Theory of 1D Bose-Liquid

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in collaboration with

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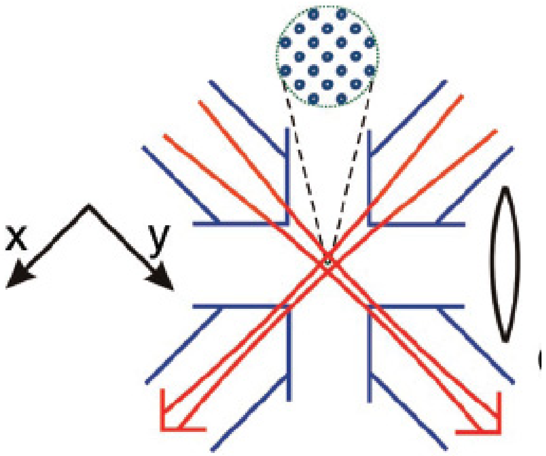
Maxim Khodas, U of M
Michael Pustilnik, Georgia Tech

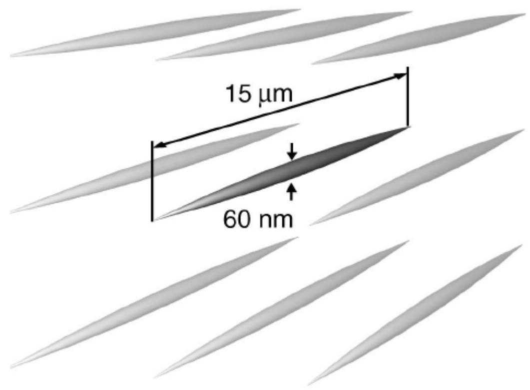
arXiv:cond-mat/0705.2015

Chernogolovka, *June, 2007*

Cold Atoms in Optical Lattices

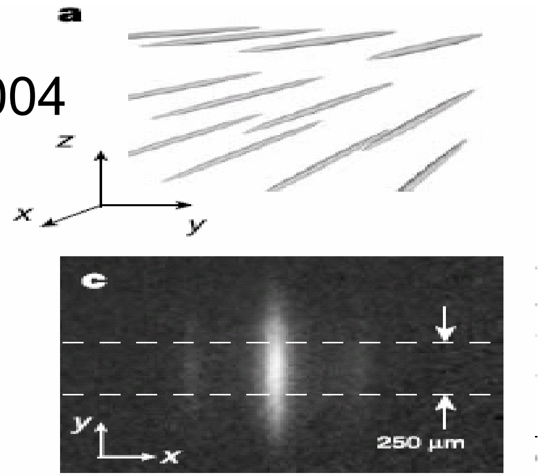


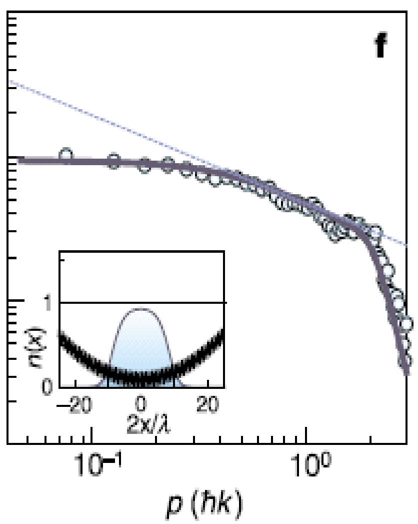
T. Kinoshita, et al 2004



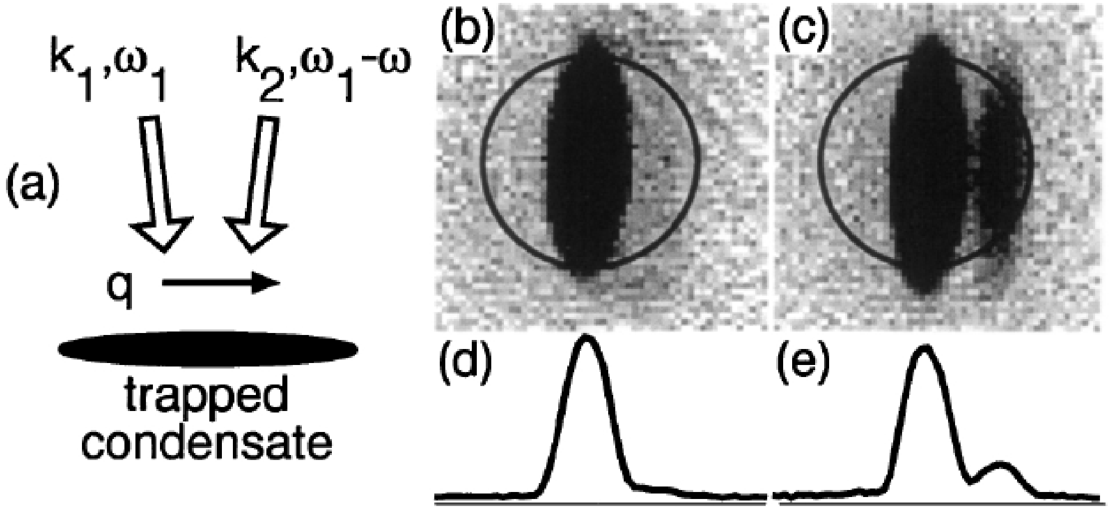
H. Moritz, et al 2003

I. Bloch, et al 2004



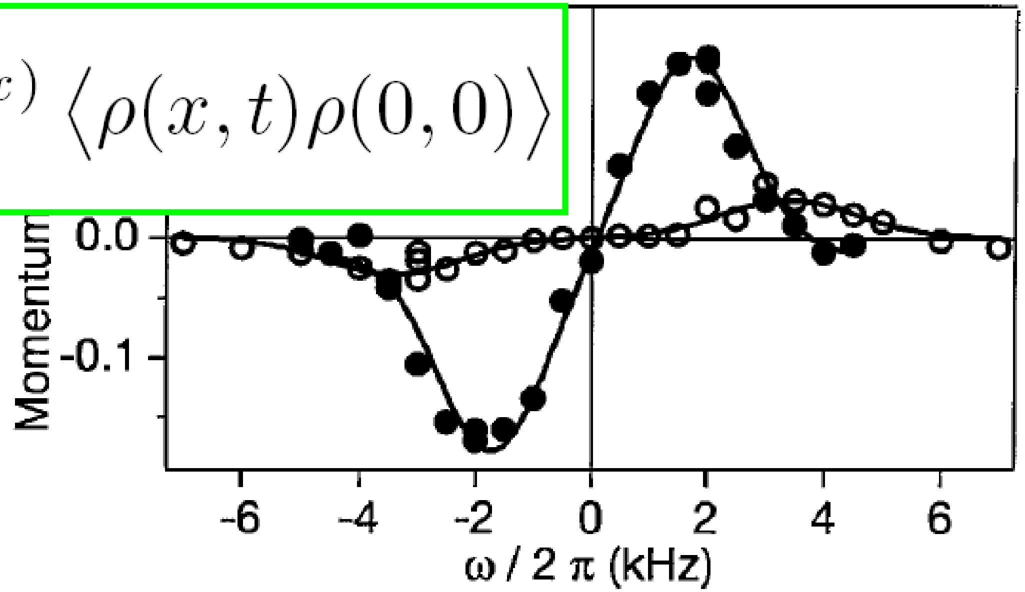


Bragg Scattering



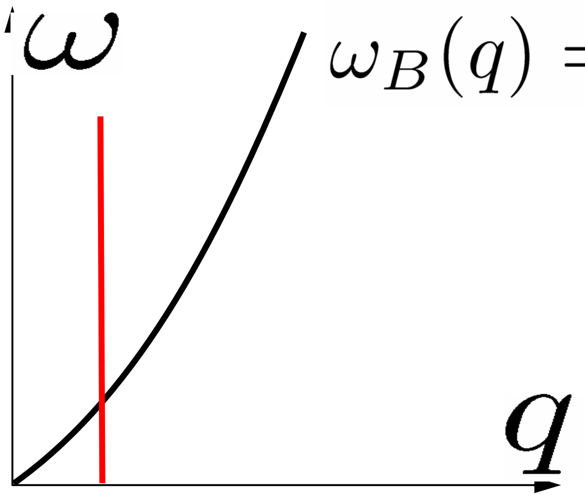
W. Ketterle, et al
2000-…

$$S(q,\omega) = \int dx\, dt\, e^{i(\omega t - qx)} \langle \rho(x,t)\rho(0,0) \rangle$$



