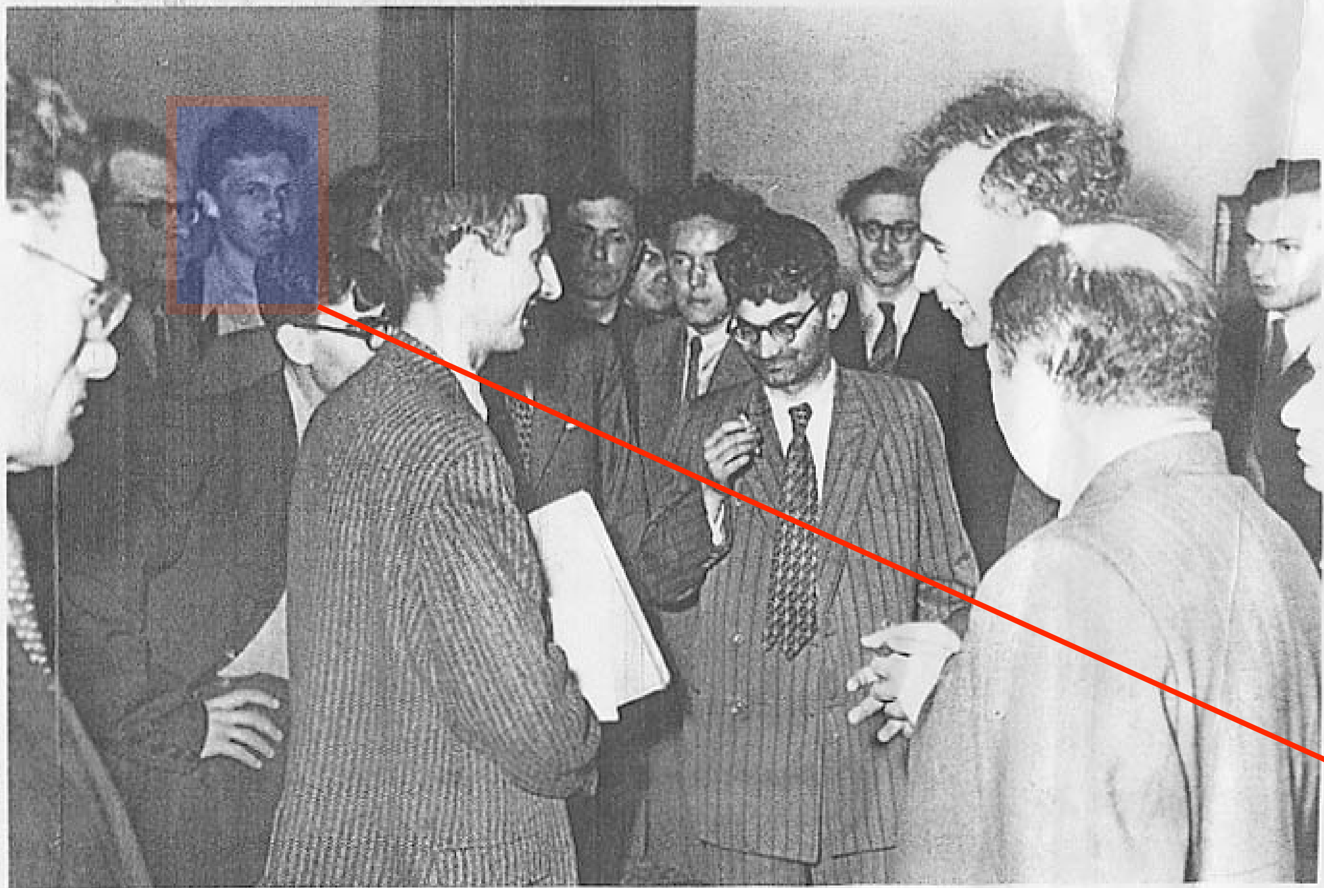
Tolya Larkin :Fifty years of pioneering science.



20. Moscow, 1956. Freeman Dyson (front, left), talking with I. Pomeranchuk and Lev Landau.

Early Critical phenomena, Many body physics, superconductivity, magnetism, particle physics, localization, gauge theories, flux lattices... and many students.

Larkin Conference: Chernogolovka, Jun 24-28 (2007)

Symply Symplectic

Putting Time reversal symmetry back into large N expansions.

Rebecca Flint
Maxim Dzero
Piers Coleman

http://www.physics.rutgers.edu/~coleman/symp.pdf

Supported by the National Science Foundation

Rutgers Center for Materials Theory

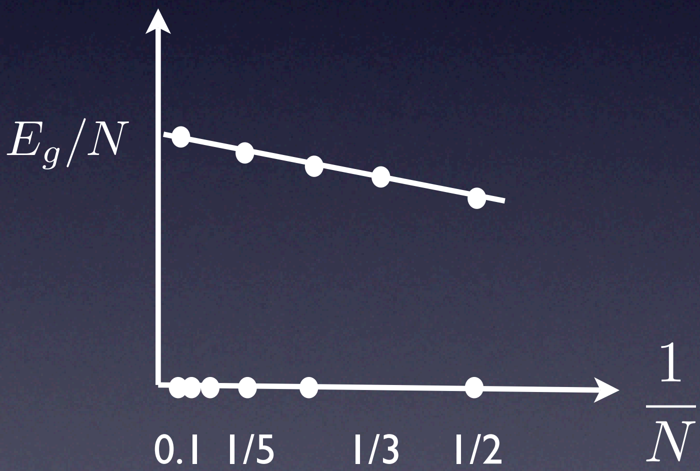
- Motivation.
- Failure of SU(N)
- Time reversal and SP(N)
- Frustrated Magnetism
- Kondo Lattice
- Future Prospects

