Infrared Hall Effect in cuprates*

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Outline

1. Introduction
2. IR Hall Effect
3. Cuprate phases
4. Optimally doped cuprates
5. Underdoped cuprates
6. Conclusions

IR Hall Effect:
Hall spectroscopy

Magneto-Transport

Boltzmann theory

$$\sigma_{xx} = \frac{e^2}{hd} \oint dk \, \vec{\ell}_k$$

$$\sigma_{xy} = \frac{e^2}{hd} \frac{eB}{\hbar c} \oint dk \bullet \vec{\ell}_k \times d\vec{\ell}_k \big/ dk$$

$DC \quad \vec{\ell}_k = v_k \tau_k$

$IR \quad \vec{\ell}_k = \dfrac{v_k}{1/\tau_k - i\omega}$

$DC: \quad \langle \ell_k \rangle$

$IR: \quad \omega_H \, , \; \left\langle 1/\tau_k \right\rangle$

Sum Rules
IR Hall effect spectroscopy

$$\int_0^{\Omega_C} \tfrac{2}{\pi}\,\mathrm{Re}(\sigma_{xx})\,d\omega \equiv \omega_p^2 \big/ 4\pi = \frac{e^2}{2}\sum_k Tr(m_k^{-1})n_k$$

$$\int_0^{\Omega_C} \mathrm{Re}(\sigma_{xy})\,d\omega = 0$$

$$\int_0^{\Omega_C} \tfrac{2}{\pi}\,\mathrm{Im}(\sigma_{xy})\,\omega\,d\omega \equiv \omega_p^2\omega_H \big/ 4\pi = \frac{e^3 B}{2c_0}\sum_k \det(m_k^{-1})n_k$$

$$\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}$$

$$\int_0^{\Omega_C} \tfrac{2}{\pi}\,\mathrm{Re}(\theta_H)\,d\omega \equiv \omega_H = \frac{2eB}{c}\frac{\sum_k \det(m_k^{-1})n_k}{\sum_k Tr(m_k^{-1})n_k}$$

Drew, Coleman, PRL (1997)

IR Hall effect:

The extended Drude model:

$$\sigma_{xx} = \frac{\omega_p^2}{4\pi}\frac{1}{\gamma - i\omega(1+\lambda)}, \quad \sigma_{xy} = \frac{\omega_p^2}{4\pi}\frac{\omega_H}{\left(\gamma - i\omega(1+\lambda)\right)^2}$$

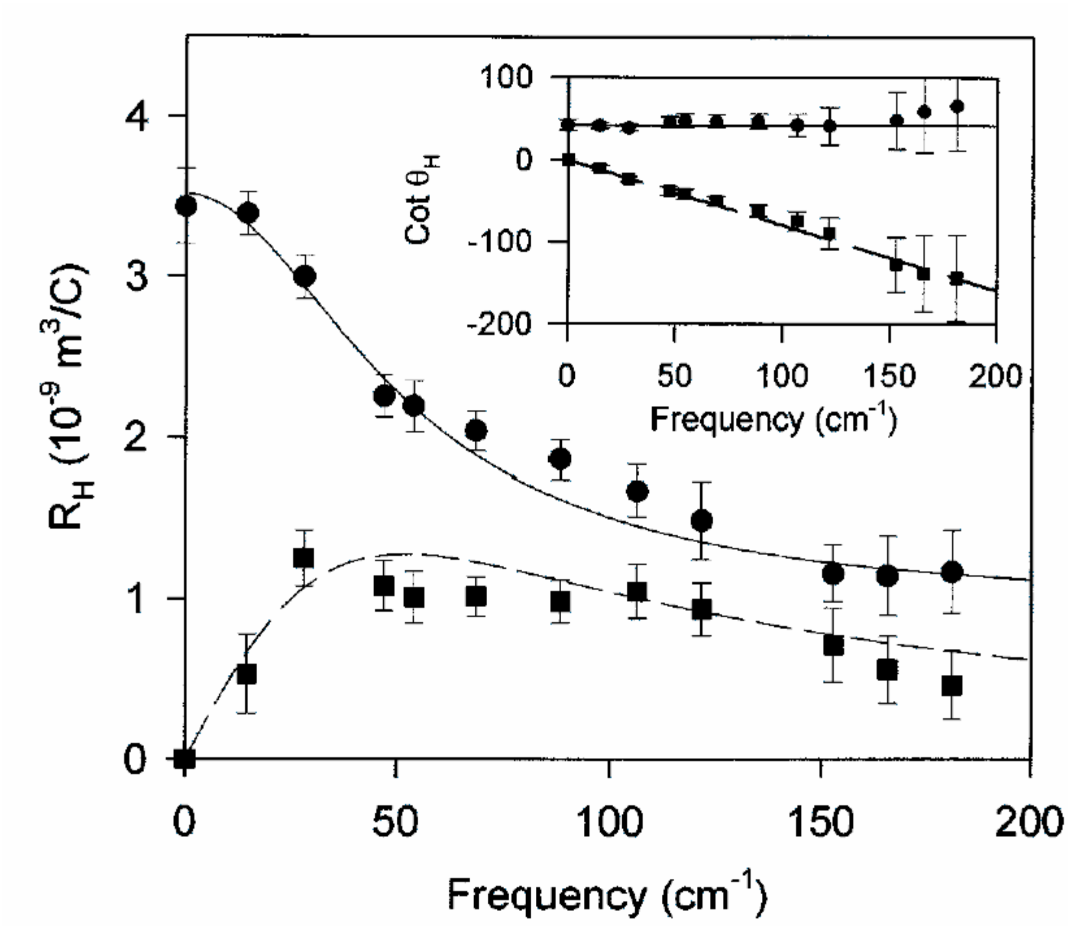
$$\theta_{xy} \equiv \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{\omega_H}{\gamma - i\omega(1+\lambda)}$$

$$R_H = \frac{1}{nec}$$

Optical self energy

$$\Sigma(\omega) \equiv \omega\lambda(\omega) + i\gamma(\omega)$$

R_H vs. ω $YBCO_7$



$$\sigma_{xx} = \frac{ne^2/m}{\gamma - i\omega}$$

$$\sigma_{xy} = \frac{(ne^2/m)(eB/mc)}{(\gamma - i\omega)^2}$$

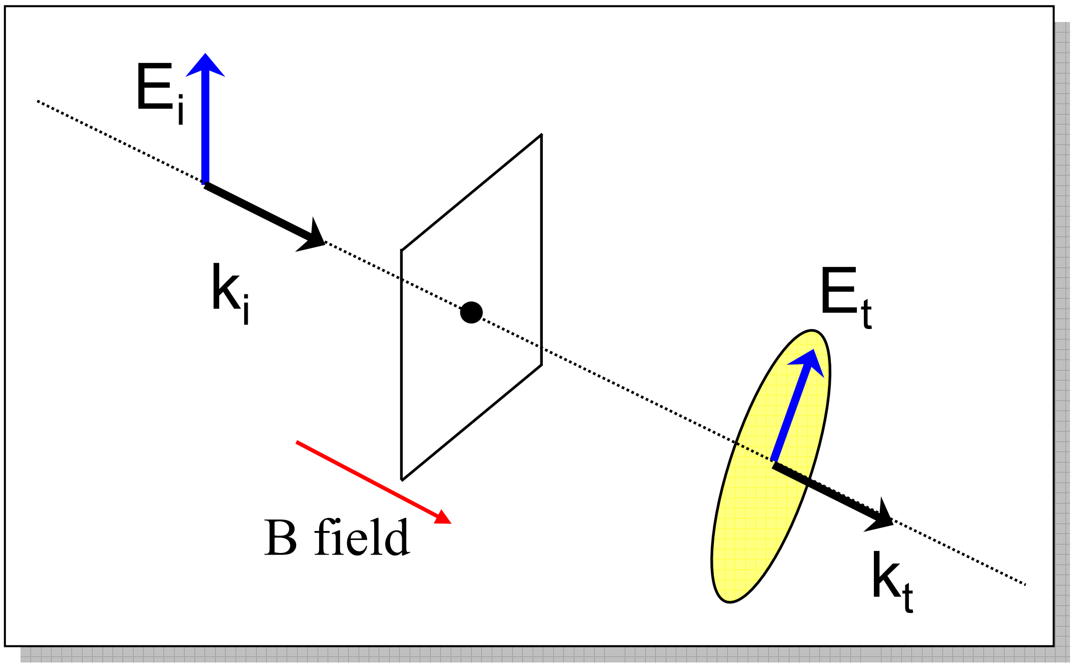
$$\rho_H = R_H B = \frac{\sigma_{xy}}{\sigma_{xx}^2} = \frac{B}{nec}$$

$$\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{eB/mc}{\gamma - i\omega}$$