3D Condensates

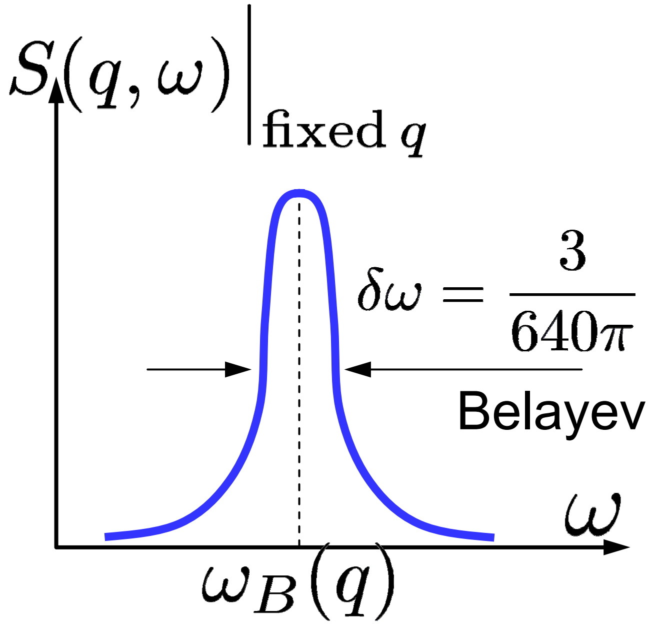
$$T < T_c$$



$$\omega_B(q) = v_s q \sqrt{1 + (q/2mv_s)^2}$$

Bogoliubov mode

✓ What about 1D?



$$\delta\omega = \frac{3}{640\pi} \frac{q^5}{\hbar^3 n} \ll \omega_B$$

Belayev 1958

PHYSICAL REVIEW VOLUME 130, NUMBER 4 15 MAY 1963

Exact Analysis of an Interacting Bose Gas. I. The General Solution and the Ground State

ELLIOTT H. LIEB AND WERNER LINIGER
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
(Received 7 January 1963)

✓ N bosons with delta-functional interactions on a 1D ring

$$H = -\frac{1}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + c\sum_{i<j}\delta(x_i - x_j)$$

✓ Two characteristic momenta: mc and $n=N/L$

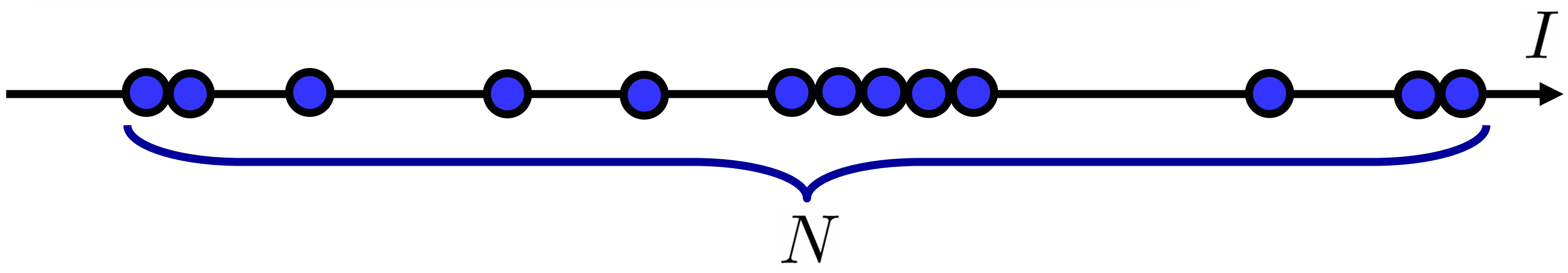
✓ Dimensionless coupling constant: $\gamma = \frac{mc}{n}$

Bethe Ansatz

$$\psi(x_1, x_2, \ldots, x_N) = \sum_P a(P)\, e^{\,i \sum_{j=1}^{N} x_j \lambda_j}$$

where $\lambda_1 < \lambda_2 < \ldots < \lambda_N$

$$\lambda_j + \frac{1}{L} \sum_k 2 \arctan \frac{\lambda_j - \lambda_k}{m\,c} = \frac{2\pi}{L} I_j$$ ← integers



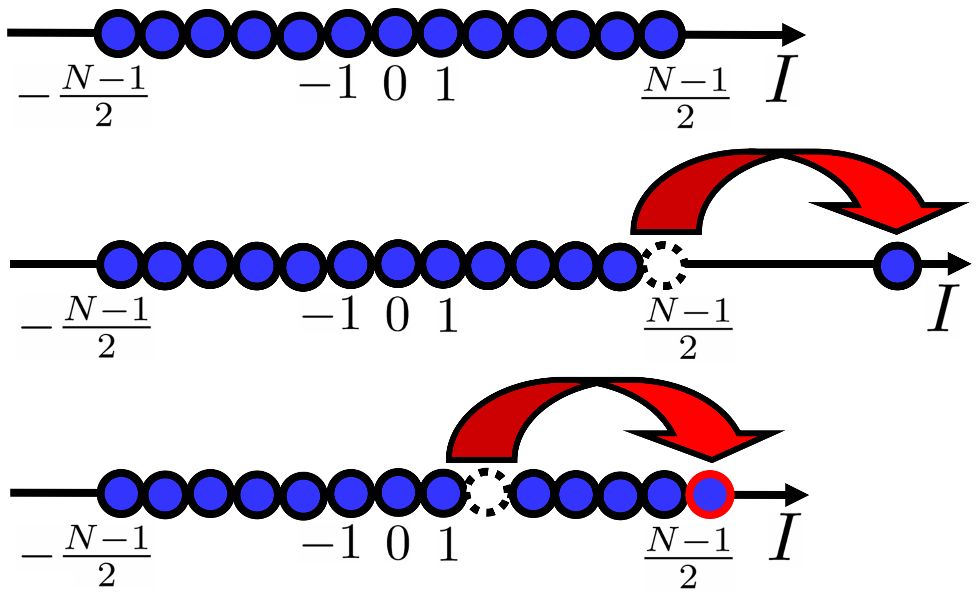
$$E = \frac{1}{2m} \sum_j \lambda_j^2 \qquad k = \sum_j \lambda_j = \frac{2\pi}{L} \sum_j I_j$$

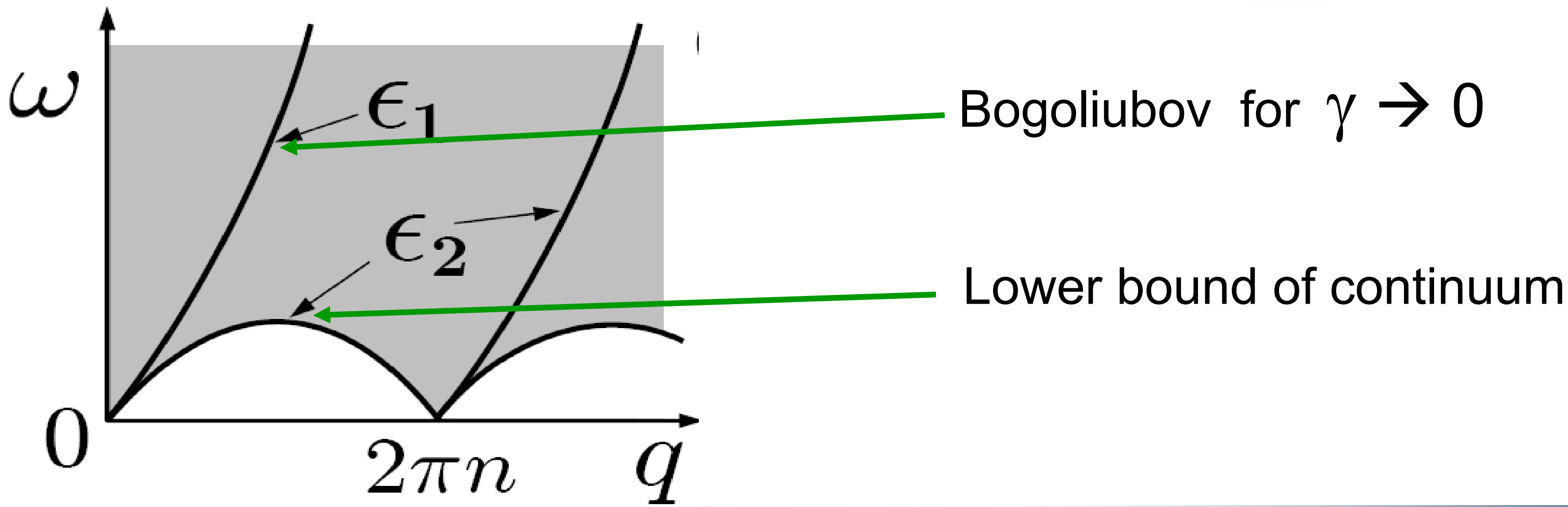
Lieb's Modes

Ground state:

