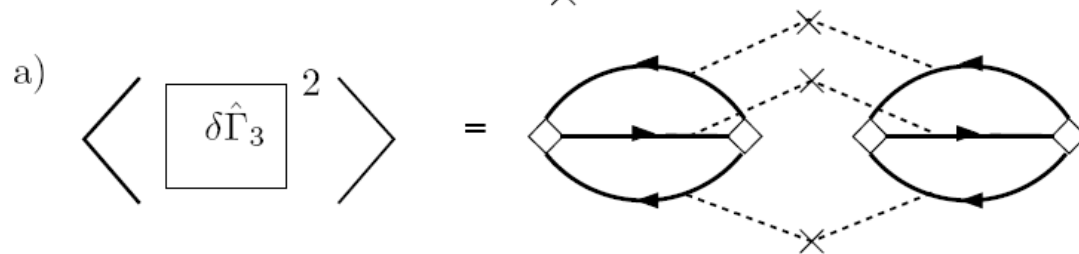
Stability of metallic phase

Assume $\Gamma_{in}(\epsilon)$ is Gaussian:



$$\left(\langle \Gamma^{(in)} \rangle = \pi \lambda^2 M T \right)^2$$

v

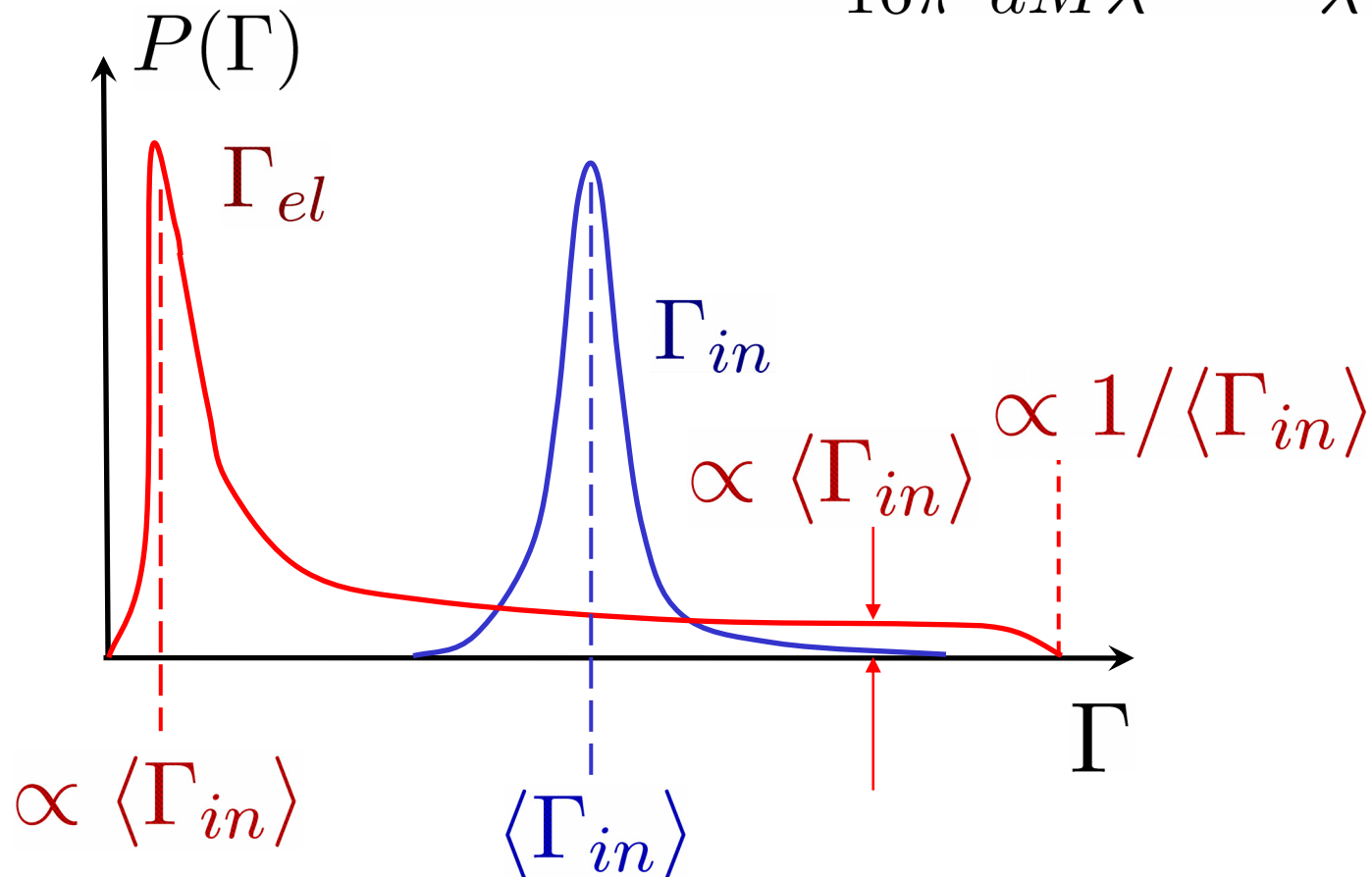