$$\gamma_{xy} \neq \gamma_{xx}$$

Kaplan *et al.*, PRL ‘96

IR Hall Effect



$\theta_{Faraday}$

σ_{xx}

θ_H

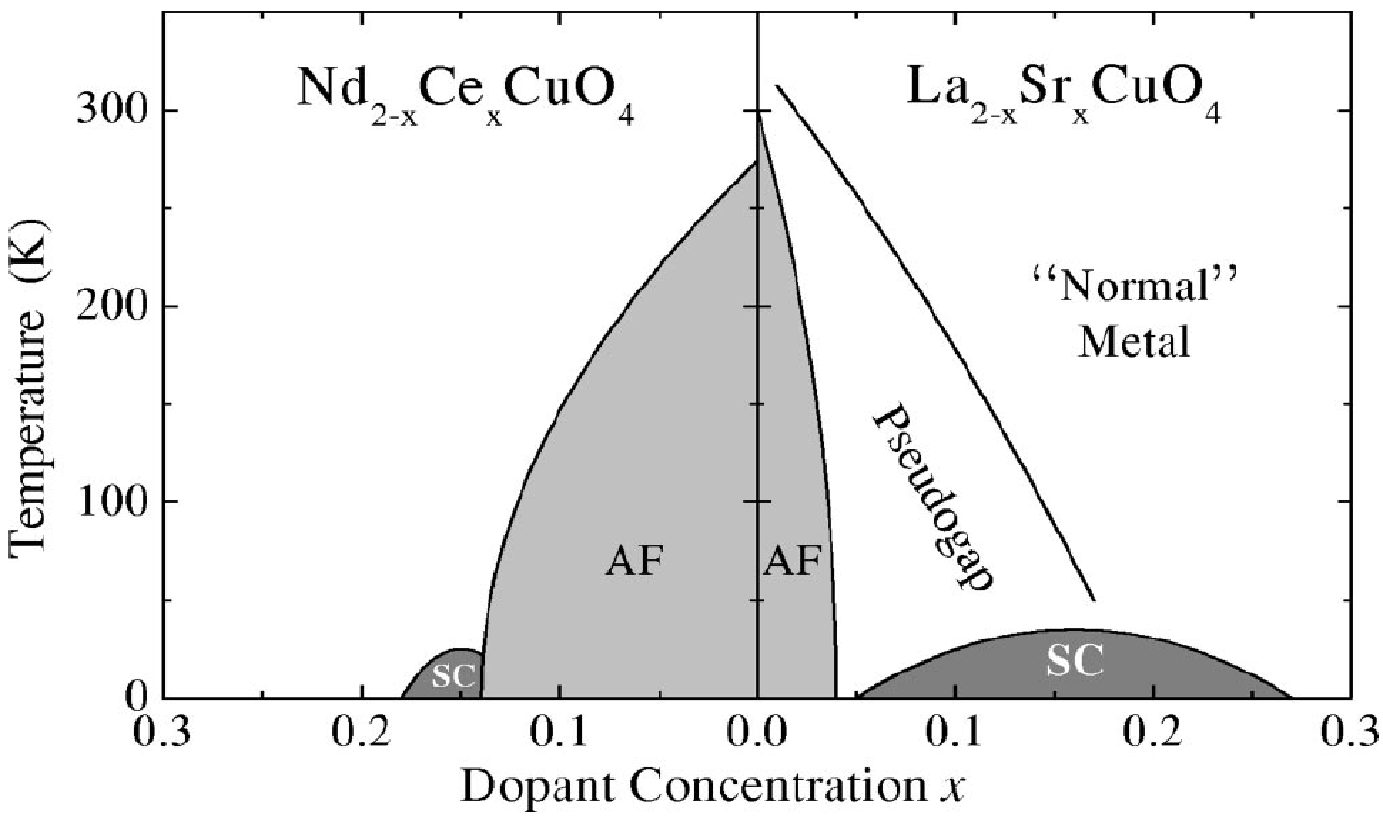
σ_{xx}

σ_{xy}

$$\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}}$$

J. Černe et al., RSI (2003)

Cuprate Phase Diagram



Fermi surfaces

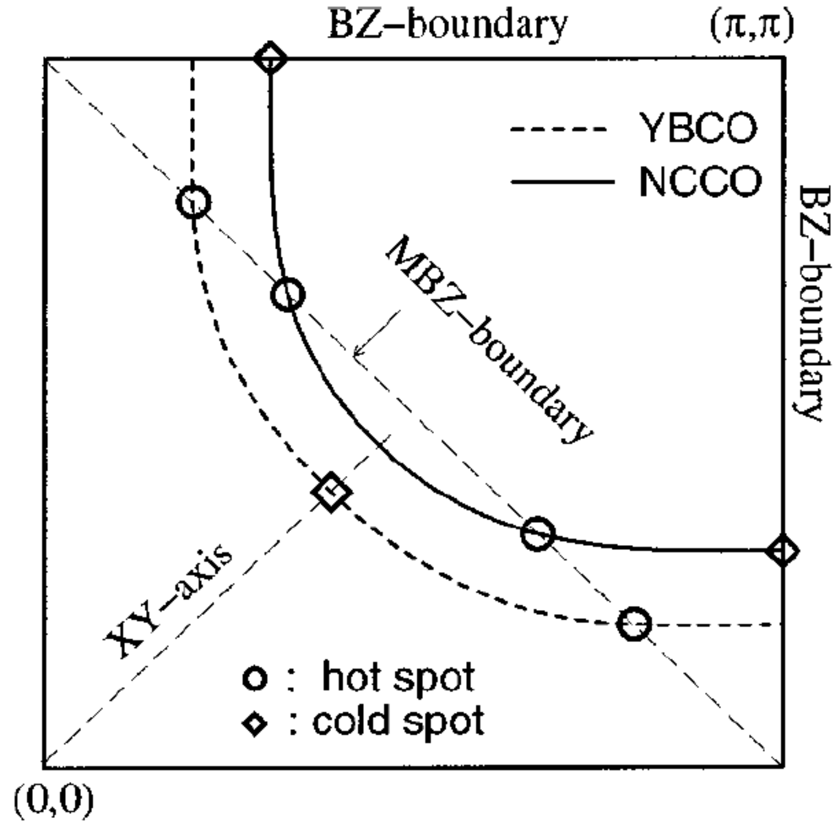


FIG. 4. The hot spots and the cold spots in YBCO and NCCO, respectively.

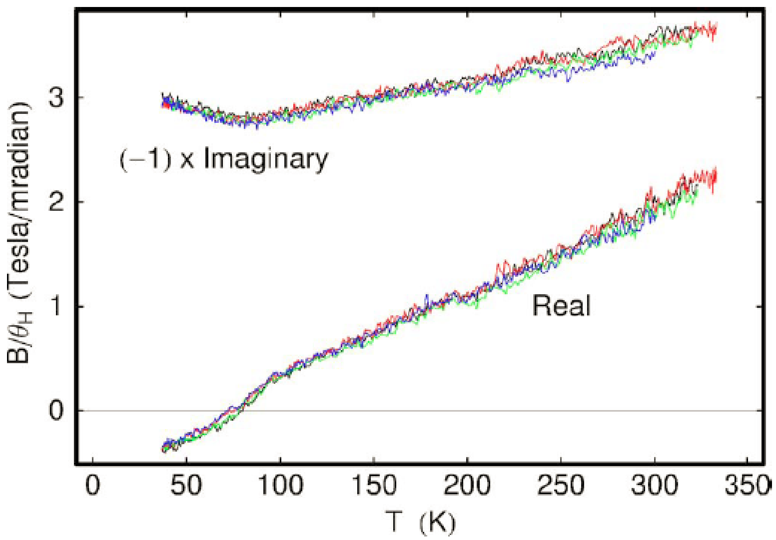
H. KONTANI, K. KANKI, AND K. UEDA PRB 59 (1999).

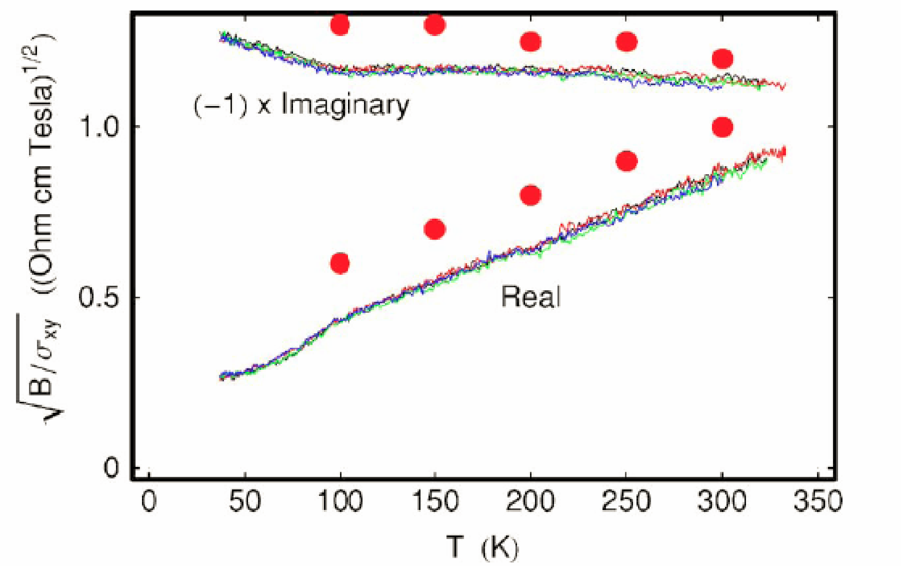
IR Hall Effect
in
Optimally doped Cuprates

Mid IR Hall in opt. Bi2212

Hall angle and Hall conductivity

ω=1000 cm^{-1}





$$\theta_H \equiv \frac{\sigma_{xy}}{\sigma_{xx}} = \frac{\omega_H}{\gamma - i\omega}, \qquad \omega_H \equiv \frac{eB}{m_H c} \qquad \sigma_{xy} = \frac{\omega_p^2}{4\pi}\frac{\omega_H}{(\gamma - i\omega)^2}$$

Schmadel, et al, PRB (2007)

Opt. Bi(2212): Far IR Hall

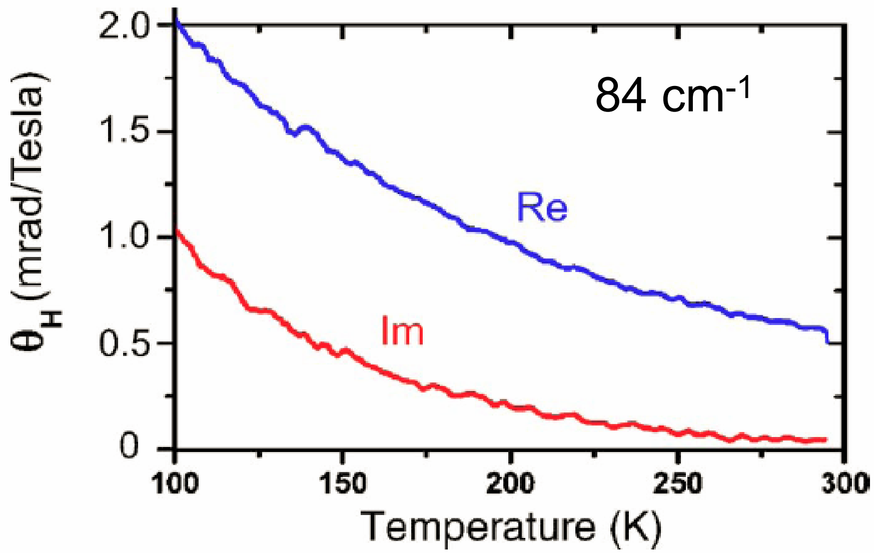


FIG. 3. (Color online) θ_H for optimally doped 2212 BSCCO versus temperature measured at 84 cm^{-1} averaged from five thermal scans and normalized to 1 T.

