The fate of the vortex lattice

A. I. Larkin, ``Effect of inhomogeneities on the structure of the mixed state of superconductors'',
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Flux Flow

A. I. Larkin, Yu. N. Ovchinnikov, ``Resistance of superconductors near the critical field strength Hc2,
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$$F = \int d^3 r \left[\frac{C}{2} (\nabla u)^2 + E_{\text{pin}} (r, u) \right]$$

replace by full
continuous elastic theory
of vortex lattice

derive from explicit
form factor of
vortices / vortex lattice

$$\langle E_{\text{pin}}(r,u) E_{\text{pin}}(r',u') \rangle = \delta(r - r') R(|u - u'|)$$

Expand in displacement $E_{\text{pin}}(r,u) = u(r) f(r)$

replacement of the random potential by the random force

leads to $C\nabla^2 u = f_{\text{pin}}(r)$

and $\left\langle [u(r) - u(0)]^2 \right\rangle = 2R''(0) \int \frac{d^3 k}{(2\pi)^3} \frac{(1 - \cos(kr))}{C^2 k^4}$

Thus $\left\langle [u(r) - u(0)]^2 \right\rangle = \frac{R''(0)}{4\pi} \frac{r}{C}$

And the long range order of the vortex lattice is destroyed!

In the ordinary crystal defects do not destroy the long range order.
Why do they destroy it in the vortex lattice?

A. I. Larkin, Onsager prize speech, 2002

"If you accidentally step on the foot of another person, you say: "excuse me, please" and it's OK. But what would happen if a lot of people step on the feet of others? The result depends on the system. For example, if while entering this room many people stepped on other people's feet, we wouldn't be too much disturbed if they are our friends from condensed-matter community. But if the offenders were all high-energy physicists, it could lead to disorder."

In the ordinary crystal impurities belong to the lattice and move with it

$\Rightarrow$ order is preserved

In the vortex lattice impurities belong to our laboratory frame and do not move with the vortices

$\Rightarrow$ they destroy the order

Other systems

Ferromagnets | Y. Imry and S.-k. Ma, (1975)

Charge Density Waves | P. A. Lee and T. M. Rice, (1979)

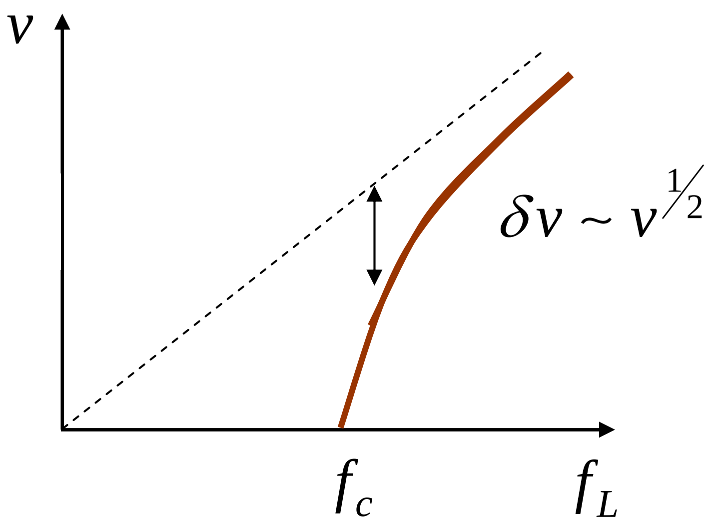
Dynamic approach

A. I. Larkin, Yu. N. Ovchinnikov, ``Electrodynamics of inhomogeneous superconductors of the second kind'', Zh. Eksp. Teor. Fiz. **65**, 1704 (1973) [Soviet Physics JETP **38**, 854].