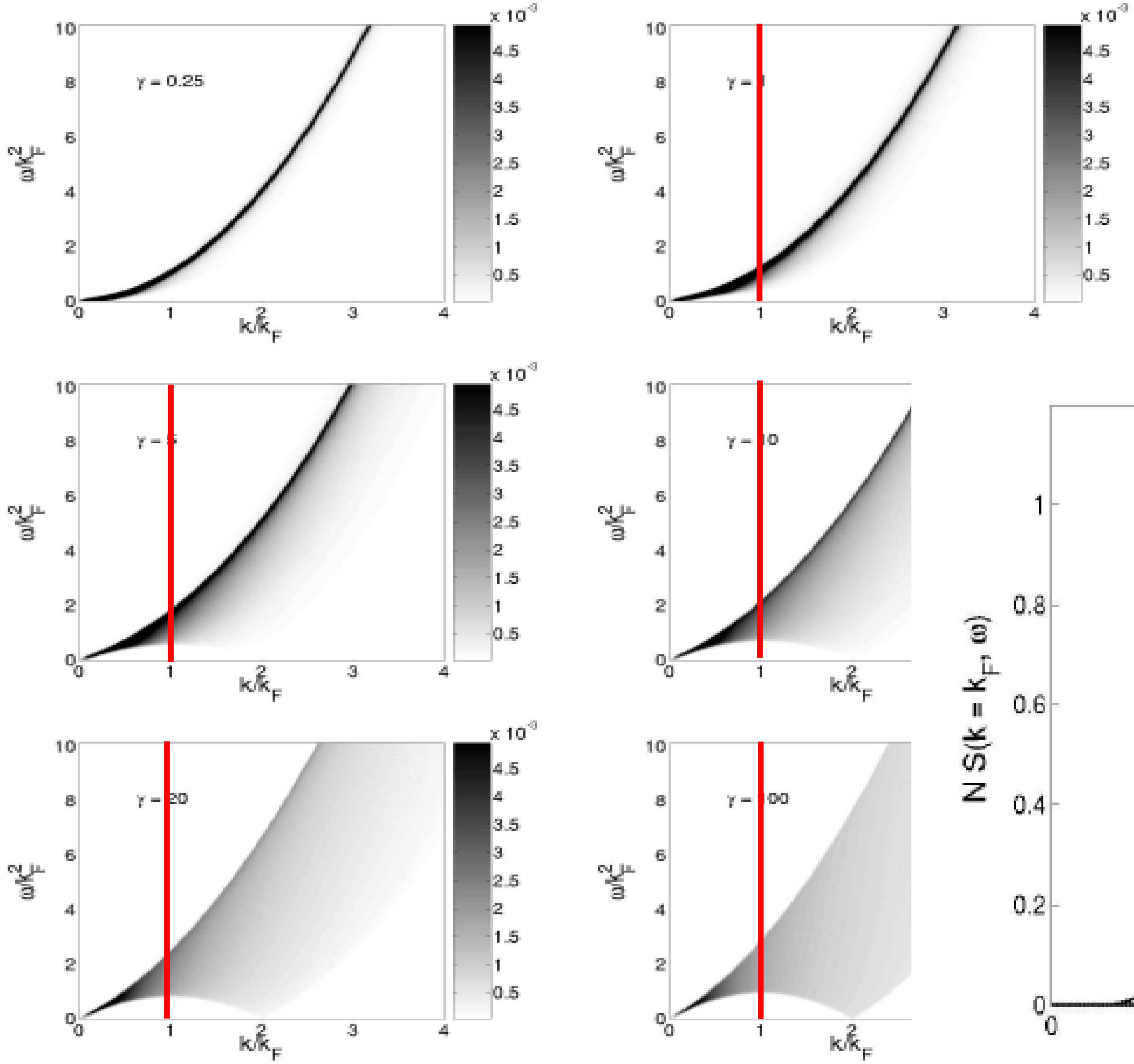
Lieb's I mode
"particles":

Lieb's II mode
"holes":