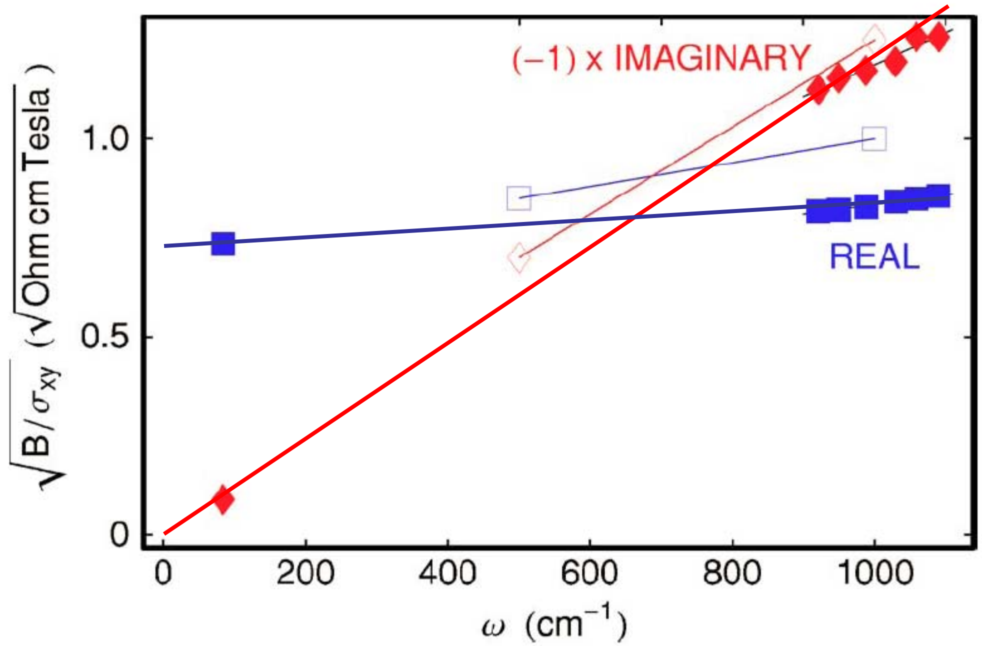
$$\theta_H = \frac{\omega_H}{\gamma - i\omega}$$

$\omega_H \sim 0.3\ cm^{-1}/T$

$\gamma_H \sim aT^{1.65}$

Schmadel, et al, PRB (2007)

Optimally doped Bi2212



Drude model

$$\sigma_{xy}^{-1/2} = \frac{2}{\omega_p}\sqrt{\frac{\pi}{\omega_H}}\left(\gamma - i\omega(1+\lambda)\right)$$

$\omega_H \sim$ 0.3 cm^{-1}/T

Nearly Drude: inelastic scattering is weak in σ_{xy}

Schmadel et al., PRB (2007)

Hall and longitudinal scattering

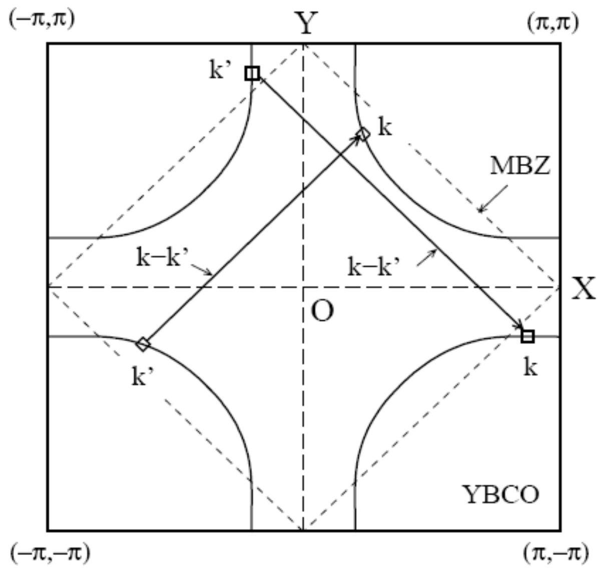
	Bi2212
γ^*_{xx}, cm^{-1}	750
γ^*_{xy}, cm^{-1}	360
m^*_{xx}	2.0

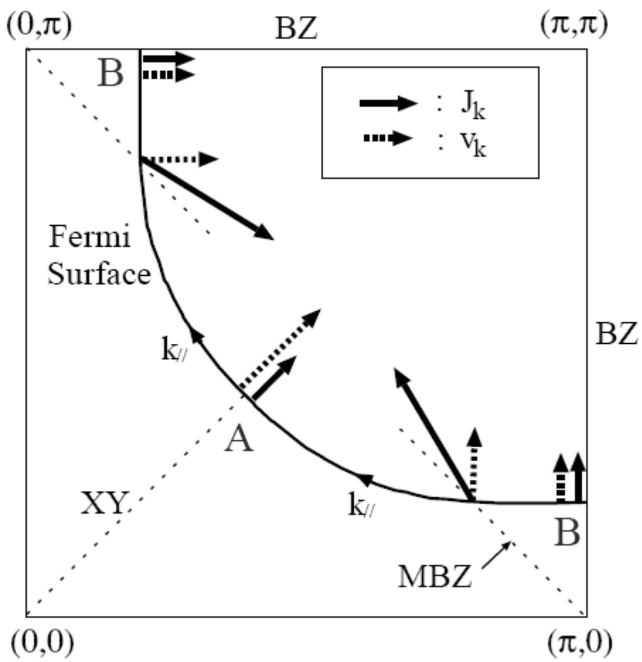
Mid IR

$$\gamma^* = \frac{\gamma}{1+\lambda} = \frac{\gamma}{m^* / m_b}$$

ω = 1000 cm^{-1}, T = 100 K

Schmadel et al., PRB (2007)





Kontani theory

vertex corrections to σ

Antiferromagnetic fluctuations →
FLEX approximation:

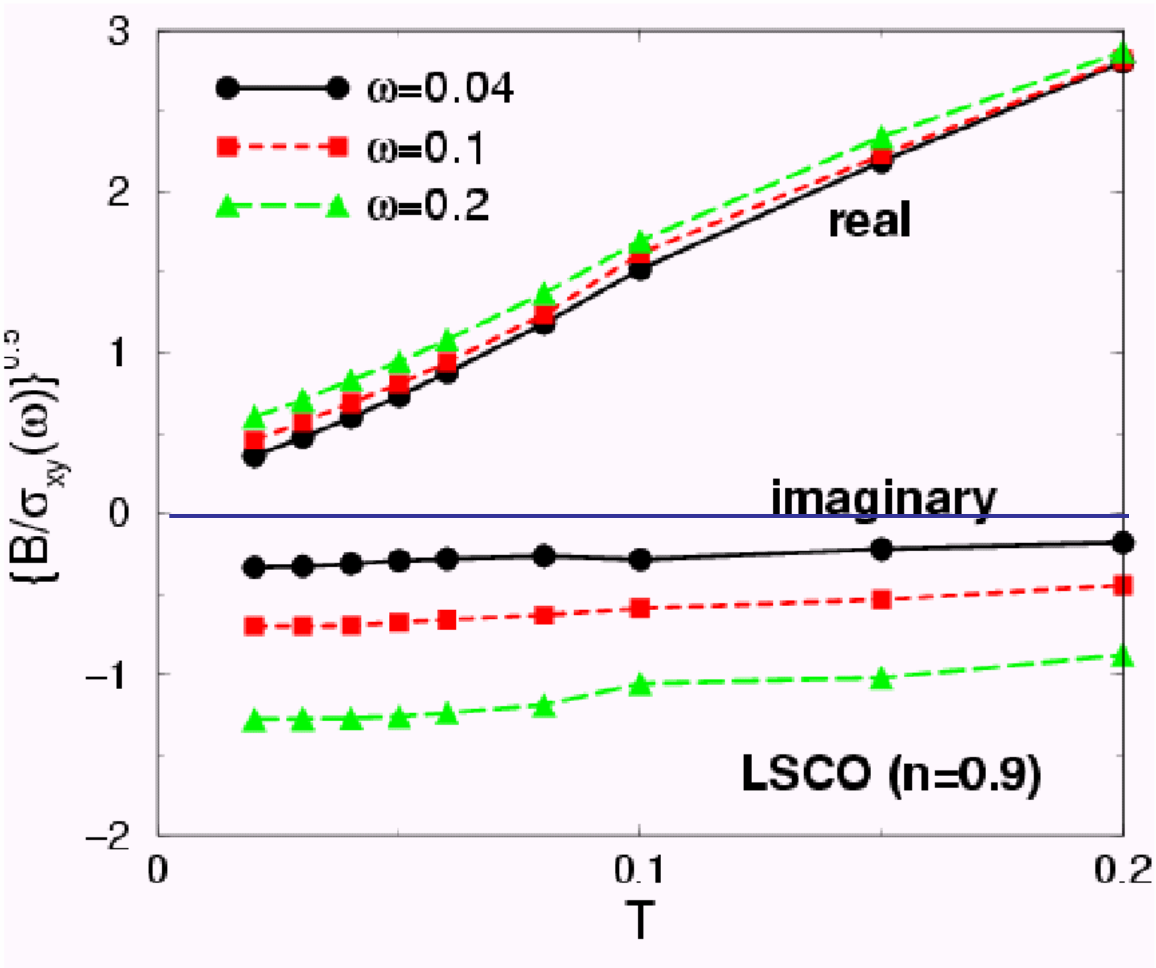
$$\chi^{s}_{\mathbf{k}}(\omega + i\delta) \approx \frac{\chi_Q}{1 + \xi^2(\mathbf{q} - \mathbf{Q})^2 - i\omega/\omega_{sf}},$$

where ξ is the AF correlation length, and Q is the nesting vector. Q = (π, π). $\chi_Q \sim 1/\omega_{sf} \sim \xi^2$, and $\xi^2 \sim T^{-1}$ in the FLEX approximation,

Conductivity calculated including Current vertex corrections

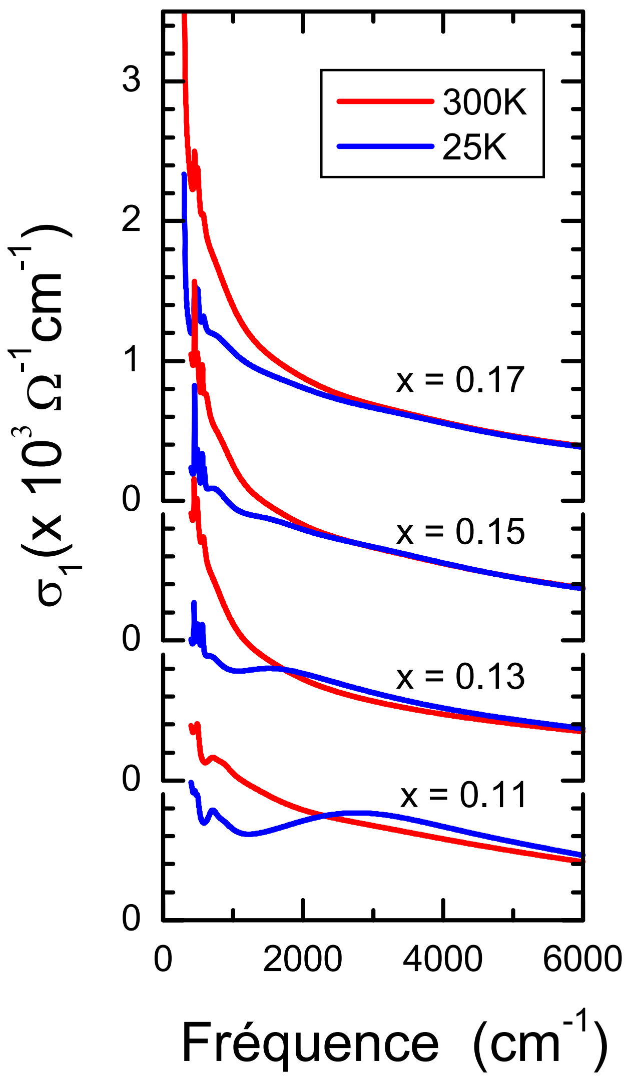
Kontani, J. Phys. Soc. Jpn. (2006)

