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L.D Landau
INSTITUTE FOR THEORETICAL PHYSICS

Nonequilibrium charge transport and current noise in quantum SINIS contacts

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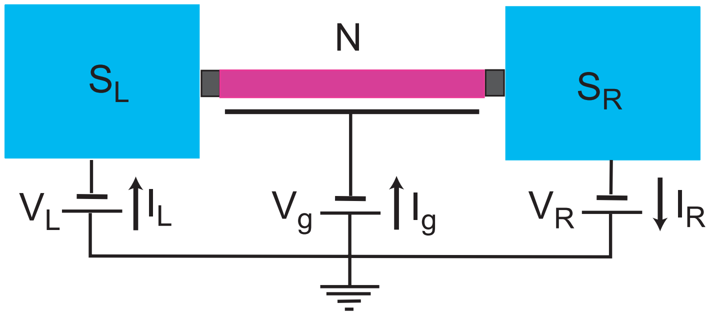
Outline

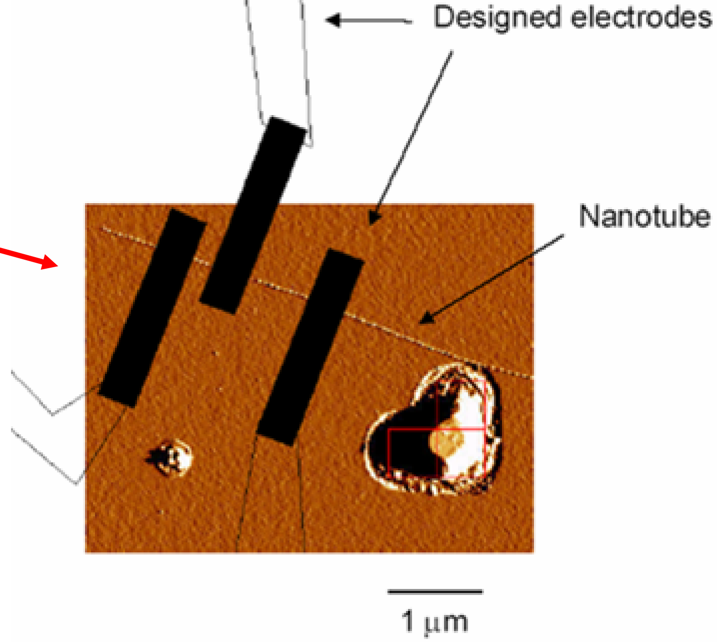
- Excitation spectrum in quantum SINIS junctions.
- Kinetic equation in presence of Landau-Zener transitions and inelastic relaxation
- Non-equilibrium DC current
- Current noise

Quantum SINIS junctions



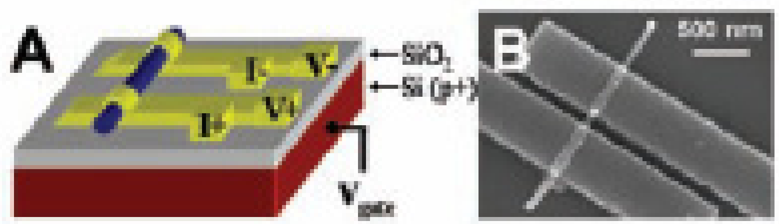
Possible realizations

Carbon nanotube-
Ti/Al electrodes



Jarillo-Herrero *et al.*, *Nature* ***439****, 953 (2006)*

InAs semiconductor nanowire-
Ti/Al electrodes

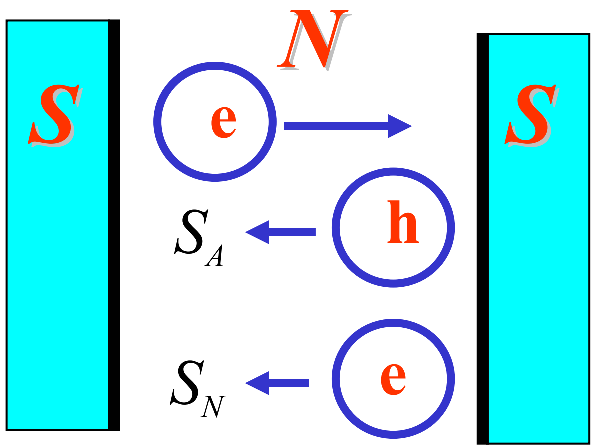


Doh *et al.*, *Science* ***309****, 272 (2005)*

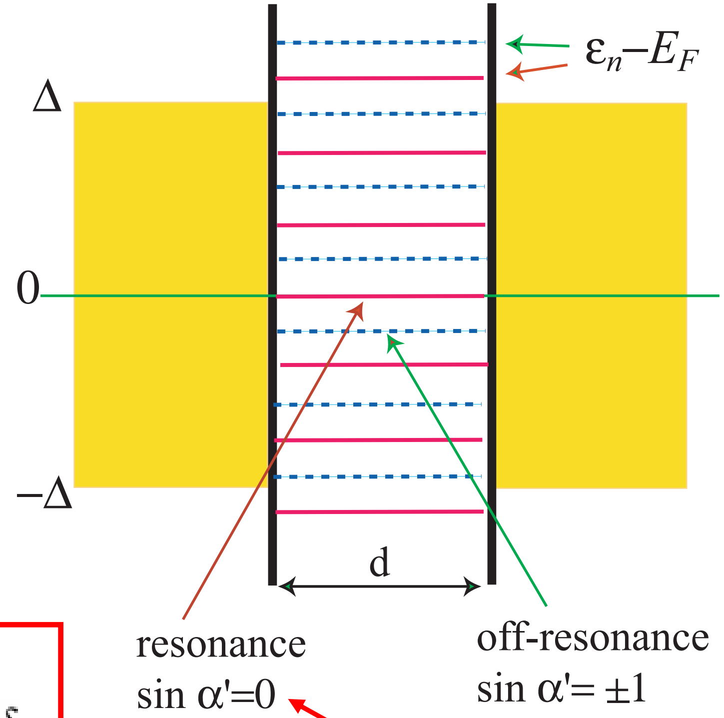
CNT: interlevel distance $\delta\varepsilon \approx 3.5$ mV
Superconducting gap: $\Delta \approx 0.25$ mV

