V.F. Gantmakher and V.T. Dolgopolov

Quantum phase transitions of "localized – delocalized electrons“ type

1. Metal-insulator transitions

3D-system of non-interacting electrons

2D-system of non-interacting electrons

2D-system of electrons with spin-orbit interaction

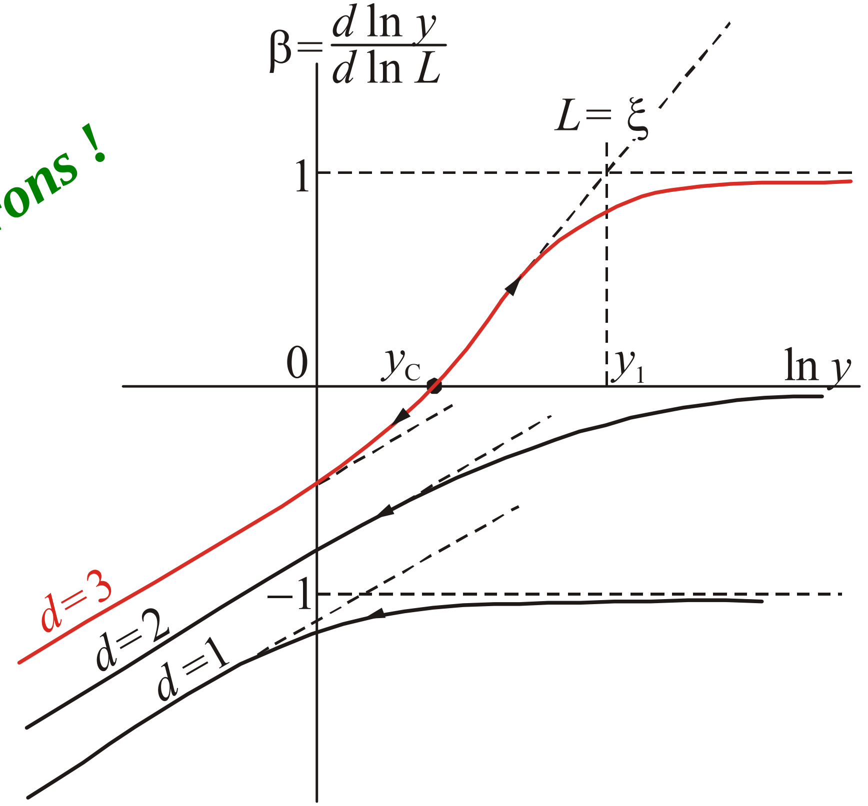
2D-system of interacting electrons

2. Superconductor-insulator transition

E.Abrahams, P.W.Anderson, D.C.Licciardello, and T.W.Ramakrishnan,
Phys.Rev.Lett. **42**, 673 (1979)

(AALR-1979)

Non-interacting electrons !

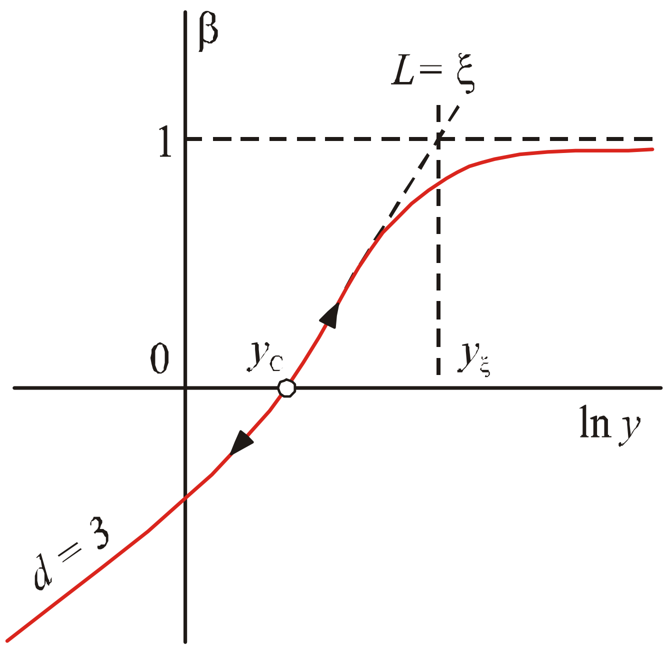


With ***d=3***,

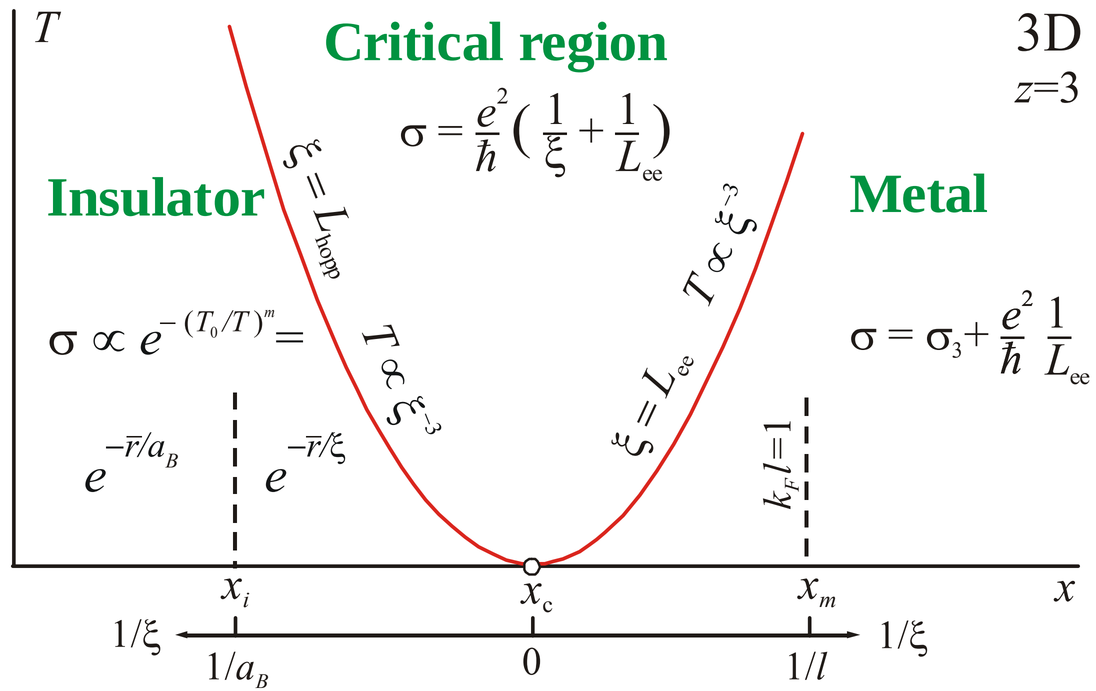
1. at y_c the metal-insulator transition takes place;
2. the value of the three-dimensional conductivity of a rather large sample may be infinitesimal;
3. the metal--insulator transition is continuous.

