Effects of Exchange Symmetry on Full Counting Statistics

G. Blatter, F. Hassler, G. Lesovik

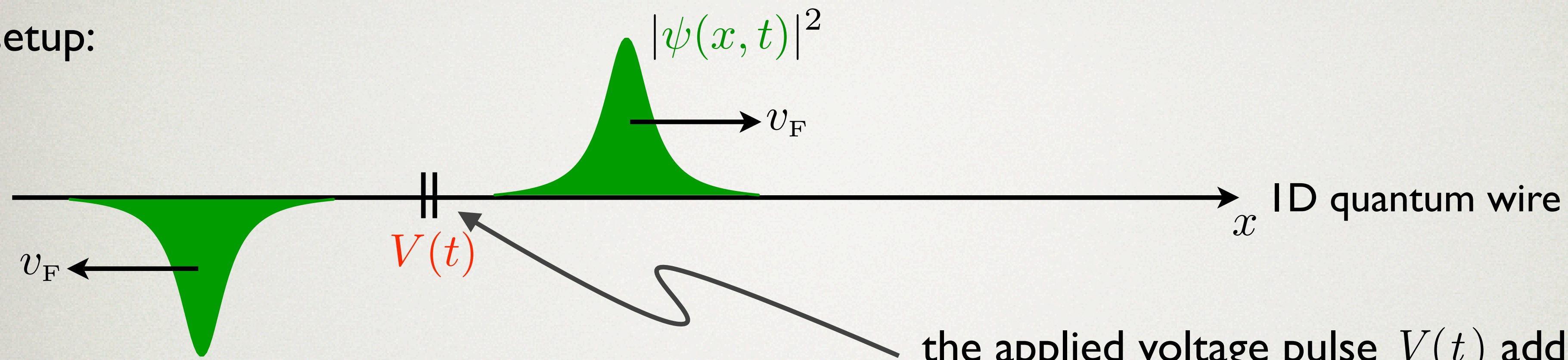
Overview

- Single Electron Source
- Full Counting Statistics
  - Fidelity $\leftrightarrow$ FCS
  - FCS with wave-packets
- Exchange Effects
  - Single Lead
  - Beam-Splitter

Single Electron Source

Single Electron Source

setup:



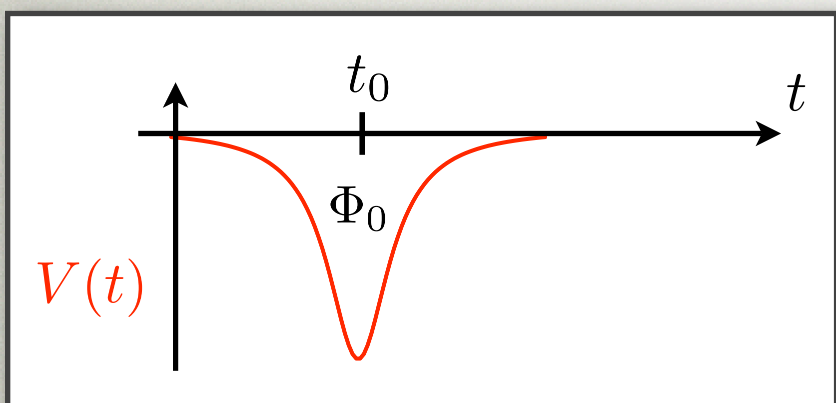
the applied voltage pulse $V(t)$ adds a phase

$$\phi(t) = -2\pi c \int_{-\infty}^{t} dt'\, V(t')/\Phi_0$$

$$\Phi_0 = hc/e$$

Unit-flux Lorentzian voltage pulse:

$$V(t) = -\frac{2\hbar\tau}{e[(t-t_0)^2+\tau^2]}$$



to the (many body) wave function and generates a single particle excitation corresponding to a wave packet with

$$|\psi(x,t)|^2 = \frac{\xi}{\pi[(x-x_0-v_{\rm F}t)^2+\xi^2]}$$

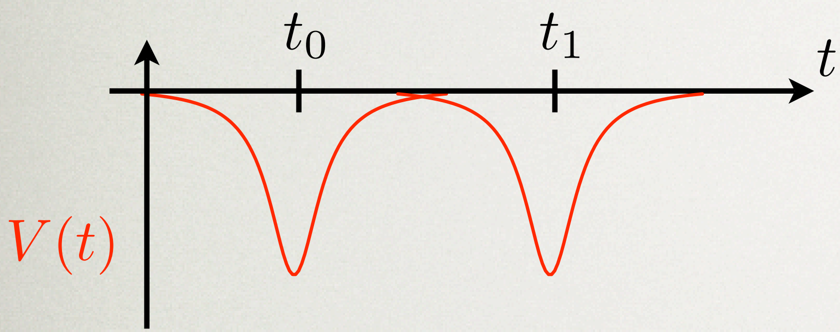
$$x_0 = v_{\rm F} t_0 \qquad \xi = v_{\rm F}\tau$$

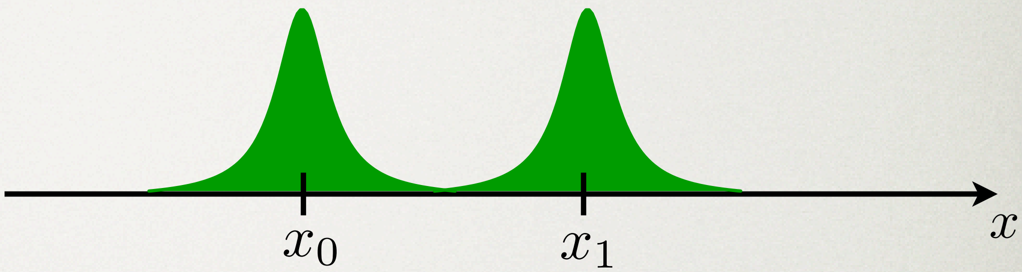
Levitov, Lee, Lesovik JMP 1996

Keeling, Klich, Levitov PRL 2006

More than One Particle

each unit-flux Lorentzian voltage pulse produces one particle





the particles are **coherent** (pure state);
their wave-function obey quantum mechanical exchange symmetry

Note on spin:
for spinful particles each voltage pulse creates two particles in a singlet

the two particles propagate independent on each other (just two particles are transmitted instead of one)

the following 2-pulse wave-function is **antisymmetric** under exchange

$$\Psi(x_1,x_2,x_3,x_4,t) = \frac{1}{\sqrt{6}}\Big[\Psi_-(x_1,x_2,t)\Psi_-(x_3,x_4,t)\big(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle\big)$$
$$+\Psi_-(x_1,x_4,t)\Psi_-(x_2,x_3,t)\big(|\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle\big) - \Psi_-(x_1,x_3,t)\Psi_-(x_2,x_4,t)\big(|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle\big)\Big].$$

Different Voltage Pulses

$$\phi(t) = -2\pi c \int_{-\infty}^{t} dt' \, V(t')/\Phi_0$$

$$\Phi_0 = hc/e$$

non-integer:

$$\phi(+\infty) \neq 2\pi\mathbb{Z}$$

→ orthogonality catastrophe

$$\langle\langle Q^2 \rangle\rangle \propto \log t$$

Levitov, Lee, Lesovik JMP 1996

integer, but

$V(t)$ not Lorentzian:

→ no clean single particle excitation is produced

average transferred particles: $\langle N \rangle = \phi(+\infty)/2\pi$

but additional particle-holes

integer, and

$V(t)$ Lorentzian:

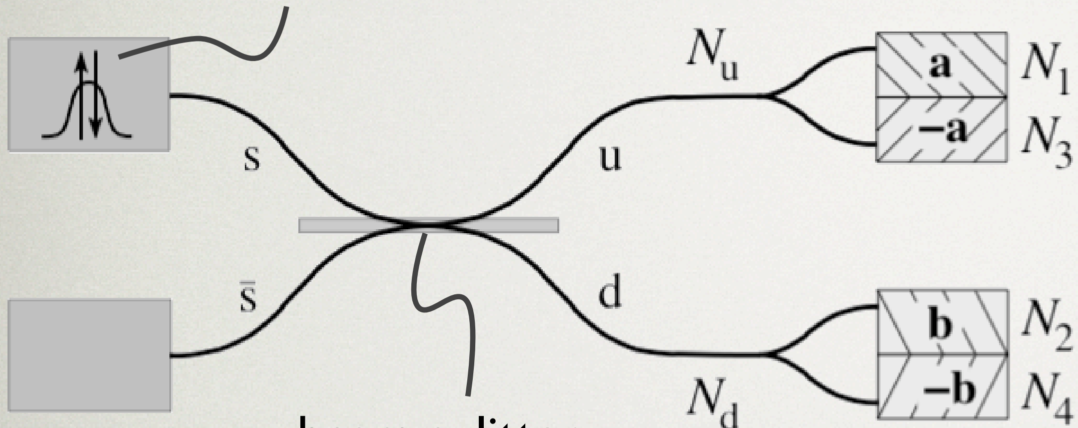
→ single particles with Lorentzian wave-functions:

$$|\psi(x,t)|^2 = \frac{\xi}{\pi[(x - x_0 - v_{\mathrm{F}} t)^2 + \xi^2]}$$

Keeling, Klich, Levitov PRL 2006

Application: Entanglement

creation of wave packets



beam splitter

measure charge-charge correlators

$$k_{nm}(a,b) = \langle\langle Q_n Q_m \rangle\rangle$$

Bell type measurement (with spin-polarizers)