A. Schmid, W. Hauger (1973)

$$C\nabla^2 u + \eta \frac{\partial u}{\partial t} = f_{\text{pin}}(r,u) + f_L$$

$$\langle \delta v(v) \rangle = ?$$



$\delta v \sim v$ Gives critical current

$$j_c \propto \frac{1}{B^3}$$

Collective pinning

A. I. Larkin, Yu. N. Ovchinnikov, ``Pinning in type II superconductors'',
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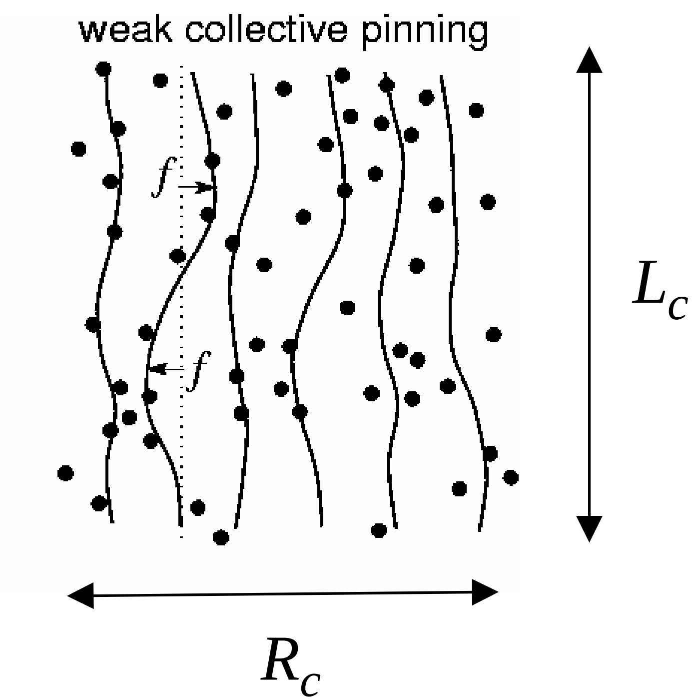


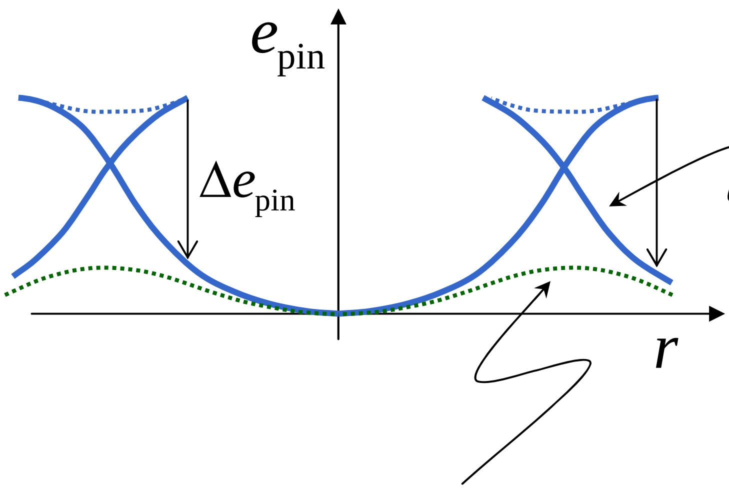
Dimensional estimates

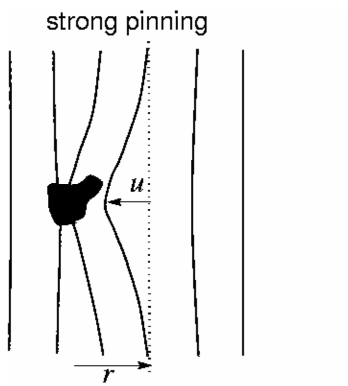
$$C_{66}\frac{\xi^{2}}{R_c^{2}}V_c \sim C_{44}\frac{\xi^{2}}{L_c^{2}}V_c \sim f\,\xi\,N^{1/2} \sim j_c B V_c / c$$

Elastic constants for the vortex lattice were derived from the microscopic theory

strong pinning (Labusch)

$$u(r) = \bar{C}^{-1} f_{\text{pin}}(r + u(r))$$



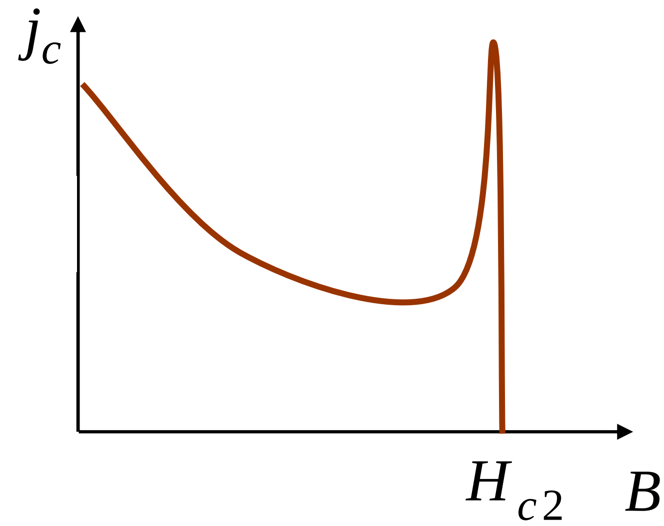


Drag the vortex lattice over the pinning landscape and **add up** all pinning forces,