Kontani calculation: σ_{xy}



$$\sqrt{\sigma_{xy}^{-1}} = \frac{\gamma - i\omega(1+\lambda)}{\sqrt{S_{xy}}}$$

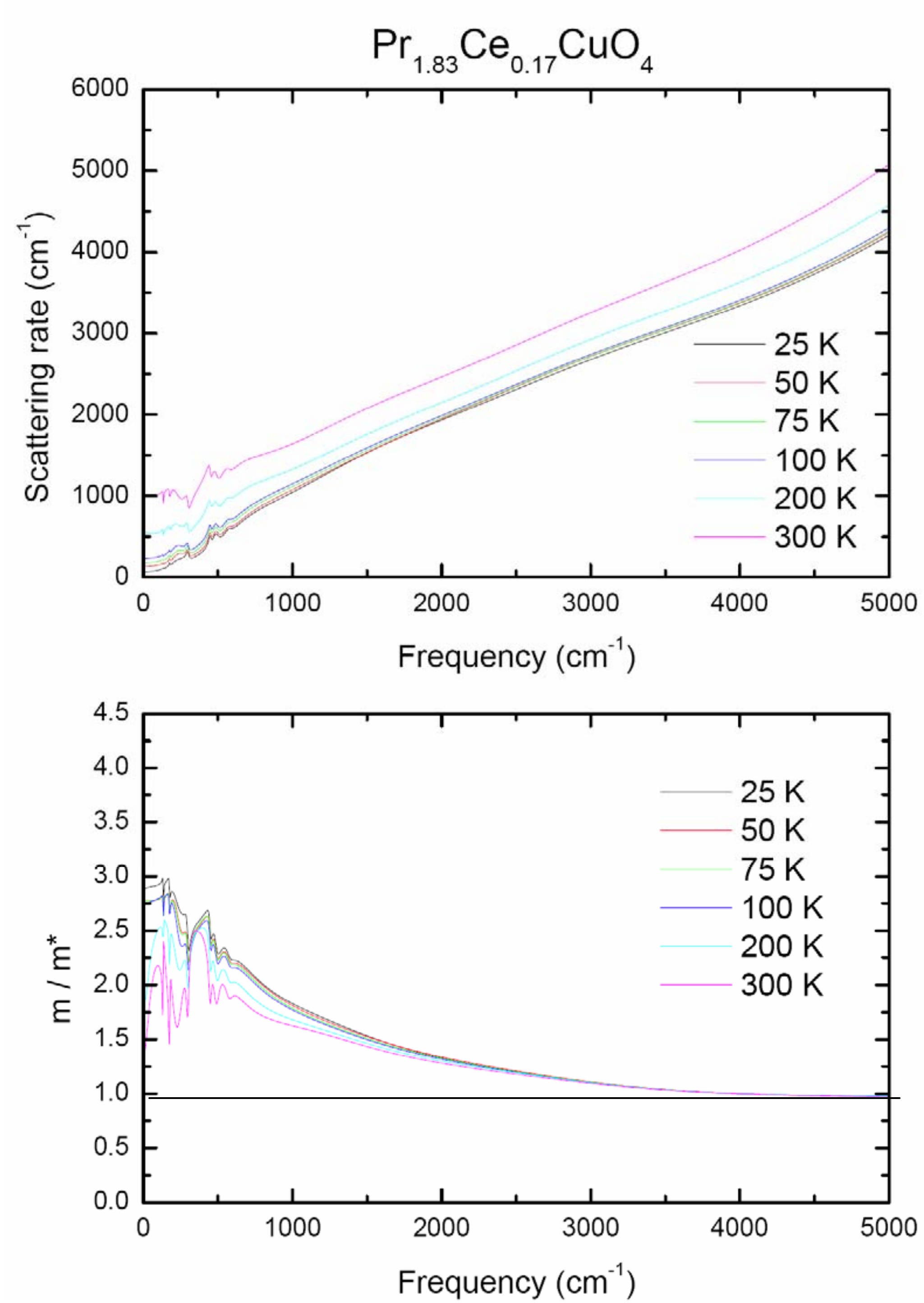
e-doped: Optics of $Pr_{2-x}Ce_xCO_4$



X=0.17
Extended
Drude analysis

$$\sigma_{xx}(\omega) = \frac{\omega_p^2 / 4\pi}{\gamma - i\omega(1+\lambda)}$$

A. Zimmers et al., Europhysics Lett. (2005)

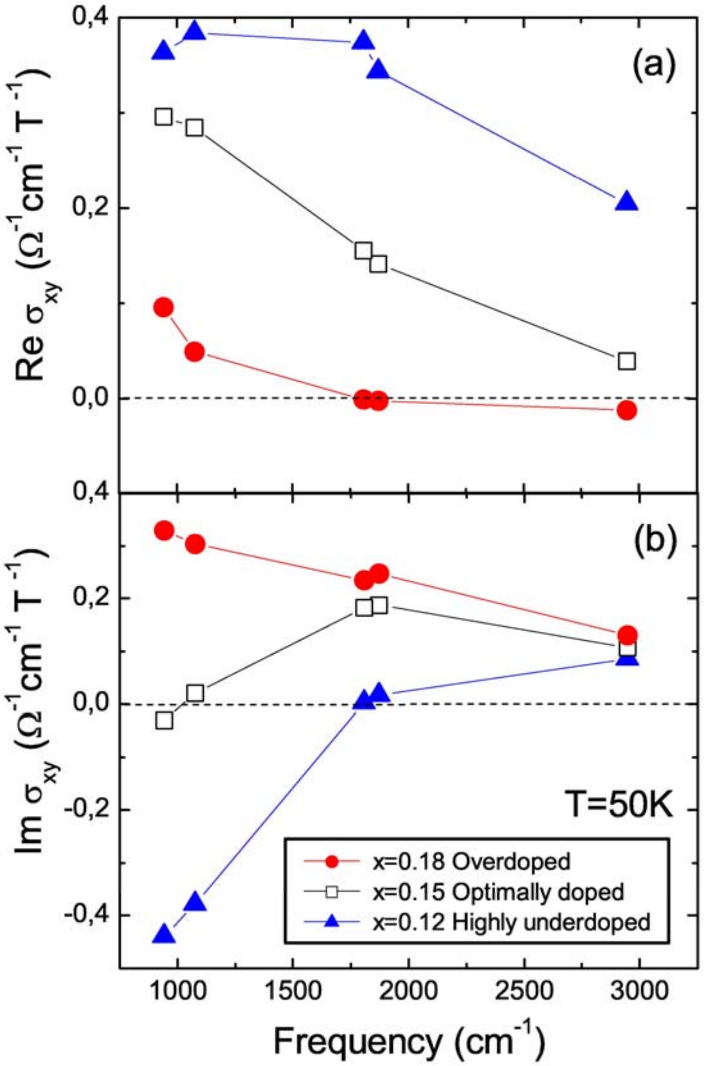


γ

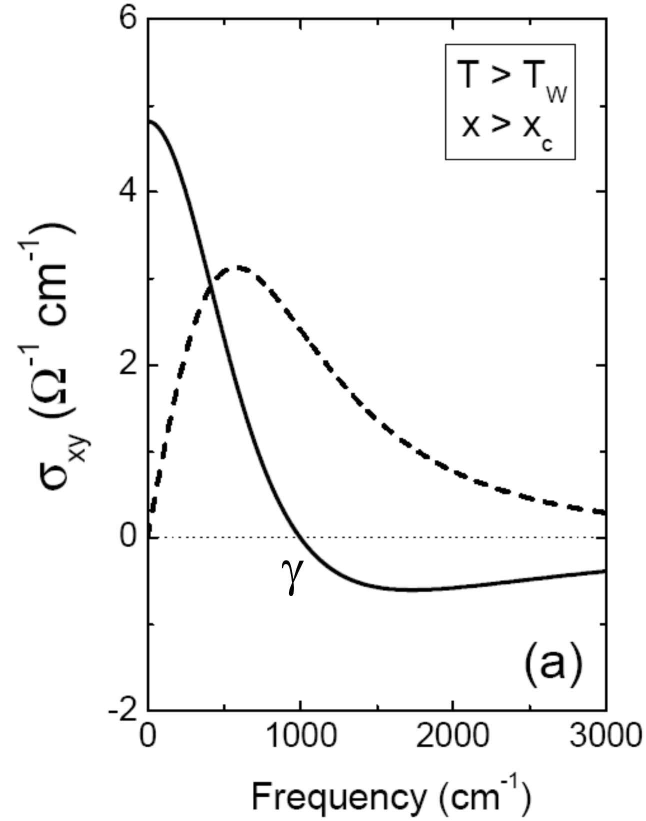
Zimmers
IR data

$1+\lambda$

e-doped: IR Hall in $Pr_{1.82}Ce_{0.18}CO_4$



Drude model



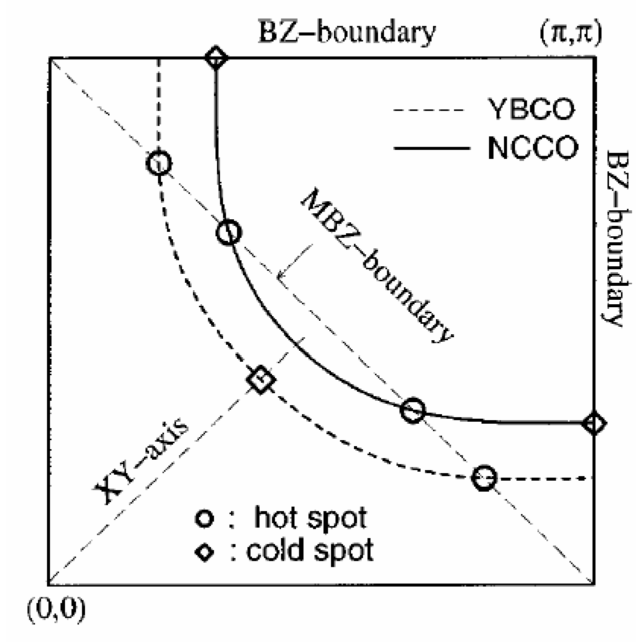
$$\sigma_{xy} = \frac{(ne^2/m^*)(eB/m^*c)}{(\gamma - i\omega)^2}$$

