Thermodynamics (specific heat) of 1D Fermi systems

the role of renormalizations at intermediate scales

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Larkin conference, June 26, 2007

In collaboration with Dmitrii Maslov (U Florida)

Sasha Finkelstein, Monday's talk

The closely related work:
I. Aleiner and K. Efetov, Phys. Rev. B 74, 075102 (2006).

I. E. Dzyaloshinskii and A. I. Larkin
JETP 34, 422 (1972).

motivations

Specific heat in Fermi liquids, in dimensions D>1

The leading term: C(T)/T = const (Fermi liquid theory)

Subleading term is universal and non-analytic

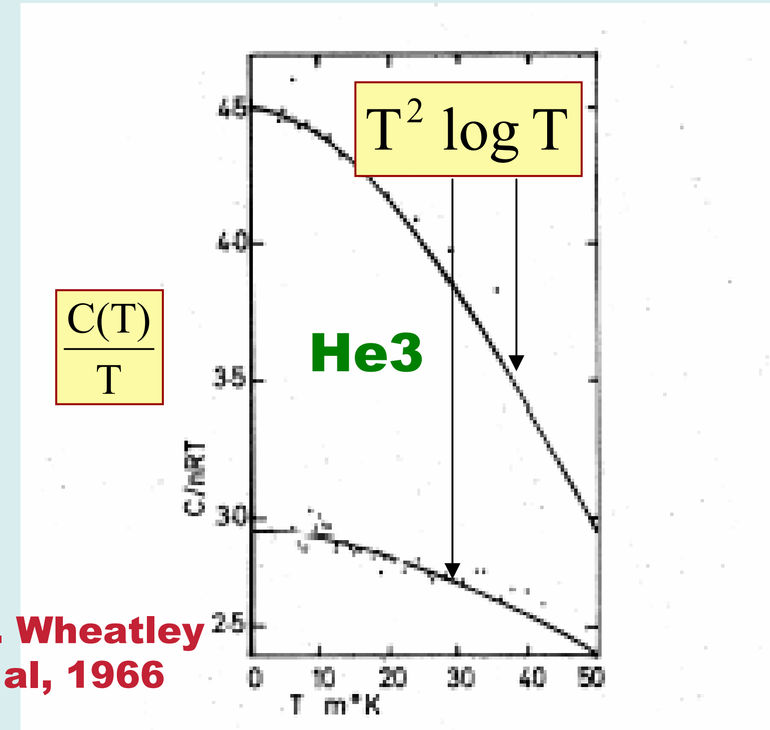
$$\boxed{\frac{\delta\, C(T)}{T} \propto T^{D-1}}$$