$$\langle (\delta\Gamma^{(in)})^2 \rangle = \frac{\pi \lambda^4 M \delta_\zeta^2 T}{36 \langle \Gamma^{(in)} \rangle}$$

$$T \gtrsim T_{in} \equiv \frac{\delta_\zeta}{6\pi\lambda M}$$

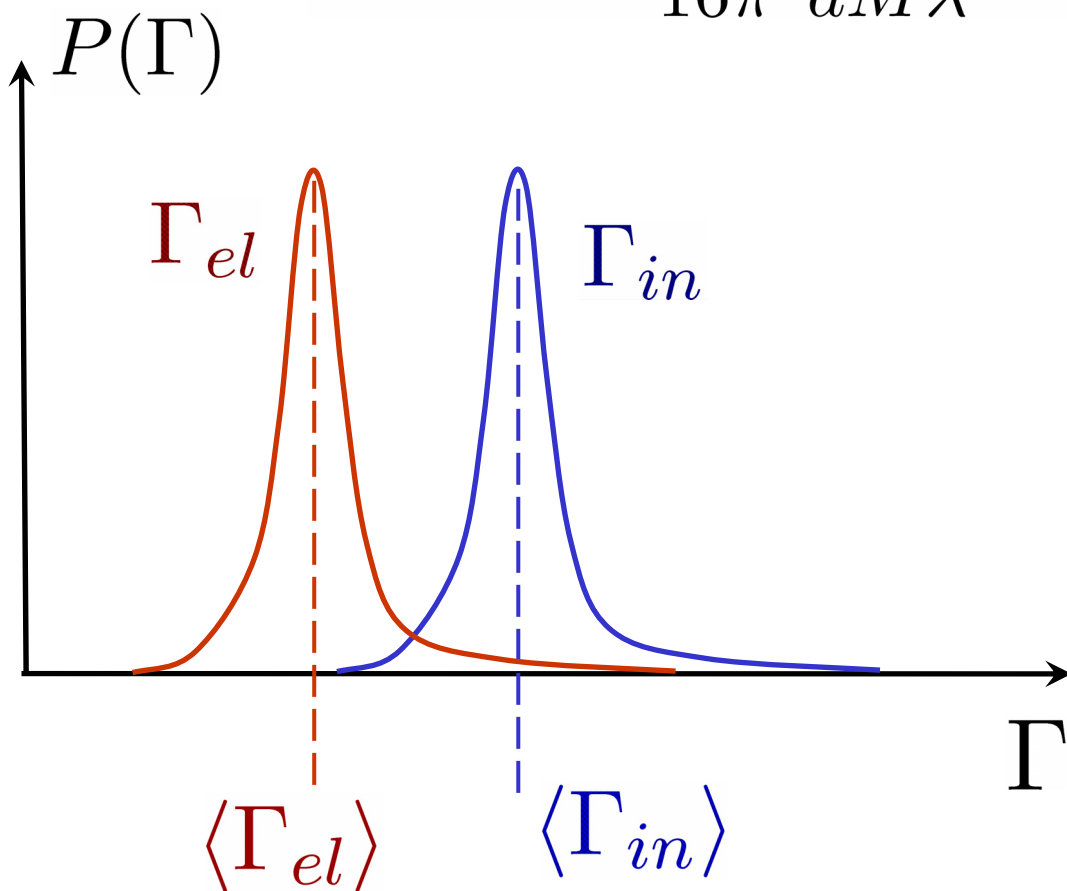
“Non-ergodic” metal [discussed first in AGKL,97]