In essence, the flow diagram plays role of existence theorem with respect to metal-insulator transition

3D-system of non-interacting electrons



Flow diagram



***(x,T)* – diagram**

$d = 3$

The function $\sigma(T)$ in the critical region

In the metallic region at $T \neq 0$ $$\sigma = \sigma_{03} + \frac{e^2}{\hbar}\frac{1}{L_T} \qquad L_T = \sqrt{D\hbar/T},$$

Near the transition at $T = 0$ $$\sigma = \frac{e^2}{\hbar}\frac{1}{\xi}$$

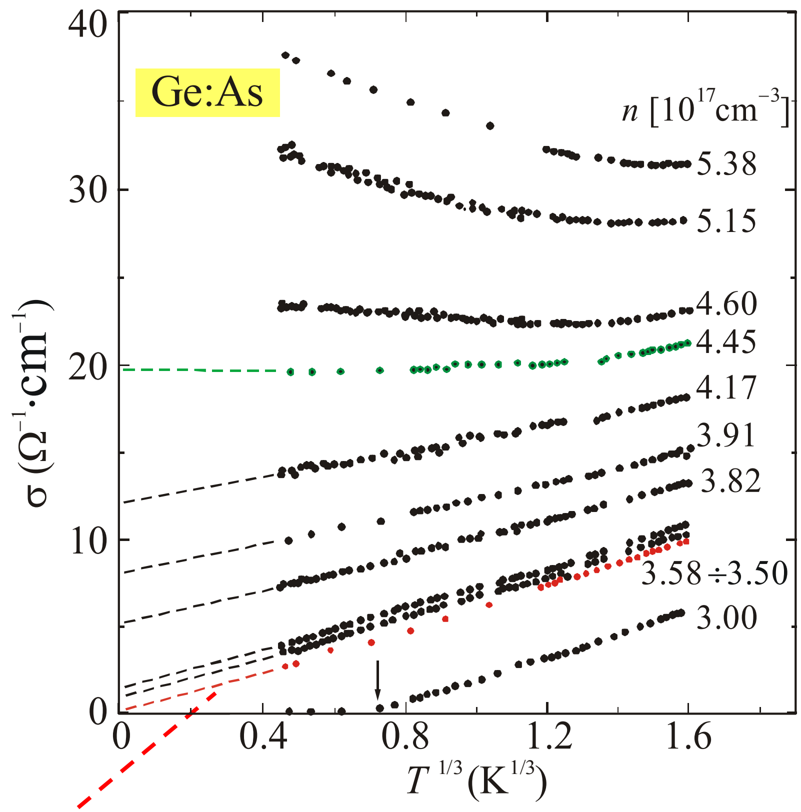
Interpolation formula $$\boxed{\sigma = \frac{e^2}{\hbar}\left(\frac{1}{\xi} + \frac{1}{L_T}\right)}$$

$$\left\{\begin{array}{l} \sigma = \dfrac{e^2}{\hbar}\left(\dfrac{1}{\xi} + \dfrac{1}{L_T}\right) \\ \sigma = e^2 g_F D \end{array}\right. \xrightarrow[\xi \to \infty]{} \sigma = \frac{e^2}{\hbar}(Tg_F)^{1/3}$$

Einstein relation

I.Shlimak, M.Kaveh, R.Ussyshkin, et al., Phys.Rev.Lett. **77**, 1103 (1996);
J.Phys.:Cond.Matt. **9**, 9873 (1997)