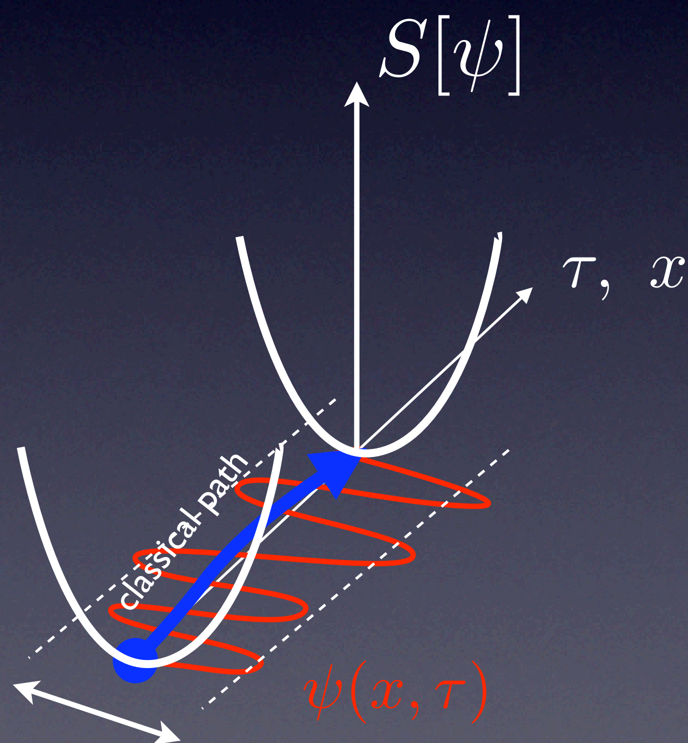
Rutgers
Center for Materials Theory

Strongly correlated electron physics: no small parameter.

Large N - family of models which retain the key physics and can be solved in the limit N $\longrightarrow \infty$

$$Z = \sum_{\text{configs}} e^{-N \times S[\psi]}$$





The large N limit is a semiclassical limit, in which 1/N plays the role of Planck's constant.

$$\frac{1}{N} \sim \hbar_{eff}$$

Example: RPA Electron gas

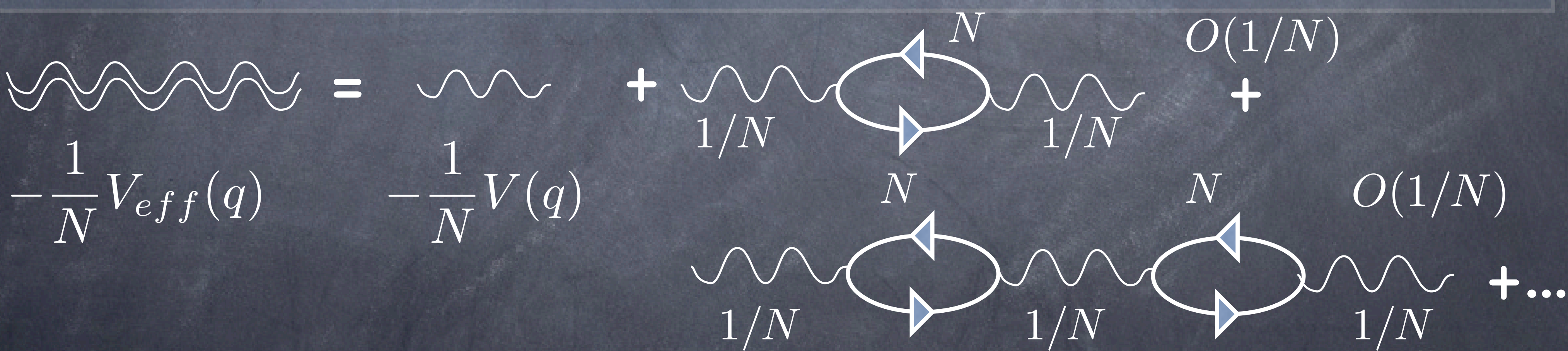
$$H_I = \frac{1}{N}\sum_q V(x-x') : \rho(x)\rho(x') :$$

$$\rho(x) = \sum_{\sigma=1,N} \psi^\dagger_\sigma(x)\psi_\sigma(x)$$

N spin components:
SU(N) Symmetry

$$\rho_{classical}(x) = \frac{1}{N}\sum_{\sigma=1,N} \psi^\dagger_\sigma(x)\psi_\sigma(x)$$

behaves as a classical variable when N is sent to infinity.