$$T_{in} \lesssim T \lesssim T_{el} = \frac{\delta_{\zeta}}{16\pi^2 d M \lambda^2} \simeq \frac{T_{in}}{\lambda}$$



Drude metal

$$T \gtrsim T_{el} = \frac{\delta\zeta}{16\pi^2 d M \lambda^2} \simeq \frac{T_{in}}{\lambda}$$

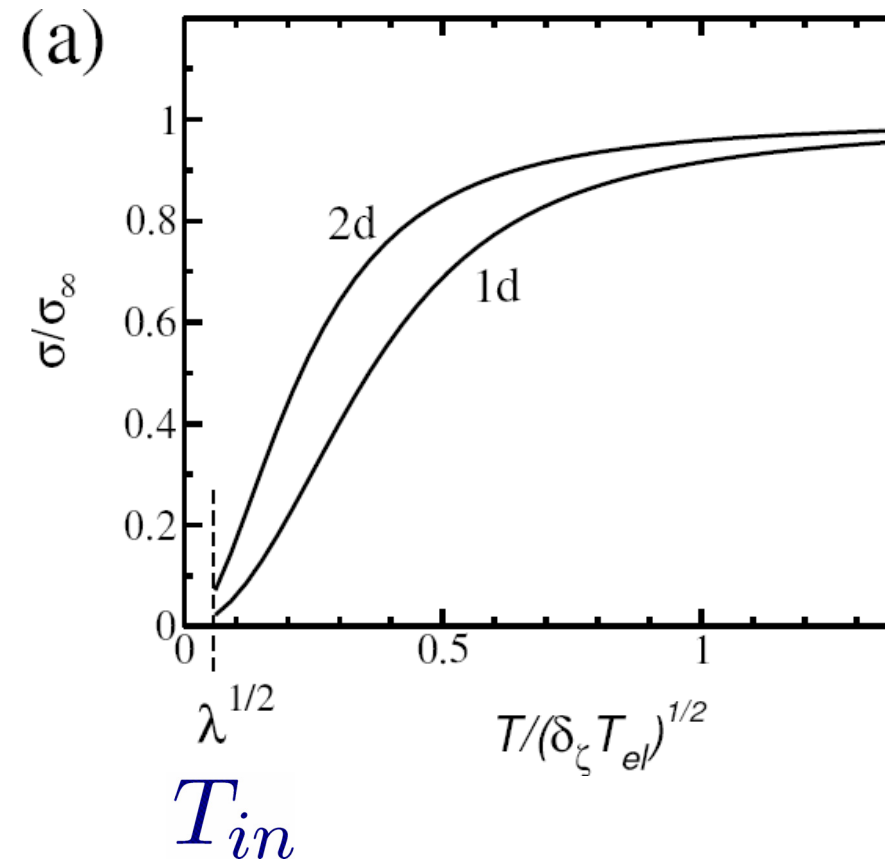


Kinetic Coefficients in Metallic Phase

$$\sigma_{\infty} \equiv \frac{2\pi e^2 I^2 \zeta_{loc}^{2-d}}{\hbar}$$

$$\sigma(T \gg \sqrt{\delta_{\zeta} T_{el}}) \approx \sigma_{\infty} \left(1 - \frac{2}{3} \frac{\delta_{\zeta} T_{el}}{T^2} \right)$$