3D $C(T)/T \propto T^2 \log T$ **Eliashberg --- el-phonon**

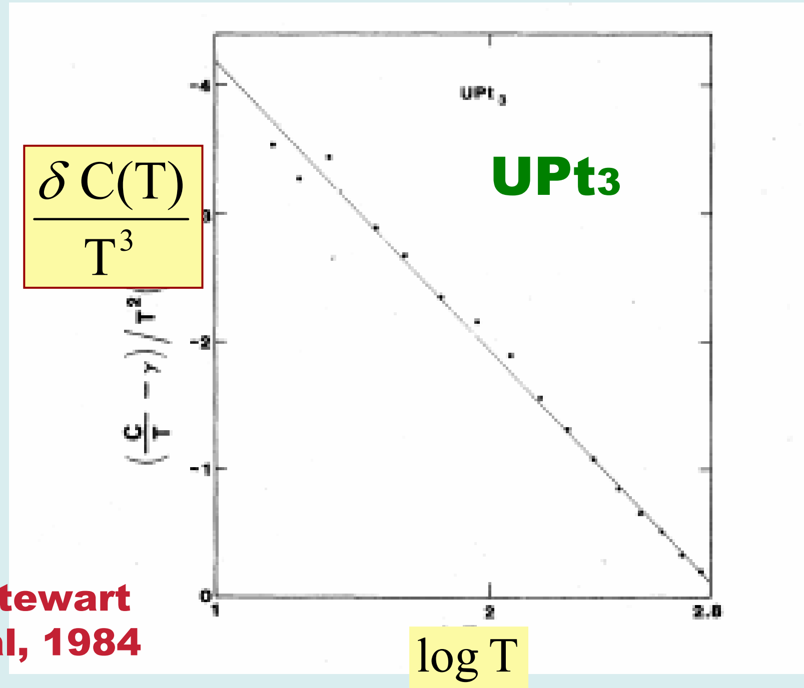
Pethick & Carneiro --- el-el

3D Fermi-liquid

Specific heat: $\delta\, C(T)\,/T \propto T^2\ \log T$



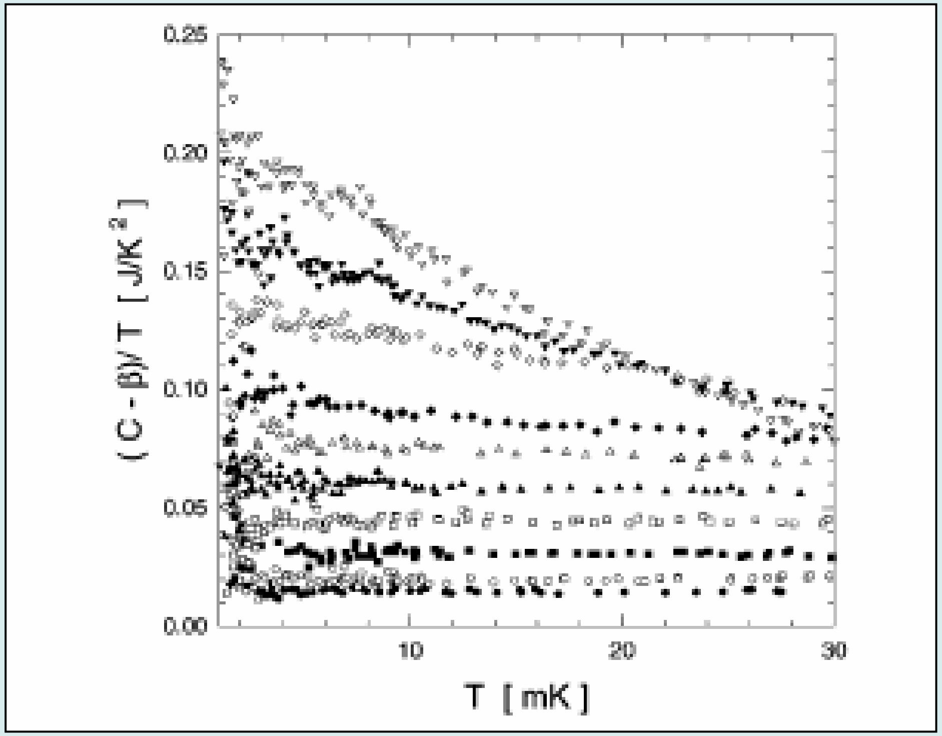
J.C. Wheatley
et al, 1966



G. Stewart
et al, 1984

2D Fermi-liquid

Monolayer of 3He



A. Casey et al.
PRL ***90,*** *115301 (2003)*

$$\frac{C(T)}{T} = \gamma - \Gamma \frac{T}{E_F}$$

Specific heat in Fermi liquids, in dimensions D>1

The leading term: C(T)/T = const (Fermi liquid theory)

Subleading term is universal and non-analytic

$$\frac{\delta C(T)}{T} =$$

$$\frac{\delta C(T)}{T} \propto T^{D-1}$$

$$\pi) + 3\, f_s^2\,(\pi)\Big)$$

charge spin

components of $f(\theta = \pi)$

A.C., Maslov, Glazman, Gangadharaiah

$f(\theta = \pi)$

$\Gamma(k,-k;k-k)$ $\Gamma(k,-k;-k,k)$

1D processes in D>1 are responsible for non-analytic δ C(T)