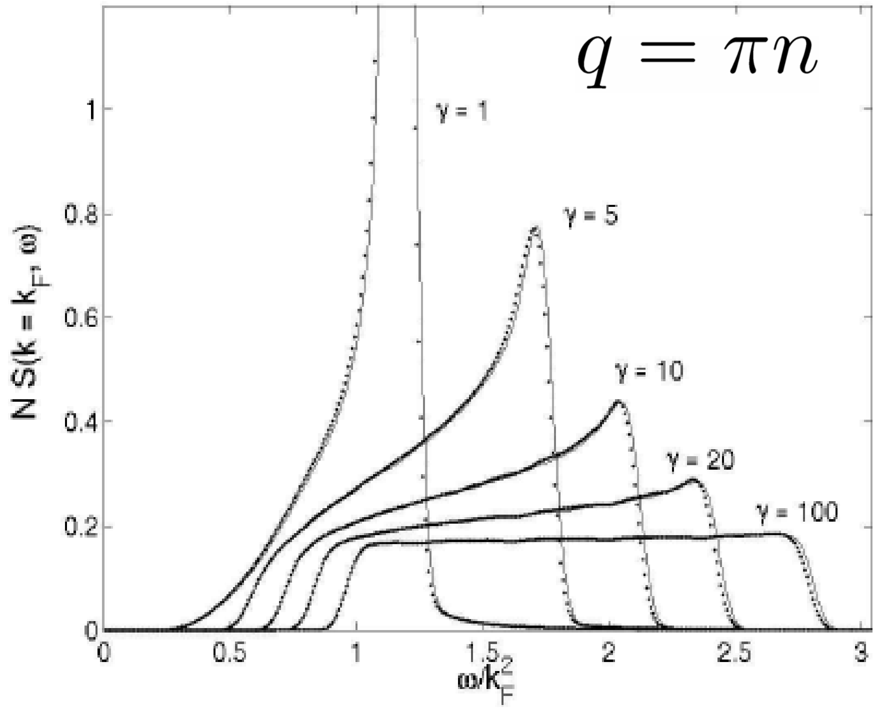




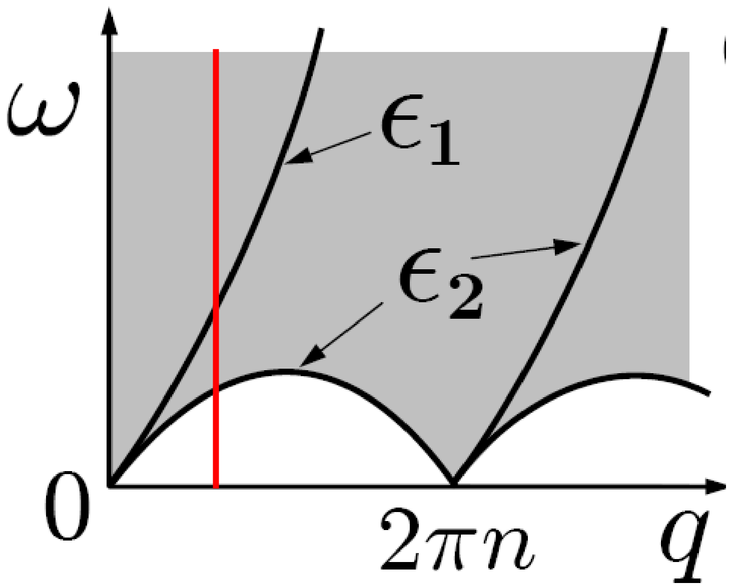
Algebraic BA exact numerics

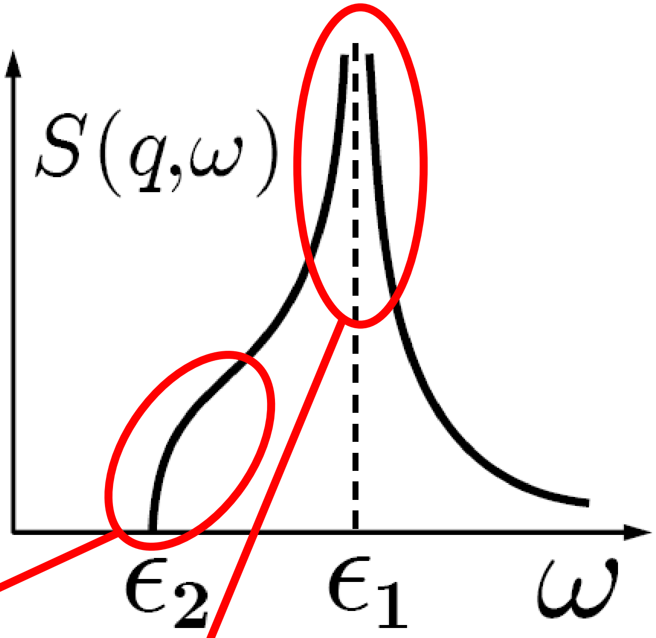


J-S. Caux, P. Calabrese, 2006
N. Slavnov, 1989



DSF singularities at Lieb's modes





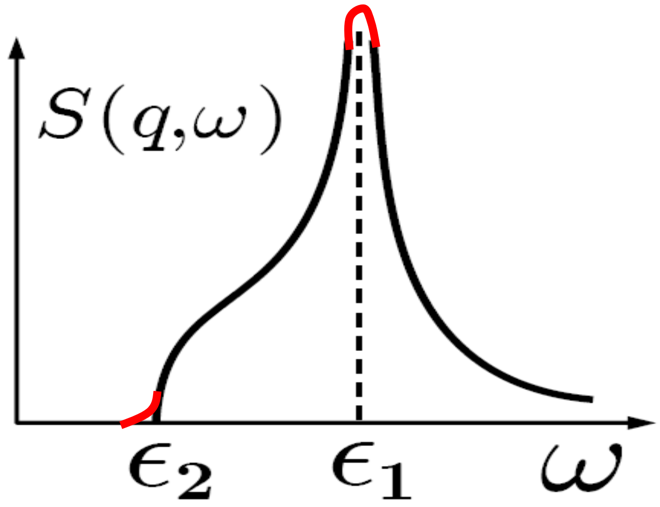
$$S(q,\omega) \sim \frac{m}{q}\left[\frac{\omega-\epsilon_2}{\delta\epsilon}\right]^{\mu_2(q)}\theta(\omega-\epsilon_2)$$

$$\delta\epsilon = \epsilon_1 - \epsilon_2$$

$$S(q,\omega) \sim \frac{m}{q}\left|\frac{\delta\epsilon}{\omega-\epsilon_1}\right|^{\mu_1(q)}\left[\theta(\epsilon_1-\omega) + \nu_1\theta(\omega-\epsilon_1)\right]$$

$$\mu_1 < 1$$

Singularities at Lieb's modes



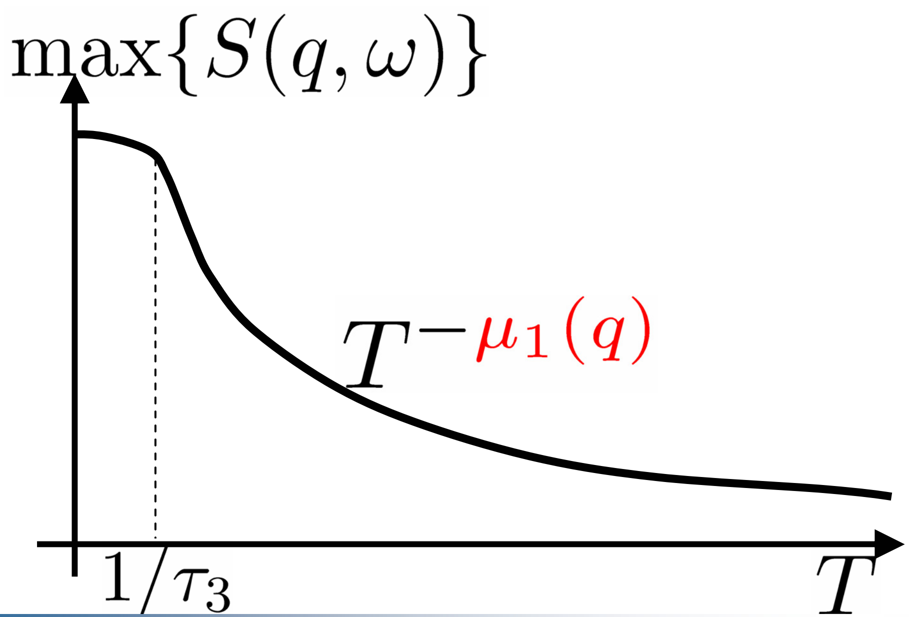
Singularities are rounded up by:

✓ a finite temperature T

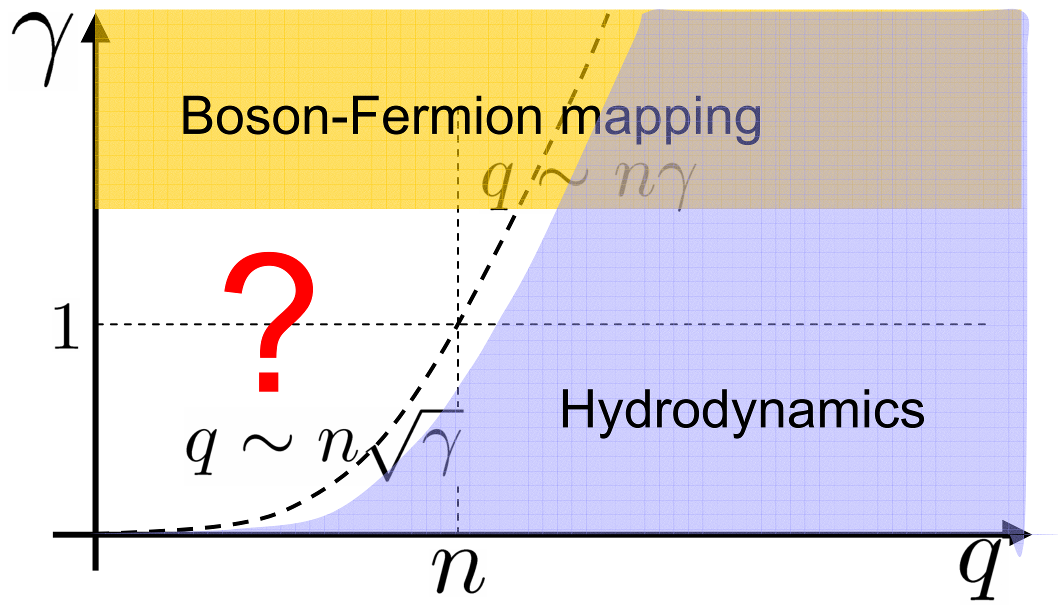
$$\max\{S(q,\omega)\}_{\text{fixed } q} \propto T^{-\mu_1(q)}$$