This is not the critical concentration



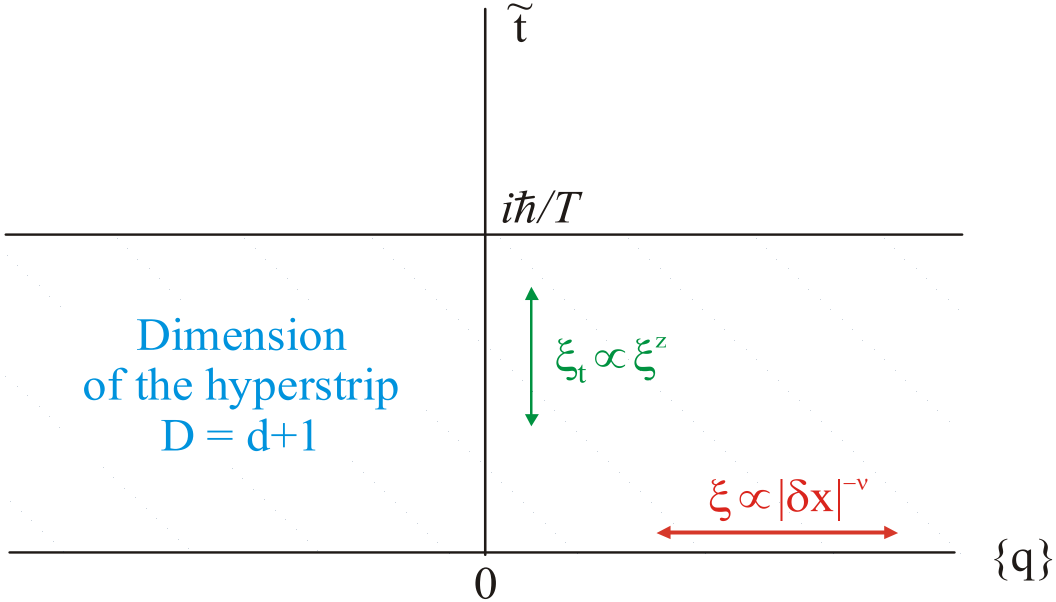
At the transition, $\boxed{\dfrac{\partial \sigma}{\partial T} \neq 0}$

QPT – 90 th

S.Sachdev, Quantum Phase Transitions (Cambridge University Press, Cambridge 2000)

The mapping

of the quantum phase transition in *d*-space to the classical phase transition in *D=d+1*–dimensional strip



The partition function

$$Z = \sum_i e^{-\varepsilon_i/T} =$$

$$= \sum_i \int \varphi_i^*(q)\varphi_i(q)e^{-\varepsilon_i/T}dq$$

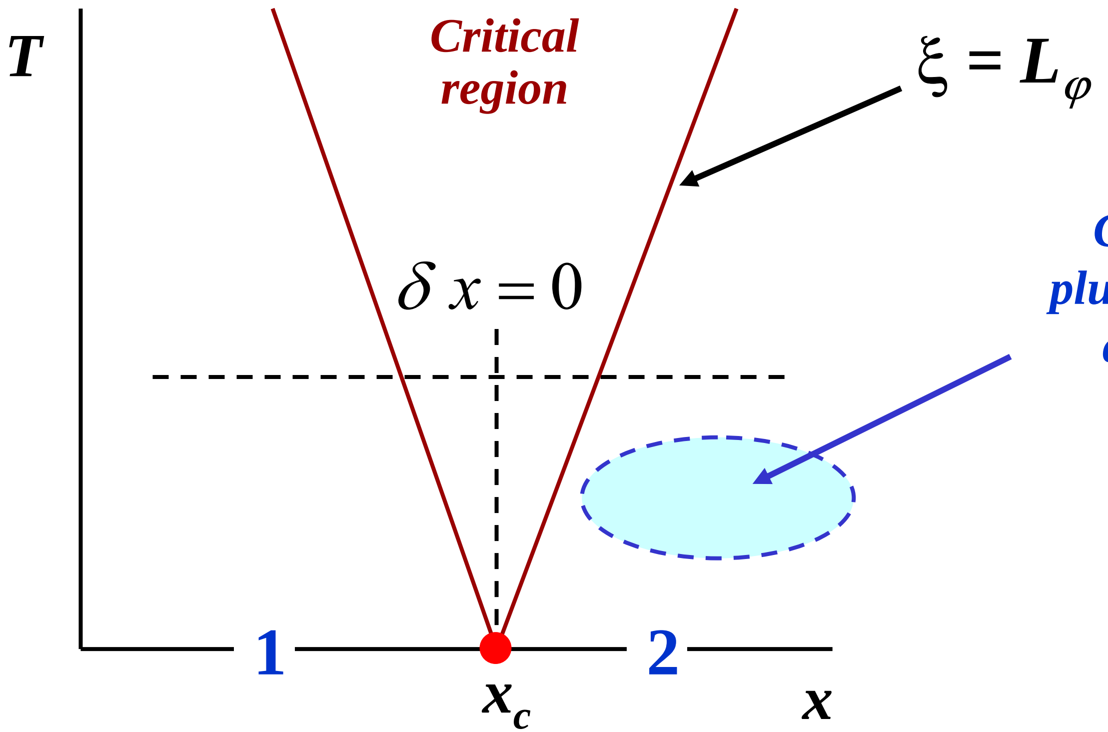
Diagonal matrix elements of time-evolution operator

$$\int \varphi_i^*(q)\varphi_i(q)e^{-i\varepsilon_i t/\hbar}dq$$

Two correlation lengths: ξ and ξ_t.

Hence, two scales: ξ *and* $L_\varphi \propto \xi_t^{1/z} \propto (\hbar/T)^{1/z}$

Vicinity of the quantum phase transition



$$\left.\begin{array}{l} \xi_t \propto \xi^{1/z} \\ 0 < \xi_t < \hbar/T \end{array}\right\}$$

In the simplest case

$$\xi \propto (\delta x)^{-\nu}$$

1 ***and*** **2** – ***two*** ***different*** ***ground states***

Two lengths: ξ and $L_\varphi \propto \xi_t^{1/z}$

Conductivity

$$\sigma \propto \xi^{2-d} f\left(\frac{L_\varphi}{\xi}\right) \propto \xi^{2-d} f_1\left(\frac{\delta x}{T^{1/z\nu}}\right)$$