$$\langle f_{\text{pin}} \rangle = -\int_0^{a_0} dx \, \frac{\partial_x e_{\text{pin}}(x)}{a_0} = \frac{\Delta e_{\text{pin}}}{a_0}$$

$$\Rightarrow \quad j_c = \frac{c}{B} \langle f_{\text{pin}} \rangle$$

Peak effect



1) Due to spatial dispersion of the tilt modulus

$$j_c \sim \exp(-AC_{66}^{3/2})$$

$$C_{66} \sim (H_{c2} - B)^2$$

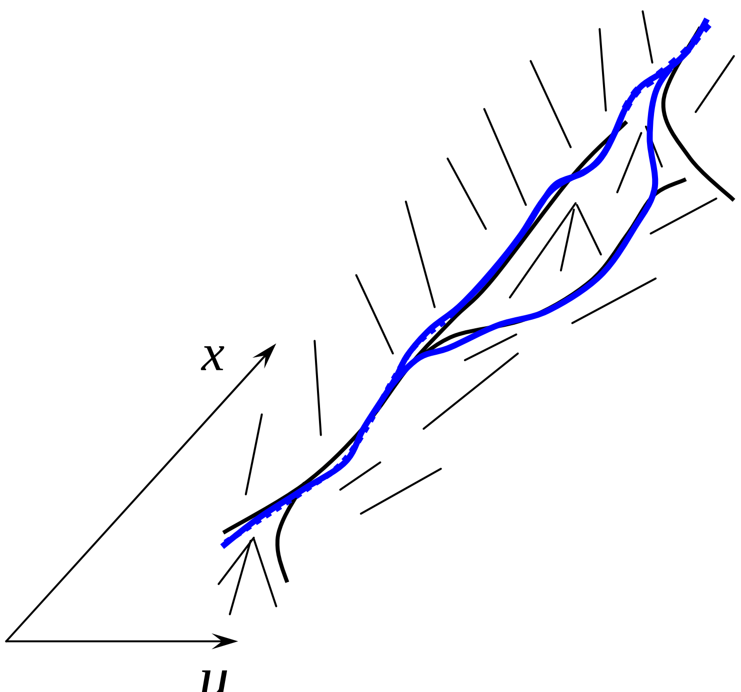
2) Due to crossover from weak to strong pinning

Where we are now?

Statics: Elastic manifolds in random media

Random force is good approximation only for $u < \xi$

For larger displacements one should consider random potential, then



Find long distance scaling of displacement field,

$$\left\langle u^2(L)\right\rangle \propto L^{2\zeta},$$

wandering exponent

$$\zeta_{1+1} = \frac{2}{3}, \ \zeta_{1+2} \cong 0.620$$

This is a generic problem, with applications in vortex matter physics, magnetism, growth, etc.

Bragg Glass

For displacements larger, than the lattice constant $u > a_0$

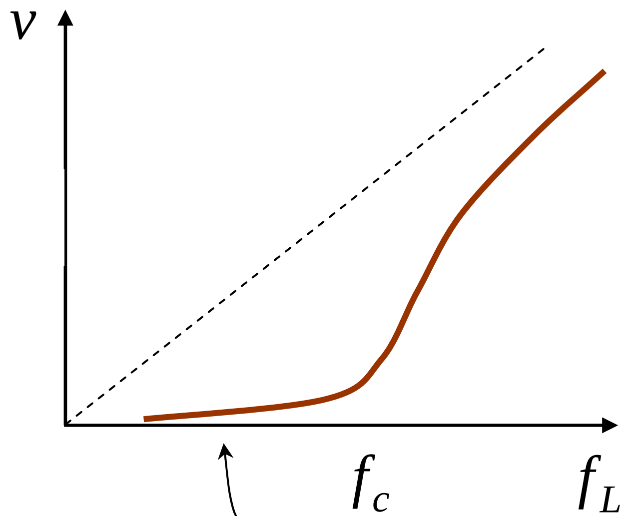
One should take into account periodicity of the lattice, then

T. Nattermann (1990), S. E. Korshunov (1993), T. Giamarchi and P. Le Doussal (1994)

$$\left\langle \left[u(r) - u(0) \right]^2 \right\rangle \simeq a_0^2 \ln(r / R_a)$$