A. Zimmers et al., cond-mat/0510085

Hall and longitudinal scattering

ω = 1000 cm^{-1}, T = 100 K

	Bi2212 Opt.	PCCO X=0.18
γ_{xx}^{*}, cm^{-1}	750	750
γ_{xy}^{*}, cm^{-1}	360	1300



$$\gamma^{*} = \frac{\gamma}{1+\lambda} = \frac{\gamma}{m^{*}/m_b}$$

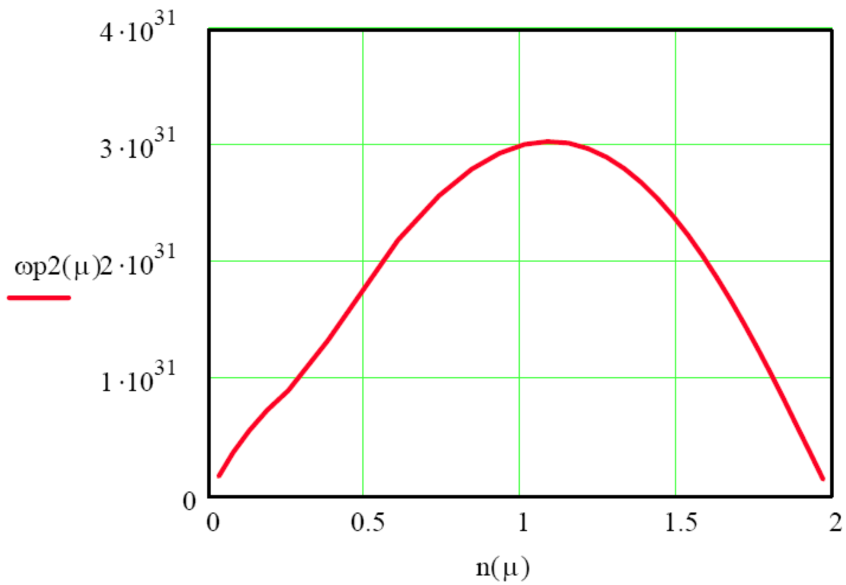
$$\sigma_{xy} = \frac{e^2}{hd}\frac{eB}{\hbar c}\oint \frac{dk \cdot v_k \times dv_k / dk}{(\gamma - i\omega)^2}$$

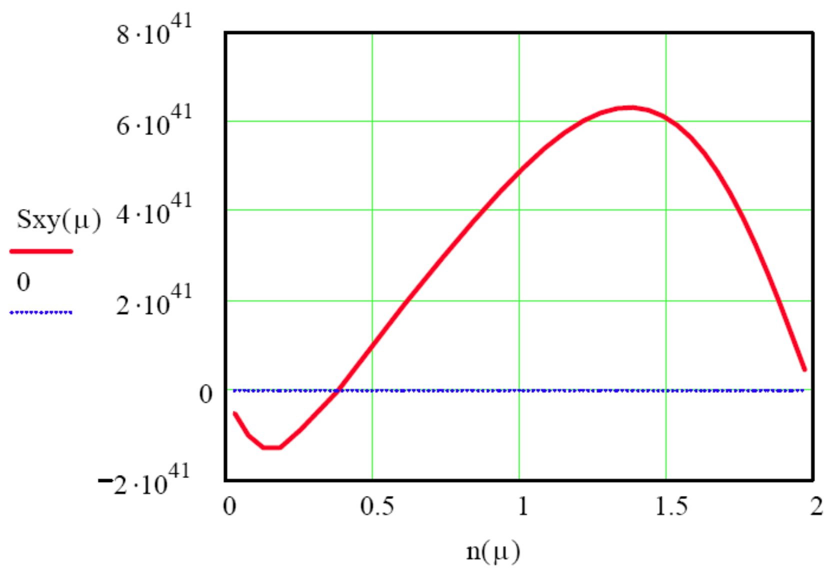
Schmadel et al., PRB (2007)

Zimmers et al., cond-mat/0510085

Conductivity Sum Rules: Cuprates

Band values





$$E(kx,ky) = -2t_1(\cos(k_x)+\cos(k_y)) +$$
$$4t_2\cos(k_x)\cos(k_y) - 2t_3(\cos(2k_x)+\cos(2k_y))$$
$$t_1 = 0.38 \text{ eV},\ t_2 = 0.32t_1,\ t_3 = 0.5t_2$$

$$S_{xx} \equiv \int_0^{\omega_C} \tfrac{2}{\pi}\mathrm{Re}(\sigma_{xx})d\omega \equiv \frac{e^2}{2c_0}\sum_k Tr(m_k^{-1})n_k$$

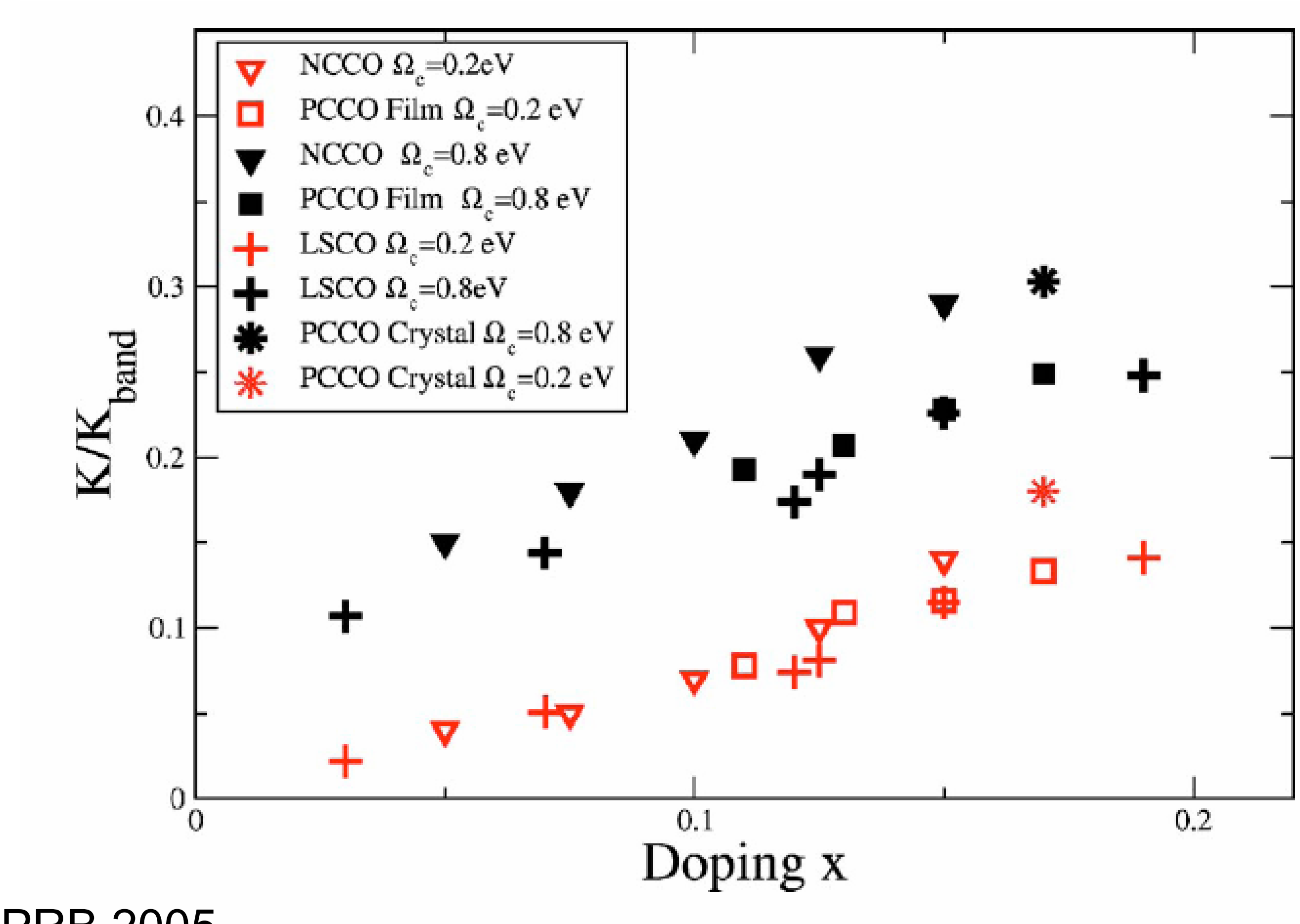
$$S_{xy} \equiv \int_0^{\omega_C} \tfrac{2}{\pi}\mathrm{Im}(\sigma_{xy})\omega d\omega \equiv \frac{e^3B}{2c_0}\sum_k \det(m_k^{-1})n_k$$

$$S_\theta \equiv \int_0^{\omega_C} \tfrac{2}{\pi}\mathrm{Re}(\theta_H)d\omega = \frac{S_{xy}}{S_{xx}} \equiv \omega_H$$

σ_{xx} partial sum rule

LSCO, NCCO, PCCO

$$S_{xx} \equiv \int_0^{\Omega_C} \frac{2}{\pi} \mathrm{Re}(\sigma) d\omega$$



Millis, PRB 2005

Conductivity Sum Rules

$$S_{xx} \equiv \int_0^{\Omega_C} \frac{2}{\pi} \mathrm{Re}(\sigma) d\omega$$

$$S_{xy} \equiv \int_0^{\Omega_C} \frac{2}{\pi} \mathrm{Im}(\sigma_H) \omega d\omega$$

$$S_{xx}^{band} = \frac{e^2}{2} \sum_k Tr(m_k^{-1}) n_k \cong \frac{ne^2}{m^*}$$

$$S_{xy}^{band} = e^3 B \sum_k \det(m_k^{-1}) n_k \cong \frac{ne^2}{m^*} \frac{eB}{m^* c}$$

Bi2212 opt. doped

$$\frac{S_{xx}^{\exp}}{S_{xx}^{band}} \simeq 0.33$$

$$\frac{S_{xy}^{\exp}}{S_{xy}^{band}} \simeq 0.09$$

Schmadel et al.,
PRB (2007)

PCCO x=0.18

$$\frac{S_{xx}^{\exp}}{S_{xx}^{band}} \simeq 0.25$$

$$\frac{S_{xy}^{\exp}}{S_{xy}^{band}} \simeq 0.09$$

Zimmers et al.,
cond-mat/0510085

Conclusions on Optimally doped Cuprates

1. $\gamma_{xy} \neq \gamma_{xx}$ vertex corrections

2. $\gamma_{xy}^{PCCO} > \gamma_{xx} > \gamma_{xy}^{Bi2212}$ Hot spots

3 $\frac{S_{xy}}{S_{xy}^{band}} < \frac{S_{xx}}{S_{xx}^{band}} < 1$ Mott physics:
e* or m*?