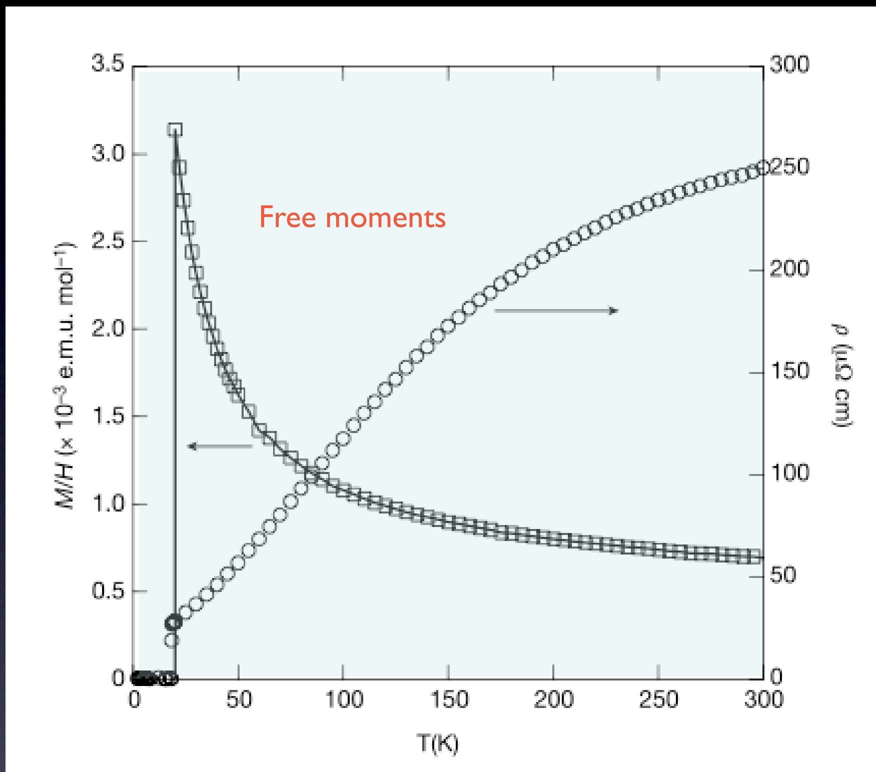
screened Coulomb interaction
(Pines and Bohm, 1951)

$$V_{eff}(q,0) = \frac{4\pi e^2}{q^2+\kappa^2}$$

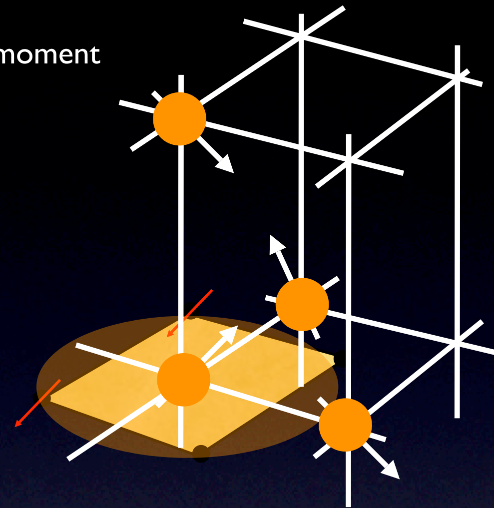
- Large S expansion   Magnetism   (Anderson, Dyson, Maleev 50s)
- Spherical model   Critical phenomena (Berlin & Kacs, Wilson 50-70's)
- SU(N) expansion   Heavy electrons (Read & Newns, PC..... 80's)
  2D Magnetism   (Affleck & Marston, Auerbach & Arovas '80s)
- SP(N) group   Read and Sachdev (1992): frustrated magnetism
  Pairing in fermionic gases
  (Nikolic and Sachdev, Veuillete, Sheehy and Radzikovsky 07)

Yet many problems remain inaccessible
e.g heavy electron superconductivity.

18K Heavy Fermion S.C
$PuCoGa_5$



J. Sarrao et al, Nature 420, 2002.

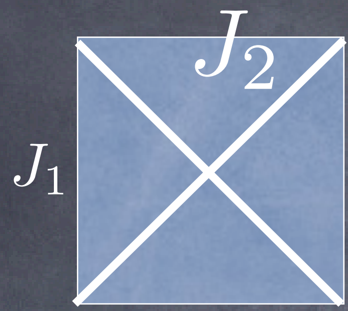


Co-operative
Kondo effect?

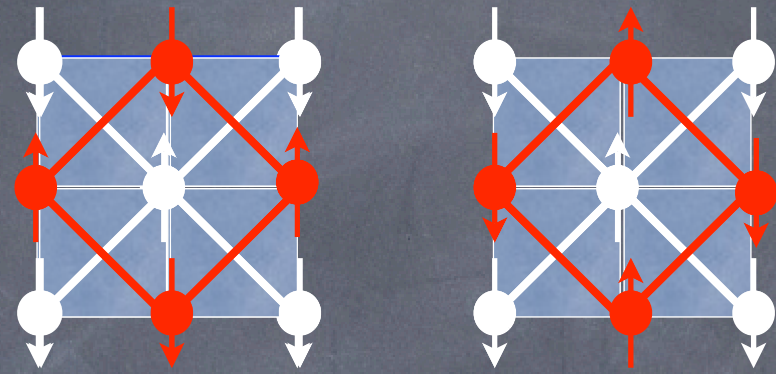
How might we describe the simultaneous quenching of moments and development of superconductivity?

Existing large N approaches can not unify the Kondo effect and superconductivity.