3D – metal - insulator transition

Theoretical scheme	AALR * experimental data	QPT
Driving factor	Disorder	Interaction
Guideline	Transport	A thermodynamical function
Features in common	* Two lengths, ξ and L_φ, in the vicinity of the transition Similar shape of the critical region	
Scaling variable	* $\sigma = \dfrac{e^2}{\hbar}\left(\dfrac{1}{\xi} + \dfrac{1}{L_\varphi}\right)$	$\sigma \propto \xi^{-1} f\left(\dfrac{L_\varphi}{\xi}\right)$
Graphical expression	Flow diagram	(x,T) – diagram x – control parameter
Area of application	Global	Vicinity of the transition

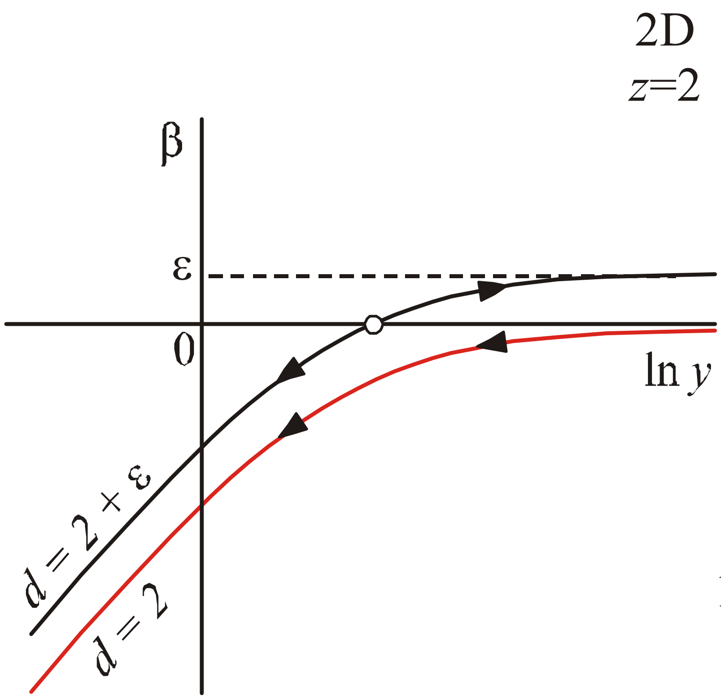
2D-system of non-interacting electrons

$$\sigma = \sigma_{02} - \frac{e^2}{\hbar}\ln\frac{L_T}{l}$$

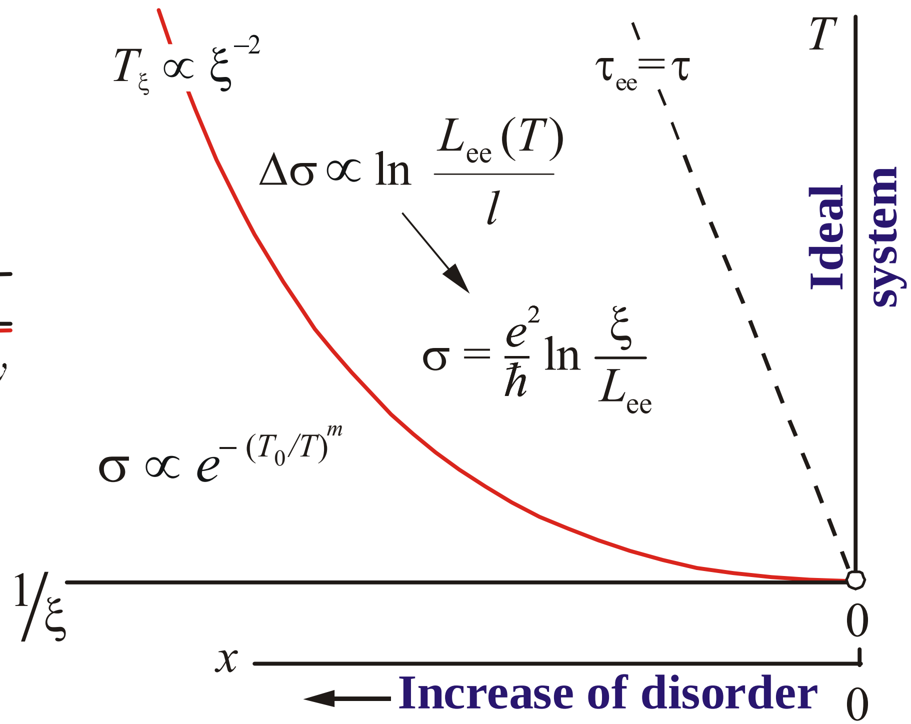
$$n \propto k_F^2 \qquad \sigma_{02} = \frac{ne^2 l}{\hbar k_F} \approx \frac{e^2}{\hbar}(k_F l)$$