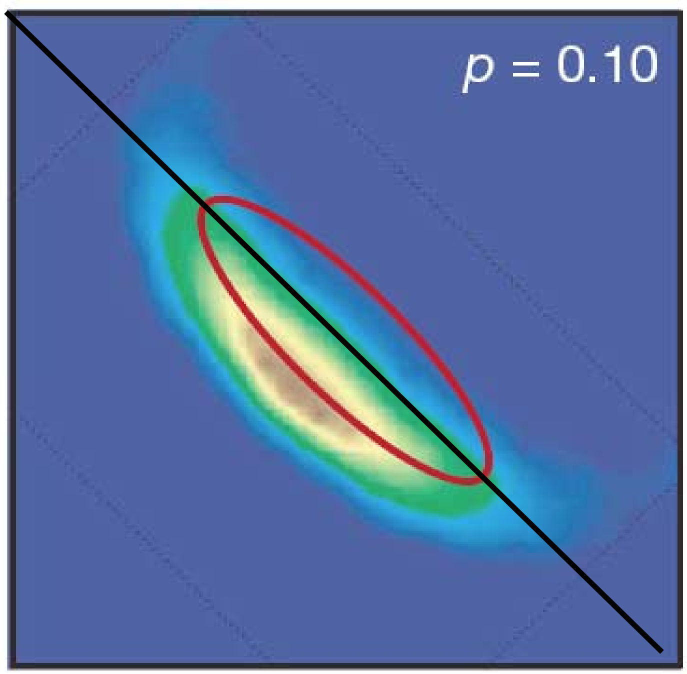
$$S_{xx} \propto \frac{e^2}{m}, \quad S_{xy} \propto \frac{e^3}{m^2}$$

IR Hall Effect
in
Underdoped Cuprates

ARPES Fermi Surface

Hole doped

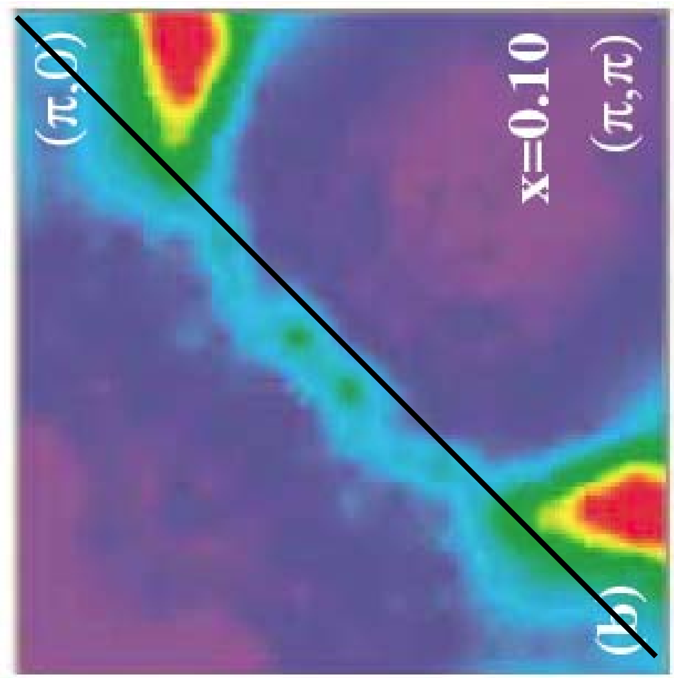
(π,π)



Γ

K. Shen, Science (2005)

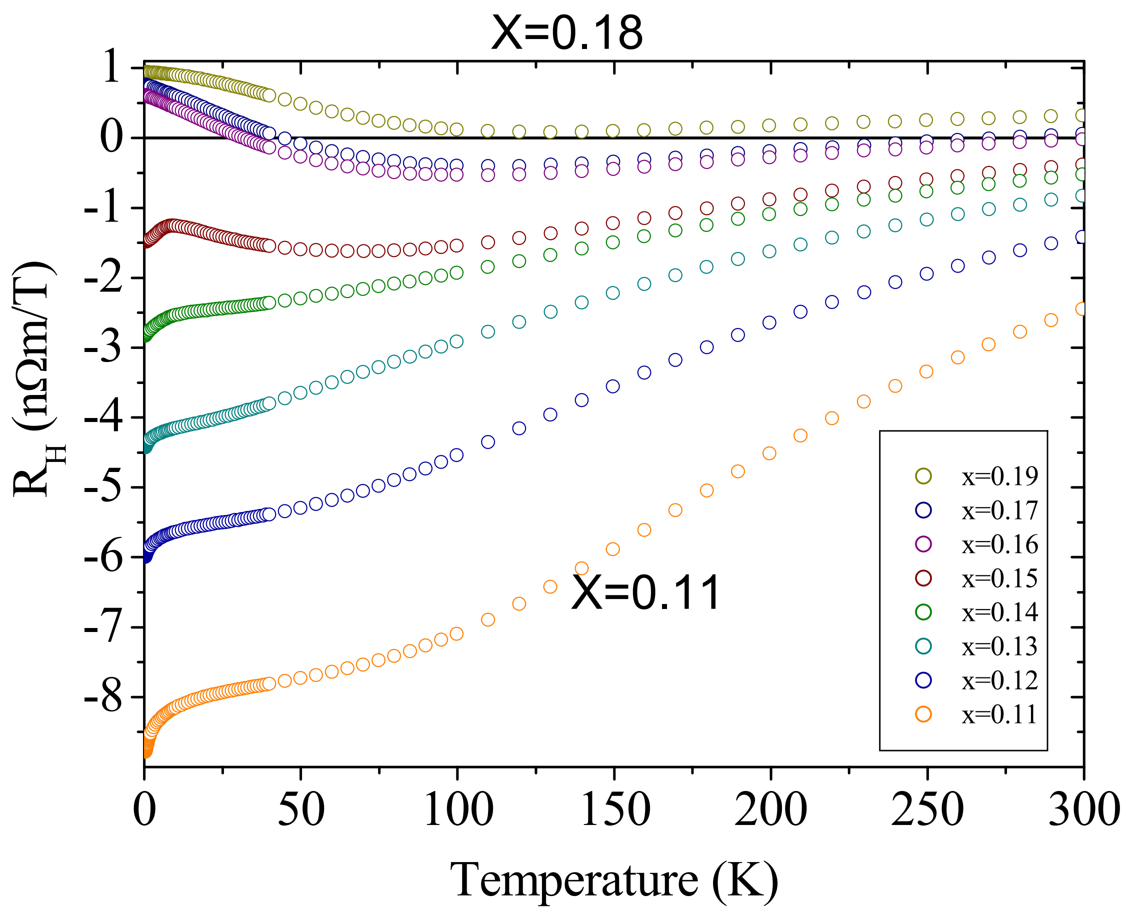
Electron doped



Γ

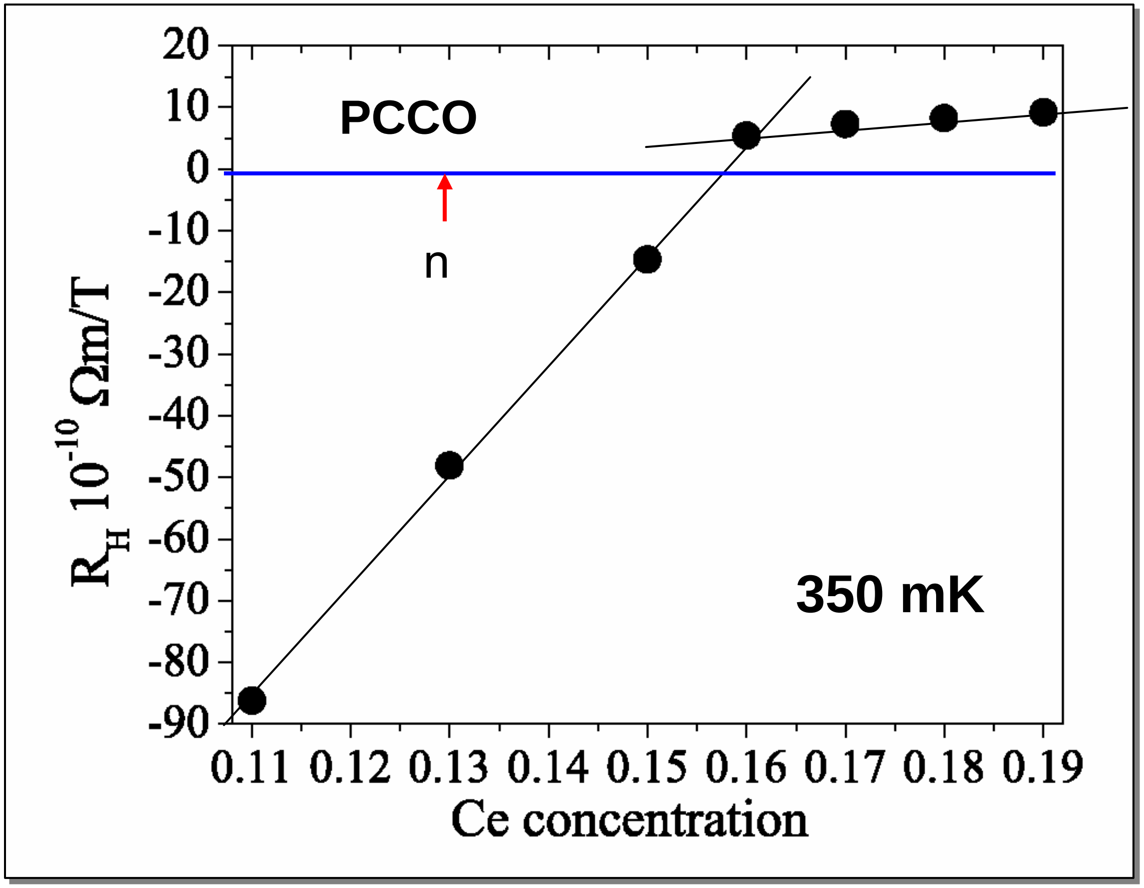
Armitage *et al.*, PRL (2002)