Within classical and local hidden variable theories, the expression

$$E_{\mathrm{BI}} = \left| \frac{2k_{ud}}{2k_{ud} + \langle Q_u \rangle \langle Q_d \rangle} \right|$$

fulfills the inequality $E_{\mathrm{BI}} \leq 1/\sqrt{2}$

For integer voltage pulses

$$E_{\mathrm{BI}} = \left| \frac{1}{2n-1} \right|$$

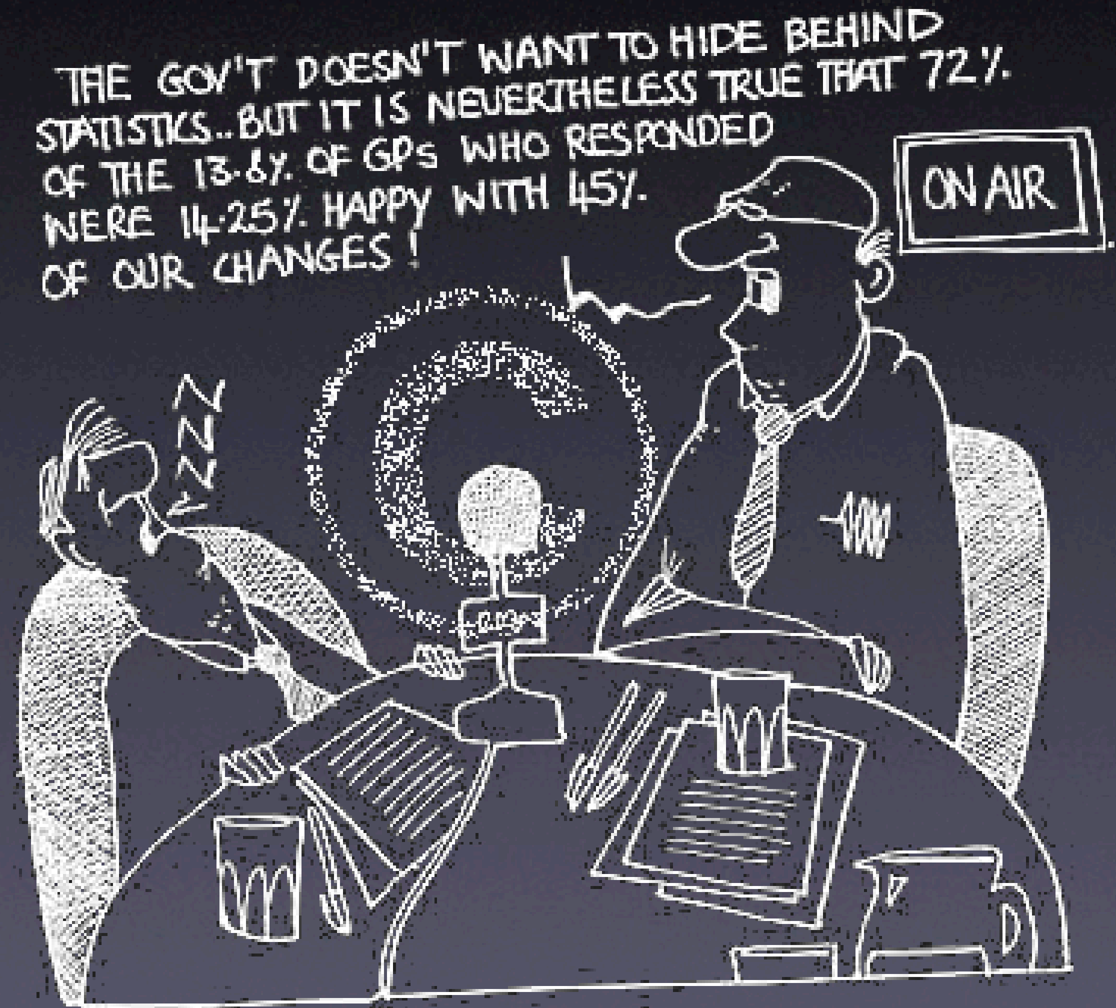
number of electrons

$n = 1$: maximal violation of Bell's-ineq

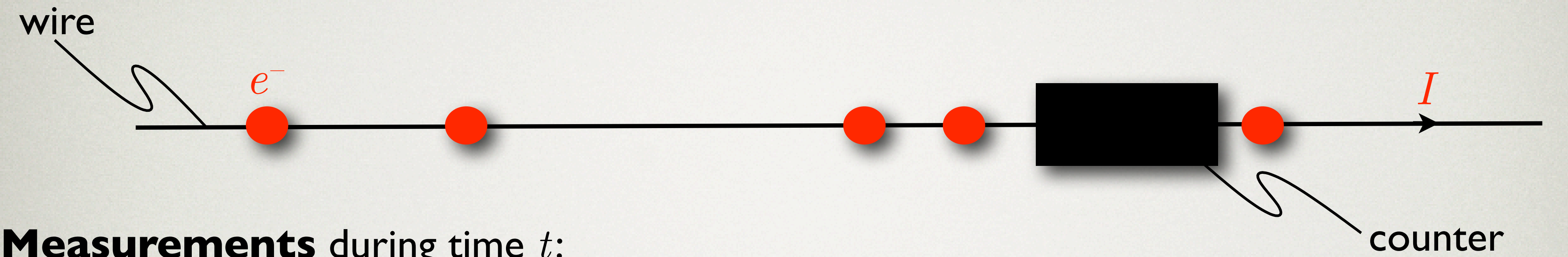
$n > 1$: no violation

$n < 1$: no consistent violation

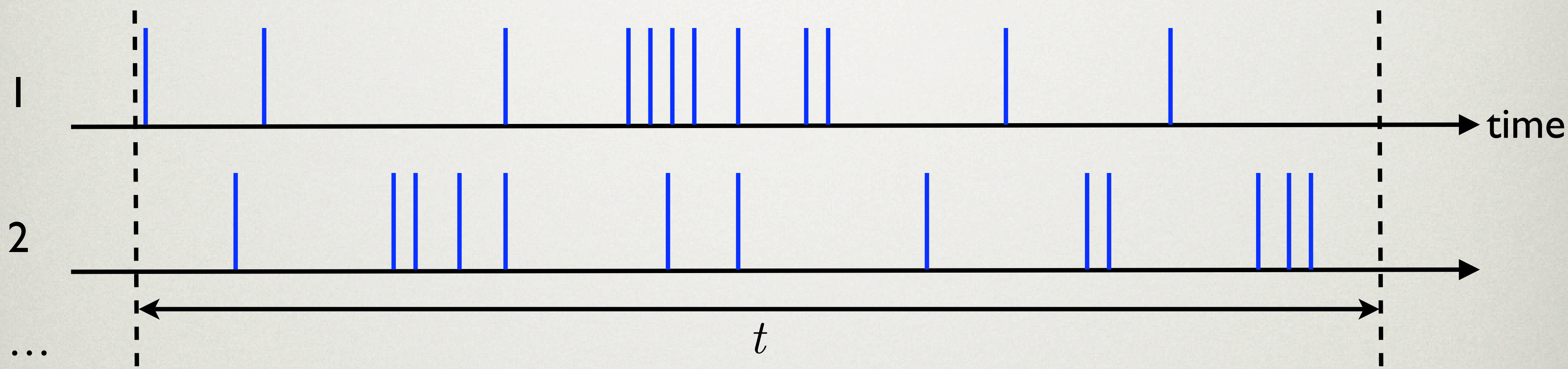
Full Counting Statistics



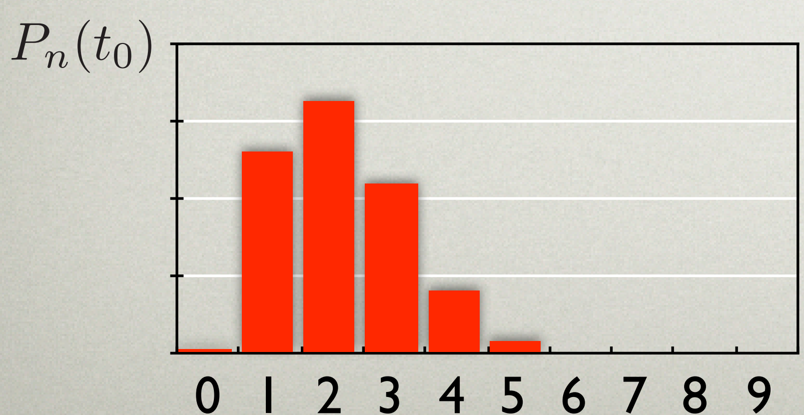
Full Counting Statistics

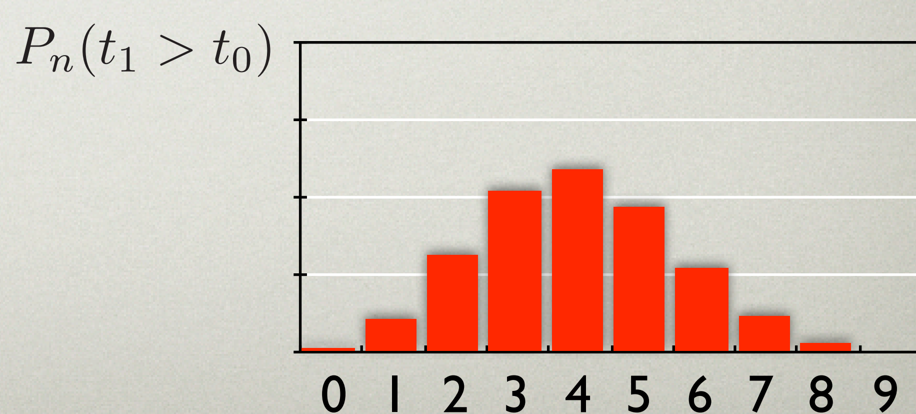


Measurements during time t:



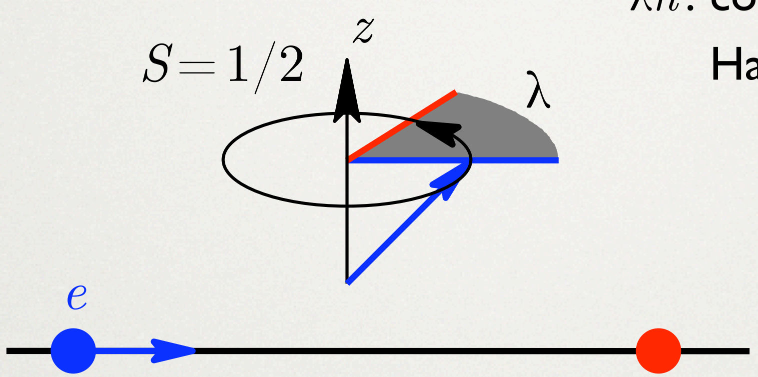
Statistical evaluation: $P_n(t)$ is the probability for the passage of n charges in the time t





Full Counting Statistics (FCS)

The FCS characterizes the charge transport through a quantum wire.



λh: coupling Hamiltonian

Levitov, Lesovik (1993)

Coupling the wire to a measurement device (e.g., a spin $1/2$ system) provides us with the generating function

$$\chi_{\text{FCS}}(\lambda, t) = \text{Tr}\left[\rho e^{i(H+\lambda h)t/\hbar} e^{-i(H-\lambda h)t/\hbar}\right]$$

Probability for the passage of n charges:

$$P_n(t) = \int_0^{2\pi} \frac{d\lambda}{2\pi} e^{-in\lambda} \chi_{\text{FCS}}(\lambda, t)$$

Fidelity

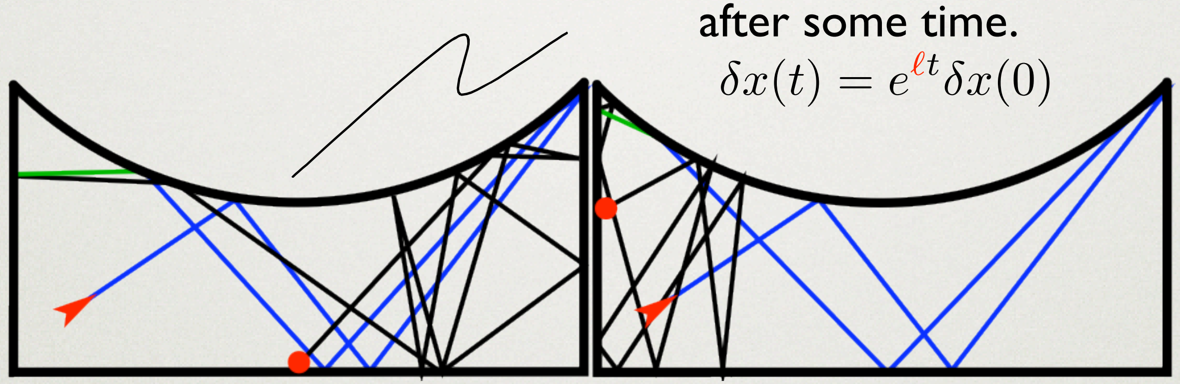
fidelity: stability of a quantum system (H) under the action of a small perturbation λh,

$$\chi_{\text{fid}}(\lambda, t) = \langle \Psi_\lambda(t) | \Psi(t) \rangle = \langle \Psi | e^{i(H+\lambda h)t/\hbar} e^{-iHt/\hbar} | \Psi \rangle$$

Peres (1984)

small changes in the initial condition produce large deviations after some time.

$$\delta x(t) = e^{\ell t} \delta x(0)$$



classical **chaotic** system: $|\chi_{\text{fid}}(t)|^2 \propto e^{-\ell t}$.

classical **regular** system: $|\chi_{\text{fid}}(t)|^2 \geq c > 0$.

FCS / Fidelity

Full Counting Statistics:

$$\chi_{\text{FCS}}(\lambda, t) = \mathrm{Tr}\left[\rho e^{i(H+\lambda h)t/\hbar} e^{-i(H-\lambda h)t/\hbar}\right]$$

Fidelity:

$$\chi_{\text{fid}}(\lambda, t) = \langle\Psi| e^{i(H+\lambda h)t/\hbar} e^{-iHt/\hbar} |\Psi\rangle$$

Correspondence between FCS / Fidelity

Second quantized formalism

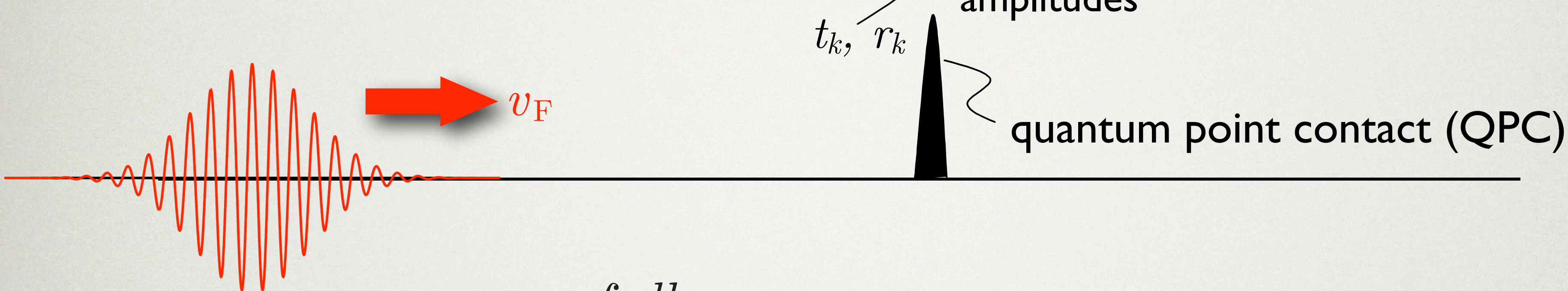
First-quantized formalism

$$\rho \leftrightarrow |\Psi\rangle\langle\Psi|$$

the spin differentiates between the forward and backpropagation and enables a measurement/calculation of the fidelity/FCS!