✓ A three-body scattering (absent in the **integrable** LL model)

$$1/\tau_3 \sim \text{interaction range}$$

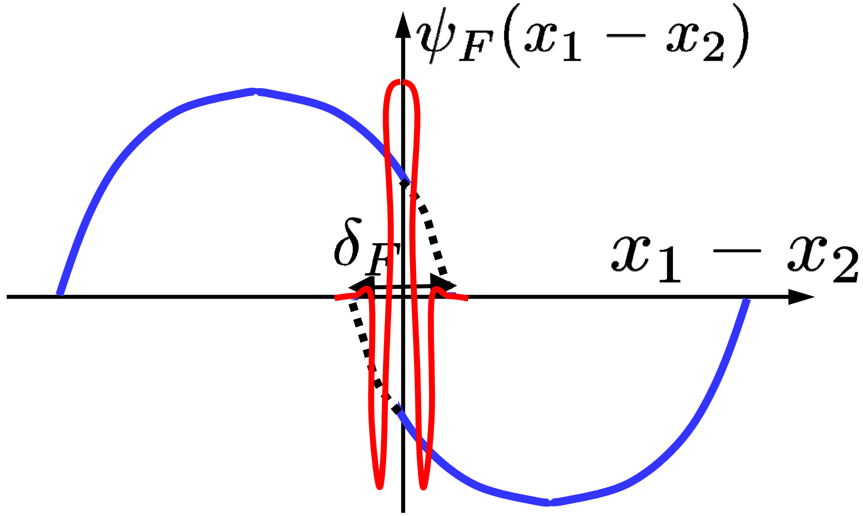


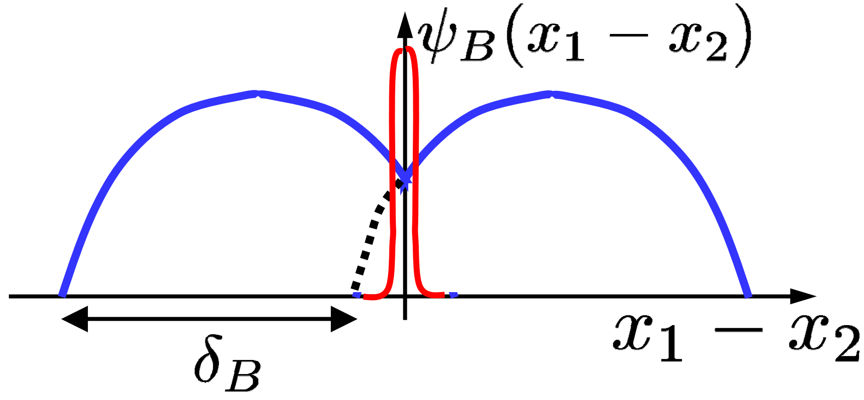
Exponents $\mu_{1,2}(q,\gamma)$



Boson-Fermion mapping

Girardeau, Olshanii, 2004





$$V_F = -\frac{2}{m^2 c}\,\delta''(x_1 - x_2)$$

$$V_B = c\,\delta(x_1 - x_2)$$

$$\delta_F = \pi - \delta_B$$

$$\underbrace{2\arctan(p/mc)}_{\delta_F} = \pi - \underbrace{2\arctan(mc/p)}_{\delta_B}$$

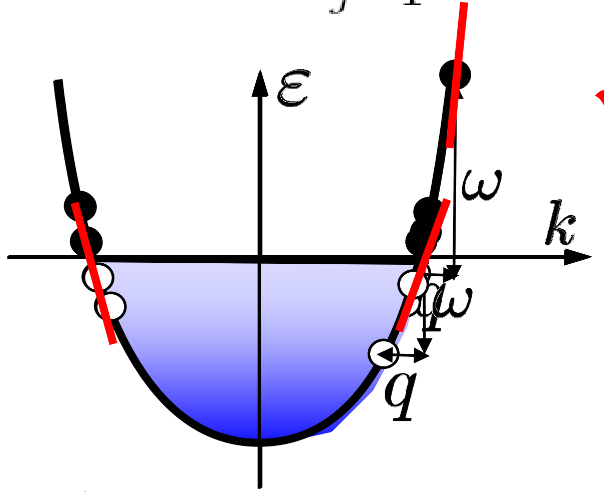
Impenetrable bosons = non-interacting fermions. Tonks-Girardeau limit

Strongly interacting bosons

= weakly interacting fermions

$$H_F = -\frac{1}{2m}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} - \frac{2}{m^2 c}\sum_{i>j}\delta''(x_i - x_j)$$

$$V(q) = \frac{2q^2}{m^2 c}$$

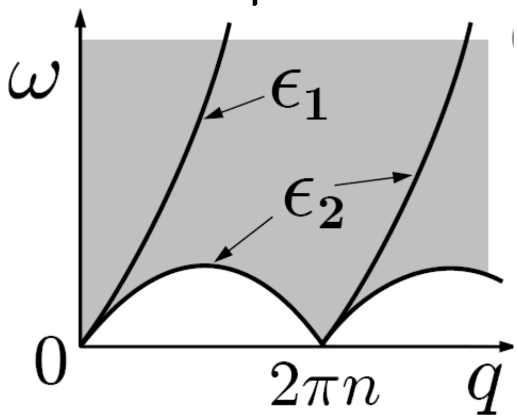


✓ Fermi-edge singularity — Schotte, Schotte 1969

$$\mu_1(q) = \frac{\delta_F(q)}{\pi} - \frac{1}{2}\left(\frac{\delta_F(q)}{\pi}\right)^2$$

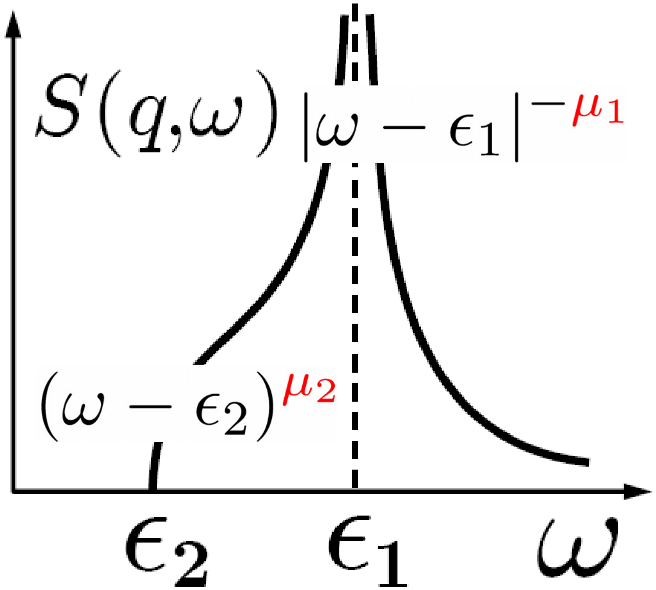
Mahan exciton — Anderson orthogonality

$$\mu_2(q) = \frac{\delta_F(q)}{\pi} + \frac{1}{2}\left(\frac{\delta_F(q)}{\pi}\right)^2$$



$$\delta_F(q) = 2\arctan(q/mc)$$

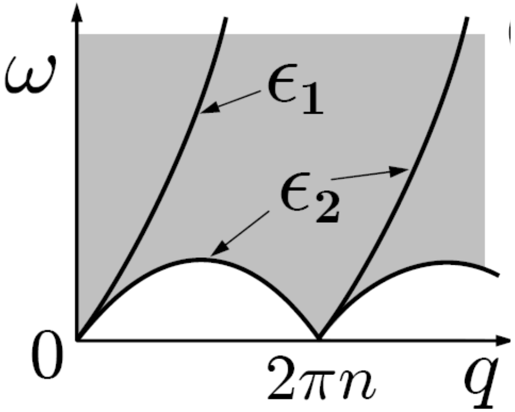
Strongly interacting bosons (cont)

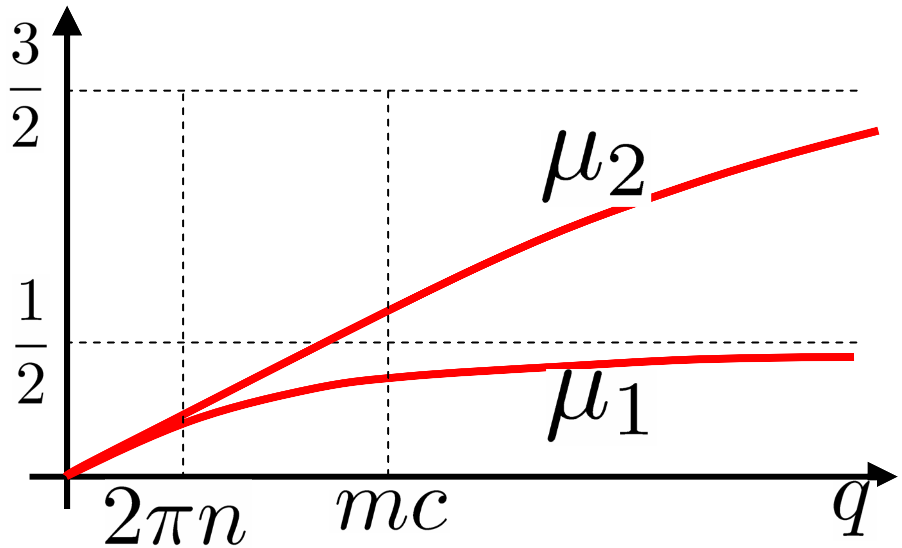


$$\gamma = \frac{mc}{n} \gg 1$$

$$\mu_{1,2}(q) = \frac{\delta}{\pi} \mp \frac{1}{2}\left(\frac{\delta}{\pi}\right)^2$$