Double-barrier SINIS junctions

Interplay between geometrical and Andreev quantization



Energy resonances,
$\delta\varepsilon = \pi\hbar v_x/d$



Amplitude of Andreev reflection = S_A

Amplitude of normal reflection = S_N

S_A and S_N depend on energy

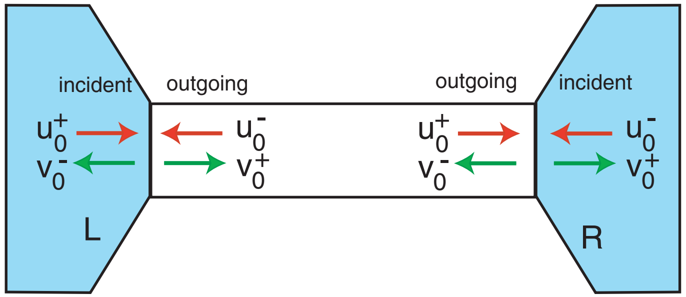
Barrier height $U_{1,2} = I\delta(x \pm d/2)$

Barrier strength $Z = mI/\hbar^2 k_x$

$$\alpha' = \alpha + \delta$$
$$\alpha = k_x d$$
$$\tan\delta = 1/Z$$

One of the resonance levels coincides with the Fermi energy

Scattering matrix approach



The incident and outgoing amplitudes are coupled by $\hat{S}_R$ and $\hat{S}_L$

$$\begin{pmatrix} u_0^- \\ v_0^+ \end{pmatrix}_R = \hat{S}_R \begin{pmatrix} u_0^+ \\ v_0^- \end{pmatrix}_R , \quad \begin{pmatrix} u_0^+ \\ v_0^- \end{pmatrix}_L = \hat{S}_L \begin{pmatrix} u_0^- \\ v_0^+ \end{pmatrix}_L , \quad \hat{S} = \begin{pmatrix} S_N e^{i\delta} & S_A e^{i\chi} \\ S_A e^{-i\chi} & S_N e^{-i\delta} \end{pmatrix}$$

$$\begin{pmatrix} u_0^\pm \\ v_0^\pm \end{pmatrix}_R = e^{\pm i k_x d} \begin{pmatrix} u_0^\pm \\ v_0^\pm \end{pmatrix}_L$$

The matrix $\hat{S}$ is unitary: $\hat{S}\hat{S}^\dagger = 1$.

Diagonal components of $\hat{S}$ $\Rightarrow$ normal reflection.

Off-diagonal components $\Rightarrow$ Andreev reflection.

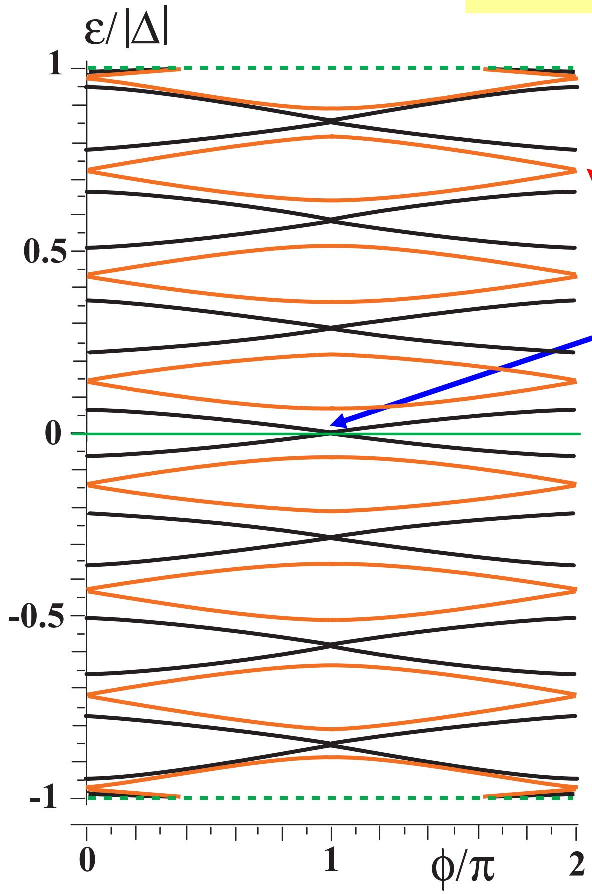
The solvability condition $\Rightarrow$ $\det\left(1 - e^{i\hat{\sigma}_z k_x d}\hat{S}_R e^{i\hat{\sigma}_z k_x d}\hat{S}_L\right) = 0$.

