Fluctuation phenomena near the magnetic-field-tuned superconducting transition

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References:

- PRB **63**, 174506 (2001), VG and A. I. Larkin
- PRB **67**, 144501 (2003), VG and S.D. Sarma
- PRL **95**, 077002 (2005), VG, G. Refael, M. Fisher, and T. Senthil

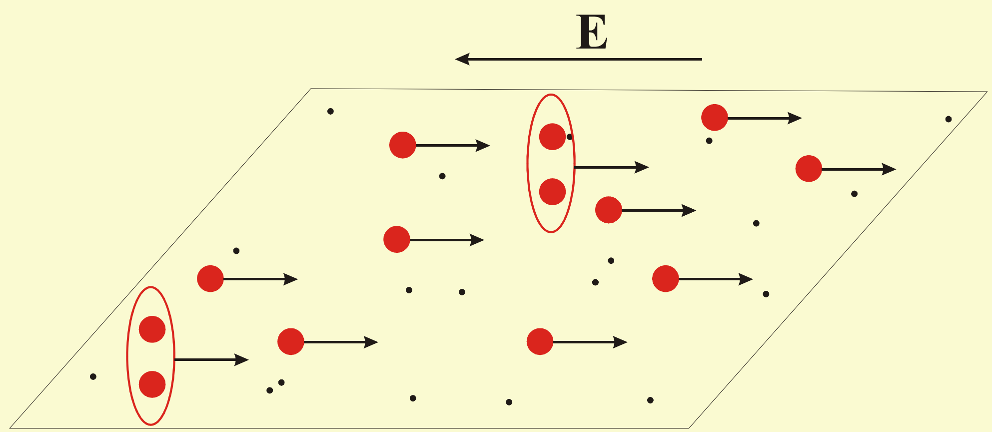
Outline

- Introduction to Aslamazov-Larkin theory
- Quantum fluctuations near $H_{c2}(0)$
- Negative fluctuation magnetoresistance, $\delta\sigma \propto -\ln\frac{1}{(H-H_{c2})}$. Higher order diagrams.
- Bosonic and fermionic mechanisms of the negative MR
- Unusual metallic phase in disordered SC films and its possible exotic scenario: Fermionic vortices (theory formally equivalent to Ioffe-Larkin uniform RVB spin liquid).

Aslamazov-Larkin theory

Superconducting fluctuations

- At high temperatures, the mean field density of Cooper pairs is zero, $\langle \Delta \rangle = 0$; but fluctuating Cooper pairs can appear even above the transition. The order parameter fluctuates, $\langle \Delta \Delta^* \rangle \neq 0$.



Effects of fluctuations on transport:

- Direct conductivity of fluctuations (Aslamazov-Larkin, 1968)
- Decrease in the electron density (DOS)
- Scattering off the fluctuations (Maki-Thomson, 1970)

Fluctuation conductivity near T_c

- Drude conductivity of a normal metal

$$\sigma_0 = \frac{n_e e^2 \tau}{m}$$

The number of Cooper pairs and their lifetime are singular quantities near the transition $\tau_{cp}(q) \propto N_{cp}(q) \propto \left(T - T_c + aq^2\right)^{-1}$.

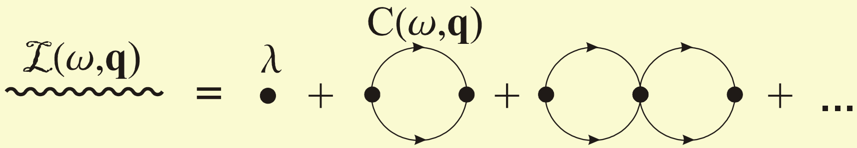
- Aslamazov-Larkin correction contains two singularities:

$$\delta\sigma_{AL} \propto \int d^2\mathbf{q} \frac{N_{cp}(\mathbf{q})(2e)^2 \tau_{cp}(\mathbf{q})}{2m} \propto \boxed{\frac{T_c}{T - T_c}}$$

- Density of states contains only one singularity:

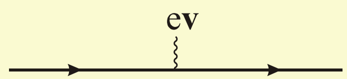
$$\delta\sigma_{DOS} \propto -\int d^2\mathbf{q} \frac{2N_{cp}(\mathbf{q}) e^2 \tau_e}{m} \propto -\ln\left[\frac{T_c}{T - T_c}\right]$$

Naïve derivation of diagrammatics for superconducting fluctuations

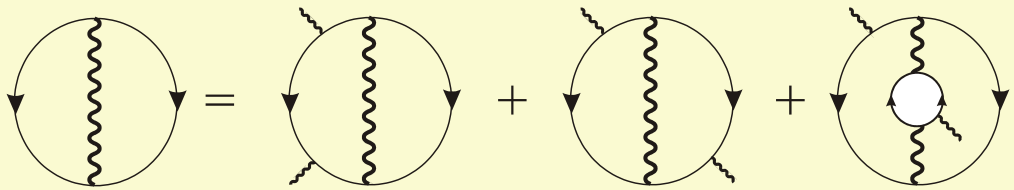


$$j_\alpha = \sigma_{\alpha\beta} E_\beta = -\frac{1}{i\omega}\sigma_{\alpha\beta} A_\beta, \qquad \sigma_{\alpha\beta} \propto \frac{1}{i\omega}\frac{\delta^2\Omega}{\delta A_\alpha A_\beta}\bigg|_{\mathbf{A}\to\mathbf{0}}$$

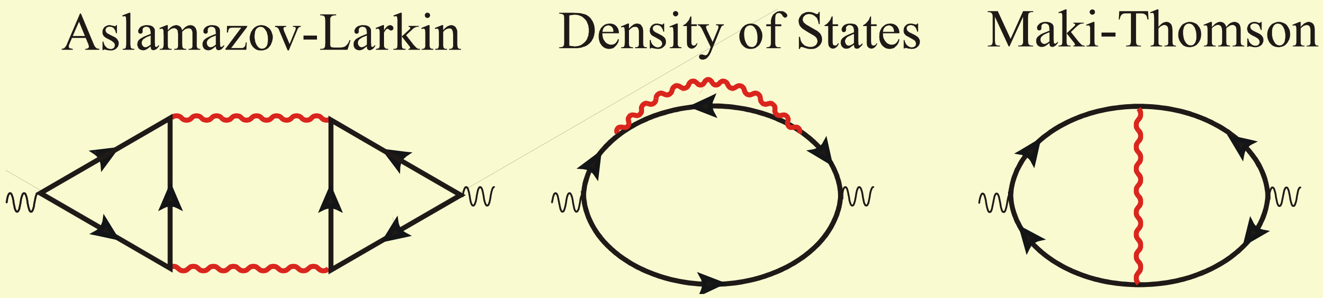
$$\frac{\delta G}{\delta \mathbf{A}}\bigg|_{\mathbf{A}\to\mathbf{0}} \propto \frac{\delta}{\delta \mathbf{A}}\frac{1}{i\varepsilon - (\mathbf{p} - e\mathbf{A})^2/(2m)} = G e\mathbf{v} G =$$



$$\frac{\delta^2\Omega}{\delta A_\alpha A_\beta} = \frac{\delta^2}{\delta A_\alpha A_\beta}$$