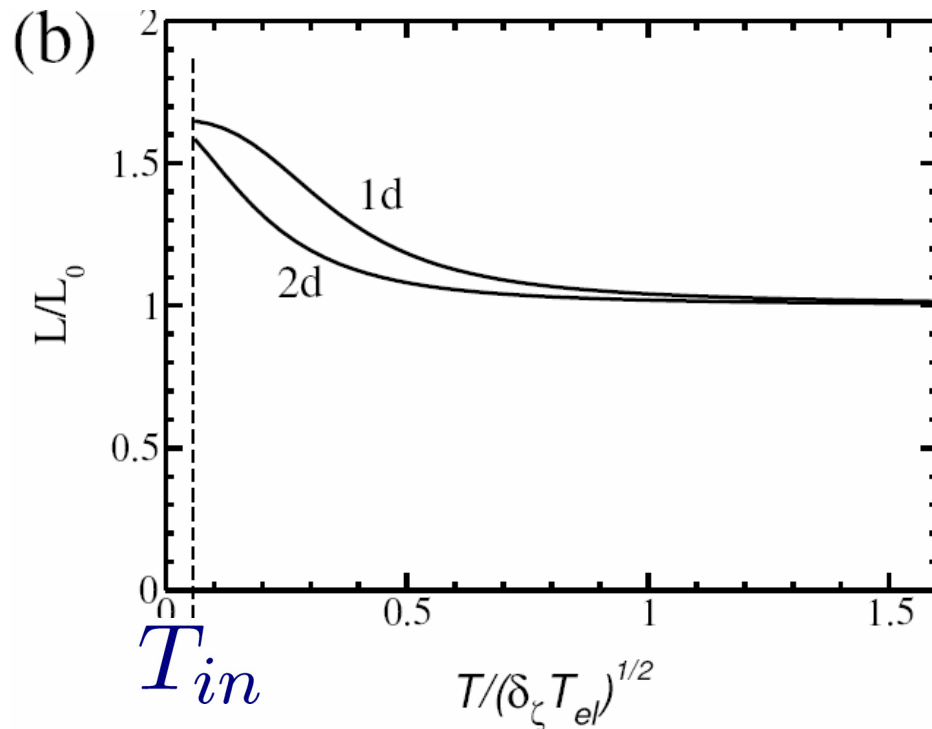
$$\sigma(T \ll \sqrt{\delta_{\zeta} T_{el}}) = \sigma_{\infty} \frac{\pi}{4} \left(\frac{T^2}{\delta_{\zeta} T_{el}} \right)$$