Long SINIS junction, Δ >>δε

$\delta\varepsilon = \pi\hbar v x/d$

Short SINIS junction, Δ <<δε

Number of levels in the contact,
$N \sim \Delta/\delta\varepsilon \sim d\Delta/\pi\hbar v_x$

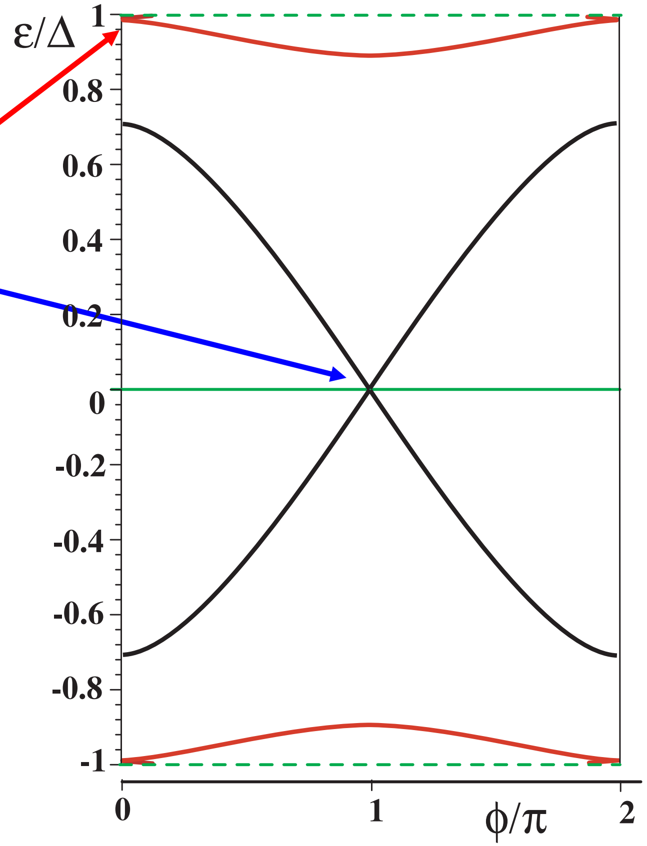


Z=0.5, $d\Delta/\hbar v_x$=10.
Black lines: sin α′=0.
Red lines: sin α′=±1

Gaps are closed

All gaps
at φ=π and φ=0
disappear for
sin α′=0 and
sin α′=±1,
respectively.

α can be controlled by gate voltage



Z=1.0, $d\Delta/\hbar v_x$=0.2.
Black lines: sin α′=0.
Red lines: sin α′=±1

Sub-gap spectrum of a SINIS junction

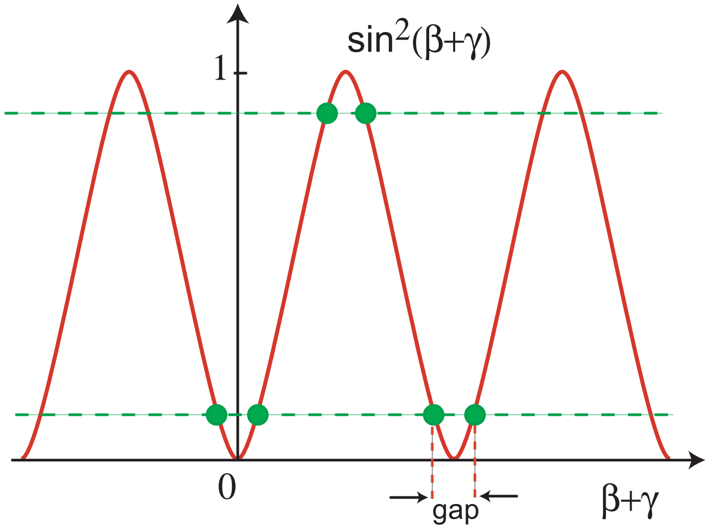
$$|S_N|^2 \sin^2 \alpha' + |S_A|^2 \cos^2(\phi/2) = \sin^2(\beta + \gamma)$$

$$\alpha = k_x d\,,\ \beta = \epsilon d/\hbar v_x$$

$$\alpha' = \alpha + \delta\,,\ \phi = \chi_2 - \chi_1$$

$$e^{2i\gamma} = S_N/S_N^*$$

$$|S_N|^2 + |S_A|^2 = 1$$



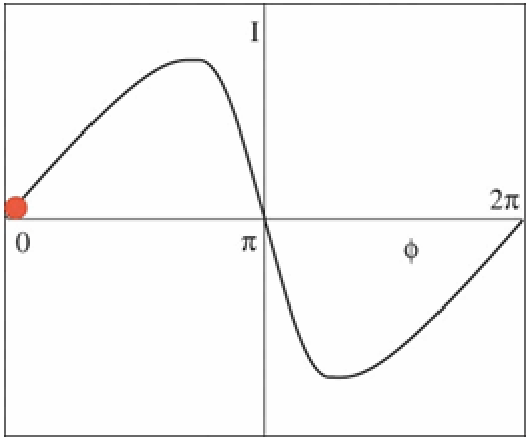
Spectrum in various limits:

e.g., A.Furusaki et al (1992)
Schüssler, Kümmel (1993)
Gogadze, Kosevich (1998)
Shumeiko, Wendin (2000)
Kuhn et al (2001)
Jakobs, Kümmel (2005)