And there is a quasi-long range order

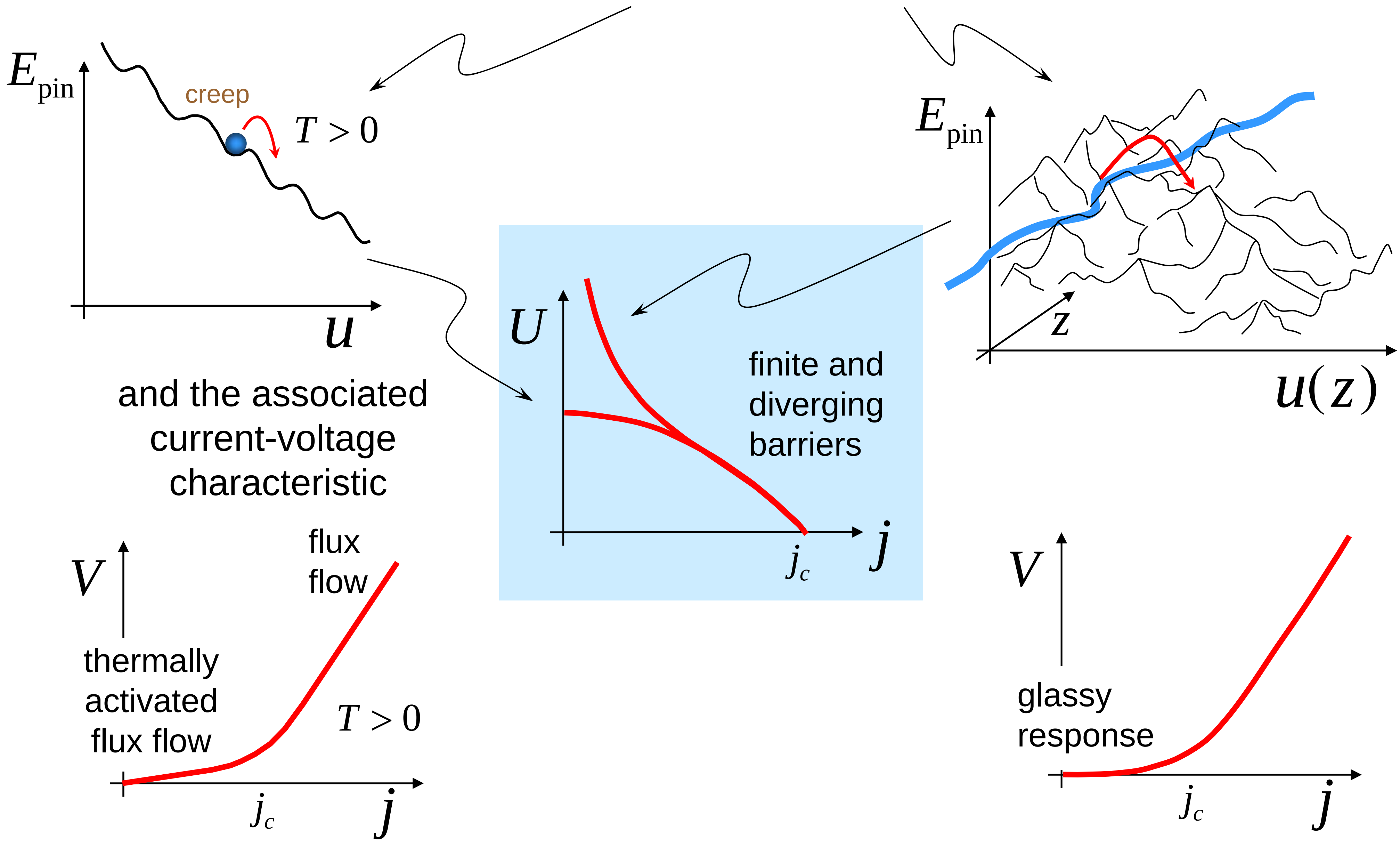
Dynamics: Creep



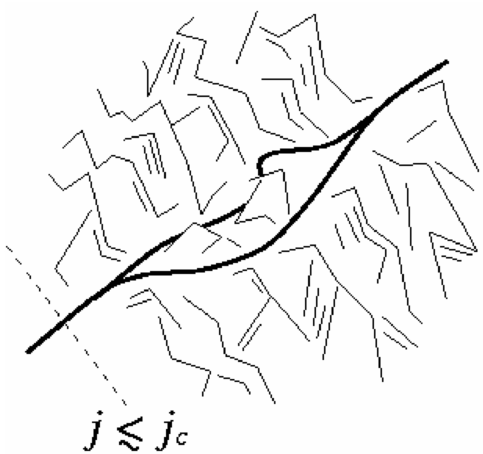
$$v \propto \exp\left(-U(f)/T\right)$$

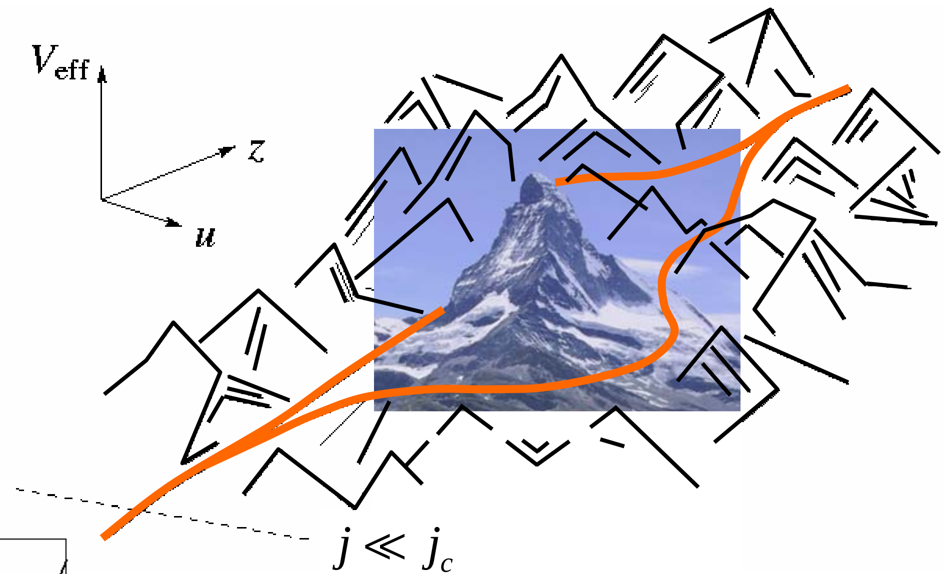
Creep, and glassyness

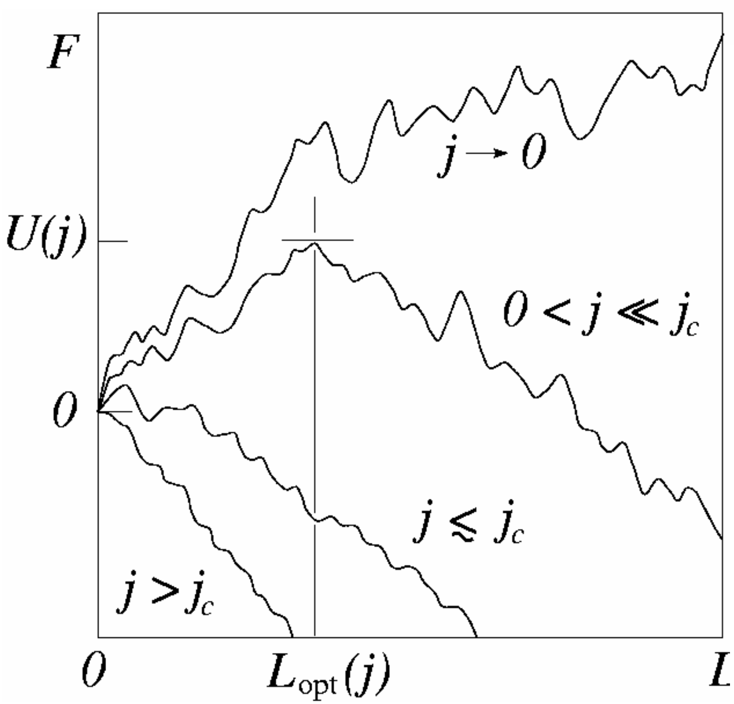
Pinning landscapes for particles and manifolds



Collective Creep Theory







With decreasing current the next favorable state is moving a larger distance away. As a consequence, thermal motion involves hops of larger segments by longer distances. This leads to the increase of the activation energy,

$$U(j) \sim U_c \left(\frac{j_c}{j} \right)^{\mu}$$

What happens for single vortex on short distances?