Alsamzaov-Larkin result



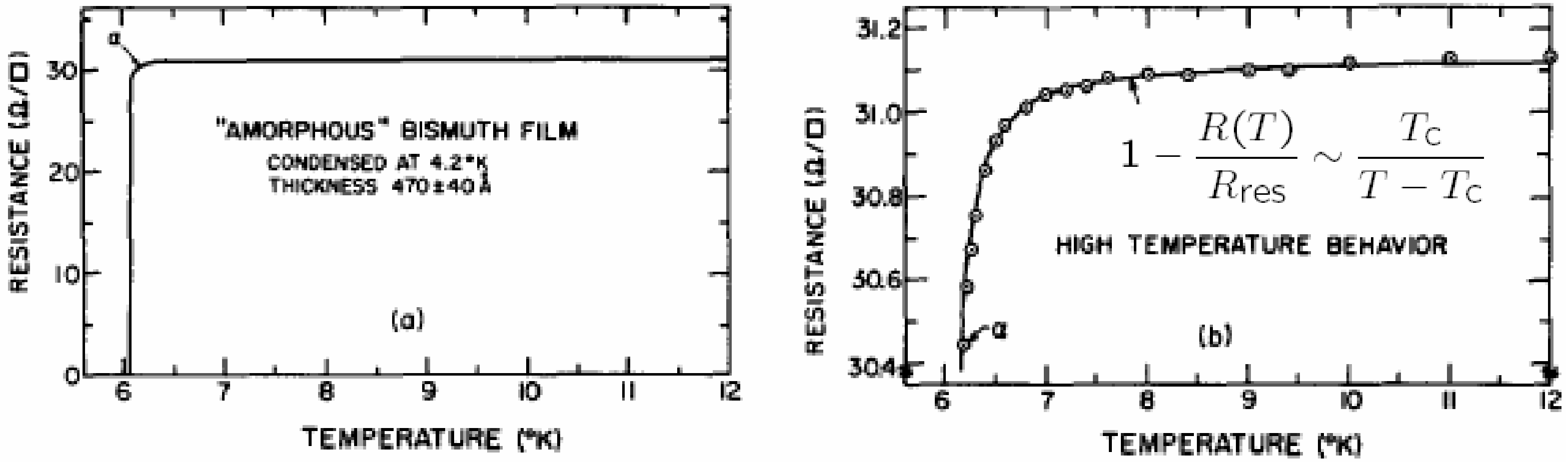
$$\sim\sim\sim\sim = \mathcal{L}(\omega_n, q) \propto \frac{1}{\gamma|\omega_n| + aq^2 + (T - T_c)}$$

AL result in two dimensions

$$\boxed{\delta\sigma = \frac{e^2}{16\hbar} \frac{T_c}{T - T_c}}$$

Experimental verification of the Alsamazov-Larkin theory

R. E. Glover, Phys. Lett. **25A**, 542 (1967)

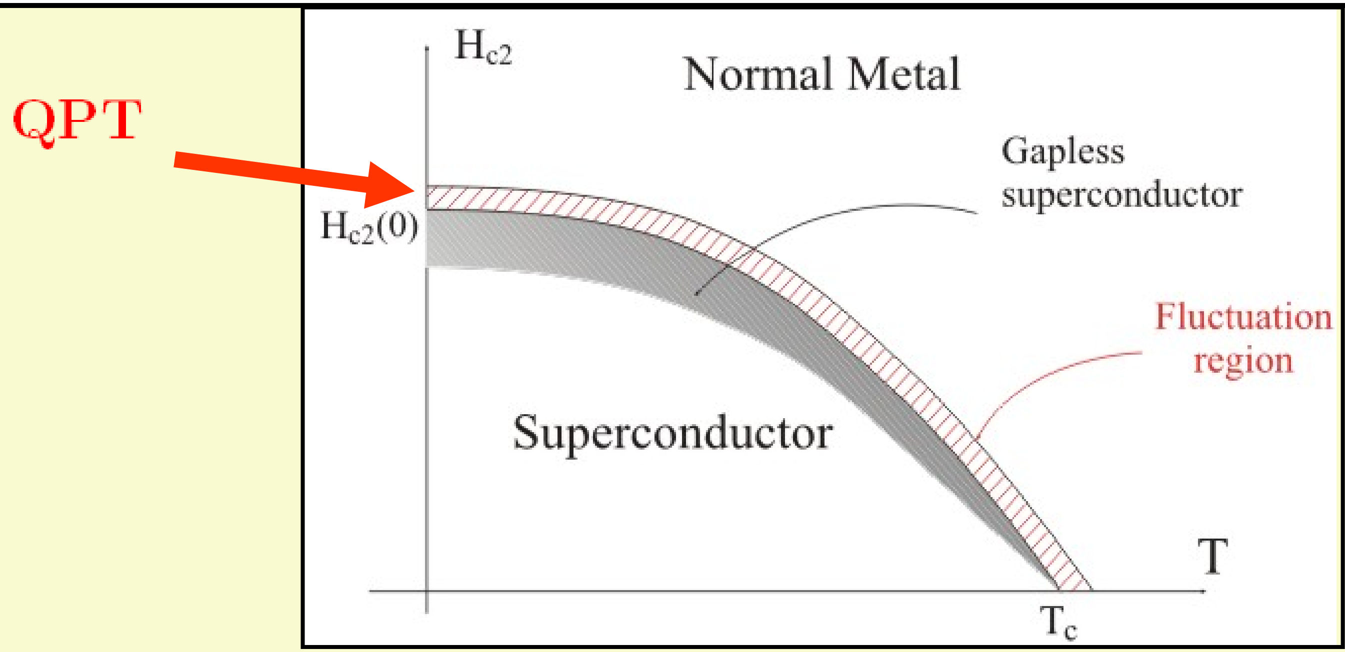


The region where the mean-field theory breaks down near the transition (Ginzburg region):

$$\frac{T - T_C}{T_C} < \text{Gi} = \begin{cases} T_C/E_F, & \text{clean system} \\ (E_F\tau)^{-1}, & \text{disordered system} \end{cases}$$