$$\delta(q) = 2\arctan(q/mc)$$

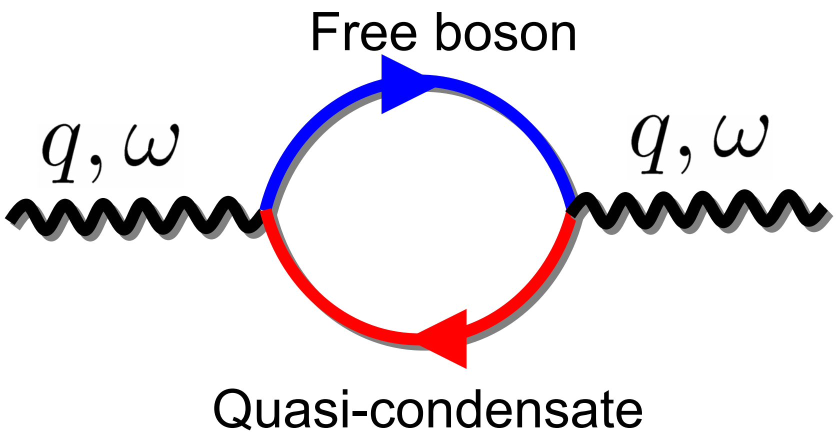




Hydrodynamic approach

= bosonization of bosons

V.N. Popov 1972
D. Haldane 1981



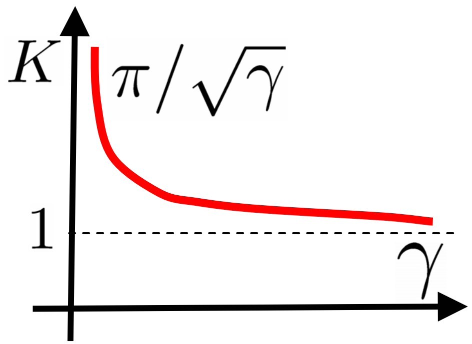
✓ Vertex corrections are small if $q \gg m v_s$

✓ Quasi-condensate is treated in hydrodynamic approximation

$$\psi(x) = \sqrt{n(x)}\, e^{i\theta(x)}\,; \quad n(x) = n + \frac{1}{\pi}\,\partial_x \phi$$

$$K = \frac{n}{\pi v_s}$$

$$H_{QC} = \frac{v_s}{2\pi}\int dx\,\left[K^{-1}(\partial_x\phi)^2 + K(\partial_x\theta)^2\right]$$



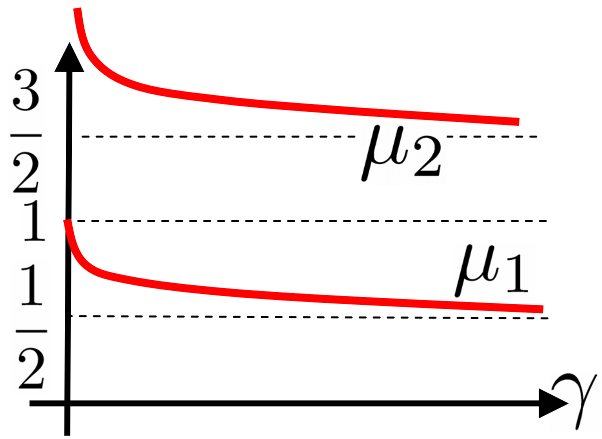
Structure factor at large momenta

$$S(q,\omega) = \int dx dt\, e^{i(\omega t - qx)} \overbrace{e^{iq^2 t/2m}\delta(x - qt/m)}^{\text{Free boson}} \overbrace{\underbrace{\left\langle e^{i\theta(x,t)}\, e^{-i\theta(0,0)} \right\rangle}_{e^{K \ln\left(x^2 + (v_s t)^2\right)}}}^{\text{Quasicondensate}}$$

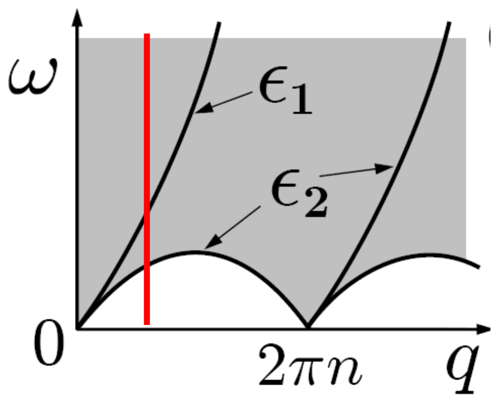
$$\propto \left(\omega - \frac{q^2}{2m}\right)^{-\mu_1}$$

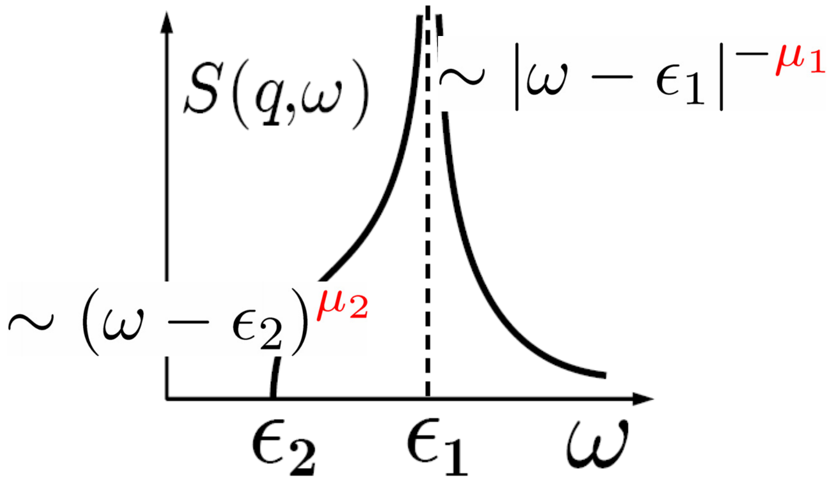
$$\mu_1 = 1 - (2K)^{-1}$$

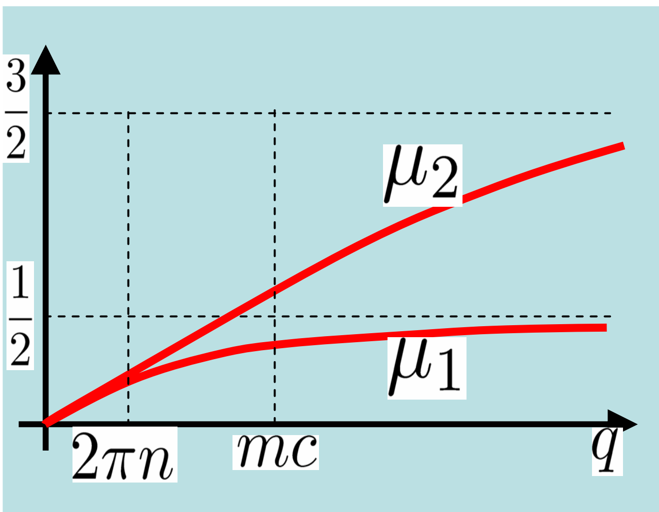
$$\mu_2 = 2K + (2K)^{-1} - 1$$

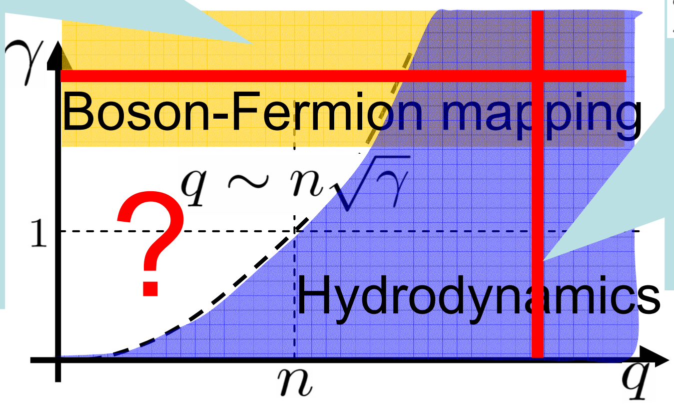


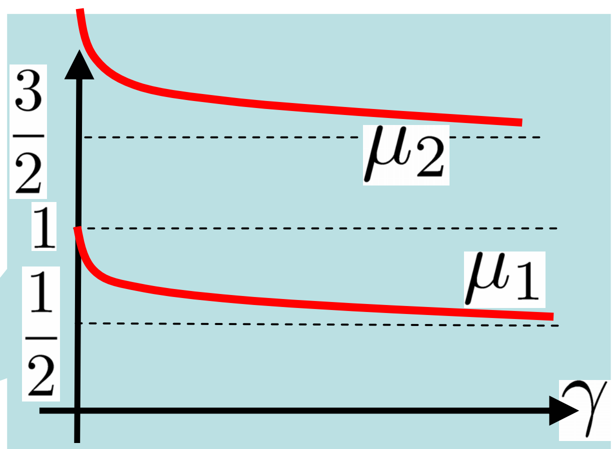
Summary of DSF exponents











$$\mu_{1,2}(q) = \frac{\delta}{\pi} \mp \frac{1}{2}\left(\frac{\delta}{\pi}\right)^2$$