Frustrated magnetism J1-J2 Heisenberg Model



$J_1 << J_2$



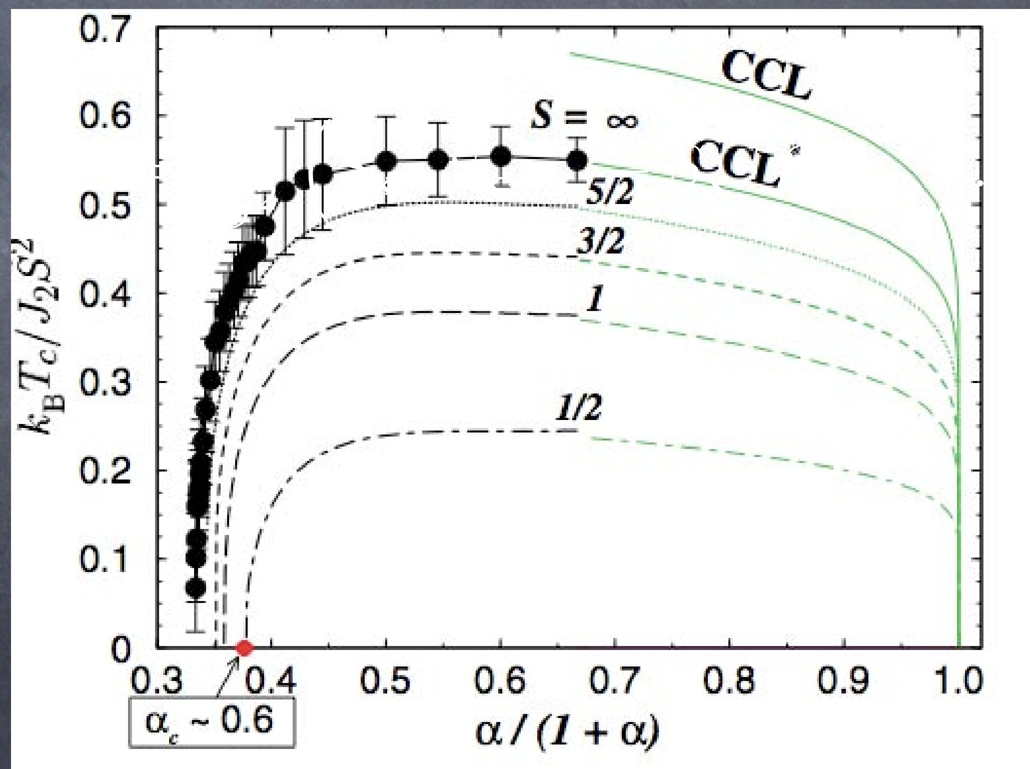
$\sigma = -1$ $\sigma = +1$

$$H = \sum_{i,j} J_{ij} (\vec{S}_i \cdot \vec{S}_j)$$

Ising Phase transition.

$$T_i = \frac{4\pi J_2 S^2}{\ln \frac{J_2}{c J_1}}, \qquad (J_2 >> J_1)$$

P. Chandra, PC, A. Larkin (90)



Capriotti et al (04)

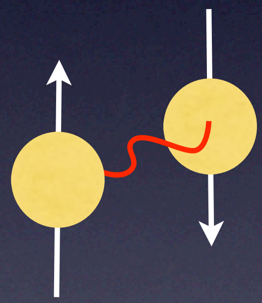
RVB instability?

SU(N)

"Designer made for particle physics"

- Singlets: Mesons $\psi^\dagger_\alpha \psi_\alpha$ (particle-hole)
- Baryons (N-particle bound-states) $\psi^\dagger_1 \psi^\dagger_2 \dots \psi^\dagger_N$ (?? Electron shells)

Not in SU(N)



but unfortunately....

- No two-particle singlets for N>2.

- no antiferromagnetism (without alternating reps)
- no Cooper pairs, no superconductivity.

Read and Sachdev (1992):
symplectic group SP(N) (N even)
Two particles can pair.

$$\sum_{\sigma=-N/2}^{\sigma=N/2} \mathrm{sgn}(\sigma)\psi_\sigma\psi_{-\sigma}$$ is a singlet under SP(N)

- Frustrated magnetism

(Read Sachdev 1992, many others)

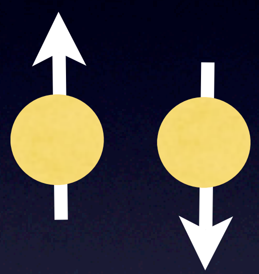
- Paired Fermionic gases

(Nikolic & Sachdev (07), Veillette et al (07).

The link with time reversal.

Inversion of spins under time reversal:
defining symmetry of magnetism

$$\vec{S} \xrightarrow{\theta} -\vec{S}$$



Spin x time reversed spin $\longrightarrow$ singlet

S - S = 0

Time reversal = antiunitary operator = U K

$$\theta = i\sigma_2 K$$

$$(Ka|\psi\rangle = a^* K|\psi\rangle)$$

e.g

$$\theta\psi(\mathbf{x},\mathbf{t}) = \mathbf{i}\sigma_{\mathbf{2}}\psi^*(\mathbf{x},-\mathbf{t})$$

$$\theta p \theta^{-1} = p^* = p^T = -p$$

$$\theta S \theta^{-1} = \sigma_2 S^* \sigma_2 = \sigma_2 S^T \sigma_2 = -S$$

Invariance of time reversal under rotations U:

$$U\theta = \theta U \qquad (U = e^{i\frac{\vec{\alpha}}{2}\cdot\vec{\sigma}})$$

So that:

$$U\theta U^\dagger = \theta$$

But

$$KU^\dagger = (U^\dagger)^* K = U^T K$$

Which implies

$$U i\sigma_2 U^T = i\sigma_2.$$

SYMPLECTIC CONDITION

Infinitesimal rotations:

$$\sigma_2 \vec{\sigma}^T \sigma_2 = -\vec{\sigma}$$

Symplectic is synonymous with time reversal

But what about SU(N)?

THE GOOD, THE BAD ....