Scattering of a wave packet

Incoming wave packet: $(t \to -\infty)$



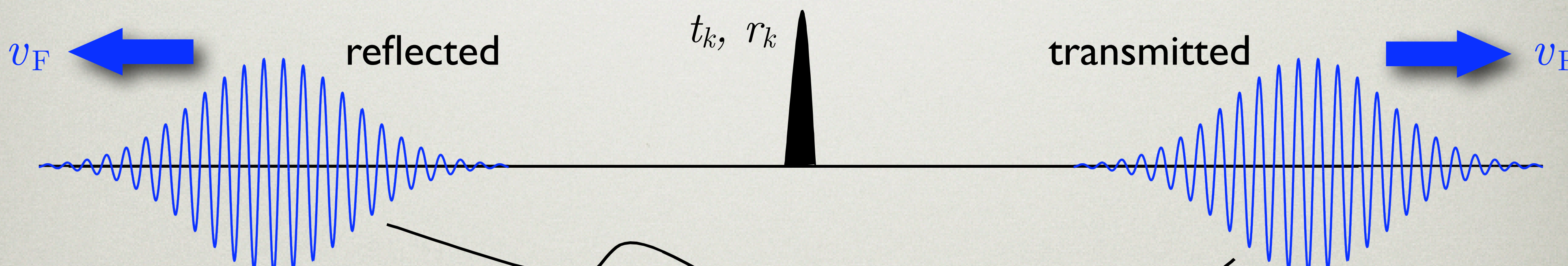
$$\Psi_\mathrm{in}(x,t) = \int \frac{dk}{2\pi} f(k) e^{i(kx - \omega_k t)}$$

Centered around $\hbar k_\mathrm{F} = m v_\mathrm{F}$,

with the dispersion $\omega_k = \hbar k^2/2m$

and normalization $\displaystyle\int \frac{dk}{2\pi} |f(k)|^2 = 1.$

Outgoing wave packet: $(t \to \infty)$



$$\Psi_\mathrm{out}(x,t) = \int \frac{dk}{2\pi} f(k) e^{-i\omega_k t} \left[r_k e^{-ikx} \Theta(-x) + t_k e^{ikx} \Theta(x) \right]$$

Fidelity/FCS with wave functions

Depending on the state $|\pm\rangle$ of the spin/qubit , the transmitted part of the wave function acquires a phase factor $\exp(\pm i\lambda/2)$,

$$\Psi^{\pm}_{\rm out}(x,t) = \int \frac{dk}{2\pi} f(k) e^{-i\omega_k t}\left[r_k e^{-ikx}\Theta(-x) + e^{\pm i\lambda/2} t_k e^{ikx}\Theta(x)\right].$$

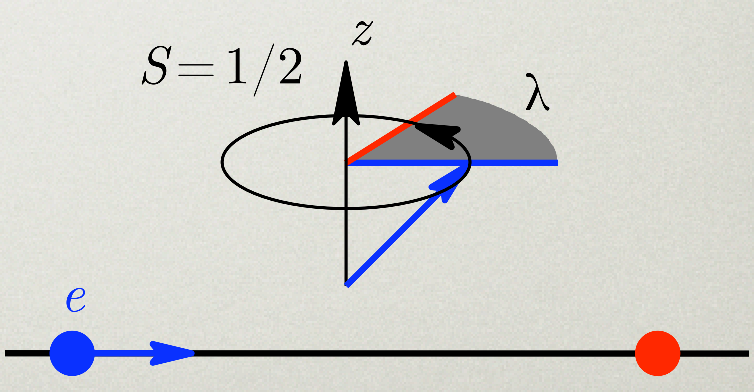
The fidelity/FCS is given by the overlap

$$\chi(\lambda,t) = \int dx \Psi^{-}_{\rm out}{}^{*}(x,t)\Psi^{+}_{\rm out}(x,t) = \int \frac{dk}{2\pi}\left(R_k + e^{i\lambda}T_k\right)|f(k)|^2$$

$$\equiv \boxed{\langle R\rangle_f + e^{i\lambda}\langle T\rangle_f.}$$

$$P_0 = \langle R\rangle_f$$

$$P_1 = \langle T\rangle_f$$



Two and more wave packets

we consider the case for the qubit placed behind the scatterer

Incoming wave packets:

$$\Psi_{\rm in}(x_1,x_2) = \Big[\Psi_{f_1}(x_1)\Psi_{f_2}(x_2) \pm (x_1 \leftrightarrow x_2)\Big]\chi_{s/t}(s_1,s_2)$$

shape of the two wave packets

spin singlet and triplet

FCS of two wave packets:

$$\chi_2(\lambda,t) = \frac{1}{N_\pm}\Bigg[\overbrace{\prod_{m=1,2}\int\frac{dk_m}{2\pi}|f_m(k_m)|^2(R_{k_m}+e^{i\lambda}T_{k_m})}^{\text{direct term}} \pm \overbrace{\prod_{m=1,2}\int\frac{dk_m}{2\pi}f_m^*(k_m)f_{n\neq m}(k_m)(R_{k_m}+e^{i\lambda}T_{k_m})}^{\text{exchange term}}\Bigg]$$

with the normalisation $N_\pm = 1 \pm |S|^2, \quad S = \int\frac{dk}{2\pi}f_1^*(k)f_2(k).$

Two and more wave packets

Fidelity of two wave packets:

$$\chi_2(\lambda,t) = \frac{1}{N_\pm}\Bigg[\overbrace{\prod_{m=1,2}\int\frac{dk_m}{2\pi}|f_m(k_m)|^2(R_{k_m}+e^{i\lambda}T_{k_m})}^{\text{direct term}} \pm \overbrace{\prod_{m=1,2}\int\frac{dk_m}{2\pi}f_m^*(k_m)f_{n\neq m}(k_m)(R_{k_m}+e^{i\lambda}T_{k_m})}^{\text{exchange term}}\Bigg]$$

with the normalisation $N_\pm = 1 \pm |S|^2, \quad S = \int \frac{dk}{2\pi} f_1^*(k) f_2(k).$

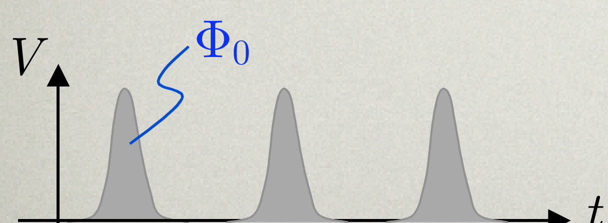
If the wave packets are **separated** either in momentum or in real space, the exchange term vanishes.

$$\chi_2(\lambda,t) = \prod_{m=1,2}\left[\langle R\rangle_{f_m} + e^{i\lambda}\langle T\rangle_{f_m}\right]$$

More wave packets:

i. separable wave packets generated by distinct voltage pulses with unit flux:

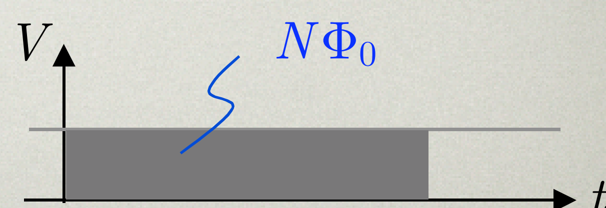
$$\chi_N(\lambda,t) = \prod_{m=1}^{N}\left[\langle R\rangle_{f_m} + e^{i\lambda}\langle T\rangle_{f_m}\right]$$



ii. many-body result at constant voltage V:

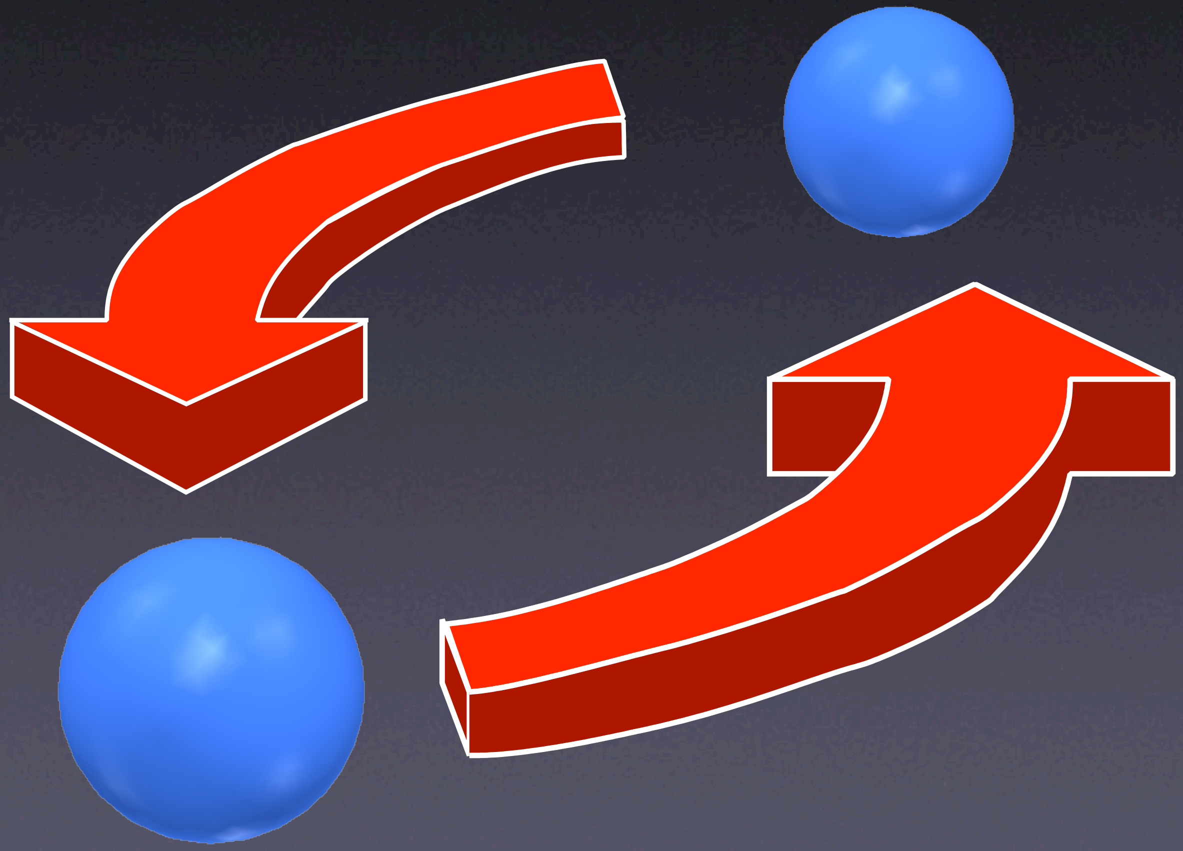
$$\chi_V(\lambda,t) = \left(R + e^{i\lambda}T\right)^{e|V|t/h}$$