Formally $$\left\langle \left[u(z) - u(0) \right]^2 \right\rangle = 2R''(0) \int \frac{dk}{(2\pi)^3} \frac{(1 - \cos(kz))}{C^2 k^4}$$

is divergent on small *k*. One can cut it only by the system size *L.*

As a result

$$\left\langle \left[u(z) - u(0) \right]^2 \right\rangle \propto L z^2$$

J. P. Bouchaud, M. Mezard, and G. Parisi (1995)

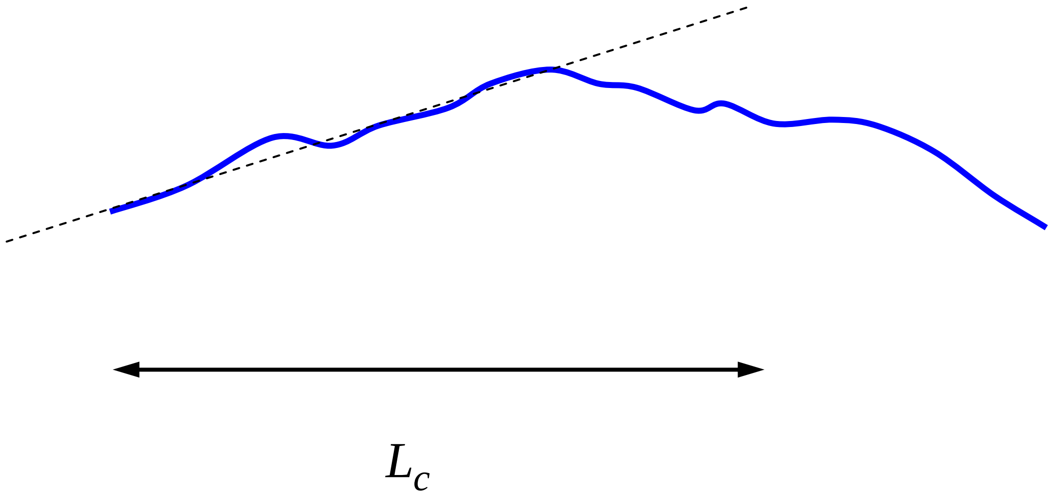
For infinite system, one should replace *L* by *Lc*

$$\left\langle \left[u(z) - u(0) \right]^2 \right\rangle \sim \xi^2 \frac{z^2}{L_c^2}, \qquad z \ll L_c$$

This looks like intermittency, but has very simple physical meaning

For $z \ll L_c$

Random force doesn't produce substantial displacement and everything is determined by initial slope



Typical slope is ξ / L_c and thus

$$u \sim \xi z / L_c$$

Unsolved problems

Vortex Glass

What happens, if disorder is so strong, that there is no lattice to start with?

M. P. A. Fisher (1989) suggested, that even for weak random potential at large distances long range order is lost and the only order, that exist is the glassy order (a la Edwards Anderson).

The model, that he proposed to describe the vortex glass, was actually solved for weak disorder by S. E. Korshunov (1993), T. Giamarchi and P. Le Doussal (1994) and leads to the Bragg Glass with quasi long range order.

Whether the phase he was looking for really exists for strong disorder, remains unclear.

FRG (Functional Renormalization Group)

D. S. Fisher (1986), K. B. Efetov and A. I. Larkin (1977)

Renormalizing of correlator gives

$$\partial_l R_l(u) = \frac{R_l''^2(u)}{2} - R_l''(0) R_l''(u)$$

Taking derivatives at *u=0* we obtain $\partial_l R_l''(0) = 0$

And since

$$\left\langle \left[u(r) - u(0) \right]^2 \right\rangle = 2R''(0) \int \frac{d^4k}{(2\pi)^3} \frac{\left(1-\cos(kr)\right)}{C^2 k^4} \propto \int dl \; R_l''(0)$$

We obtain that the lowest order Larkin result from the perturbation theory gives exact answer

K. B. Efetov and A. I. Larkin (1977)

However, $$\partial_l R_l^{IV}(0) = \left(R_l^{IV}(0)\right)^2$$

And $R_l^{IV}(0)$ diverges at finite *Lc* and the flow leads towards strong coupling

K. B. Efetov and A. I. Larkin (1977)

D. S. Fisher (1986) argued, that for larger *L>Lc* flow will eventually arrive to the weak coupling fixed point with *R''(u)* developing cusp at *u=0.*

It is not clear, whether this scenario is realized, since around *Lc* higher order terms are important. And it is not clear how to go through the strong coupling region.