SU(N), spins divide into two groups according to whether they flip under time reversal.

$$\sigma_2 {S_\lambda}^T \sigma_2 = \begin{cases} -S_\lambda & \text{"Good" (A)} \\ +S_\lambda & \text{"Bad" (B)} \end{cases}$$

e.g SU(4)

The good spins form a subgroup called SP (N), or symplectic N.

$$\underline{\sigma}_2 = \begin{bmatrix} \sigma_2 & \\ & \sigma_2 \end{bmatrix},$$

$$S_{good} = \left\{ \begin{bmatrix} \vec{\sigma} & \\ & \pm\vec{\sigma} \end{bmatrix}, \begin{bmatrix} & \vec{\sigma} \\ \vec{\sigma} & \end{bmatrix}, \begin{bmatrix} & i\underline{1} \\ -i\underline{1} & \end{bmatrix} \right\}$$

6+3+1= 10 good

$$S_{bad} = \left\{ \begin{bmatrix} \underline{1} & \\ & -\underline{1} \end{bmatrix}, \begin{bmatrix} & i\vec{\sigma} \\ -i\vec{\sigma} & \end{bmatrix}, \begin{bmatrix} & \underline{1} \\ \underline{1} & \end{bmatrix} \right\}$$

1+3+1 = 5 bad

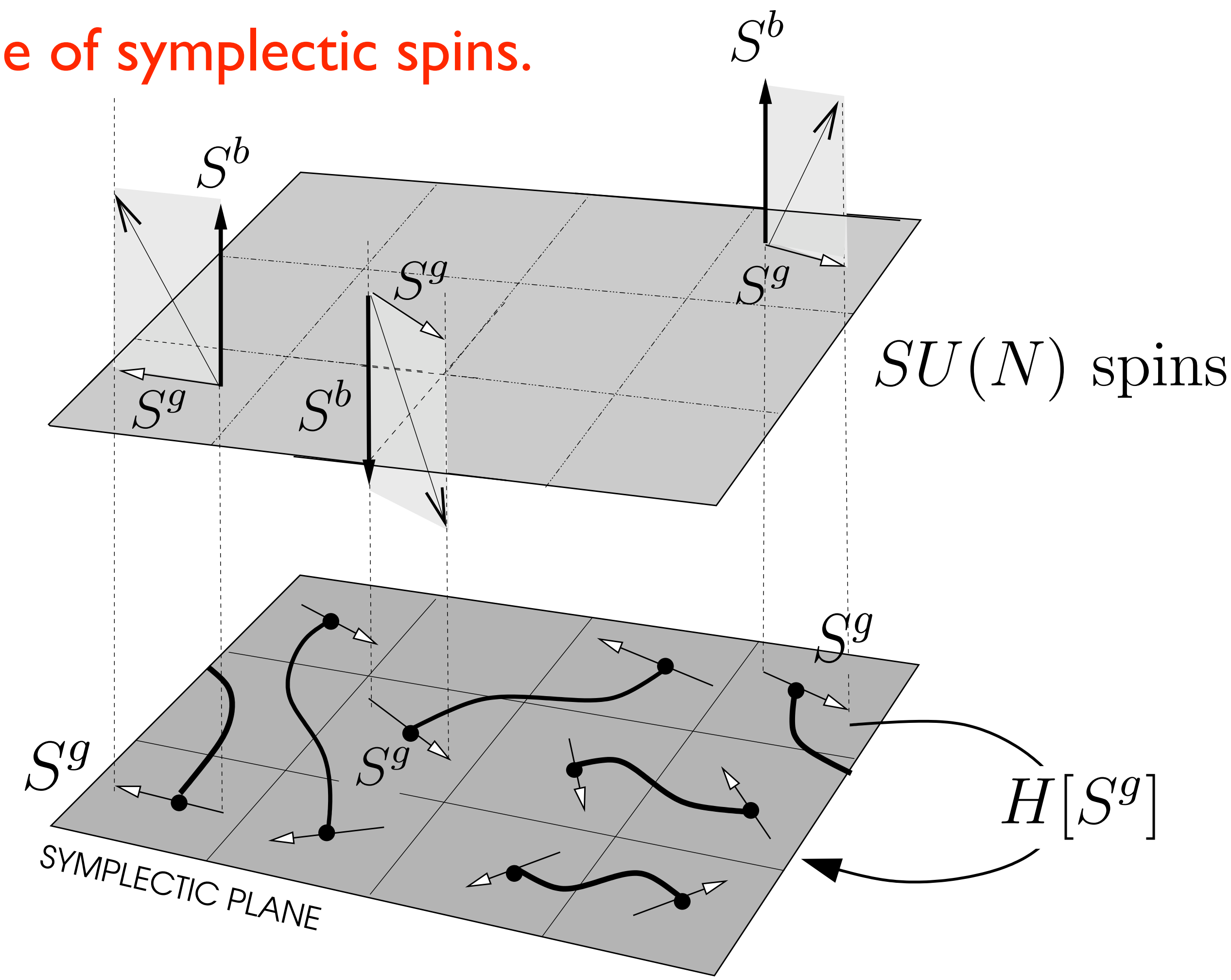
(c.f Wu and Zhang (2005))

Symplectic N: construct large N Hamiltonians exclusively from "good" spins

Since $[S^g,\ S^{g'}] \sim S^g$, the dynamics of a Hamiltonian

$$H = H[S^g]$$

is closed within the plane of symplectic spins.



$$[S^{pq}]_{\alpha\beta} = [\delta^p_\alpha \delta^q_\beta - \tilde{\alpha}\tilde{\beta}\delta^{-p}_\alpha \delta^{-q}_\beta]$$

$(\tilde{\alpha} \equiv \mathrm{sgn}(\alpha))$

Schwinger boson: $(n_b = NS)$,

$$S^{pq} = b^\dagger_\alpha S^{pq}_{\alpha\beta} b_\beta = [b^\dagger_p b_q - \tilde{p}\tilde{q} b^\dagger_{-q} b_{-p}].$$