Another consequence of "one-dimensionality"

$$\frac{\delta C(T)}{T} = A_D\, T^{D-1} \left(f_c^2(\pi) + 3 f_s^2(\pi) \right)$$

$$f_c(\pi), \quad f_s(\pi)$$

are combinations of interactions with zero total momentum

$$\Downarrow$$

$$f_c(\pi), \quad f_s(\pi) \propto 1/\log T$$

due to renormalization in the Cooper channel

Aleiner & Efetov
Finkelstein & Shekhter
Efetov & Schwiete
A.C. & Maslov

Now we move to actual 1D

1.

$$\frac{\delta\, C(T)}{T} = A_D\, T^{D-1}\left(f_c^2(\pi) + 3\, f_s^2(\pi)\right), \qquad T^{D-1} \Rightarrow \mathrm{const}$$

2.

In 1D, charge backscattering amplitude remains finite

Spin backscattering amplitude scales as $f_s(T) = \dfrac{f_s}{1 - 2\, f_s\, |\log T|}, \quad f_s < 0$

Bychkov, Gor'kov & Dzyaloshinskii
Dzyaloshinskii & Larkin

One might expect that

- **charge component** $\dfrac{C_c(T)}{T} = \mathrm{const}$
- **spin component** $\dfrac{C_s(T)}{T} \sim \left(\dfrac{f_s}{1 - 2\, f_s\, |\log T|}\right)^2$

By this logics, T log T should appear at the third order

The actual result is different:

- **T log T only appears at the fourth order**
- **the spin component** $$\frac{C_s(T)}{T} \sim \left(\frac{f_s}{1 - 2 f_s |\log T|} \right)^3$$

Aleiner & Efetov -- "bosonization"
Wiegmann & Tsvelik – Kondo model
Luk'yanov -- XXZ S=1/2 model

What's special about backscattering in 1D?

We did “brute force” perturbative calculations for 1D fermions with short-range interaction.

g-ology in 1D:

Giamarchi

g_4 – forward scattering $\Gamma(k,k;k,k)$

g_2 – charge part of backscattering $\Gamma(k,-k;k-k)$

g_1 – $2p_F$ scattering (spin part of backscattering) $\Gamma(k,-k;-k,k)$

The spin contribution to the specific heat comes from $2p_F$ scattering (g_1 processes)

I will only be talking about $2k_F$ processes

In essence, the issue is the following:

In perturbation theory in 1D, C(T) is expanded in powers of g1, beginning from the first order.

$$\frac{C(T)}{T} = \frac{2\pi}{v_F}\left(a_1\, g_1 + a_2\, g_1^2 + a_3\, g_1^3 + a_4\, g_1^4 + \ldots\right)$$

$$a_i = a_i\,(\log T)$$

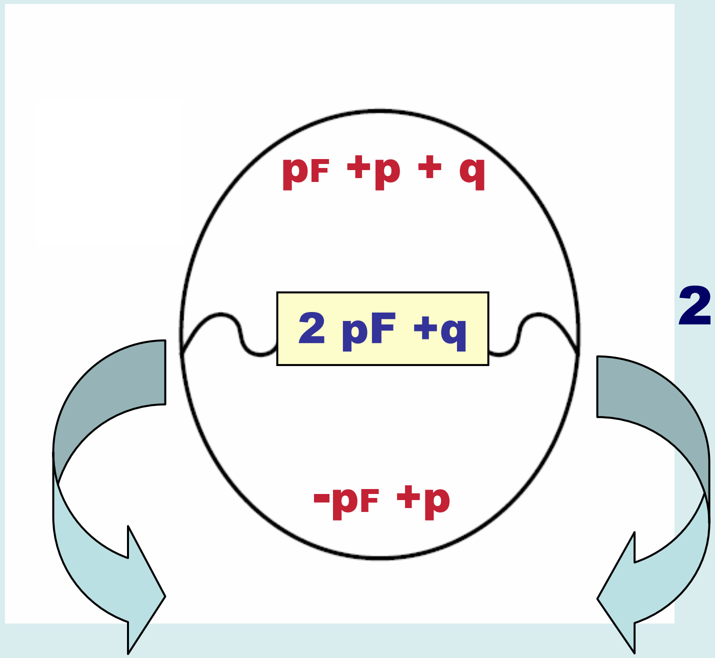
$$g_1\,(E) = \frac{g_1}{1 + 2\, g_1\, |\log E\,|}$$