$$\delta = 2\arctan(q/n\gamma)$$

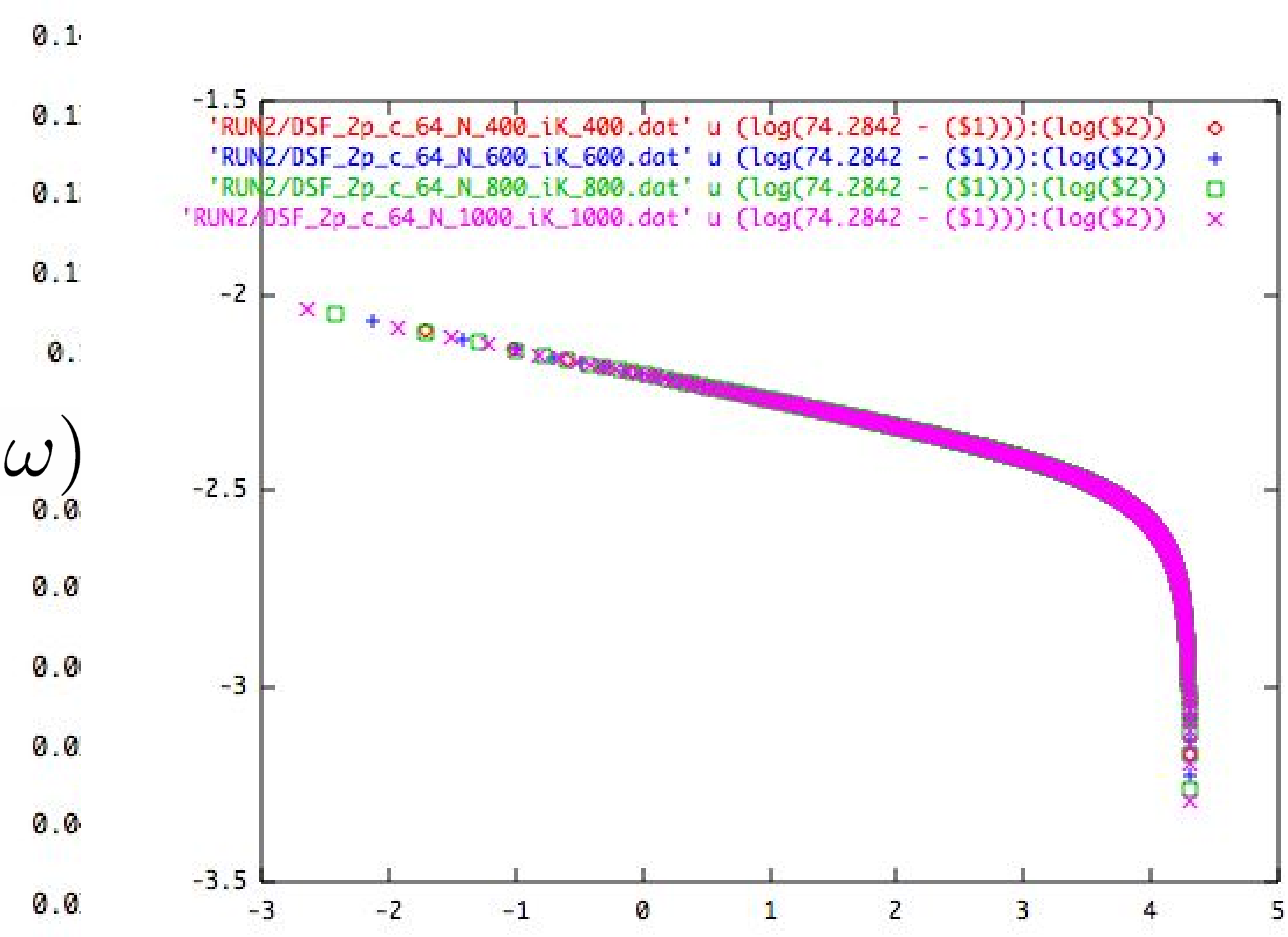
$$\mu_1 = 1 - (2K)^{-1}$$

$$\mu_2 = 2K + (2K)^{-1} - 1$$

Numerics (preliminary)