Quantum SC fluctuations near $H_{c2}(0)$

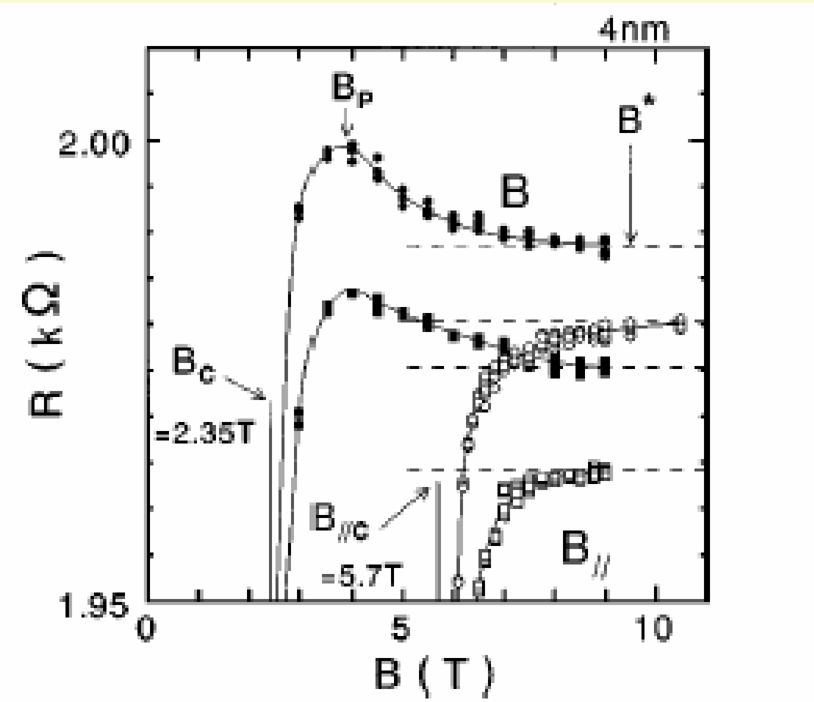
Suppression of T_c

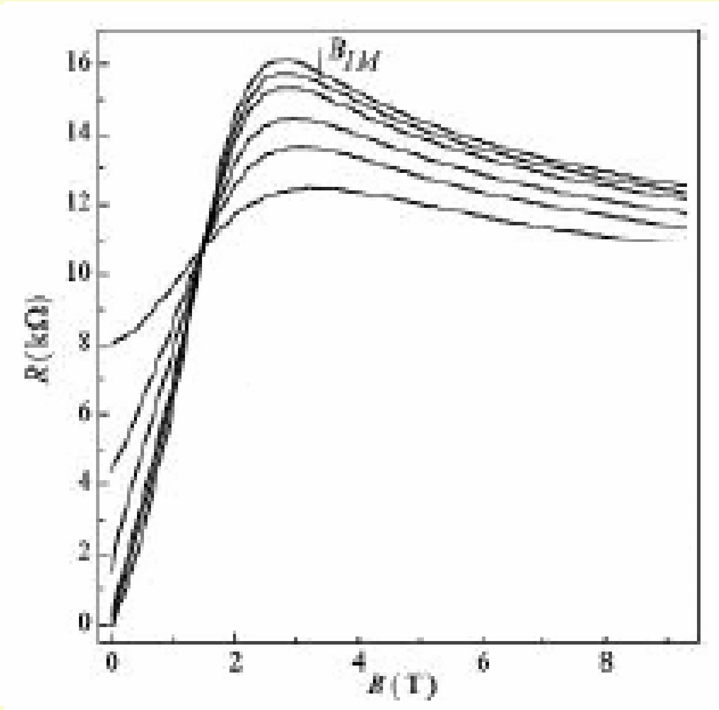


- Strong fields; Gapless excitations (de Gennes):
- Weak or zero field; Gap in the spectrum (BCS):

$$E(\mathbf{p}) = \sqrt{\xi^2(\mathbf{p}) + \Delta^2}$$

Unusual fluctuations near $H_{c2}(0)$





Mo_xSi_{1-x}: S. Okuma *et al.* (1998)

TiN_x: V. F. Gantmakher *at al.* (2002)

Ginzburg-Landau expansion in strong fields

The usual Ginzburg-Landau theory is based on a long wave-length expansion, but in strong fields, $(\mathbf{p} - e\mathbf{A})^2$ is not in any sense small.

$$S[\Delta] = \int_{1,2} \Delta_1^* A(1,2)\Delta_2 + \int_{1,2,3,4} \Delta_1^*\Delta_2^* B(1,2,3,4)\Delta_3\Delta_4$$

$\hat{A}$ is a non-local integral operator, but it is diagonal in the Landau basis (Helfand & Wethamer):

$$\langle n \left| \hat{A} \right| m \rangle = A_n \delta_{n,m}$$

The transition point is determined by the condition

$$\mathcal{L}_{n=0}(\omega = 0) = A_0^{-1} = \infty$$

Quantum fluctuation propagator and diagrammatics near $H_{c2}(0)$

Fluct. propagator is a diagonal operator in the Landau basis, $\widehat{\mathcal{L}}(\omega)$.

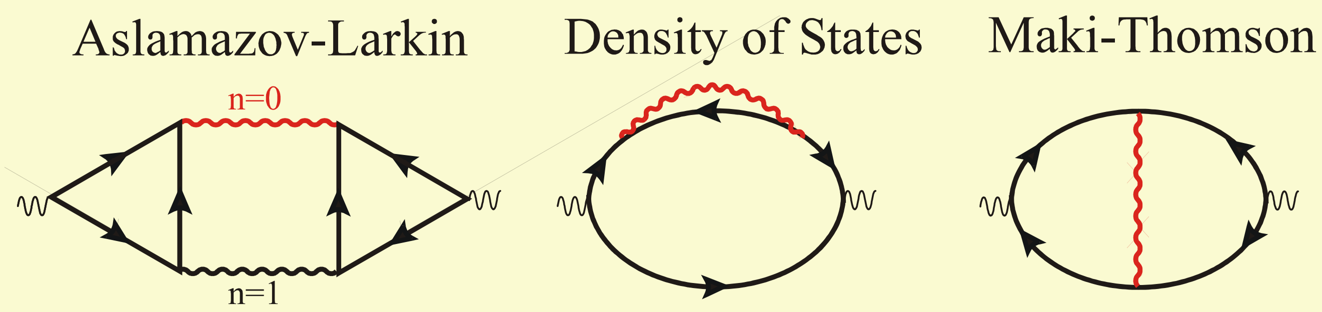
- At the LLL its matrix element is singular

$$\mathcal{L}_0(\omega) \propto \frac{1}{\gamma|\omega| + \left[H - H_{c2}(0)\right]/H}$$

- At all higher levels, $\mathcal{L}_{n>0}(\omega = 0) = \text{const} < \infty$

The conductivity has velocity operators in the vertices

$$\langle n \left| \widehat{v}_\alpha \right| m \rangle \propto \delta_{n+1,m}, \text{ thus}$$