The coupling g1 in C(T) can be → **at the scale of T, E ~ T, g1 is the RG- running coupling**

If, at a given order, g1 in C(T) is the coupling at the scale of T, one should obtain log T at the next order.

What it means that g1 is at a particular scale?

$$\frac{C(T)}{T} = -\frac{\partial^2 \Xi}{\partial T^2}$$



1st order
2pF contribution

$$g_1 = \Gamma(p_F + p + q, -p_F + p; -p_F + p, \ p_F + p + q)$$

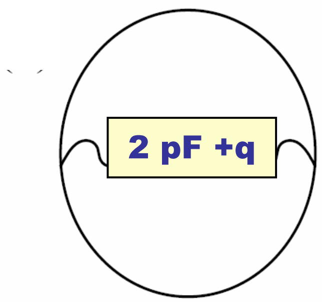
If p, or q = O(cutoff)

$g_1 \approx \Gamma(p_F, -p_F; -p_F, p_F)$ **is the vertex at the scale of the cutoff**

g_1 **is the running vertex at the scale O(T)**

Generally, contributions from momenta of the order of the cutoff scale as powers of the cutoff and can be easily separated from universal contributions from momenta O(T).

However, in 1D, we have “anomalies” – the contributions which are cutoff independent, but can be obtained as low-energy contributions OR as high-energy contributions.



$$\Xi_1(2p_F) = -g_1\, T\,\Sigma_\Omega \int_{-\Lambda_b}^{\Lambda_b} dq\, \Pi_{2p_F}(q,\Omega)$$

$$\Pi_{2k_F}(q,\Omega) = \frac{1}{2\pi}\left(\log\frac{\Omega^2+q^2}{4\Lambda_f^2} - 8\int_0^\infty n_F(x)x\,dx\left(\frac{1}{(q-i\Omega)^2-4x^2}+\frac{1}{(q+i\Omega)^2-4x^2}\right)\right)$$

$$T\,\Sigma_\Omega \int_{-\Lambda_b}^{\Lambda_b} dq\, \Pi_{2p_F}(q,\Omega) = \frac{\pi}{3}T^2$$

Take a symmetrized combination: the answer $\Xi_1(T) = \frac{\pi}{3}T^2$ **comes solely from q,** $\Omega = O(\Lambda_b)$

The model:

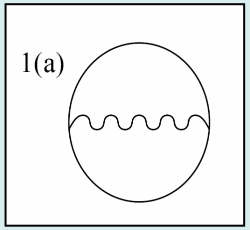
Fermions with linearized dispersion $\varepsilon_p = v_F\,(p - p_F)$ **up to a cutoff at** $|p - p_F| = \Lambda_f$

2pF interaction $U(2p_F + q)$ **extending up to a cutoff at** $|q| = \Lambda_b$

Assume $\Lambda_b << \Lambda_f$ **, the same condition as in conventional bosonization.**

If we set $\Lambda_b \sim T$ **, the contribution to** $\Xi_1 = \pi T^2 / 3$ **from "high-energy cutoff at** $q \sim \Lambda_b$ **becomes the contribution from** $q \sim T$

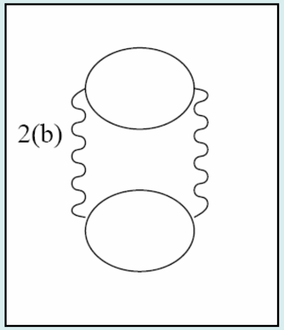
The results:



1. First order

$$\Xi_1 = -\frac{\pi}{3}\, T^2\, g_1$$

2. Second order



$$\Xi_2 = \frac{\pi}{3}\, T^2\, g_1 \underbrace{\left(2\, g_1 \log \frac{\Lambda_f}{\Lambda_b} \right)} - \underbrace{\frac{\pi}{3}\, T^2\, g_1^2}$$

1st order renormalization of the backscattering vertex

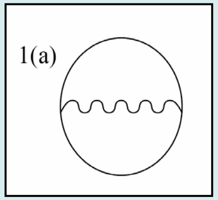
this term is $T^2\, f_s^2\, (\pi)$

RG

$$g_1\,(E) = \frac{g_1}{1 + 2\, g_1\, |\log E\,|}$$

the coupling g_1 is at the intermediate, but still a finite energy scale $E = \Lambda_f / \Lambda_b$

The results:



1. First order

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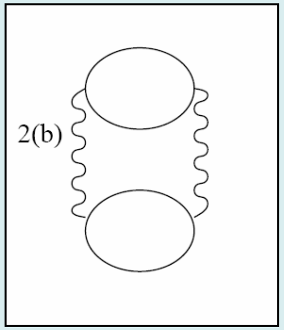
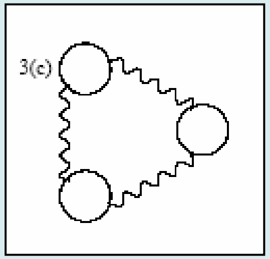


$$\Xi_2 = \frac{\pi}{3}\, T^2\, g_1 \left(2\, g_1 \log \frac{\Lambda_f}{\Lambda_b} \right) - \frac{\pi}{3}\, T^2\, g_1^2$$

3. Third order



$$\Xi_2 = -\frac{\pi}{3}\, T^2\, g_1 \underbrace{\left(4\, g_1^2 \log^2 \frac{\Lambda_f}{\Lambda_b} \right)}_{\text{2nd order renormalization of the backscattering vertex}} + \frac{\pi}{3}\, T^2\, g_1^2 \underbrace{\left(4\, g_1 \log \frac{\Lambda_f}{\Lambda_b} \right)}_{\text{1st order renormalization of the backscattering vertex}}$$

g_1 in C(T) is the backscattering at the scale of the cutoff set by the interaction

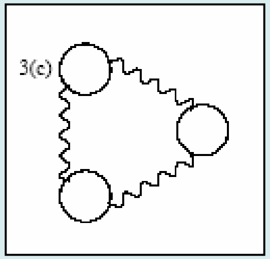
Is there a coupling at the scale of T?

Third order, but more carefully

$$\Xi_2 = -\frac{\pi}{3}\, T^2 g_1 \left(4 g_1^2 \log^2 \frac{\Lambda_f}{\Lambda_b} \right) + \frac{\pi}{3}\, T^2 g_1^2 \left(4 g_1 \log \frac{\Lambda_f}{\Lambda_b} \right) + 0 * (T^2 g_1^3)$$

$$0 * (T^2 g_1^3) = \underbrace{\pi T^2 g_1^3}_{\text{same as at first and second order } g_1(\Lambda)} - \underbrace{\pi T^2 g_1^3}_{\text{comes from } q,\Omega \to 0}$$

$$-\pi T^2 g_1^3 \Leftarrow \pi T \, g_1^3 \, T \Sigma_{\Omega_1} T \Sigma_{\Omega_2} (1 - \delta_{\Omega_1,0}\, \delta_{\Omega_2,0})$$



The same type of frequency sums gives rise to the non-analyticity in the spin susceptibility