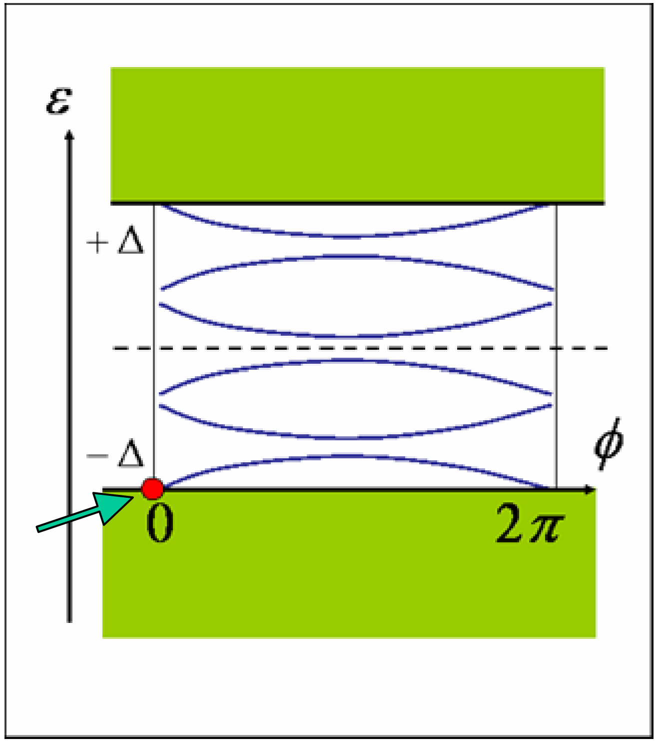
All gaps at φ=π and φ=0 disappear for sin α′=0 and sin α′=±1, respectively

Supercurrent

$$I_s = -\frac{2e}{\hbar}\sum_{\epsilon_n>0}^{\Delta}\frac{\partial\epsilon_n}{\partial\phi}(1-2f_n)$$



Example 1: Equilibrium SINIS contact under a small bias eV<<δε



Gaps or strong inelastic relaxation

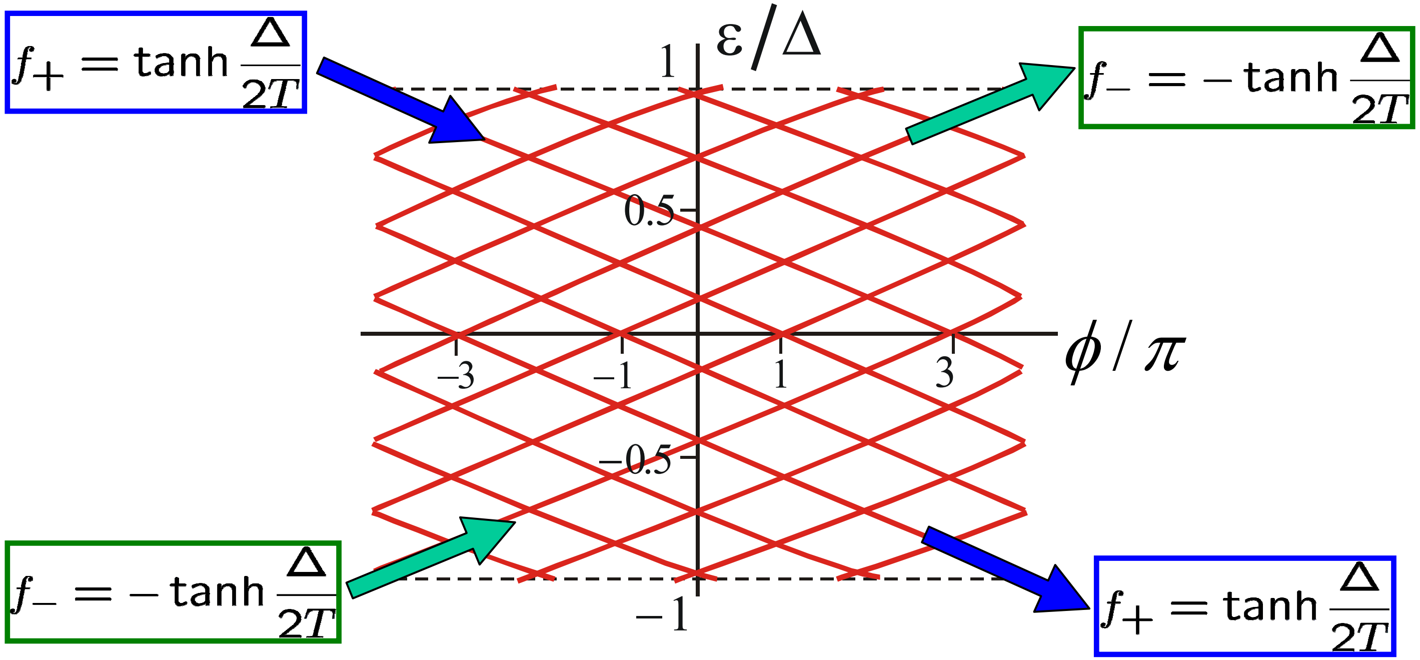
Adiabatic variation along the Andreev levels:

$$d\phi/dt = 2eV/\hbar \equiv \omega_J$$

Bloch oscillations of supercurrent

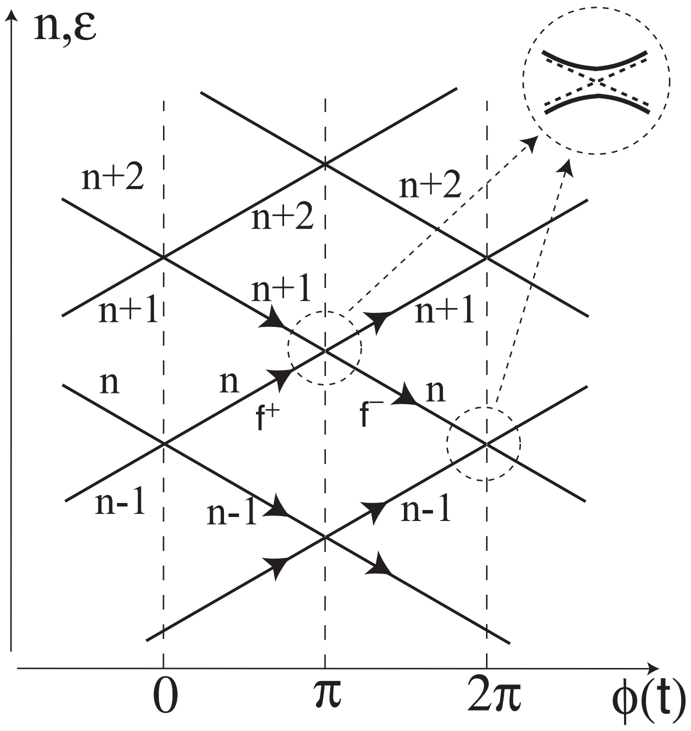
NO dc current without the interlevel Landau-Zener transitions

Example 2: Charge transfer for continuous spectral flow



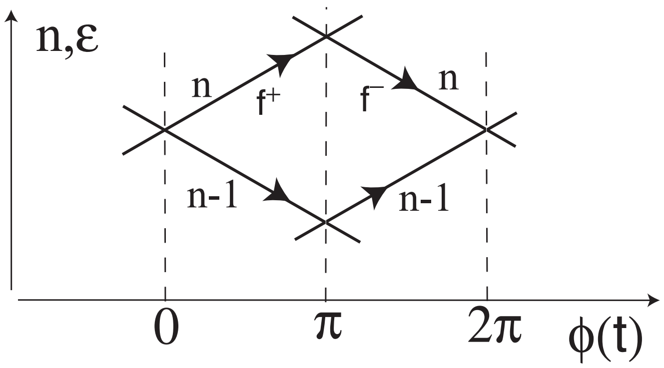
Maximum dissipation

Distribution in the presence of weak inelastic relaxation and voltage-driven Landau-Zener transitions ?



Kinetic equation

Single loop



$$\frac{1}{\hbar}\int(\epsilon_n-\epsilon_{n-1}) \sim \frac{v_x}{\omega_J d} \gg 1$$

Incoherent inter-level transitions due to the low-voltage condition

$$\begin{aligned}
f_n^+(0+) &= p_0 f_{n-1}^+(0-) + (1-p_0) f_n^-(0-) \\
f_n^-(\pi+) &= p_\pi f_{n+1}^-(\pi-) + (1-p_\pi) f_n^+(\pi-) \\
f_{n+1}^-(0+) &= p_0 f_{n+2}^-(0-) + (1-p_0) f_{n+1}^+(0-) \\
f_{n+1}^+(\pi+) &= p_\pi f_n^+(\pi-) + (1-p_\pi) f_{n+1}^-(\pi-)
\end{aligned}$$

Boundary conditions:

$$f^+_{\epsilon=-|\Delta|} = f^{(0)}(-|\Delta|)\,,\ f^-_{\epsilon=+|\Delta|} = f^{(0)}(+|\Delta|)$$

Model of weak intra-level inelastic relaxation

$$\frac{\partial f_n}{\partial t} = -\frac{f_n - f_n^{(0)}}{\tau_{\rm in}}\,, \qquad \tau_{\rm in} v_x \gg d$$

Low temperatures: $T \ll \hbar v_x / d \;\Rightarrow\; f_n^{(0)} = \Theta(-\epsilon_n)$

$$f_n^{\pm} - f_n^{(0)} = C_{\pm} \exp\left[-r|n|\right]$$

$\gamma = \dfrac{\pi\hbar}{2eV\tau_{\rm in}}$ - Degree of inelastic relaxation during Josephson half-period

The effective relaxation rate

Strong inelastic relaxation $\gamma \gg 1 \;\Rightarrow\; r \approx \gamma$

Weak inelastic relaxation $\gamma \ll 1$

$$\sinh^2(r) = \gamma(\gamma + p_0 + p_\pi - 2p_0 p_\pi)/p_0 p_\pi\,.$$

$$r = \begin{cases} \gamma \ll 1\,, & p_0, p_\pi = 1 \\ \gg 1\,, & p_0, p_\pi \to 0 \end{cases}$$