$\sigma = 0$

$$k_F l = \ln L_T/l \qquad L_T \equiv \xi = l\exp(k_F l)$$

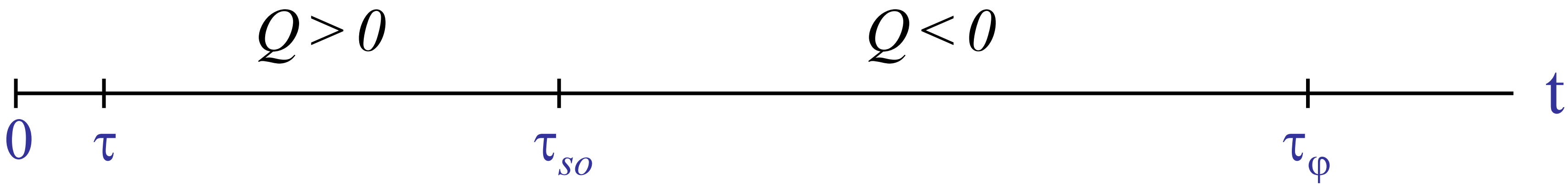


Flow diagram

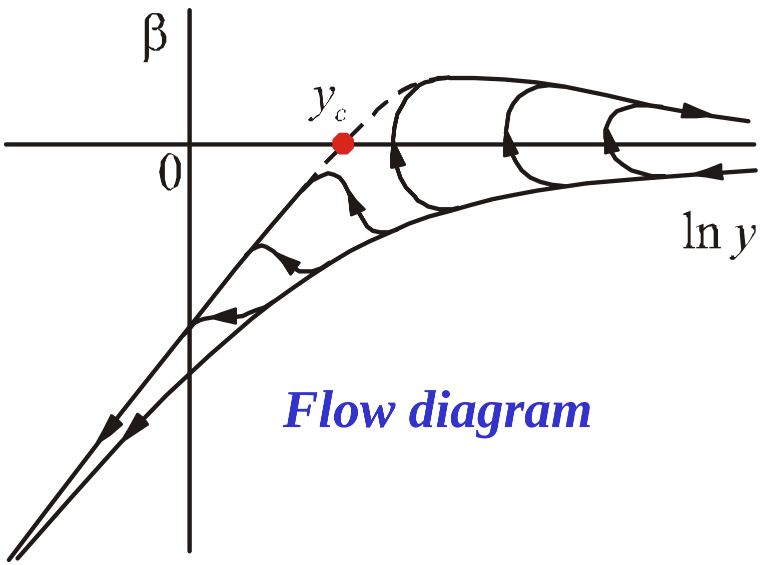


(x,T) – diagram

2D-system of electrons with spin-orbit interaction



$$\frac{\Delta\sigma}{\sigma} \approx -\int_{\tau}^{\tau_\varphi} \frac{v_F \lambda^2 dt}{(Dt)b} \underbrace{\left(\frac{3}{2} e^{-t/\tau_{so}} - \frac{1}{2} \right)}_{Q}$$



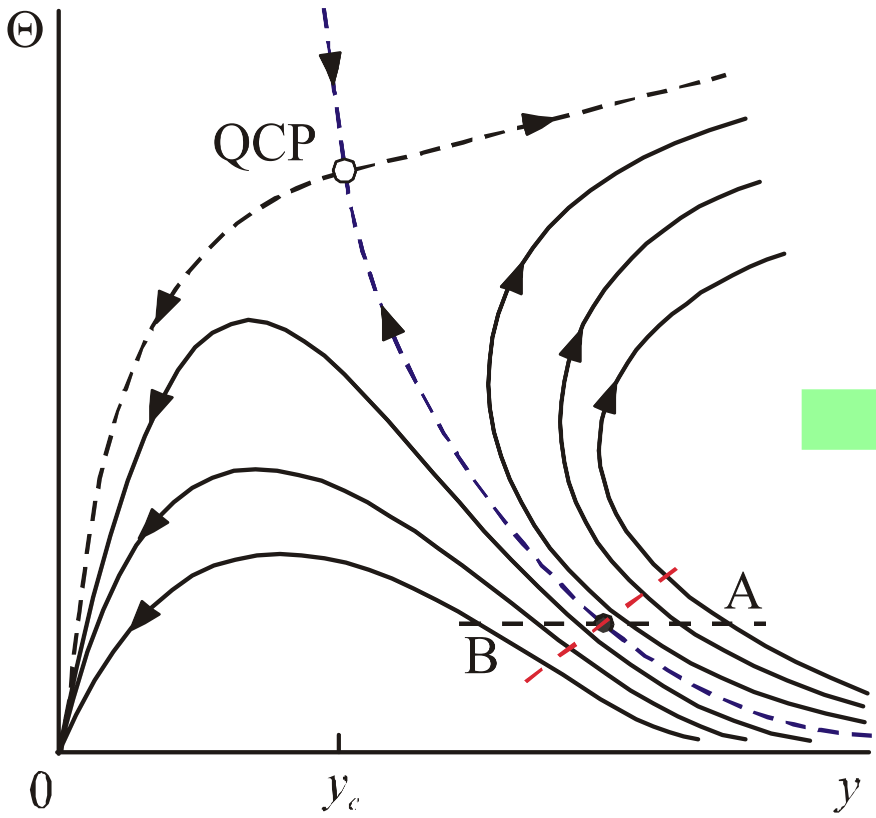
Flow diagram

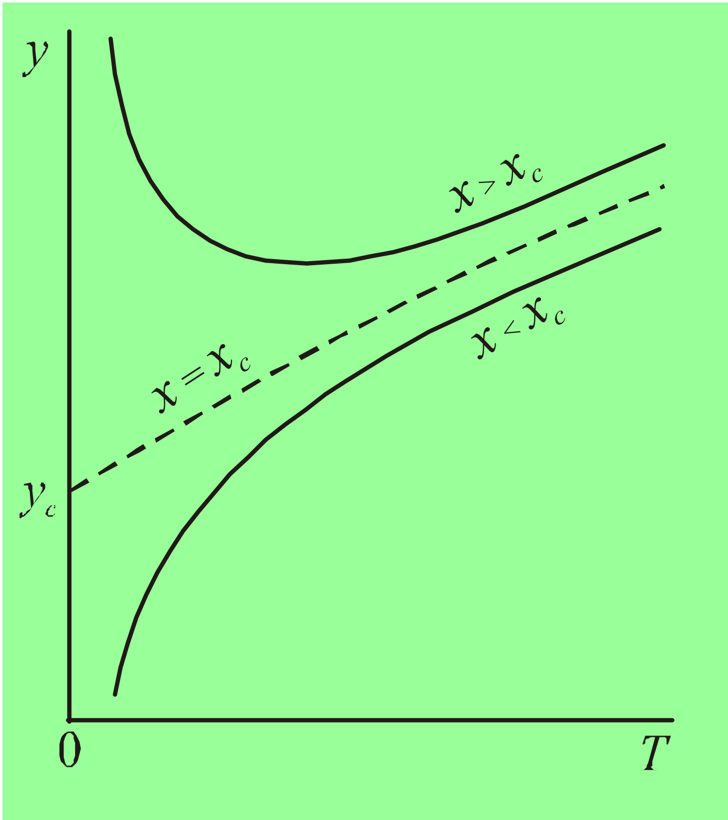
Estimate for the transition point y_c

$$k_F l = \ln \tau^*/\tau \qquad \tau^* = \tau \exp(k_F l)$$

If $\tau \ll \tau_{so} \ll \tau^* = \tau\, e^{k_F l}$, **the film remains metallic**