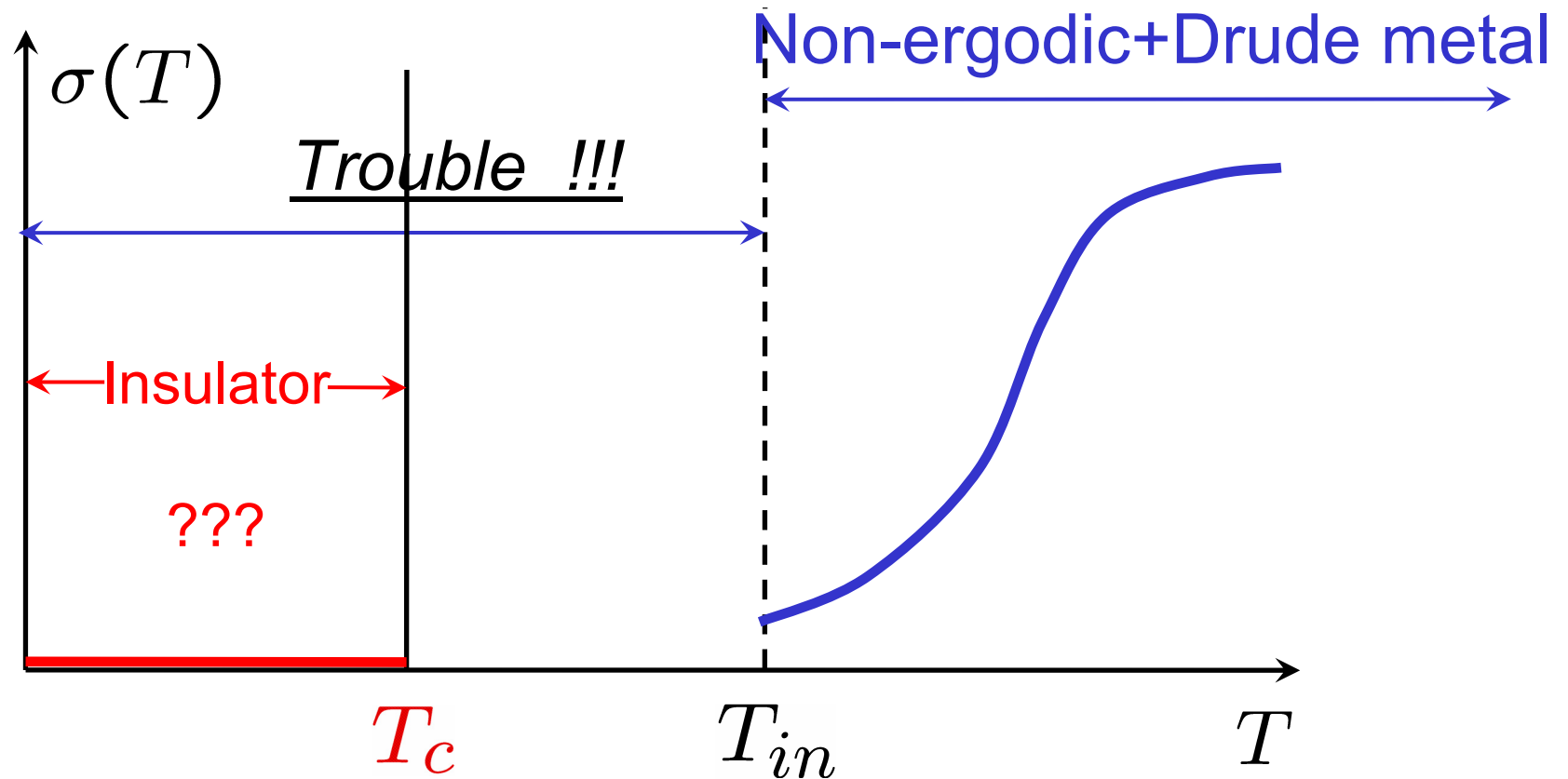
Kinetic Coefficients in Metallic Phase

Wiedemann-Frantz law ?

$$\frac{L(T)}{L_0} \equiv \frac{3e^2 \kappa(T)}{\pi^2 \sigma(T) T} = \begin{cases} 1 + 0.3 \left(\frac{\delta_\zeta T_{el}}{T^2} \right), & T \gg \sqrt{\delta_\zeta T_{el}}, \\ \frac{192G^2}{\pi^4} \approx 1.65 \dots, & T \ll \sqrt{\delta_\zeta T_{el}}. \end{cases}$$



So far, we have learned:



Stability of the insulator

Nonlinear integral equation with random coefficients

$$\Gamma_l(\epsilon) = \Gamma_l^{(el)}(\epsilon) + \Gamma_l^{(in)}(\epsilon) + \eta;$$

$$\Gamma_l^{(el)}(\epsilon, \rho) = \pi I^2 \delta_\zeta^2 \sum_{l_1, \mathbf{a}} A_{l_1}(\epsilon, \rho + \mathbf{a});$$

$$\Gamma_l^{(in)}(\epsilon) = \pi \lambda^2 \delta_\zeta^2 \sum_{l_1, l_2, l_3} Y_{l_1, l_2}^{l_3, l} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 A_{l_1}(\epsilon_1) A_{l_2}(\epsilon_2) A_{l_3}(\epsilon_3) \delta(\epsilon - \epsilon_1 - \epsilon_2 + \epsilon_3) F_{l_1, l_2; l_3}^{\rightarrow}(\epsilon_1, \epsilon_2; \epsilon_3);$$

$$A_l(\epsilon) = \frac{\pi^{-1} \Gamma_l(\epsilon)}{[\epsilon - \xi_l]^2 + [\Gamma_l(\epsilon)]^2}$$