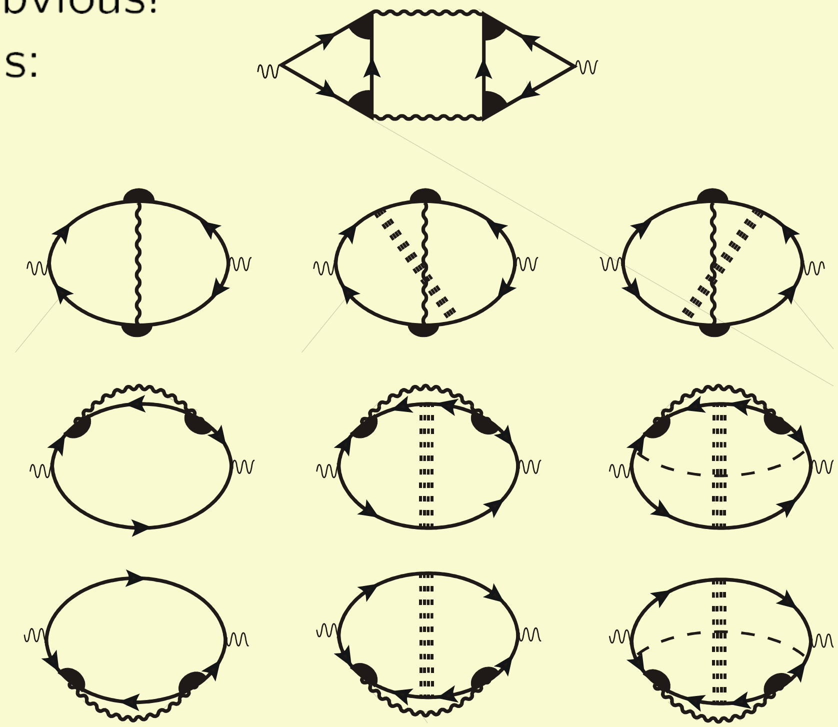
Quantum fluctuation conductivity

Naïve qualitative estimate of the fluctuation conductivity

$$\delta\sigma \propto \int d\omega N_{\text{cp}}(\omega) = \int \frac{d\omega}{\gamma|\omega| + (H - H_{c2})/H} \propto \ln\left[H - H_{c2}(0)\right]$$

The sign of the correction is not obvious!

One needs to calculate all diagrams:



Fluctuation conductivity

$$\delta\sigma_{\text{tot}} = -\frac{2e^2}{3\pi^2\hbar} \ln\left[\frac{H}{H - H_{\text{c}2}(0)}\right]$$

Fluctuation magnetization

$$\delta\chi = \frac{2e^2\mathcal{D}}{\pi^2\hbar c^2} \frac{H}{H - H_{\text{c}2}(0)} \gg \chi_{\text{Landau}}$$

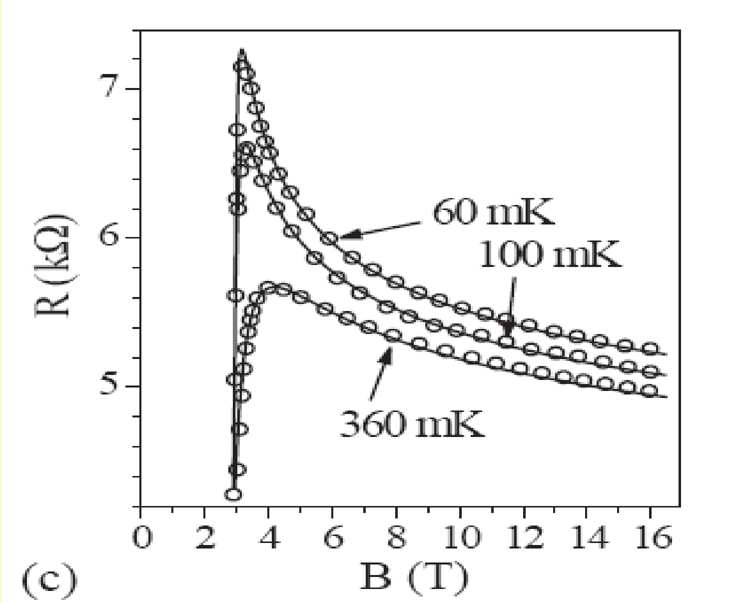
Quantum-to-classical crossover
Experimental verification

General expression at low but finite temperatures

$$\delta\sigma = \frac{2e^2}{3\pi^2\hbar}\left\{-\ln\frac{r}{h}-\frac{3}{2r}+\psi(r)+4\left[r\,\psi'(r)-1\right]\right\},\ r=\frac{1}{2\gamma}\frac{H-H_{c2}(0)}{H}\frac{T_{c0}}{T}$$

Quant.-to-class. crossover

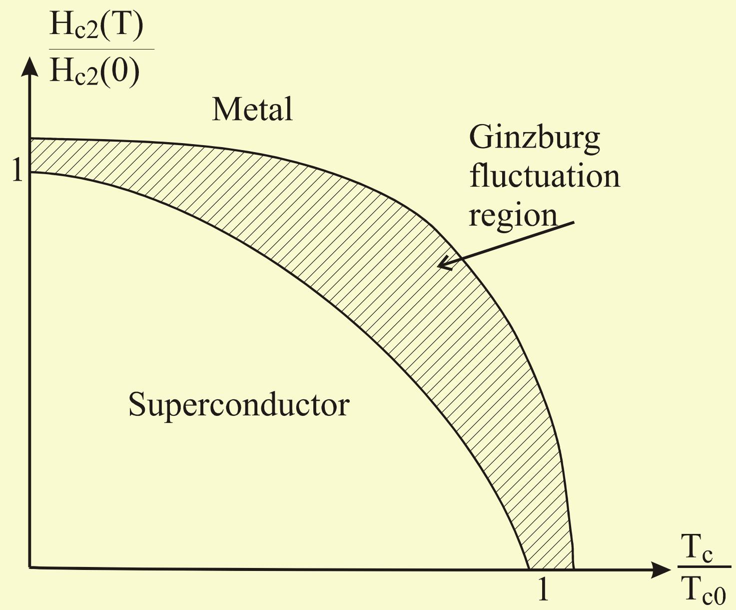
$$\delta\sigma \propto \begin{cases} [T-T_c(H)]^{-1}, & \text{if } t \gg h; \\ -\ln\frac{H}{H-H_{c2}(0)}, & \text{if } t \ll h. \end{cases}$$

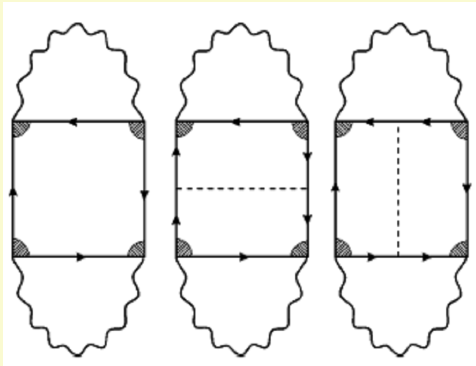


Baturina *et al.*, Physica B **359**, 500 (2005)

Ginzburg region

How to define the Ginzburg region? Compare the leading and sub-leading terms.