when $N = e\,|V|t/h$ the results agree!



Why there is no exchange term in the constant voltage case?

Exchange Effect in FCS



Energy Independent Transmission

the generating function

$$\chi_\pm(\lambda) = \frac{1}{1 \pm |S|^2}\left[\prod_{m=1,2}\int \frac{dk_m}{2\pi}|f_m(k_m)|^2(R_{k_m} + e^{i\lambda}T_{k_m}) \pm \prod_{m=1,2}\int \frac{dk_m}{2\pi} f_m^*(k_m) f_{n\neq m}(k_m)(R_{k_m} + e^{i\lambda}T_{k_m})\right]$$

with the definition $\langle m|T|n\rangle = \int \frac{dk}{2\pi} f_m^*(k) T_k f_n(k)$

can be written as

$$\chi_\pm(\lambda) = \frac{\left[1 + (e^{i\lambda} - 1)\langle 1|T|1\rangle\right]\left[1 + (e^{i\lambda} - 1)\langle 2|T|2\rangle\right]}{1 \pm |S|^2}$$

$$\pm \frac{\left[S + (e^{i\lambda} - 1)\langle 1|T|2\rangle\right]\left[S^* + (e^{i\lambda} - 1)\langle 2|T|1\rangle\right]}{1 \pm |S|^2}$$

for energy independent transmission: $\langle 1|T|2\rangle = S\langle T\rangle$

and the exchange term cancels with the denominator!!!

in this limit the wave-packet formalism also yields no exchange contribution different from the **classical** binomial results

Energy Independent Transmission

the generating function

$$\chi_\pm(\lambda) = \frac{1}{1 \pm |S|^2} \left[\prod_{m=1,2} \int \frac{dk_m}{2\pi} |f_m(k_m)|^2 (R_{k_m} + e^{i\lambda} T_{k_m}) \pm \prod_{m=1,2} \int \frac{dk_m}{2\pi} f_m^*(k_m) f_{n \neq m}(k_m) (R_{k_m} + e^{i\lambda} T_{k_m}) \right]$$

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$$\pm \frac{\left[S + (e^{i\lambda} \right.}{}$$

for energy dependent transmission there are exchange effects

for energy independent trans ... $S\langle T\rangle$

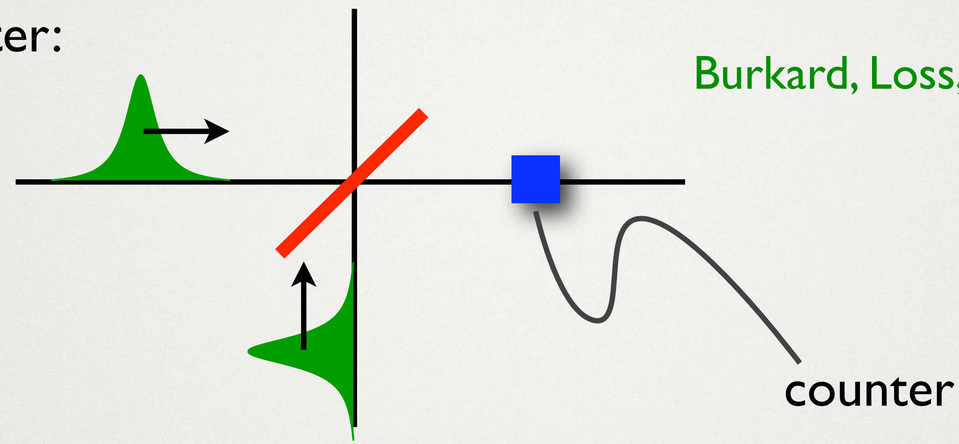
and the exchange term cancels with the denominator!!!

in this limit the wave-packet formalism also yields no exchange contribution different from the **classical** binomial results

Other Effects of Exchange

symmetric beam splitter:

Burkard, Loss, Sukhorukov PRB 2000



$$\langle\langle Q^2 \rangle\rangle = 2e^2(1 \pm |S|^2)T(1-T)$$

singlet/triplet

overlap

noise is increased for singlet

noise is reduced for triplet

measurement of the singlet/triplet state

constant voltage and energy dependent transmission

$$\langle\langle Q^2 \rangle\rangle = \frac{e^2}{\pi\hbar} \int dE \, \{T_E[f_{\rm L}(1 \mp f_{\rm L}) + f_{\rm R}(1 \mp f_{\rm R})] \pm T_E(1-T_E)(f_{\rm L} - f_{\rm R})^2\}$$

Büttiker PRB 1992

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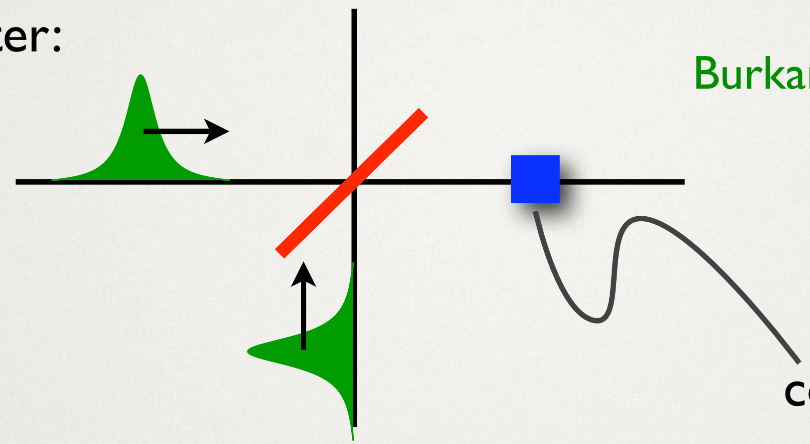

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noise is increased for singlet
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singlet/triplet

...rement of the singlet/triplet state

no effect on the
average transmitted charge

constant voltage and en...

$$\langle\langle Q^2\rangle\rangle = \frac{e^2}{\pi\hbar}\int dE\,\{T_E[f_{\rm L}(1\mp f_{\rm L}) + f_{\rm R}(1\mp f_{\rm R})] \pm T_E(1-T_E)(f_{\rm L}-f_{\rm R})^2\}$$

Büttiker PRB 1992

Energy Dependent Transmission

generating function for the FCS:

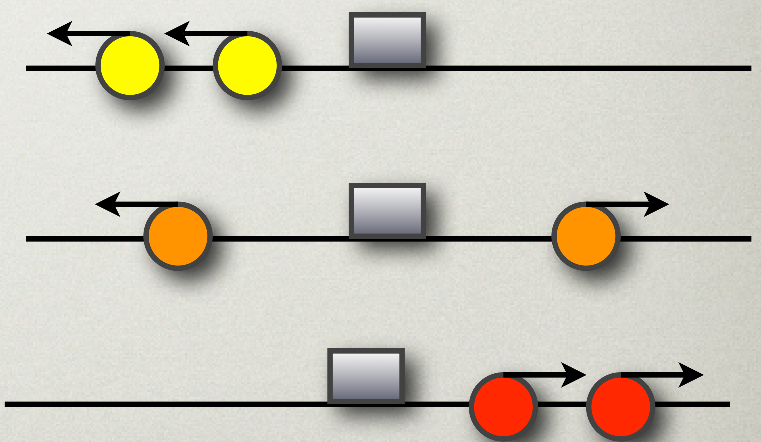
$$\chi_\pm(\lambda) = \frac{\left[1 + (e^{i\lambda} - 1)\langle 1|T|1\rangle\right]\left[1 + (e^{i\lambda} - 1)\langle 2|T|2\rangle\right]}{1 \pm |S|^2} \pm \frac{\left[S + (e^{i\lambda} - 1)\langle 1|T|2\rangle\right]\left[S^* + (e^{i\lambda} - 1)\langle 2|T|1\rangle\right]}{1 \pm |S|^2}$$

probabilities for transmission:

$$P_{0,\pm} = \frac{(1 - \langle T\rangle)^2 \pm |S - \langle 1|T|2\rangle|^2}{1 \pm |S|^2}$$

$$P_{1,\pm} = 2\frac{\langle T\rangle(1 - \langle T\rangle) \pm \left[\mathrm{Re}(\langle 1|T|2\rangle S^*) - |\langle 1|T|2\rangle|^2\right]}{1 \pm |S|^2}$$

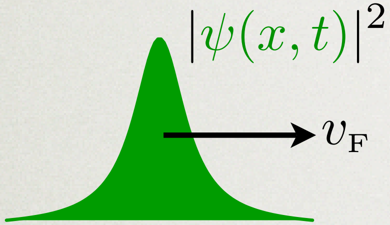
$$P_{2,\pm} = \frac{\langle T\rangle^2 \pm |\langle 1|T|2\rangle|^2}{1 \pm |S|^2}$$



Lorentzian Wave-Packet

wave-packets is of Lorentzian shape:

$$f_1(k) = \sqrt{4\pi\xi}\, e^{-\xi(k-k_{\mathrm{F}})-ikx_1}\Theta(k-k_{\mathrm{F}})$$

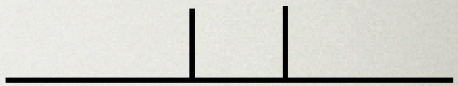


the transmission-probability is:

QPC

$$T_k \approx \Theta(k-k_0)$$

resonance



$$T_k \approx \delta(k-k_0)$$

generating functions of the FCS:

$$\chi_{\pm}^{\mathrm{res}} = 1 + \langle Q/e\rangle_{\pm}^{\mathrm{res}}(e^{i\lambda}-1) + \langle T^{\mathrm{res}}\rangle^2 \frac{(1\pm 1)[(\delta x/2\xi)^2+1]}{(\delta x/2\xi)^2+1\pm 1}(e^{i\lambda}-1)^2$$

$$\chi_{\pm}^{\mathrm{qpc}} = 1 + \langle Q/e\rangle_{\pm}^{\mathrm{qpc}}(e^{i\lambda}-1) + \langle T^{\mathrm{qpc}}\rangle^2(e^{i\lambda}-1)^2$$

with the average transmitted charge:

$$\langle Q/e\rangle_{\pm}^{\mathrm{res}} = 2\langle T^{\mathrm{res}}\rangle \frac{1+(\delta x/2\xi)^2 \pm [\cos(k_0\delta x) + (\delta x/2\xi)\sin(k_0\delta x)]}{1+(\delta x/2\xi)^2\pm 1}$$

$$\langle Q/e\rangle_{\pm}^{\mathrm{qpc}} = 2\langle T^{\mathrm{qpc}}\rangle \frac{1+(\delta x/2\xi)^2 \pm \cos(k_0\delta x)}{1+(\delta x/2\xi)^2\pm 1}$$

oscillations

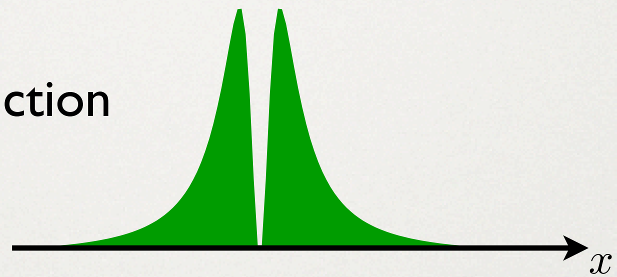
exchange effects

Exchange Effect on Average Charge

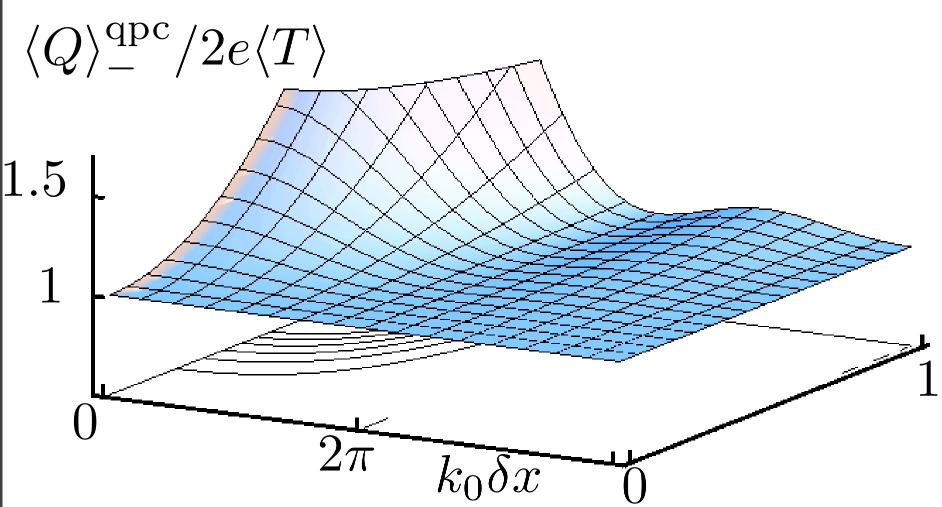
weak overlap : exchange corrections decay with separation $\propto 1/\delta x^2$

strong overlap: no exchange corrections for symmetric wave function

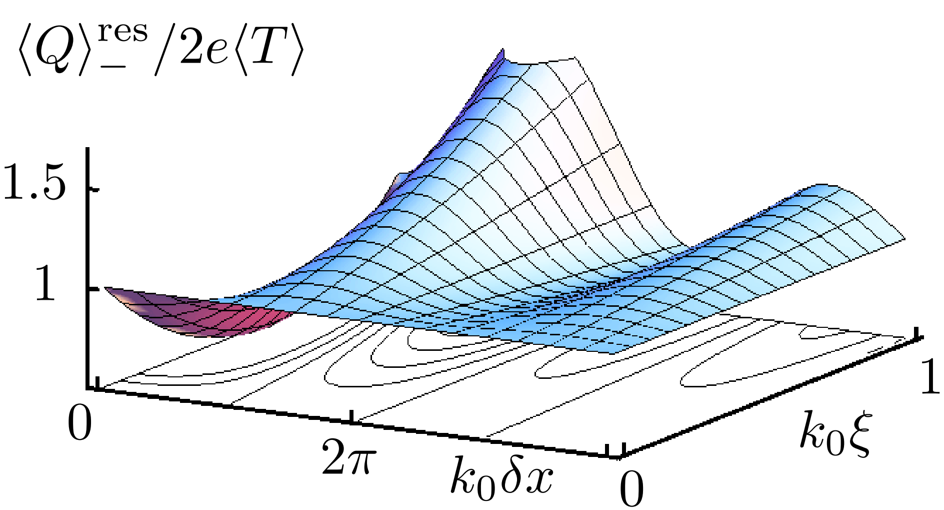
large effect for
antisymmetric wave function