Completeness:

resonanation

$$\sum_{a \in g} S^a_{\alpha\beta} S^a_{\gamma\eta} = [\delta_{\alpha\delta}\delta_{\beta\eta} + \tilde{\alpha}\tilde{\eta}\delta_{\alpha -\gamma}\delta_{\eta\ -\beta}].$$

pairing

Applications to Quantum Magnetism

$$\hat{S}^a(j) = b^\dagger_{j\alpha} S^a_{\alpha\beta} b_{j\beta}, \qquad (a \in \{\text{good}\})$$

$$J\vec{S}^g_1 \cdot \vec{S}^g_2 = J[A^\dagger_{21} A_{21} - B^\dagger_{21} B_{21}],$$

$$B^\dagger_{21} = \sum_\sigma \tilde{\sigma} b^\dagger_{2\sigma} b^\dagger_{1-\sigma} \qquad A_{21} = (b^\dagger_2 b_1) \equiv \sum_\sigma b^\dagger_{2\sigma} b_{1\sigma}$$

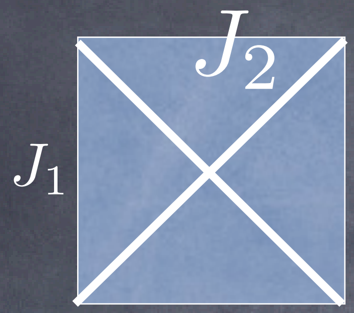
Bond pair

Bond resonance

$$-2B^\dagger_{12} B_{12} = \sum_{\alpha \in g} S^\alpha_1 S^\alpha_2 - \sum_{\beta \in b} S^\beta_1 S^\beta_2 .$$

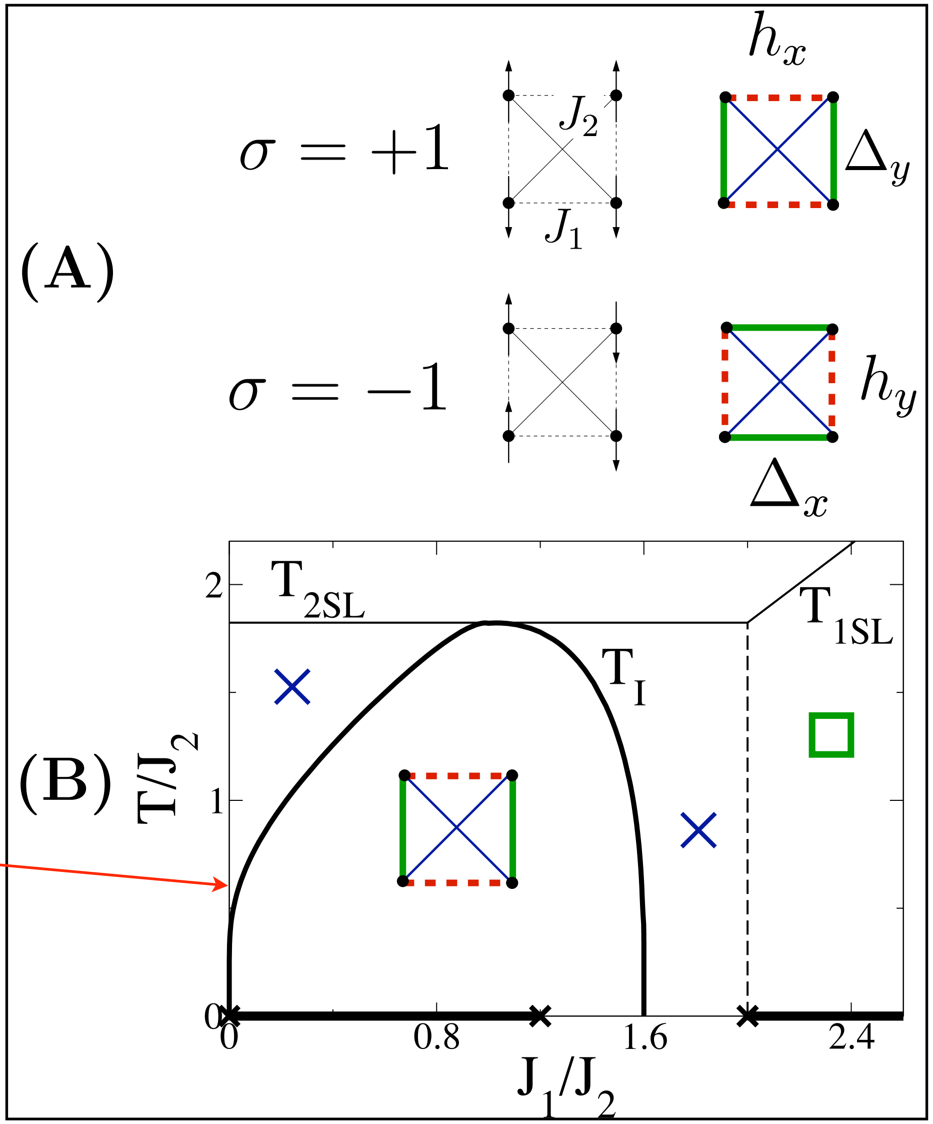
"SP(N)" decoupling scheme is a mix of symplectic and antisympletic spins

$$J\vec{S}_1 \cdot \vec{S}_2 = \sum_\sigma \left(b^\dagger_{2\sigma}, \tilde{\sigma} b_{2\ -\sigma} \right) \begin{bmatrix} h & \Delta \\ \bar{\Delta} & \bar{h} \end{bmatrix} \begin{pmatrix} b_{1\sigma} \\ \tilde{\sigma} b^\dagger_{1\ -\sigma} \end{pmatrix} + N \left(\frac{\bar{\Delta}\Delta - \bar{h}h}{J} \right) .$$