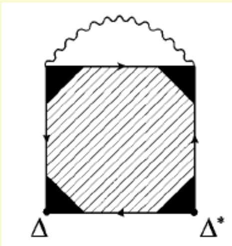


- At $H = 0$, $(T - T_{c0})/T \gg \text{Gi} \sim 1/(E_F\tau)$
- At $T = 0$, $[H - H_{c2}(0)]/H \gg \text{Gi}$
- In the intermediate T-regime, we have $[H - H_{c2}(T)]/H \gg \sqrt{\text{Gi}}$

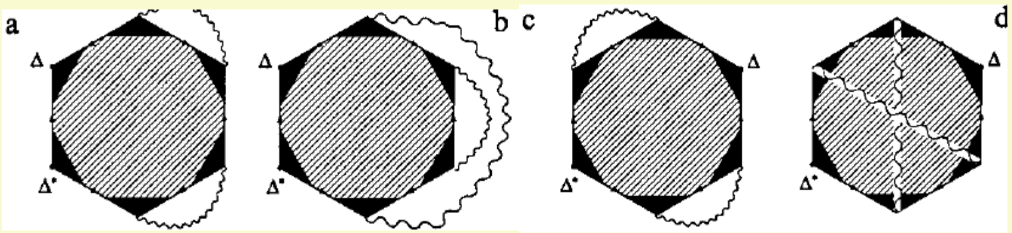
Is it possible to collect the logs within an RG/parquet scheme?

Corrections to the quadratic coefficient A of GL:

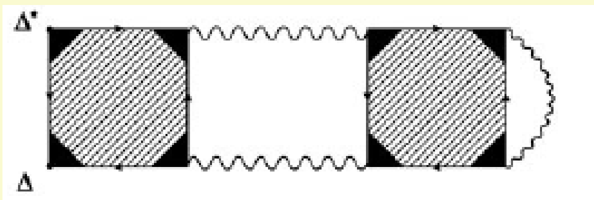
$$F[\Delta] = \hat{A}\Delta^2 + \hat{B}\Delta^4$$



$\propto \text{Gi} \log (H - H_{c2})$



$\propto \text{Gi}^2 \log^2 (H - H_{c2})$



$$\propto \text{Gi}^3 \frac{\log (H - H_{c2})}{H - H_{c2}}$$

An earlier “dirty-boson” theory

Duality transformation

$$\hat{\mathcal{H}} = -\rho_s \sum_{\langle ij \rangle} \cos\left(\hat{\phi}_i - \hat{\phi}_j - A_{ij}\right) + U \sum_i \hat{n}_i^2$$

It is possible to re-write the theory in terms of vortices (defects in the phase field): $\hat{b}_V^\dagger$ creates a vortex and $\hat{b}_V$ annihilates a vortex.

$$\hat{b}_V = \sqrt{\hat{n}_V}\, e^{i\hat{\theta}}, \quad \left[\hat{n}_V, \hat{\theta}\right] = i$$

Dual Hamiltonian

$$\hat{\mathcal{H}} = -t_V \sum_{\langle ij \rangle} \cos\left(\hat{\theta}_i - \hat{\theta}_j - a_{ij}\right) + \frac{1}{2} \sum_{ij} \left(\hat{n}_{V,i} - B\right) V_{ij} \left(\hat{n}_{V,j} - B\right) + \ldots$$

- $\cos(\hat{\theta}_i - \hat{\theta}_j)$: Vortex hopping
- a_{ij}: Dual gauge field
- B: Cooper pair density
- V_{ij}: Inter-vortex interaction $\sim \ln R_{ij}$
- $\ldots$: Gauge field fluctuations

Boson-vortex duality

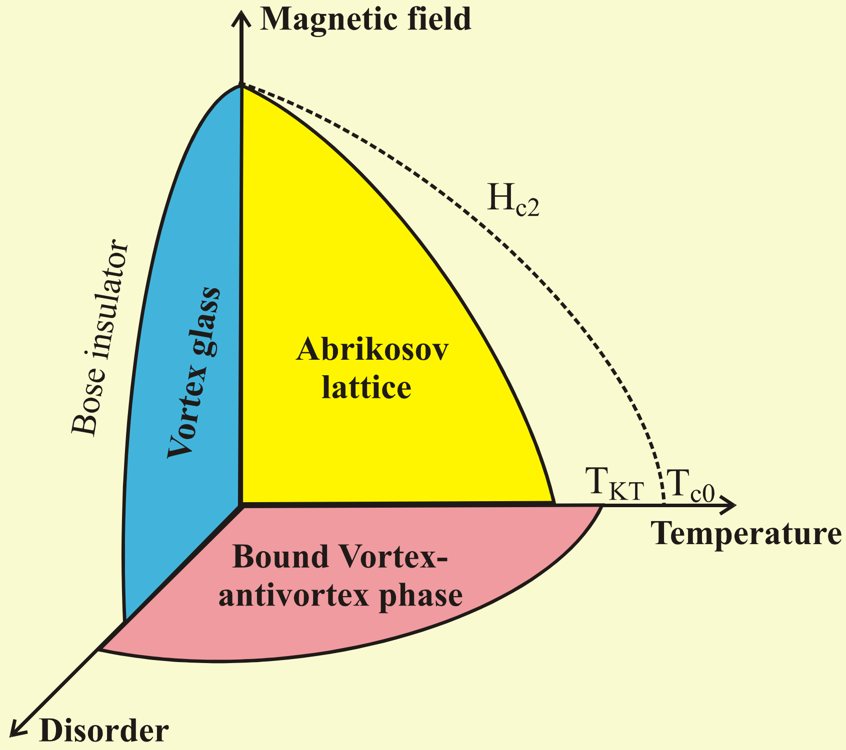
Field seen by Cooper pairs	$\longrightarrow$	Density of vortices
Field seen by vortices	$\longrightarrow$	Density of cooper pairs
Cooper pair condensation	$\longrightarrow$	Vortex localization
Vortex condensation	$\longrightarrow$	Cooper pair localization

Vortex conductivity = Particle resistivity

Schematic phase diagram

Ref.: M. P. A. Fisher, Phys. Rev. Lett. **65**, 923 (1990).

- Superconducting phase has delocalized pairs and localized vortices.
- Insulating phase has localized pairs and delocalized vortices.
- The phase boundary is a *"Bose metal"*.