Notice: $\Gamma(\epsilon) = 0$; **for** $\eta = 0$ **is a solution**

Linearization:

$$A_l(\epsilon) \approx \delta(\epsilon - \xi_l) + \frac{\Gamma_l(\epsilon)}{\pi(\epsilon - \xi_l)^2}$$

of interactions

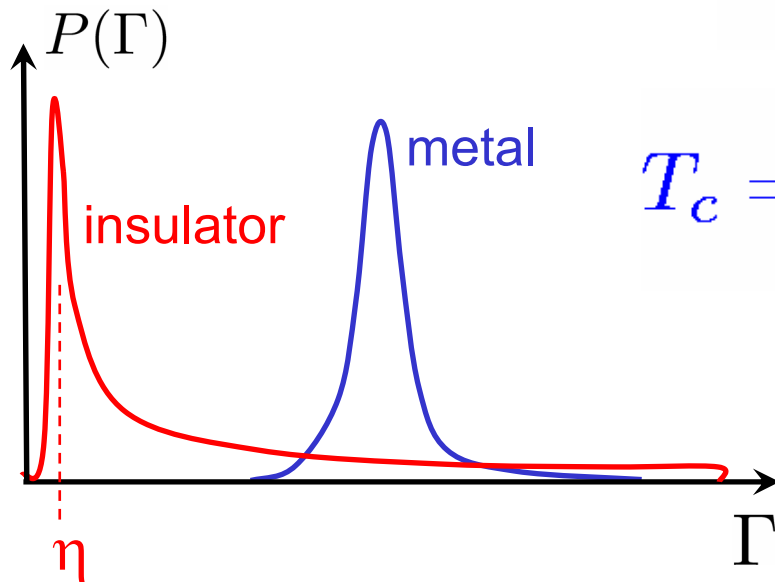
of hops in space

$$\Gamma = \sum_{n,m} \Gamma^{n,m}$$

$$P(\Gamma^{n,m}) = \sqrt{\frac{\gamma^{n,m}}{\pi [\Gamma^{n,m}]^3}} \exp\left(-\frac{\gamma^{n,m}}{\Gamma^{n,m}}\right)$$

Recall:

$$\gamma^{n,m} \leq \eta \left(\frac{T}{T_c}\right)^n$$



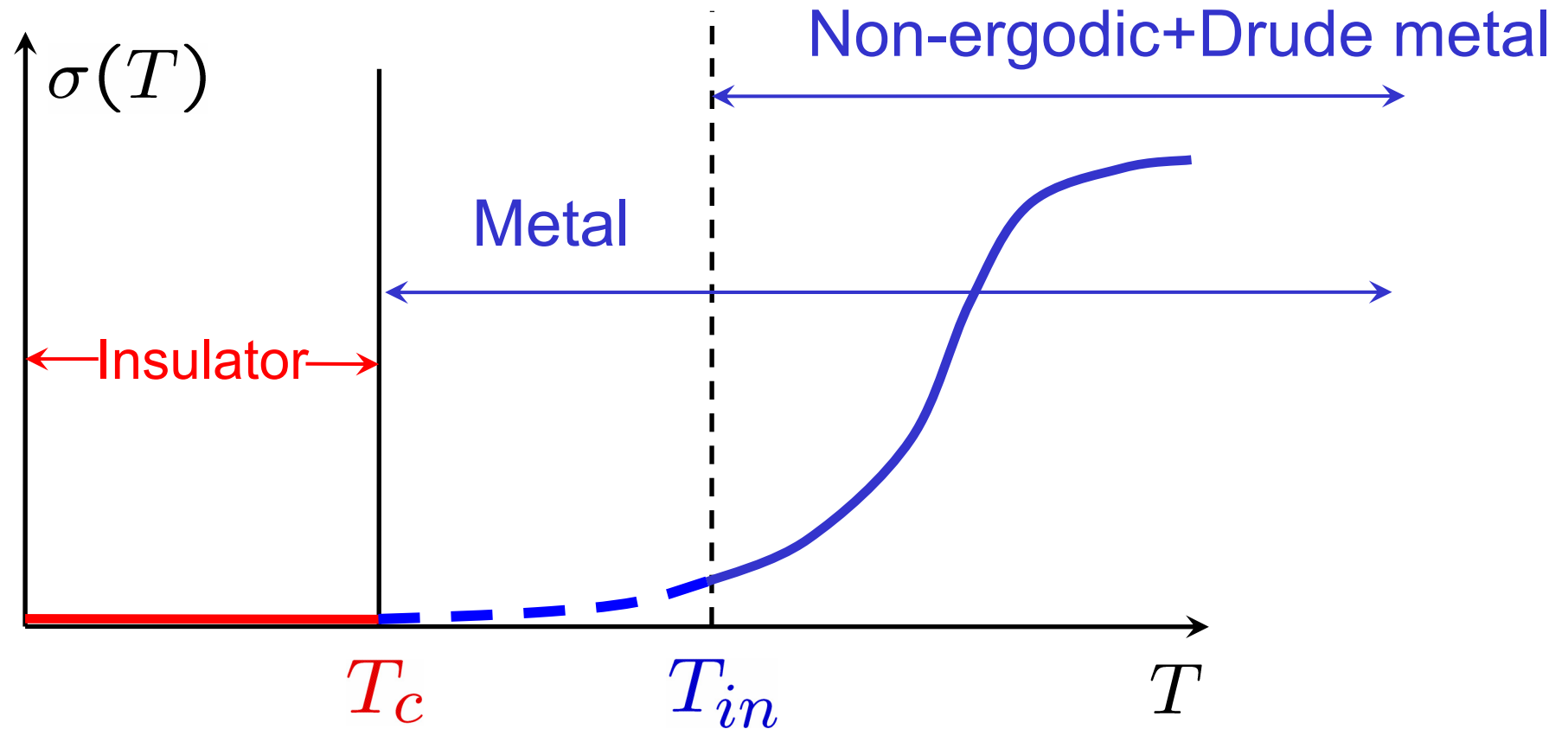
$$T_c = \frac{\delta_\zeta}{12\lambda M |\ln \lambda|} [1 + \mathcal{O}(\lambda M \ln I)]$$

$T < T_c$ **STABLE**

$T > T_c$ **unstable**

probability distribution
for a fixed energy

So, we have just learned:



$$T_c = \frac{\delta_\zeta}{12\lambda M |\ln \lambda|}$$

$$T_{in} = \frac{\delta_\zeta}{6\pi\lambda M}$$

Conclusions:

- Existence of the many-body mobility threshold is established.
- The many body metal-insulator transition is *not* thermodynamic phase transition.
- It is associated with the vanishing of the Langevine forces rather the divergences in energy landscape

and speculations:

- Stronger interactions: this is the only phase transition feasible for the **pinned Wigner crystal**
- Phonons: Cascades. Divergence of the cascade size at the mobility threshold.
- Non-linear I-V. Bistability. Noise enhancement

Arxive: 0704.1479

Instead of Conclusions - Some speculations

Conductivity exactly vanishes below some temperature. Is it an ordinary thermodynamic phase transition (I do not think so.-I.A.) or low temperature phase is a glass?

We considered weak interaction.

What about strong electron-electron interactions?

Melting of a pinned Wigner crystal?

What if we now turn on phonons?

Cascades.

Is conventional hopping conductivity picture ever correct?

NEW

Some more notes

**Is the metal to insulator transition irrelevant?
Are there experimental proposals?**

Finite electric field E (finite current J)

*) $T_c = T - eE\zeta$ i.e. insulating phase survives if E is small.

***) insulator– hopping conductivity – no heating $T = T_{ph}$

***) (bad, non-ergodic) metal – heating $T = T_{ph} + eEL_{ph}$

Therefore in the interval $T_c - eEL_{ph} < T_{ph} < T_c - eE\zeta$
both metal and insulator are stable.

Bistability !