Fourth and higher orders:

$$\Xi \Rightarrow -\pi\, T^2 \left(g_1^3(T) - g_1^3(\Lambda)\right)$$

$$g_1(T) = \frac{g_1}{1 + 2\, g_1 \, |\log \Lambda/T\,|}$$

$$\frac{C(T)}{T} \sim g_1^4 \quad \log T \; + \ldots.$$

Final result (including g4, g2)

$$\frac{C(T)}{T} = \frac{\pi}{3}\left(\frac{1}{v_\rho} + \frac{1}{v_\sigma}\right) + \frac{\pi}{3} g_1^2 + 2\pi \frac{g_1^3}{(1 + 2\, g_1 \,\log \Lambda/T)^3}$$

$$g_1 = g_1(\Lambda_b)$$

bosonization result, v_υ, v_σ are charge and spin velocities

$$v_\rho = \left((1 + 2\, g_4 - g_1)^2 - (2\, g_2 - g_1)^2\right)^{1/2}$$

$$v_\sigma = \left((1 - g_1)^2 - g_1^2\right)^{1/2}$$

h the

(T)

Efetov

Conclusions:

The appearance of the backscattering coupling at some order in C(T) with apparently universal,cutoff-independent prefactor does NOT imply that this coupling is the running one at the scale of T.

The running coupling at the scale of T does appear in the specific heat at the third order.

$$\frac{C(T)}{T} = \frac{\pi}{3}\left(\frac{1}{v_\rho} + \frac{1}{v_\sigma}\right) + \frac{\pi}{3} g_1^2 + 2\pi \frac{g_1^3}{(1 + 2 g_1 \log \Lambda/T)^3}$$

N E W

g1 is renormalized down to the scale set by the interaction

an additional constant backscattering contribution to C(T)

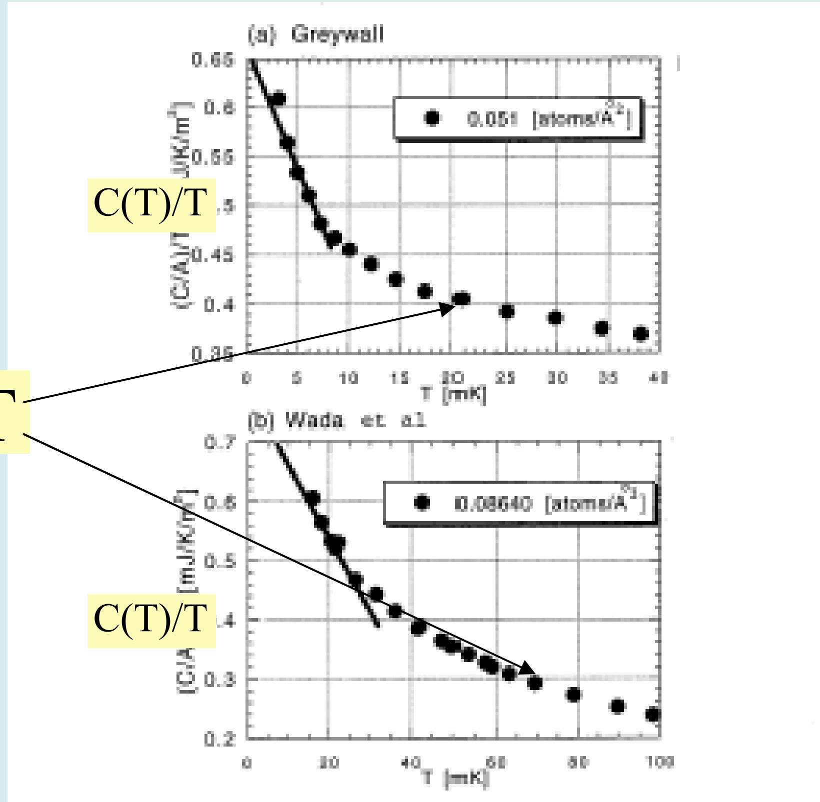
THANK YOU!

Monolayer of 3He

Greywall

Wada

Experiments in mid-80th



$$\boxed{f_c^2(\pi) + 3 f_s^2(\pi) \sim 15}$$