The distribution function

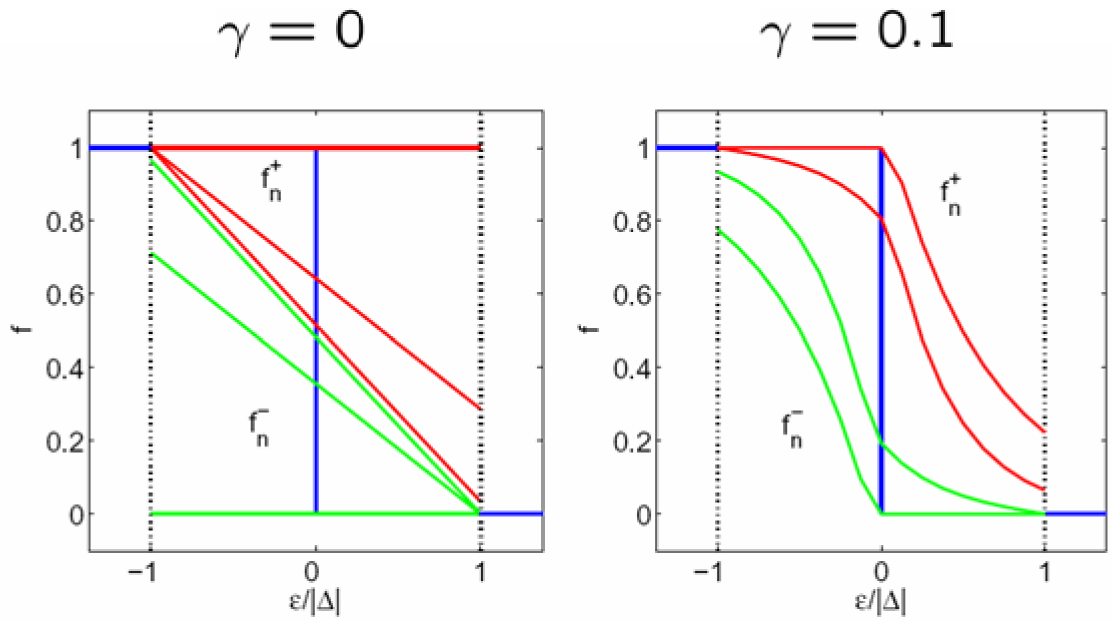


Figure 4.2: Distribution functions for absent, $\gamma = 0$, and small, $\gamma = 0.1$, relaxation on left and right, in this order. Values for tunneling probabilities are $(p_0, p_\pi) = \{(1,1), (0.9, 0.95), (0.1, 0.1)\}$ from top to bottom for f_n^+ functions (from bottom to top for f_n^-). In the right figure, the last pair is omitted for

Number of excited levels $$N_{\text{eff}} = \begin{cases} r^{-1}, & Nr \gg 1 \\ N, & Nr \ll 1 \end{cases}$$

Here $N = 2d\Delta/\pi\hbar v_x \gg 1$ is the total number of levels with positive energies

Effective temperature $$T_{\text{eff}} = r^{-1}(d\epsilon_n/dn) = (\pi\hbar v_x/2rd)$$

DC current

Nonzero average is due to non-equilibrium current carried by the Andreev states

$$\overline{I_{\text{neq}}} = -\frac{2e}{\hbar}\sum_{\epsilon_n>0}^{\Delta}\overline{\frac{\partial \epsilon_n}{\partial \phi}\tilde{f}_n} = -\frac{ev_x}{d}\sum_{k=1}^{N/2}\left[\tilde{f}^{+}_{2k} - \tilde{f}^{-}_{2k-1}\right]$$

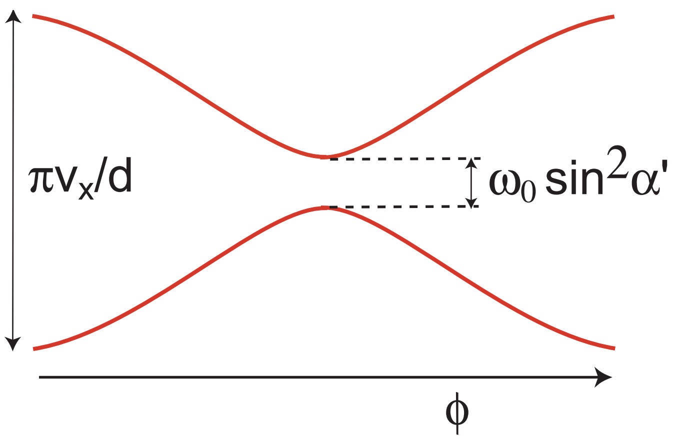
The average current $\bar{I} = \frac{ev_x}{2d}\Phi(p_0, p_\pi)$

For $r \ll 1$

$$\Phi(p_0, p_\pi) = \begin{cases} \dfrac{2p_0 p_\pi}{(p_0 + p_\pi - 2p_0 p_\pi) + \gamma}\,, & N\gamma \gg 1 \\ \dfrac{2p_0 p_\pi}{(p_0 + p_\pi - 2p_0 p_\pi) + 1/N}\,, & N\gamma \ll 1 \end{cases}$$

The dc current increases when LZ probabilities grow $p_0\,,\ p_\pi \to 1$

LZ probability



$$p_\pi = \exp\left[-\frac{\omega_0}{\omega_J}\sin^2\alpha'\right]$$

$$p_0 = \exp\left[-\frac{\omega_0}{\omega_J}\cos^2\alpha'\right]$$

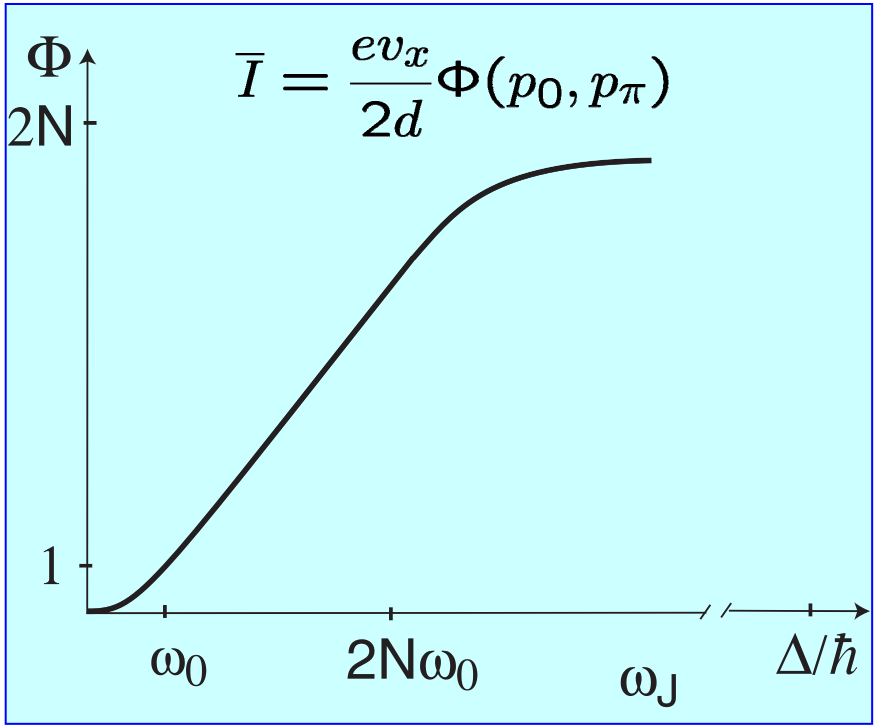
$$\omega_0 = \pi v_x(1-\mathcal{T}^2)/\mathcal{T}d$$