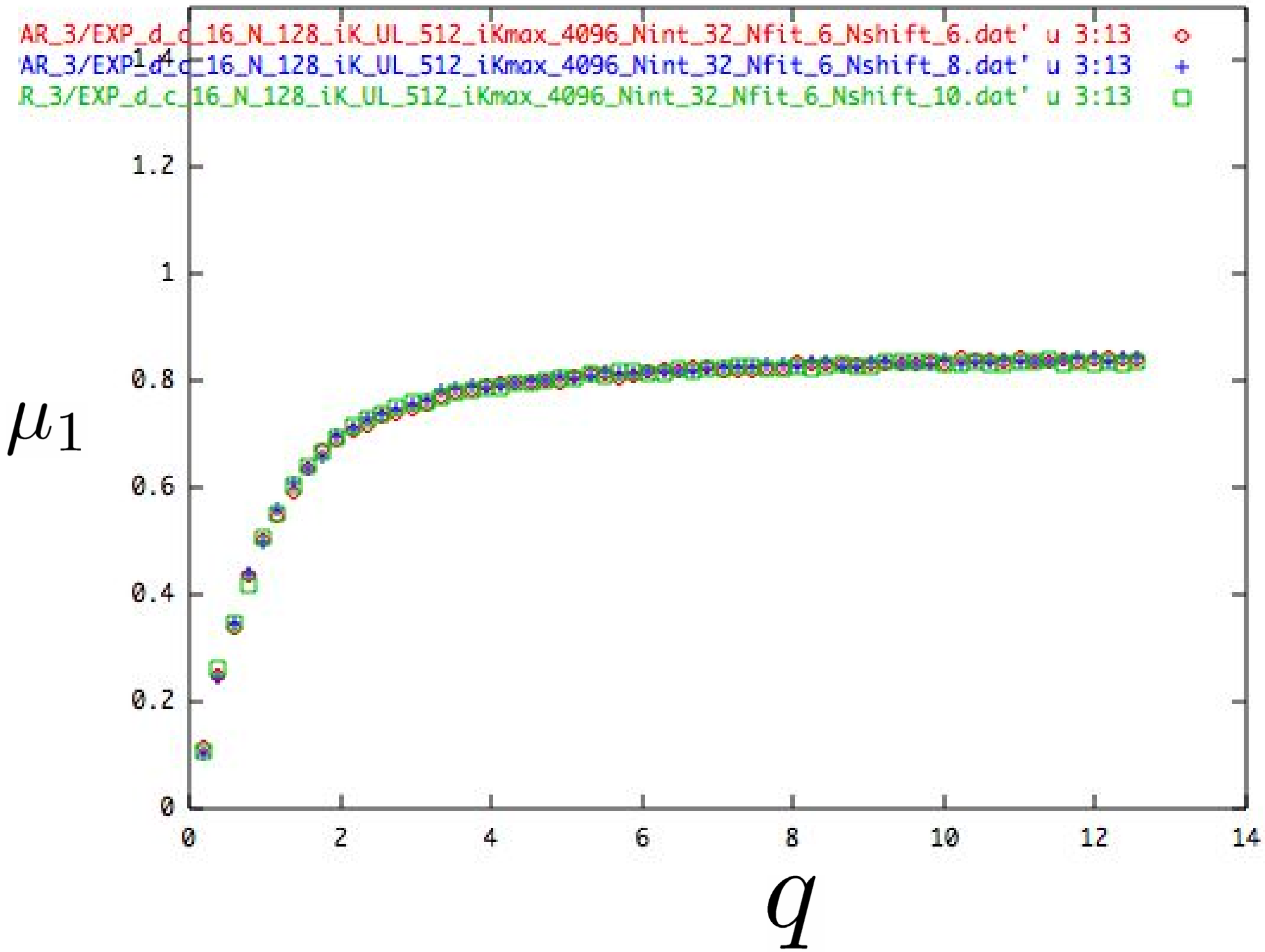
Courtesy of J-S. Caux

$S(2\pi n, \omega)$

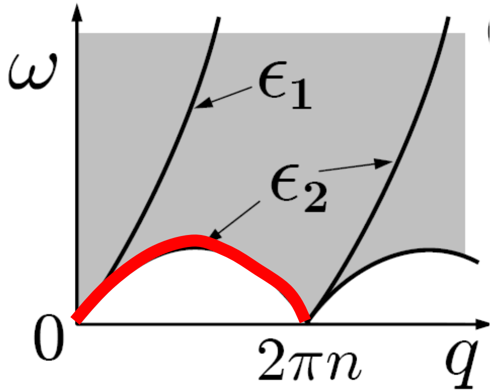


Numerics (preliminary)

Courtesy of J-S. Caux



Dark Solitons



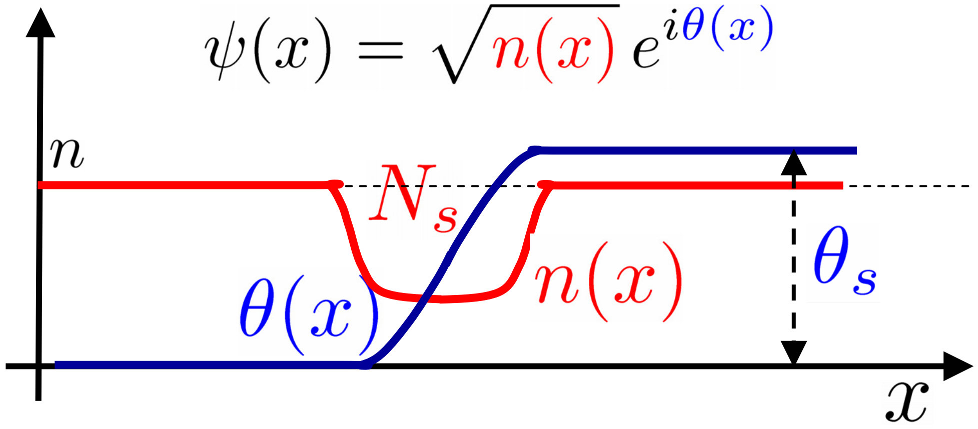
$$\gamma \ll 1$$

P.P. Kulish, S.V. Manakov and L.D. Faddeev 1976

$$i\partial_t \psi = -\frac{1}{2m}\nabla^2\psi + c|\psi^2|\psi - \mu\psi$$

Gross-Pitaevskii equation

$$\psi(x) = \sqrt{n(x)}\, e^{i\theta(x)}$$



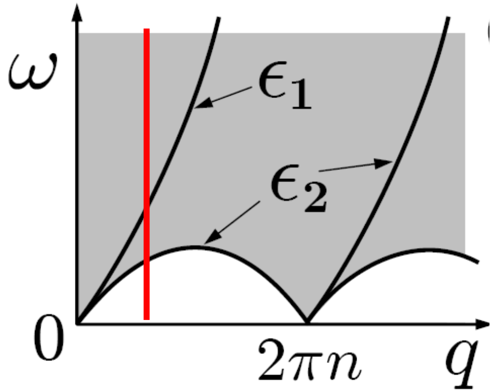
$$\epsilon_s(q) = \epsilon_1(q)$$

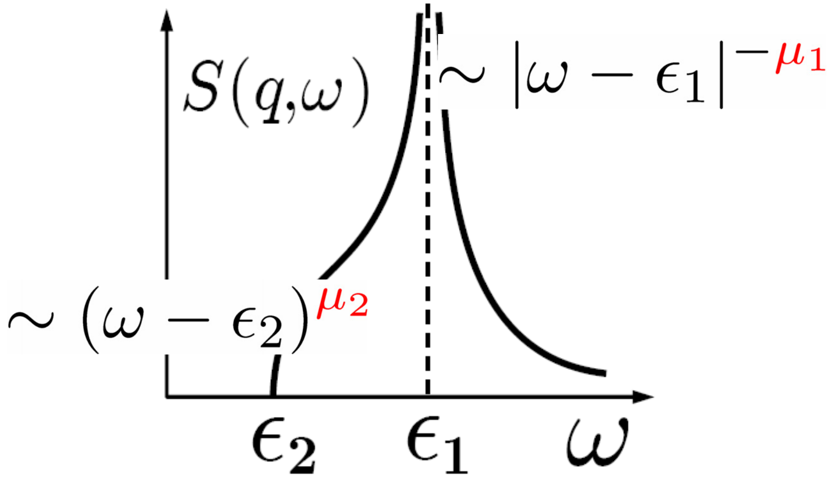
$$\theta_s\,; \qquad \phi_s = \pi N_s$$

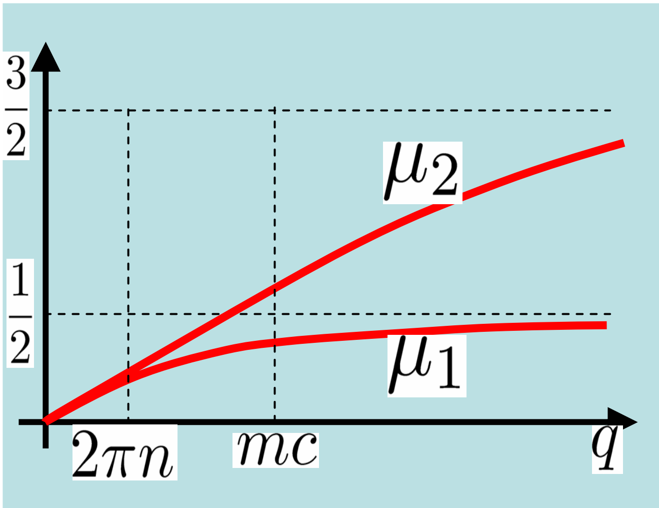
Soliton creation operator:

$$\hat{\psi}_s(x) \propto e^{i\theta_s\hat{\phi}(x) - i\phi_s\hat{\theta}(x)}$$

Summary of DSF exponents







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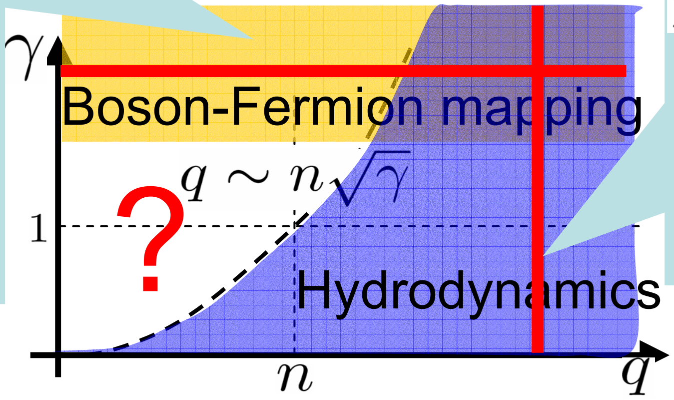

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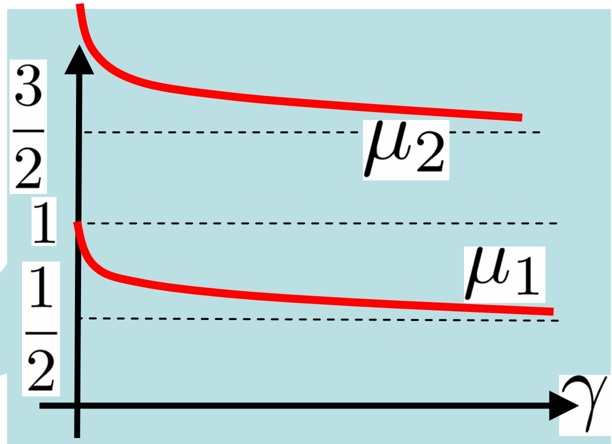

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