$Pr_{2-x}Ce_xCuO_4$ DC R_H



Y. Dagan *et al.*, Phy. Rev. Lett. **92**, 167001 (2004).

Hall coefficient $Pr_{2-x}Ce_xCuO_4$

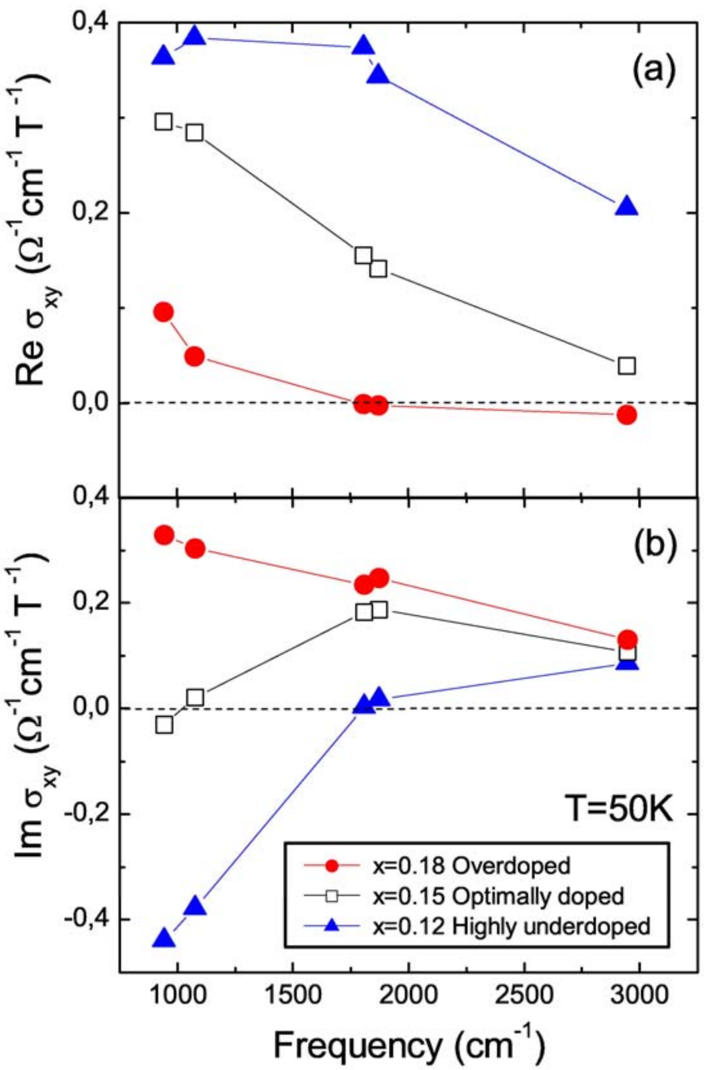


Quantum critical point?

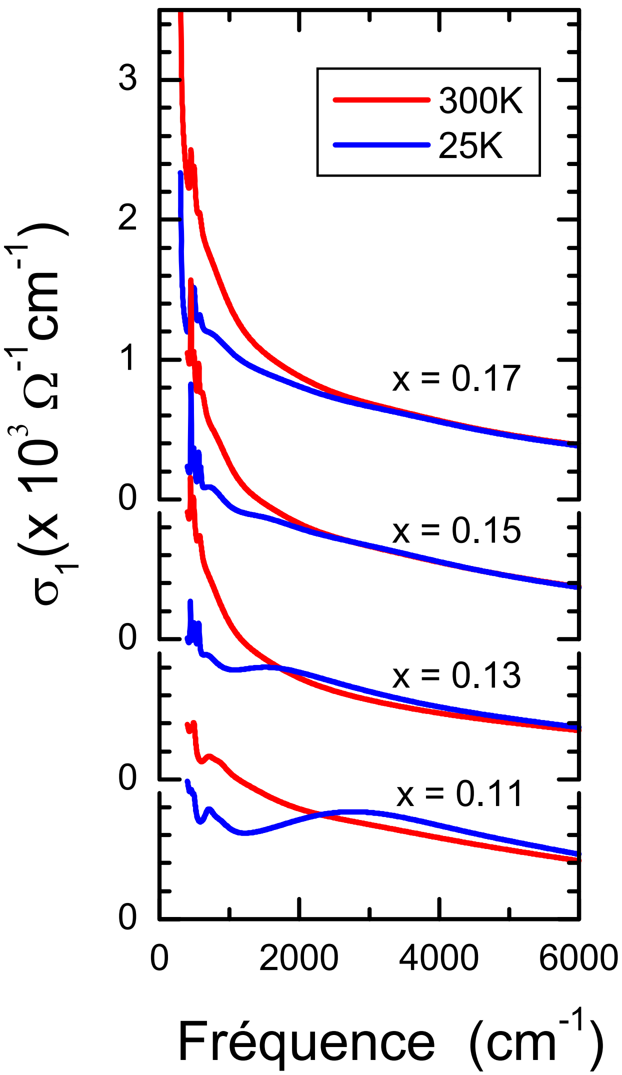
Hall - Dagan et al., PRL 92 (2004)

Neutrons - Motoyama, et al., Nature (2007)