2D-system of interacting electrons

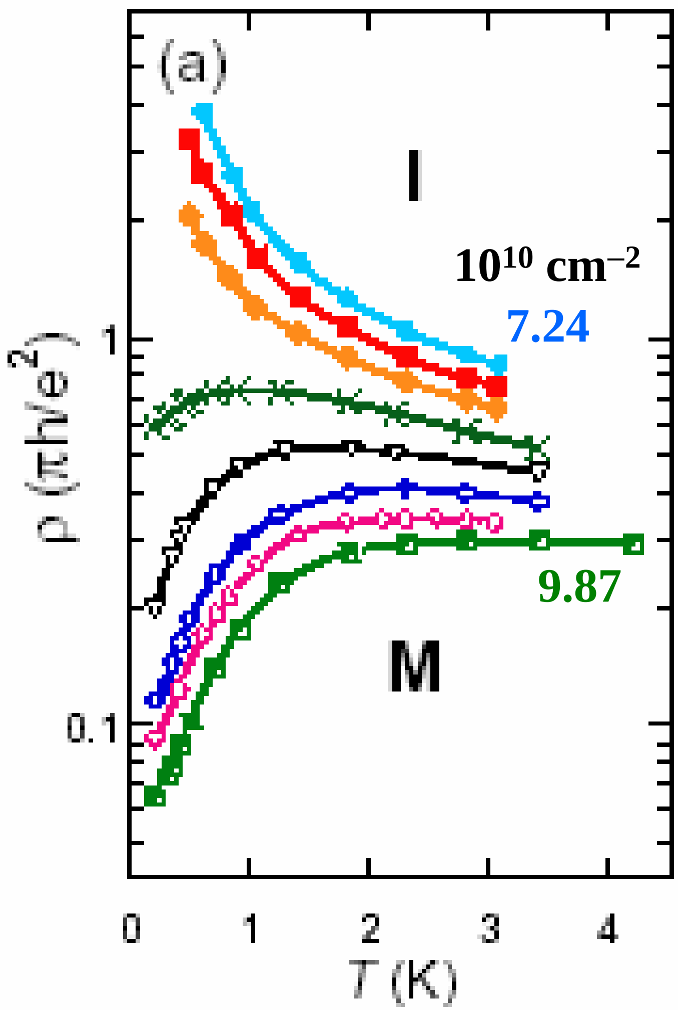




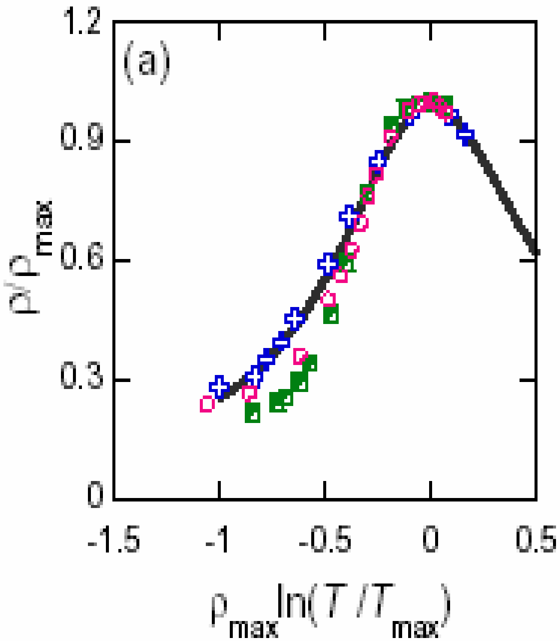
Flow diagram

A.Punnoose and A.Finkel'stein,
Science **310**, 289 (2005)

Si-MOSFET



Comparison between
theory (lines)
and
experiment (symbols from the left plot)



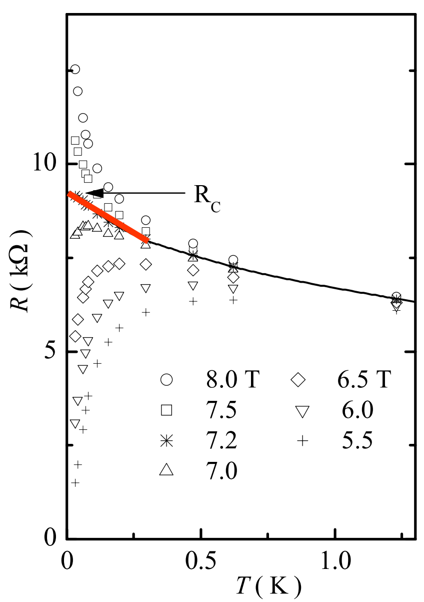
S.Anissimova, S.V.Kravchenko, A.Punnoose, A.Finkel'stein, and T.M.Klapwijk
cond-mat/0609181