$J_1 << J_2$

$$T_I \equiv T_{RVB} = \frac{4\pi J_2 S^2}{\log\left(\frac{2J_2 S}{J_1\sqrt{2\gamma}}\right)}.$$



Decoupling the interactions: fermions

- e.g. Kondo Model

$$f_\alpha = \begin{pmatrix} f_{1\uparrow} \\ f_{1\downarrow} \\ \vdots \\ f_{k\uparrow} \\ f_{k\downarrow} \end{pmatrix}, \qquad SP(2k)$$

$$\begin{aligned} H_I &= \frac{J_K}{N} \sum_A S^A (\psi_c^\dagger \sigma_{cd}^A \psi_d) \\ &= -\frac{J_K}{N} \Big[(f^\dagger \psi)(\psi^\dagger f) + \big(f^\dagger \sigma_2 \psi^\dagger\big)(\psi \sigma_2 f)\big) \Big] \end{aligned}$$

SU(2) gauge symmetry
(Affleck et al. '89) for all N.

$$f_{a\sigma} \longrightarrow \cos\phi \;\, f_{a\sigma} + \sin\phi \;\, \sigma f^\dagger_{a-\sigma}$$

$$H_I \quad \rightarrow \quad \Big[(f^\dagger \psi)V + V^*(\psi^\dagger f) \Big] + \Big[\big(f^\dagger \sigma_2 \psi^\dagger\big)\Delta + \Delta^* (\psi \sigma_2 f)\big) \Big] + N \left(\frac{V^*V + \Delta^* \Delta}{J_K} \right.$$

Single channel KL : Pairing term can always be gauged away.
Two channel KL, pairing term in second channel converts the second channel into a relevant variable.

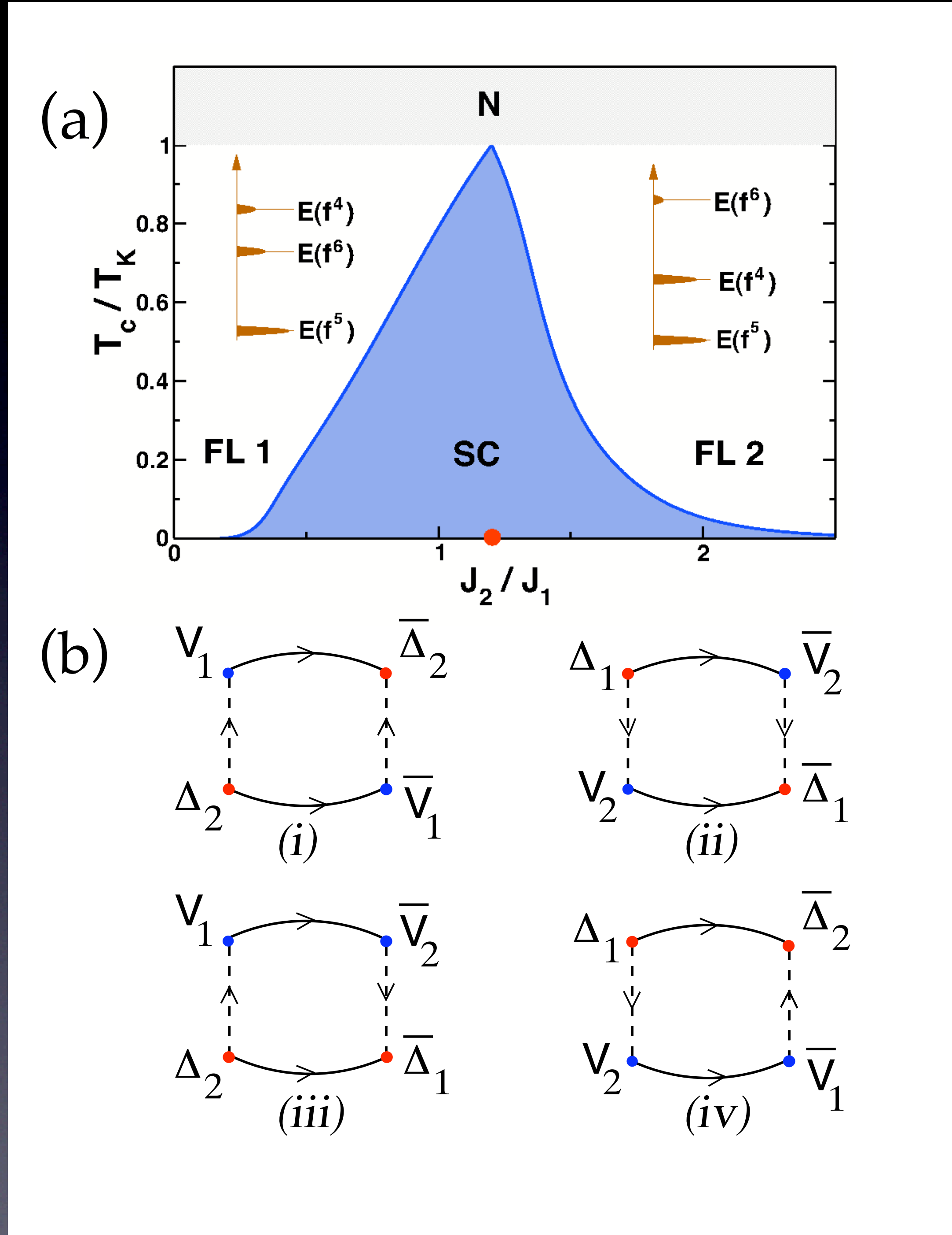
Pu: 115 $f^5 \rightleftharpoons \begin{cases} f^6 + h^{+}_{\Gamma=1} \\ f^4 + e^{-}_{\Gamma=2}, \end{cases}$