e-doped: IR Hall in $Pr_{2-x}Ce_xCO_4$

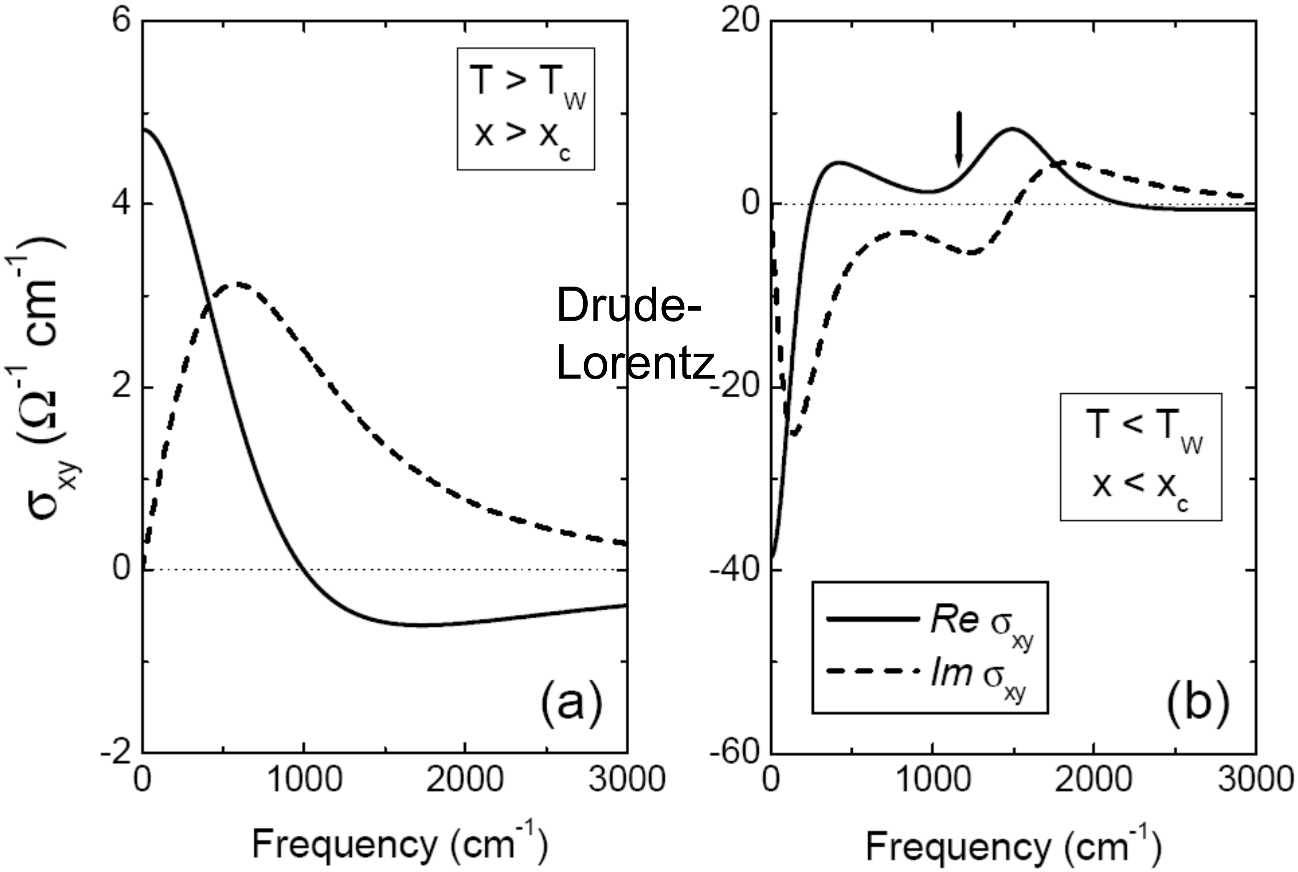


A. Zimmers et al., cond-mat/0510085



A. Zimmers et al., Europhysics Lett. (2005)

σ_{xy} in SDW: Drude-Lorentz model

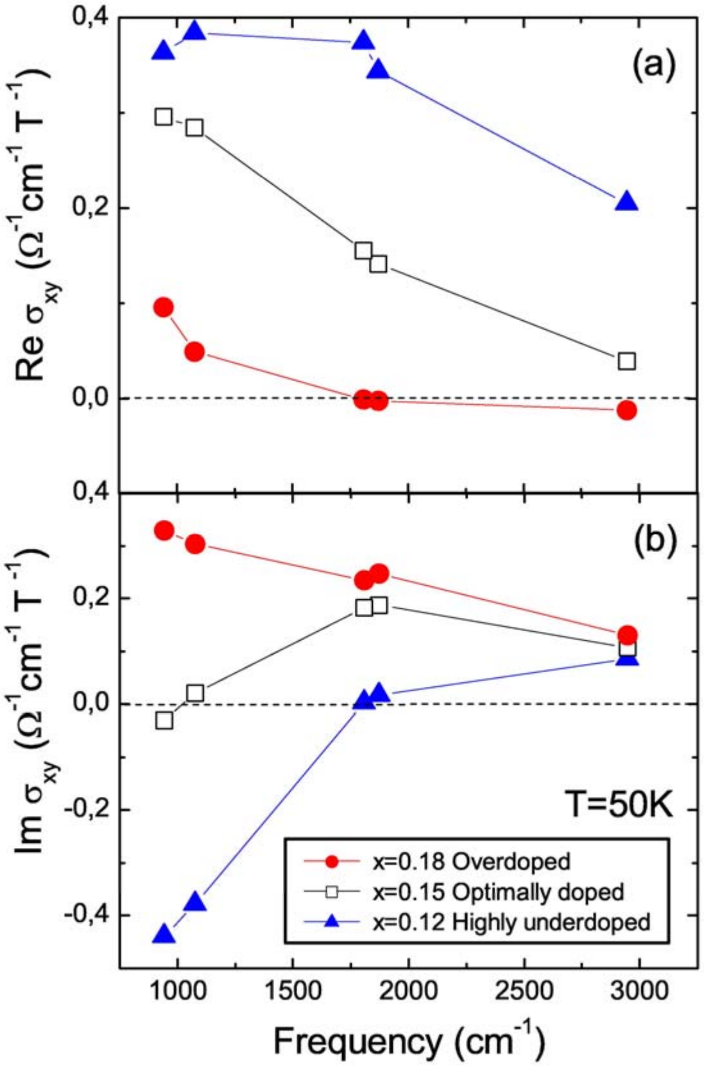


Drude-
Lorentz

Drude

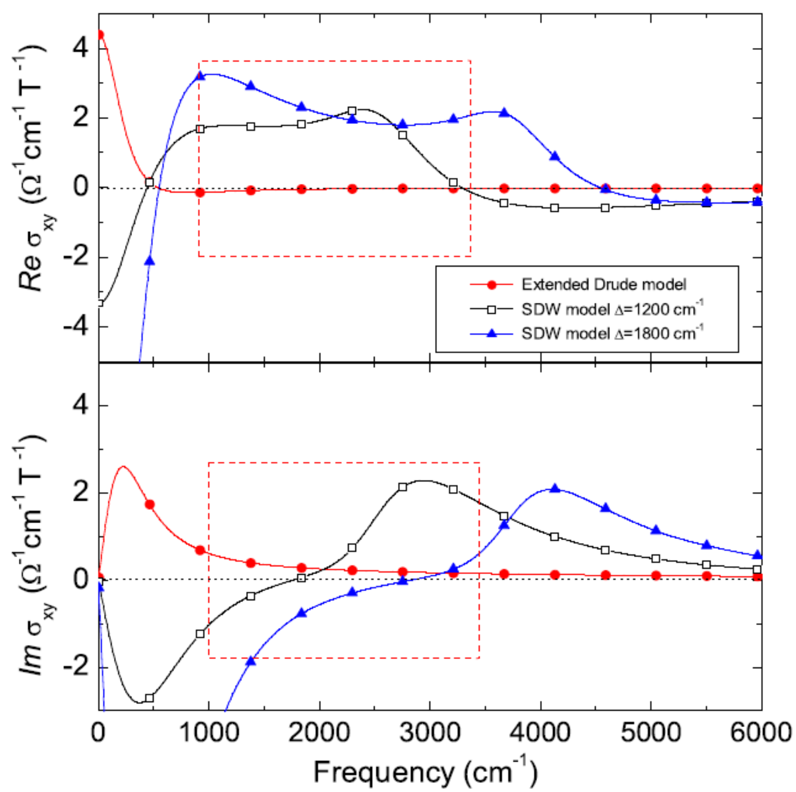
$$\sigma_{xy} = \frac{(ne^2/m^*)(eB/m^*c)}{(\gamma - i\omega)^2}$$

Mid IR σ_H for PCCO vs. x



A. Zimmers et al., cond-mat/0510085

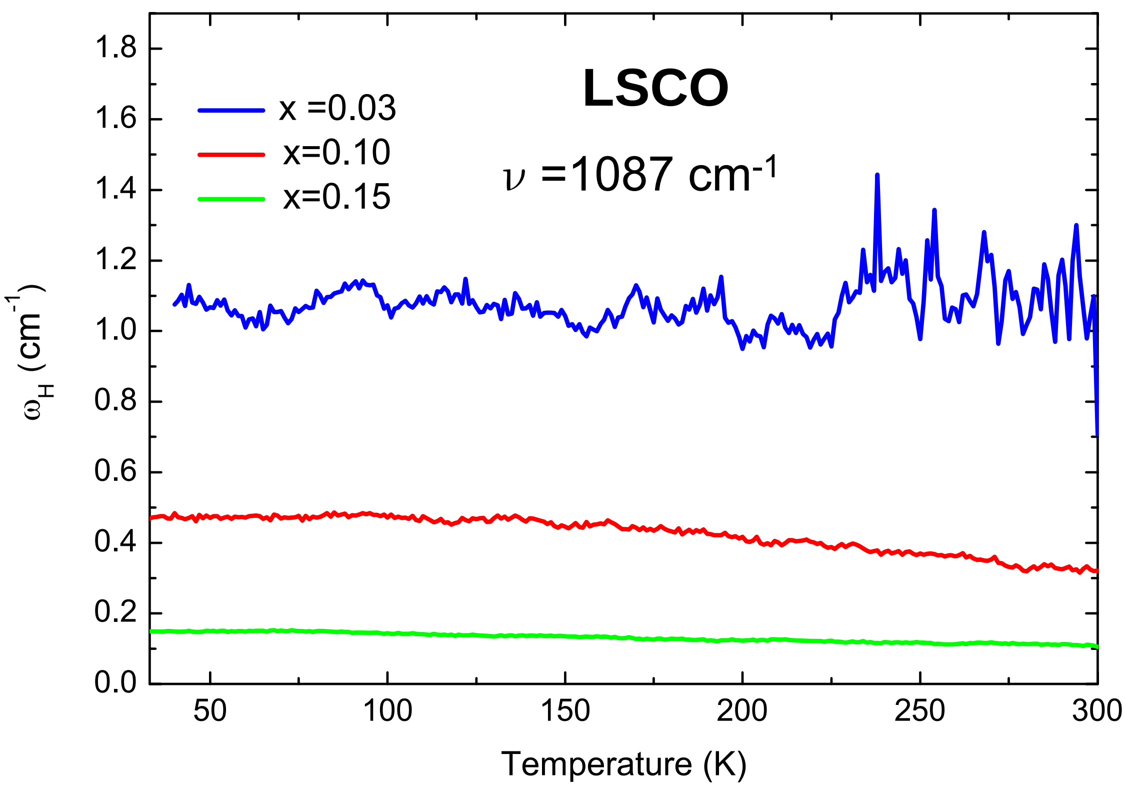
SDW model: J. Lin and A. Millis

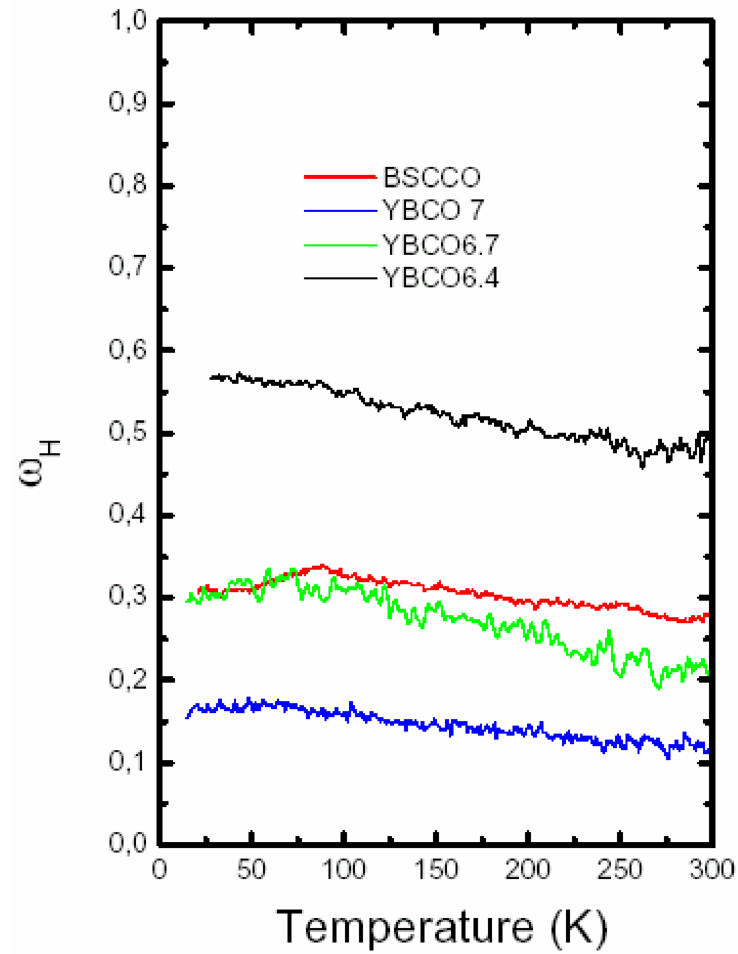


x	2Δ(meV)
0.11	450
0.15	300
0.18	0

Underdoped h-cuprates: Hall Frequency: ω_H

$$\theta_H^{-1} = \frac{\gamma_H - i\omega}{\omega_H}, \qquad \omega_H \equiv \frac{eB}{m_H c}$$



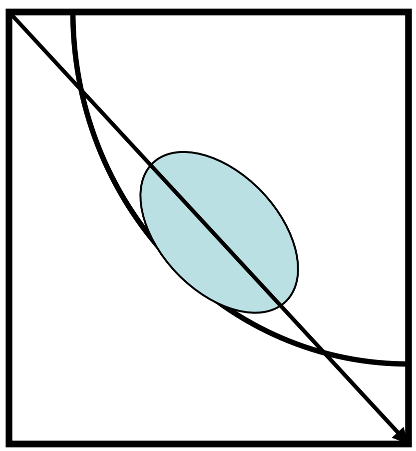


L. Shi, et al., cond-mat/0510794

L. Rigal et al., PRL (04)

Fermi Pocket Hall Frequency

Partially gapped Fermi surface



$$\theta_H = \frac{\sigma_{xy}}{\sigma_{xx}} \cong \frac{\omega_H}{\gamma - i\omega} \qquad \omega_H \equiv \frac{eB}{m_H c}$$

$$\omega_H = \frac{eB}{\hbar c^2} \frac{\oint dk \bullet v \times dv/dk}{\oint dk\,|v|} = \frac{eB}{\hbar c^2} \frac{\oint dv \times v}{\oint dk\,|v|}$$

Chakravarty et al., Phys. Rev. (2004).

Konatani theory also predicts increase in ω_H

Taillefer et al., Nature (2007)

Kanigel et al., Nature Physics, (2006)

Conclusions

Underdoped cuprates

1. e-doped: SDW gap evidence in σ_{xx} and σ_{xy}

   2Δ ~300 meV- 450 meV

2. h-doped: evidence for Fermi surface reconstruction:

   if gap $\rightarrow$ 2Δ > 500 meV

3. Is the reconstructed FS real, fluctuating or only effective?

4. Hall conductivity spectroscopy gives important information on strongly interacting electron materials