$$\omega_J = 2eV/\hbar$$

Important: High transparency

$$\omega_0 \ll \pi v_x/d \;\Rightarrow\; 1-\mathcal{T} \ll 1$$

The I-V curve



For $\omega_0 \ll eV/\hbar \ll v_x/d$ **when**

$$p_0, p_\pi \to 1$$

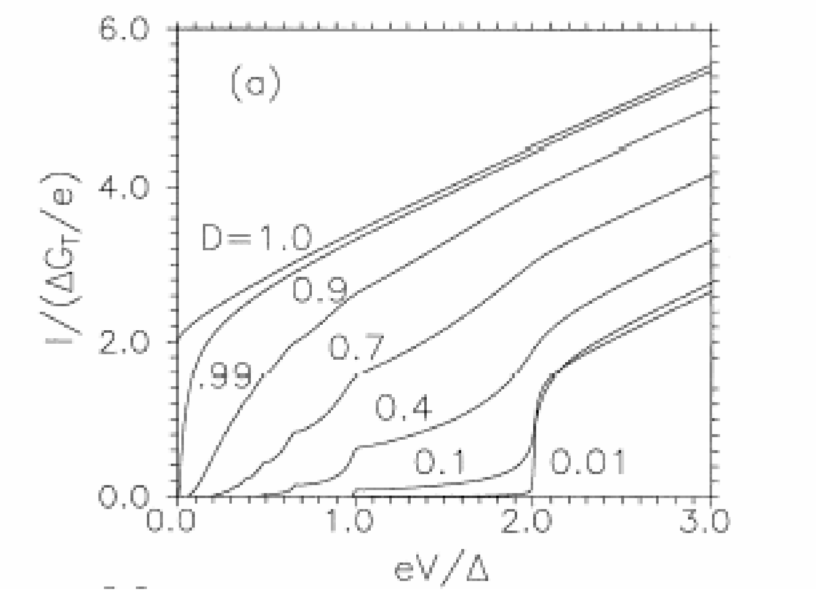
$$\Phi(p_0, p_\pi) \approx \frac{\omega_J}{\omega_0 + \pi/2\tau + \omega_J/2N}$$

The current is N times larger than the critical Josephson current

$$I_c \sim \frac{ev_x}{\pi d}$$

I-V curves. Theory

Point contact, various transparencies D, T=0



Averin & Bardas 1995

Long multi-channel bridge, D=1

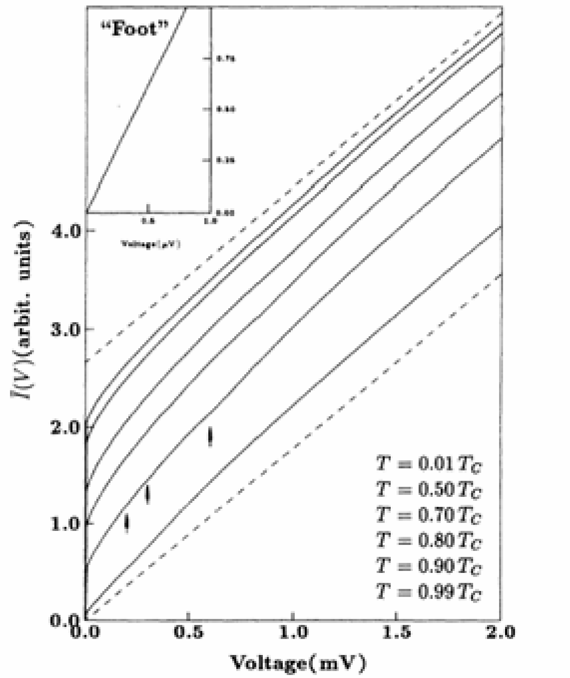


FIG. 1. The solid curves represent the current-voltage characteristics of a SNS junction at different temperatures (the lowest temperature corresponds to the uppermost curve), $\ell_{\rm in}/2a \approx 10^4$. The lower and the upper dashed lines represent the Sharvin current and the extrapolation of the excess current at $T = 0$ K to $V = 0$, respectively. The arrows indicate the subharmonic gap structures at voltages $V_n = 2\Delta(T)/ne$, $n = 1, 2, 3$. The inset shows the steep rise of the current at voltages $V \ll (2a/\ell_{\rm in})\Delta/e$ corresponding to the high-low voltage conductance of Eq. (70), $T = 0$ K.