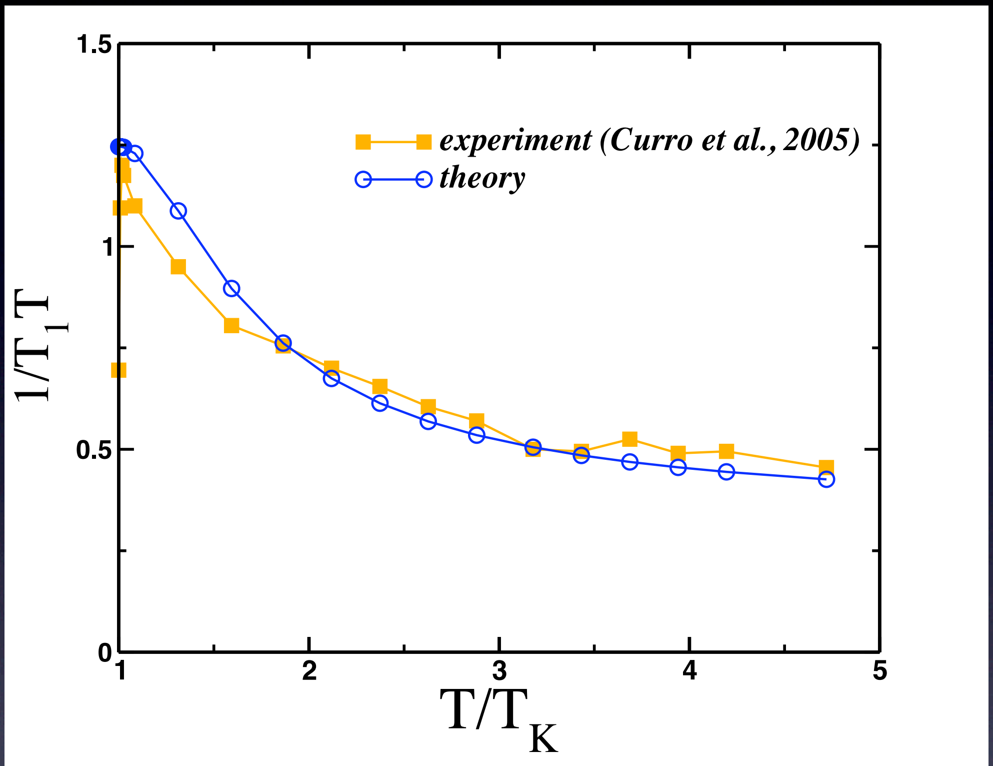
Two channels?

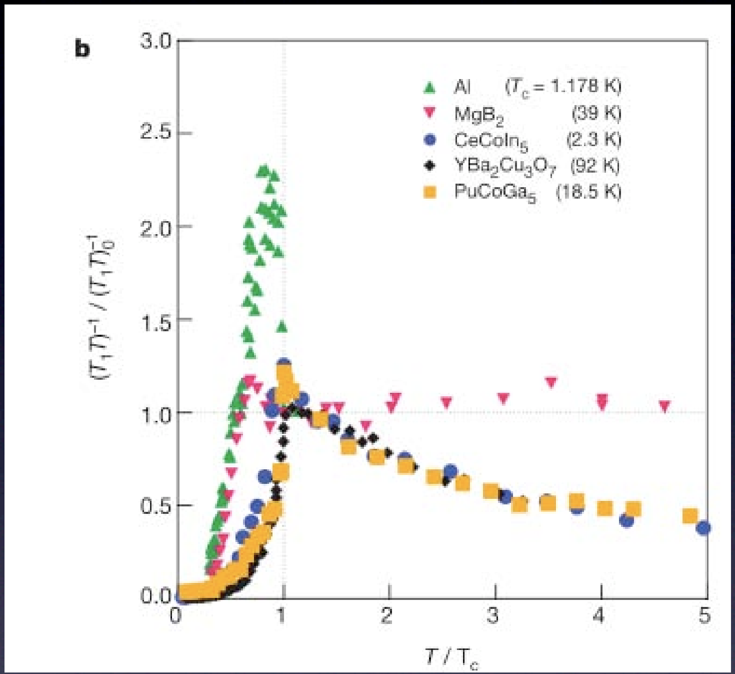
Co-operative Kondo Effect
Coleman, Andrei, Tsvelik, Kee
(97)

$$\Lambda_s(x) = \langle \Psi_{N-2} | \psi_{1\downarrow}(x) \psi_{2\downarrow}(x) S_f^+(x) | \Psi_N \rangle.$$

Symplectic N
provides a controlled
realization
(Dzero et al)







Curro et al, Nature, 434,622 (2005).

Future prospects

- t-J model.

  Wen and Lee SU(2) formulation of Hubbard operators form a symplectic algebra, first controlled treatment of RVB superconductivity.

- Electron gas (short range interaction)

  (Symplectic N = RPA in p-h and p-p)

- Long range quantum spin glasses.