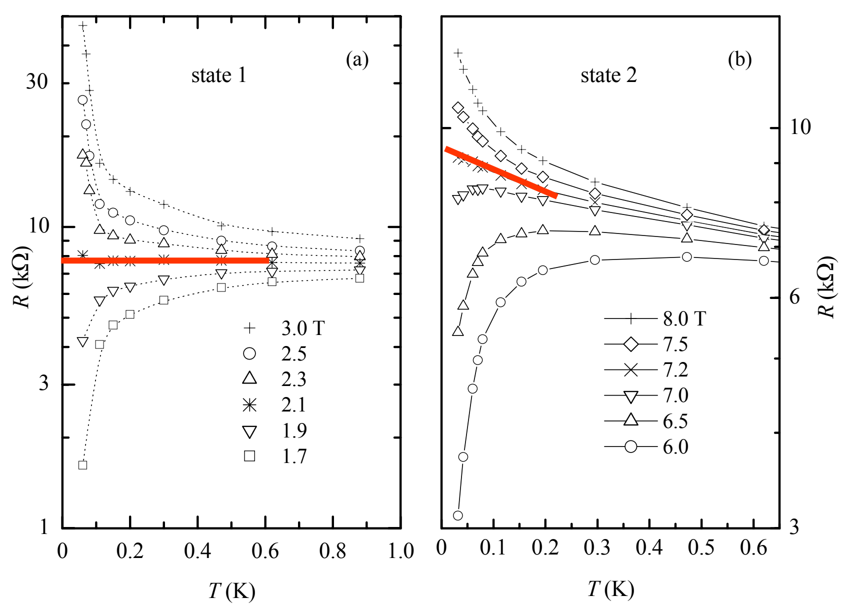
M.P.A. Fisher, PRL ***65****, 923 (1990)*

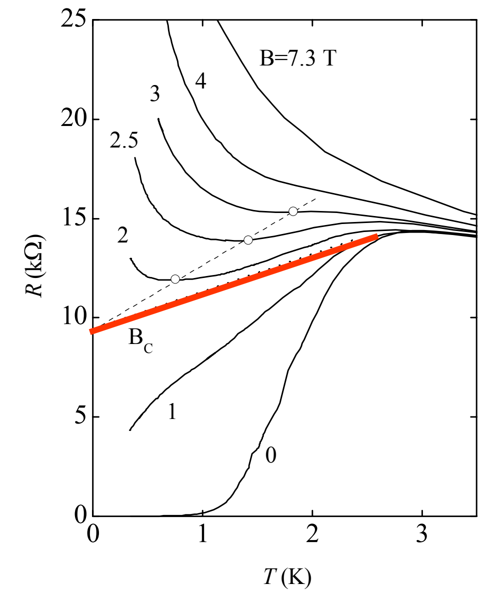
Phenomenological description in the frame of QPT of 2D transitions based on 2*e*-bosons – vortex duality

Predicts scaling relations with $|B - B_{c0}| / T^{1/z\nu}$ ***as scaling variable***