VOLUME 65, NUMBER 7 PHYSICAL REVIEW LETTERS 13 AUGUST 1990

Magnetic-Field-Tuned Superconductor-Insulator Transition in Two-Dimensional Films

A. F. Hebard and M. A. Paalanen
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
(Received 26 February 1990)

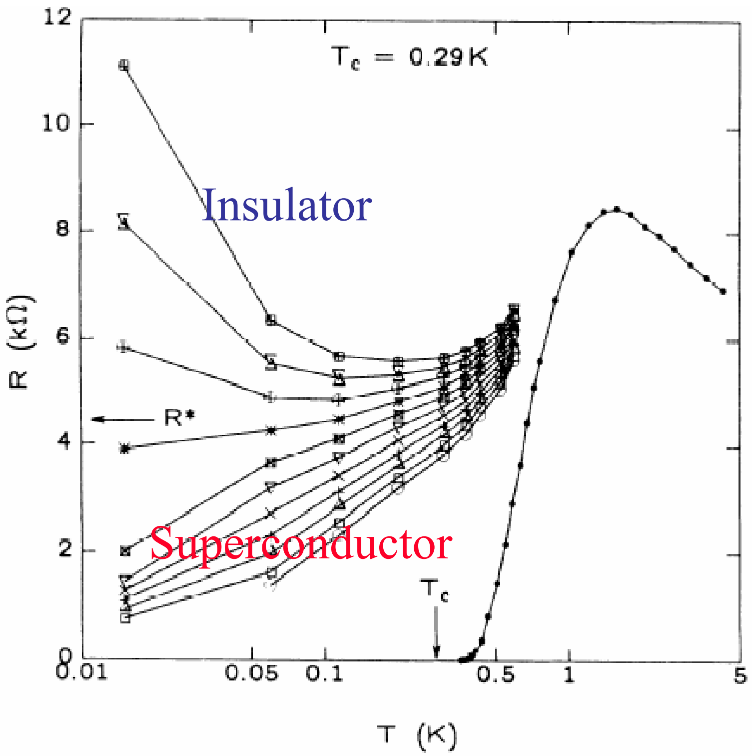


FIG. 1. Logarithmic plots of the resistance transitions in zero field (●) and nonzero field (open symbols) for a film with T_c = 0.29 K. The isomagnetic lines range from B = 4 kG (○) to B = 6 kG (□) in 0.2-kG steps. The horizontal and vertical arrows identify R^* and T_c, respectively.

Which theory describes the experiment?

In Ann. Phys.(Leipzig) **8**, 785 (1999), A. I. Larkin posed a similar question in the context of disorder-induced SC-INS transition and his suggestion was:

- If Gi $\ll$ 1, the "fermionic theory" works
- If Gi $\gg$ 1, the "bosonic theory" works

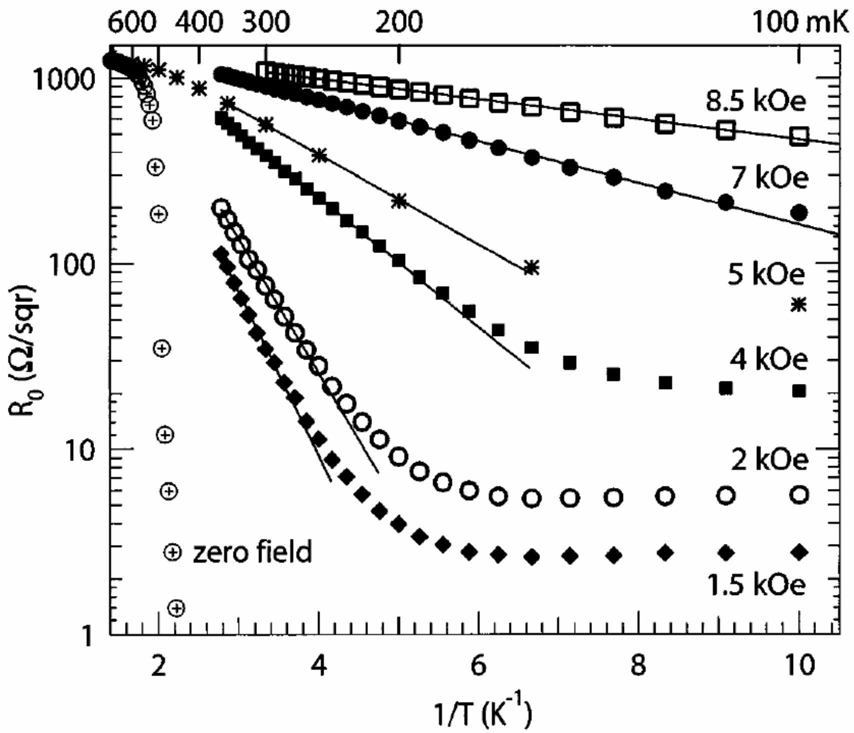
Unusual metallic phase in dirty SC films

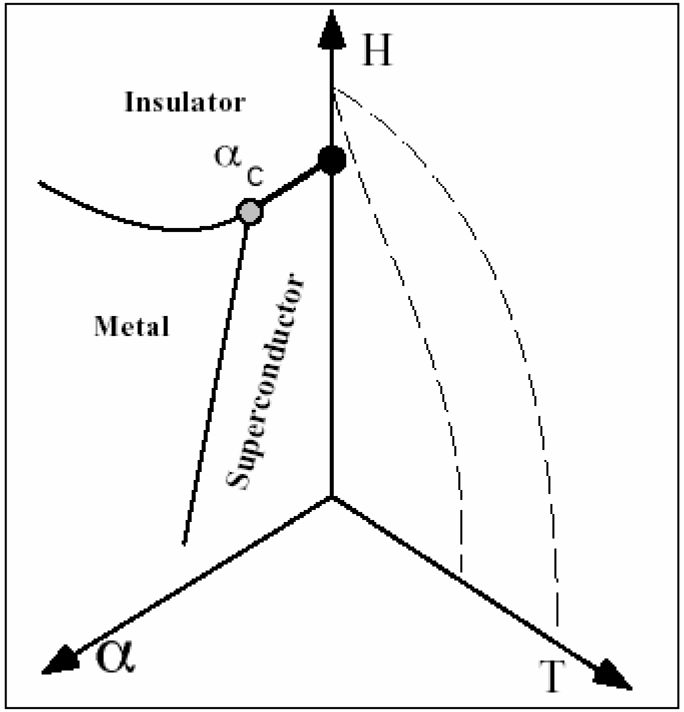
and its possible theoretical scenario

Metallic phase

N. Mason and A. Kapitulnik, PRL **82**, 5341-5344 (1999)

MoGe films:

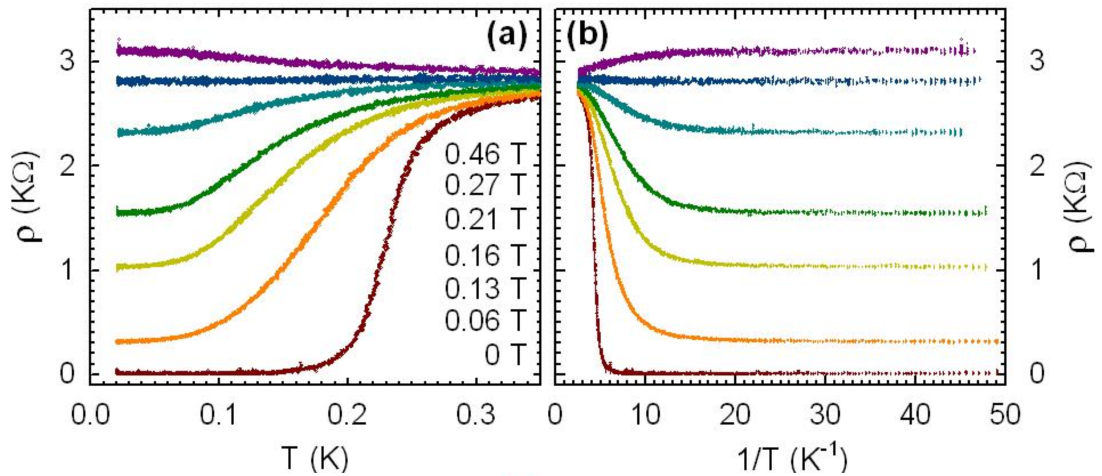


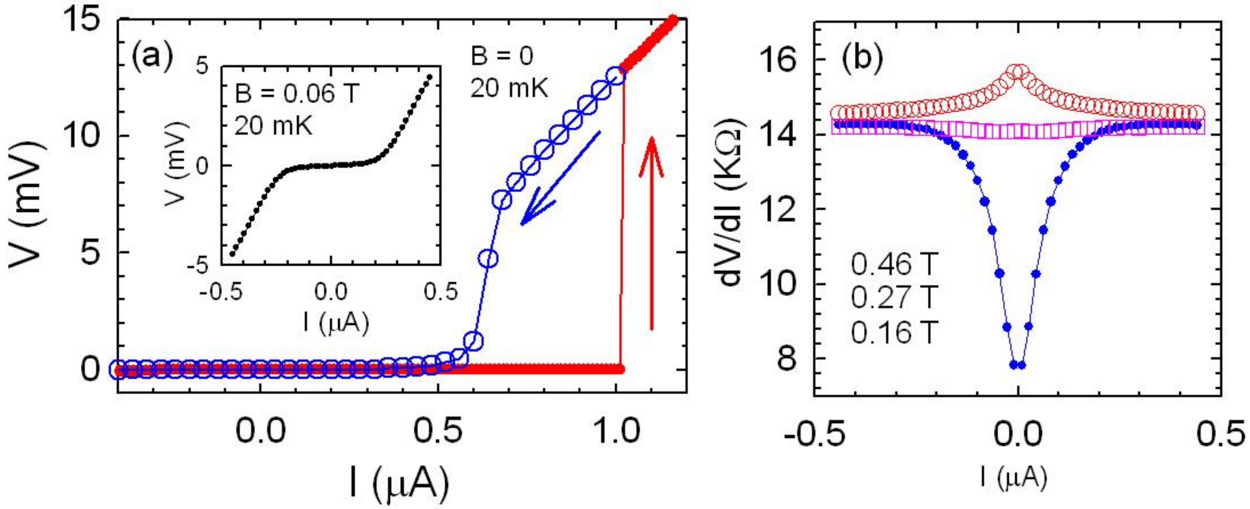


Metallic phase

Ta films:

(Jongsoo Yoon)

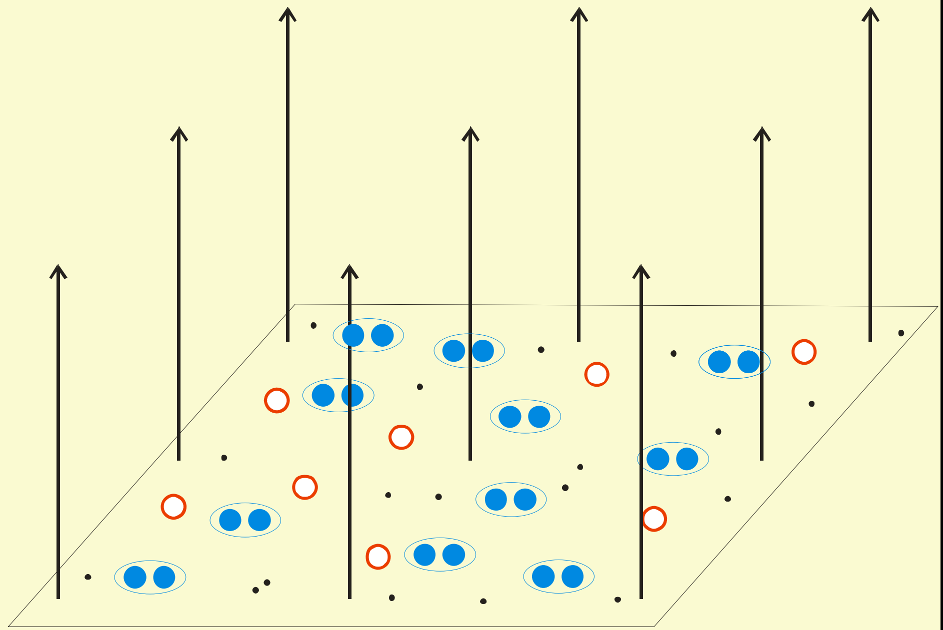




Natural descripton

Our system includes:

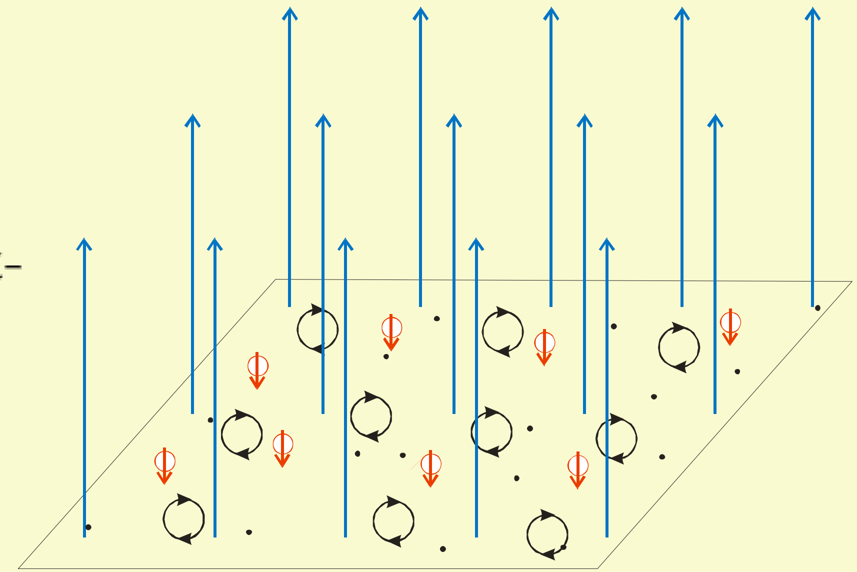
- Cooper pairs
- Gapless excitations
- A magnetic field
- Disorder



Dual descripton

In dual language:

- Quantum vortices
- Gapless excitations
- Dual magnetic field acting on the vortices
- Disorder



If vortices are bosons they are either localized or condensed at zero temperature!