Gunsenheimer & Zaikin, 1994

Noise spectrum

Spectral density of fluctuations

$$S(\omega) = \int e^{i\omega\tau} d\tau \overline{\left\langle \left\{ \Delta\hat{I}(t - \frac{\tau}{2}, x), \Delta\hat{I}(t + \frac{\tau}{2}, x) \right\} \right\rangle}$$

$$\Delta\hat{I} \equiv \hat{I} - \langle\hat{I}\rangle \ , \ \{\hat{A}, \hat{B}\} \equiv (\hat{A}\hat{B} + \hat{B}\hat{A})/2$$

$$S(\omega) = -\frac{\pi\hbar^3 e^2}{4m^2} \overline{\sum_{n,m} \delta(\epsilon_n - \epsilon_m + \hbar\omega) L_{nm}[1 - f_1(\epsilon_n) f_1(\epsilon_m)]}$$

Matrix elements in terms of the BdG wave functions

$$L_{nm}(x,t) = \left\{ \left(\nabla_1 - \nabla_1'\right) \left(\nabla_2 - \nabla_2'\right) \left[u_n^*(1) u_m(1') + v_n^*(1) v_m(1')\right] \left[u_n(2') u_m^*(2) + v_n(2') v_m^*(2)\right] \right\}_{1=1'=2=2'} .$$

Distribution function

$$f_1 = 1 - 2f(\epsilon)$$

Low frequency $\hbar\omega \ll \delta\epsilon \sim \hbar v_x/d$

Only diagonal elements are important

$$S(\omega) = -\frac{\pi\hbar^2 e^2}{8m^2}\delta(\omega)\sum_n \overline{L_{nn}\left[1 - [f_1^{(0)}]^2 - 2f_1^{(0)}\tilde{f}_1\right]}.$$

Here

$$f_1 = f_1^{(0)} + \tilde{f}_1 \; , \; f_1^{(0)} = \tanh\frac{\epsilon}{2T}$$

Due to inelastic relaxation

$$\delta(\omega) \Rightarrow \Lambda(\omega) = \frac{1}{2\pi}\frac{\tau^{-1}}{\omega^2 + 1/4\tau^2}$$

Equilibrium thermodynamic noise

High transparency, $Z \to 0$

$$S_{\mathrm{th}}(\omega) = \frac{\pi v_x^2 e^2}{d^2} \Lambda(\omega) \overline{\sum_{n>0} \cosh^{-2} \frac{\epsilon_n}{2T}}$$

If $\hbar v_x/d \ll T \ll \Delta$,

$$S_{\mathrm{th}}(\omega) = \frac{4 T v_x e^2}{\hbar d} \Lambda(\omega) \ , \ S_{\mathrm{th}}(0) = \frac{8 T e^2 v_x \tau}{\pi \hbar d}$$

Number of excited levels $N_{eff} = 2Td/\pi\hbar v_x$

Low transparency, $Z \to \infty$

$$S_{\mathrm{th}}(\omega) = \frac{T v_x e^2}{8 Z^8 \hbar d \sin^2(2\alpha')} \Lambda(\omega)$$

Thermodynamic noise: Due to fluctuations in the occupation of Andreev levels.

For point contacts:
Averin & Imam (1996)
Martin-Rodero et al. (1996)
Naveh & Averin (1999)

Non-equilibrium low-voltage noise, T=0

$$S(\omega) = -\frac{\pi v_x^2 e^2}{d^2}\delta(\omega)\sum_{k=1}^{N/2}\overline{\left[\tilde{f}_{2k}^{+} + \tilde{f}_{2k-1}^{-}\right]}$$

Number of excited levels

$$N_{eff} = \begin{cases} r^{-1}, & 1/N \ll r \ll 1 \\ \sim N, & r \ll 1/N \end{cases}$$

Effective relaxation rate decreases with increase in LZ probabilities

$$r = \sqrt{\frac{\gamma(\gamma + p_0 + p_\pi - 2p_0 p_\pi)}{p_0 p_\pi}}$$

Effective temperature increases with the LZ probabilities

$$T_{\rm eff} = r^{-1}(d\epsilon_n/dn) = (\pi\hbar v_x/2rd)$$

Zero-frequency noise increases

$$S_{\rm ne}(0) = \frac{8T_{\rm eff}e^2 v_x \tau}{\pi\hbar d}$$

Fano factor and the effective charge

Note that

$$\frac{\bar{I}}{S(0)} \propto \frac{\tilde{f}^{+}_{2k} - \tilde{f}^{-}_{2k-1}}{\tilde{f}^{+}_{2k} + \tilde{f}^{-}_{2k-1}} \equiv \lambda$$

The efficiency of Landau-Zener processes in producing current:

$$\lambda \equiv \frac{\gamma}{r} = \sqrt{\frac{p_0 p_\pi}{1 + (p_0 + p_\pi - 2p_0 p_\pi)/\gamma}} \leq 1 \; ; \quad \lambda = \begin{cases} 1 \, , & p_0 = p_\pi = 1 \\ 0 \, , & p_0 = p_\pi = 0 \end{cases}$$

Fano factor $F = S(0)/2eI$

Effective charge $q = eF = 2en/\lambda \gg e$

For point contacts:
Averin & Imam (1996)
Naveh & Averin (1999)

Here $n = v_x \tau / d \gg 1$ is the number of times which a particle can fly along the Andreev level before it escapes due to inelastic scattering

For $p_0, p_\pi \to 1 \; , \; \omega_J \gg \omega_0 \gg 1/\tau$

$$q = 2en\sqrt{1 + \omega_0 \tau / \pi} \gg 2en$$

CONCLUSIONS

- ❑ The DC current and noise spectrum in quantum SINIS contacts are sensitive to non-equilibrium distribution on the Andreev levels established by:
  - Voltage-driven Landau-Zener inter-level transitions
  - Inelastic intra-level relaxation
- ❑ DC current can considerably exceed the critical Josephson current.
- ❑ The Fano factor and the effective charge are enhanced by MAR and by Landau-Zener transition probabilities.