Superconductor-insulator transition in In-O





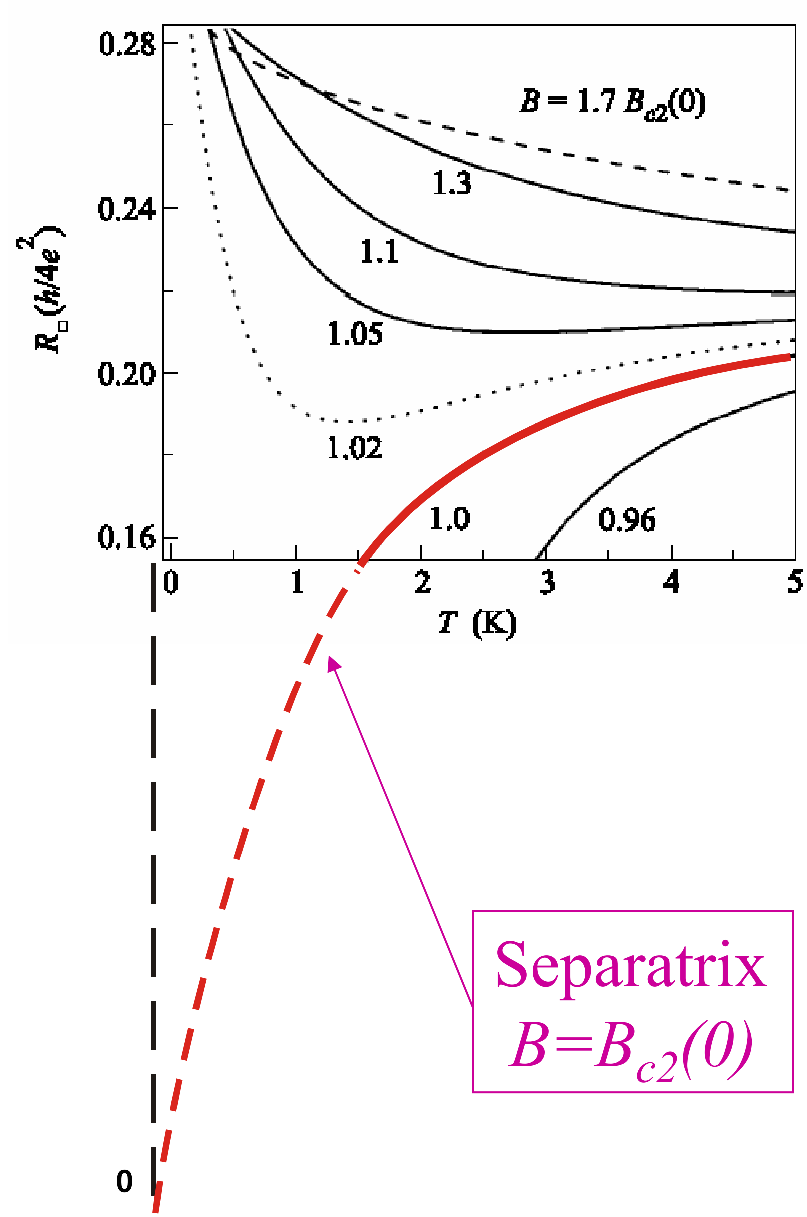


The separatix may have non-zero slope

V.M. Galitski and A.I. Larkin,
Phys.Rev. B ***63****, 174506 (2001)*

Superconducting fluctuations

at $T \ll T_{c0}$ in magnetic field

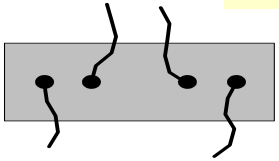


Rigid calculations of the superconducting fluctuations

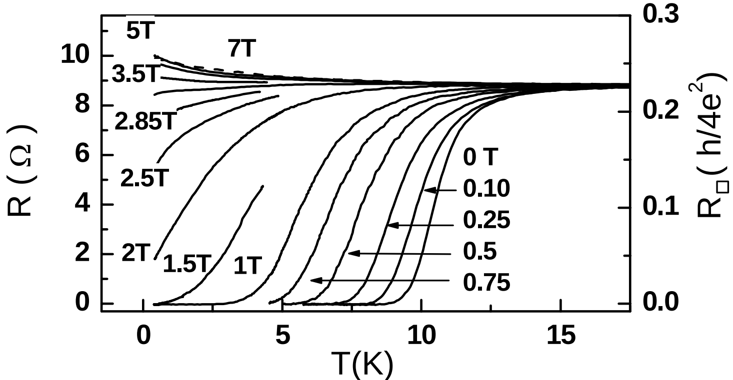
How the magnetic field destroys the supercoducting state (dirty limit).

Film $Nd_{2-x}Ce_xCuO_{4+y}$

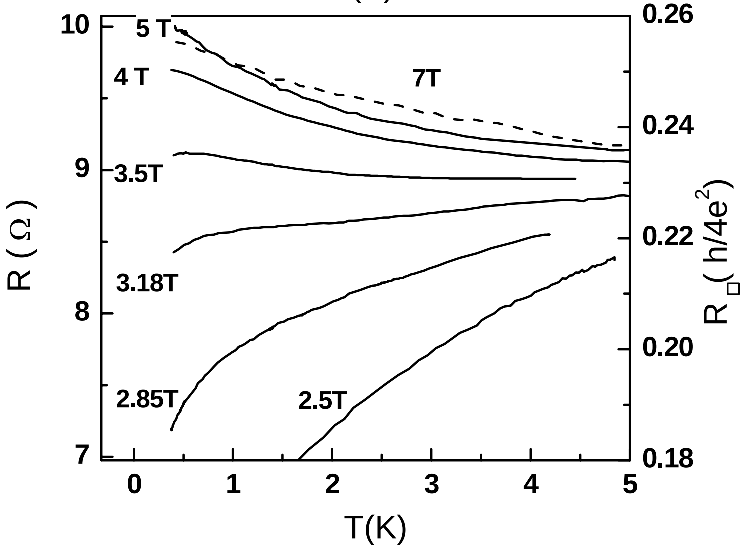
V.F. Gantmakher et al., JETPL ***77****, 424 (2003)*

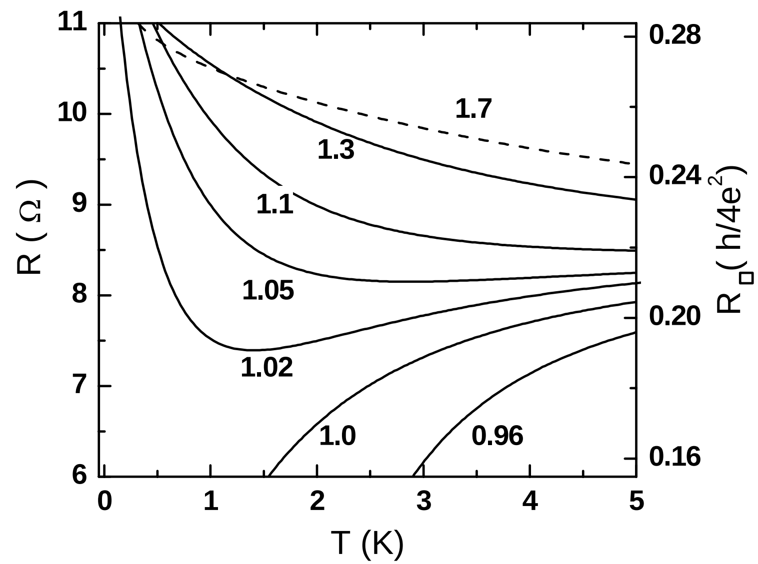


d = 1000 Å, c = 12 Å, T_C = 11.5 K, ΔT = 2K, $\sigma_0 = 17\ e^2/h$



$$R(B,T) = 1/[\sigma_0 - \alpha \frac{e^2}{\hbar}\ln(T) + \Delta\sigma_{\rm fl}(B,T)]$$





EXPERIMENT

CALCULATION

Conclusions

1. Metal-insulator transitions

3D-system of non-interacting electrons

2D-system of non-interacting electrons

2D-system of electrons with spin-orbit interaction

2D-system of interacting electrons

There is consensus between different theoretical approaches and between theory and experiment

2. Superconductor-insulator transition

Apparently, there are different scenarios but the main questions remain unresolved. Neither experiment, not theory have answers concerning 3D materials.