quantum point contact

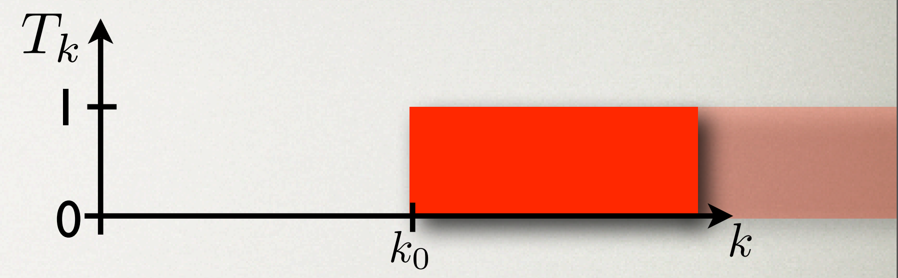


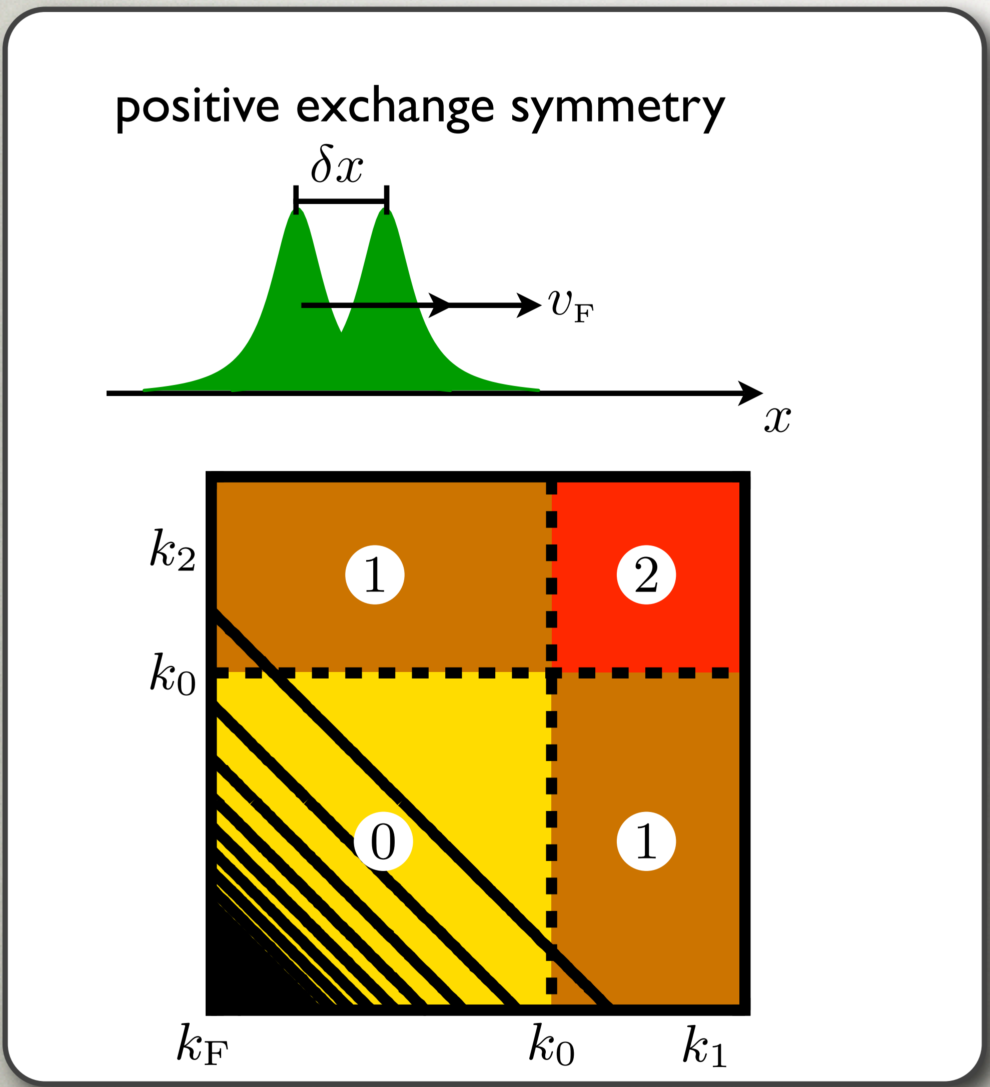
transmission resonance

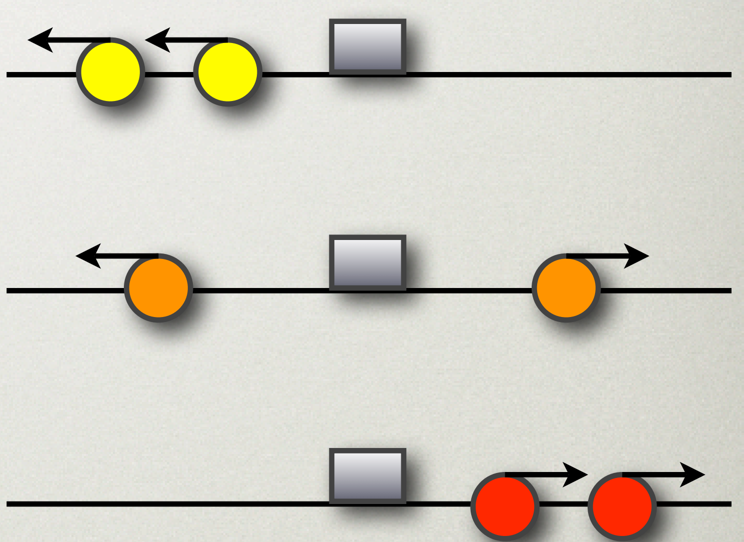


Why Exchange Effects?

Consider two strongly overlapping particles incident on a quantum point contact with transmission probability







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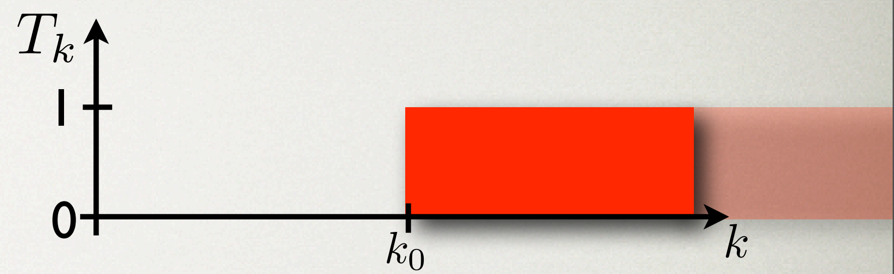
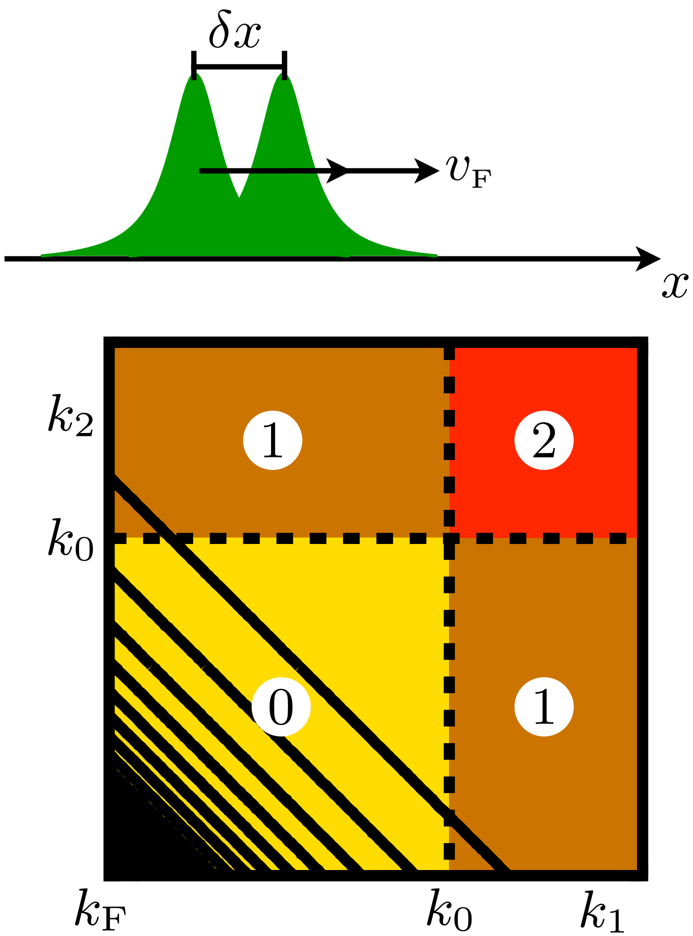
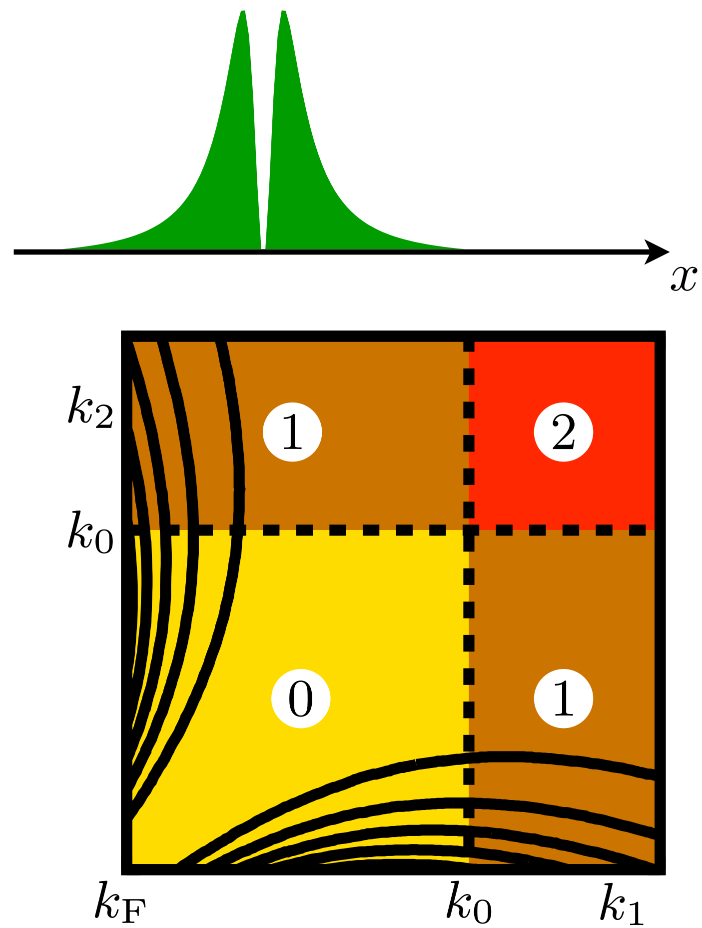


positive exchange symmetry



negative exchange symmetry



Coulomb Effects

incoming leads:

- Fermi-/Luttinger liquid
- renormalized v_{F}
- excitations confined to a region around k_{F} ($\xi \gg \lambda_{\mathrm{F}}$)
- Coulomb effect already incorporated in the Fermi-/Luttinger liquid picture

scatterer:

QPC

L length of QPC

charging energy

$$E_C = e^2/\epsilon L$$

energy spread of the wave-packet

$$E_\xi = \hbar v_{\mathrm{F}}/\xi$$

Coulomb effects can be neglected if:

$$E_\xi \gg E_C \iff \xi/L \ll 137\epsilon\, v_{\mathrm{F}}/c$$

resonance

L length of res.
d dist. to ground

charging energy

$$E_C = e^2/\epsilon L \log(L/d)$$

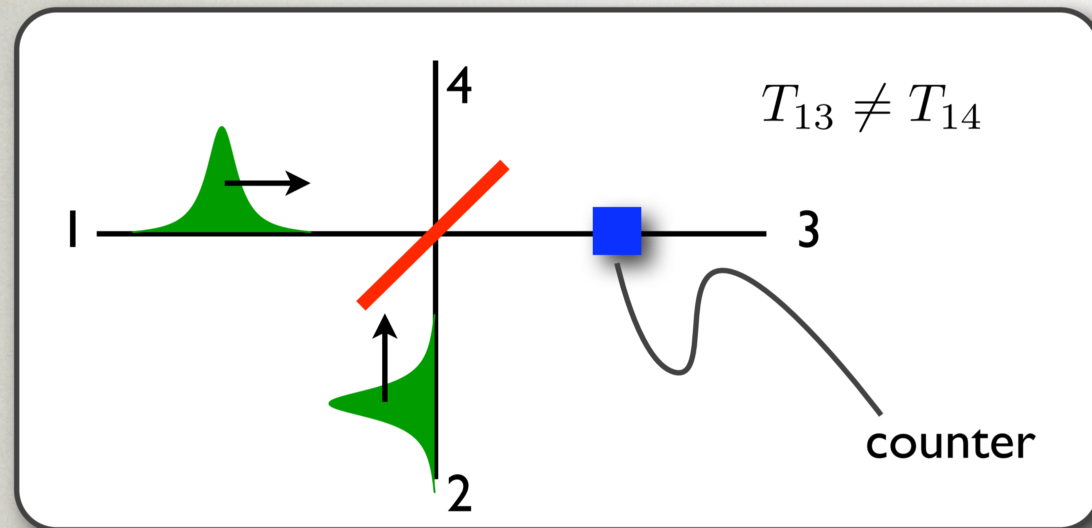
level spacing

$$\delta E = \hbar v_{\mathrm{F}}/L$$

Coulomb effects can be neglected if:

$$\delta E \gg E_C \iff 1 \ll 137 \log(L/d)\epsilon\, v_{\mathrm{F}}/c$$

Asymmetric Beam-Splitter



the average current in lead three depends on the exchange symmetry (singlet/triplet) of the incoming pair of electrons

$$\langle Q/e \rangle = T_{13} + T_{23} \pm |S|^2 (T_{13} + T_{23} - 1)$$

for symmetric beam-splitter:

$$T = T_{13} = T_{24}, T_{23} = T_{14} = 1 - T_{24} = 1 - T$$

and exchange term is absent!!!

$$\langle\langle (Q/e)^2 \rangle\rangle_{\pm} = (1 \pm |S|^2)\left[(1 - T_{13})T_{13} + (1 - T_{23})T_{23} \mp |S|^2 (T_{13} + T_{23} - 1)^2\right]$$

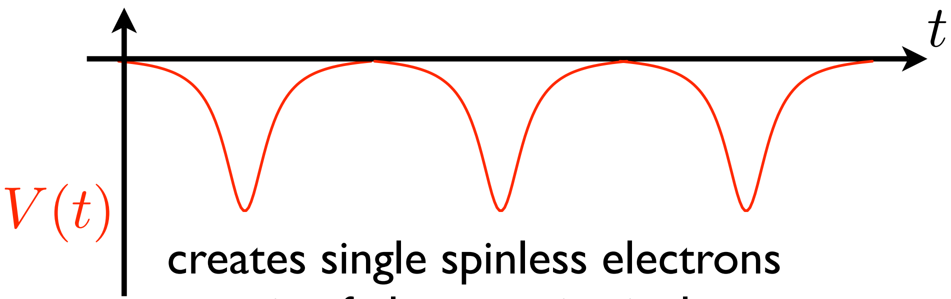
for symmetric beam splitter

$$\langle\langle Q^2 \rangle\rangle = 2e^2 (1 \pm |S|^2) T(1 - T)$$

Burkard, Loss, Sukhorukov PRB 2000

Experiment

Test for nonlinearity in the voltage-current response

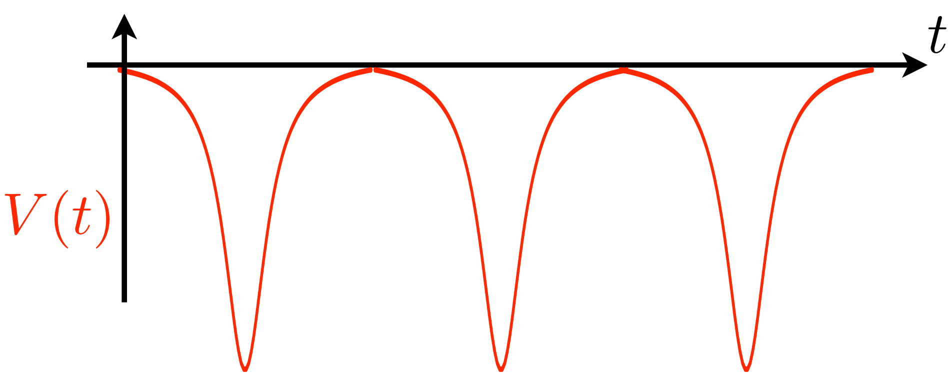


creates single spinless electrons
or pair of electrons in singlet state

average transmitted charge

$$\langle Q^{(1)} \rangle$$

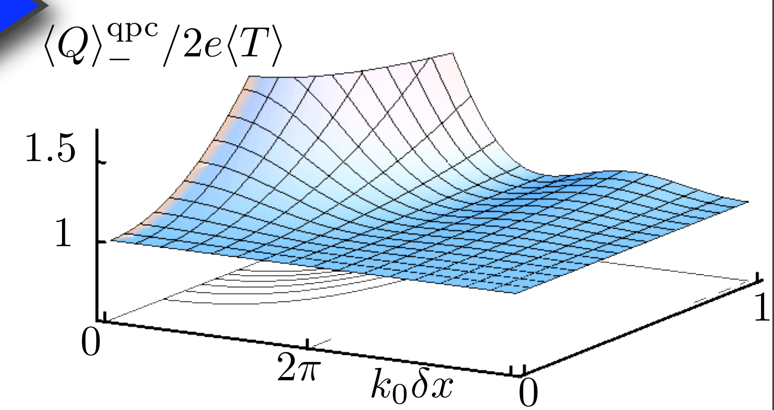
doubling the voltage:



creates pair of strongly overlapping spinless electrons
or pair of strongly overlapping electron singlets

average transmitted charge

$$\langle Q^{(2)} \rangle > 2\langle Q^{(1)} \rangle$$



Summary

- Single flux Lorentzian voltage pulse → e^-
- Connection fidelity ↔ FCS
- Exchange effects are absent for energy-independent transmission probability
- Nonlinearity in the transport due to exchange effects (without interaction)