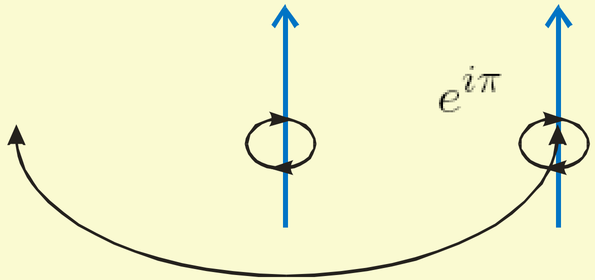
Composite vortices

- The dual magnetic field acting on the vortices is related to the densities of Cooper pairs and quasiparticles:

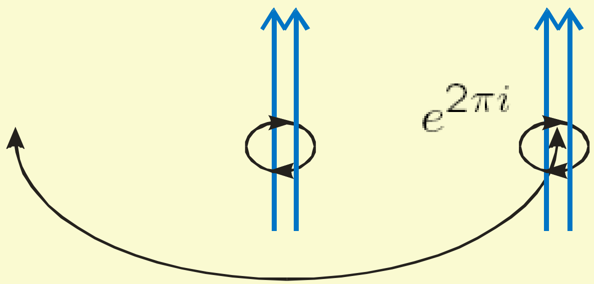
$$B_{\text{dual}} = 2\pi N_{\text{CP}} - \pi N_{\text{qp}}$$

- In a disordered SC, the dual field is not known but it can be very large ("dual quantum Hall" physics is possible).
- Formation of composite particles (bound state of a vortex and fluxes) is possible.

Boson + 1 flux = fermion



Boson + 2 fluxes = boson



Statistics vs. interactions

- Field theory for fermionized vortices

$$\mathcal{L} = \psi^\dagger \left[-\frac{1}{2m_V} (\boldsymbol{\nabla} - i\mathbf{a} + i\mathbf{A})^2 + (\partial_\tau - ia_0 + iA_0) \right] \psi + \frac{1}{2C} (\nabla \times a)^2 + \frac{1}{4\pi} A \nabla \times A$$

- Let us shift variables $a_\mu - A_\mu \longrightarrow a_\mu$ and integrate out the Chern-Simons field:

$$\mathcal{L} = \psi^\dagger \left[-\frac{1}{2m_V} (\boldsymbol{\nabla} - i\mathbf{a})^2 + (\partial_\tau - ia_0) \right] \psi + \frac{1}{2C} \left(\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \right)^2 + \frac{1}{4C^2} (\nabla \times a)(\nabla \times \nabla \times a)$$

The Chern-Simons terms drop out of the action. Statistics is less important than interactions!!!

Singular $(2+1)$-electrodynamics

The appropriate language is a gauge theory.

$$\mathcal{L} = \psi^\dagger \left[-\frac{1}{2m_V} (\nabla - i\mathbf{a})^2 + (\partial_\tau - ia_0) \right] \psi + \frac{1}{2C} \left(\epsilon_{\mu\nu\lambda} \partial_\nu a_\lambda \right)^2 + \text{disorder}$$

Integrate out diffusive fermions:

$$\mathcal{E} \sim \int d\omega d^2\mathbf{q} \left[\varepsilon(\omega, \mathbf{q}) \mathbf{E}^2 + \mu B^2 \right]$$

Electric and magnetic fields:

$$E_i = -\partial_0 a_i - \partial_i a_0, \quad B = \text{curl}\, \mathbf{a}$$

Diffusive fermionic vortices lead to the dielectric constant:

$$\varepsilon(\omega, \mathbf{q}) \propto |\omega|^{-1}$$

It is an *anisotropic* $(2+1)$ electrodynamics.

Tunneling into the vortex metal

- Tunneling conductance:

$$G \propto \int_0^\infty dtt \langle [I(t),\, I(0)] \rangle\,, \quad I(t) = 2eJ \sin\phi(t)$$

- We need to calculate the correlation function:

$$C(t) = \left\langle e^{-i\phi(\mathbf{0},0)} e^{i\phi(\mathbf{0},t)} \right\rangle$$

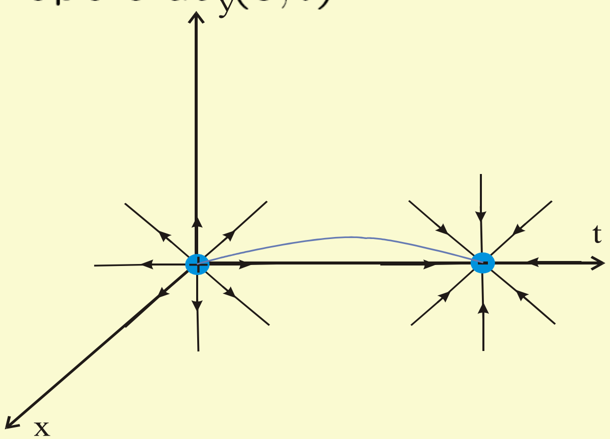
In the language of the (2+1)-electrodynamics, it means inserting a monopole at the origin and anti-monopole at $(\mathbf{0}, t)$.

- Energy of this configuration:

$$\mathcal{E} \sim \sigma_{\mathrm{v}} \ln^2 (t/\tau)$$

- Tunneling conductance:

$$G(T) \propto \exp\left[-\frac{\sigma_{\mathrm{v}}}{2} \ln^2 (T\tau)\right]$$



Summary

- Quantum superconducting fluctuations near $H_{c2}(0)$ lead to negative magnetoresistance in two dimensions.
- The Ginzburg region becomes wider at intermediate fields/temperatures.
- The bosonic and fermionic theories are not necessarily competing theories. Their applicability is controlled by the width of the fluctuation region.
- A possible scenario of a low-temperature metallic phase in dirty superconducting films is a statistical transmutation in the system of vortices, which may be converted into fermions.