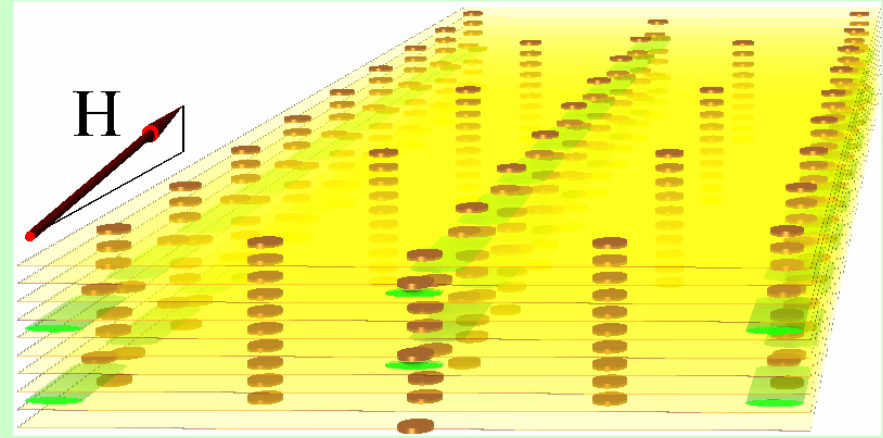


Vortex Lattices in Tilted Magnetic field



Collaborations:

C. Van der Beek, M. Konczykowski (Ecole Polytechnique, France)

Yu. Latyshev (IRE, Moscow)

V. Vlasko-Vlasov (MSD, ANL)

A. Grigorenko (Univ. of Manchester, UK) and S. Bending (Univ. of Bath, UK)

A. I. LARKIN MEMORIAL CONFERENCE

June 24-28, 2007, Chernogolovka, Russia

Paramagnetic moment in field-cooled superconducting plates: Paramagnetic Meissner effect

A. E. Koshelev

Material Science Division, Argonne National Laboratory, Argonne, Illinois 60439
and Institute of Solid State Physics, Chernogolovka, Moscow District, 142432, Russia

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(Received 16 June 1995)

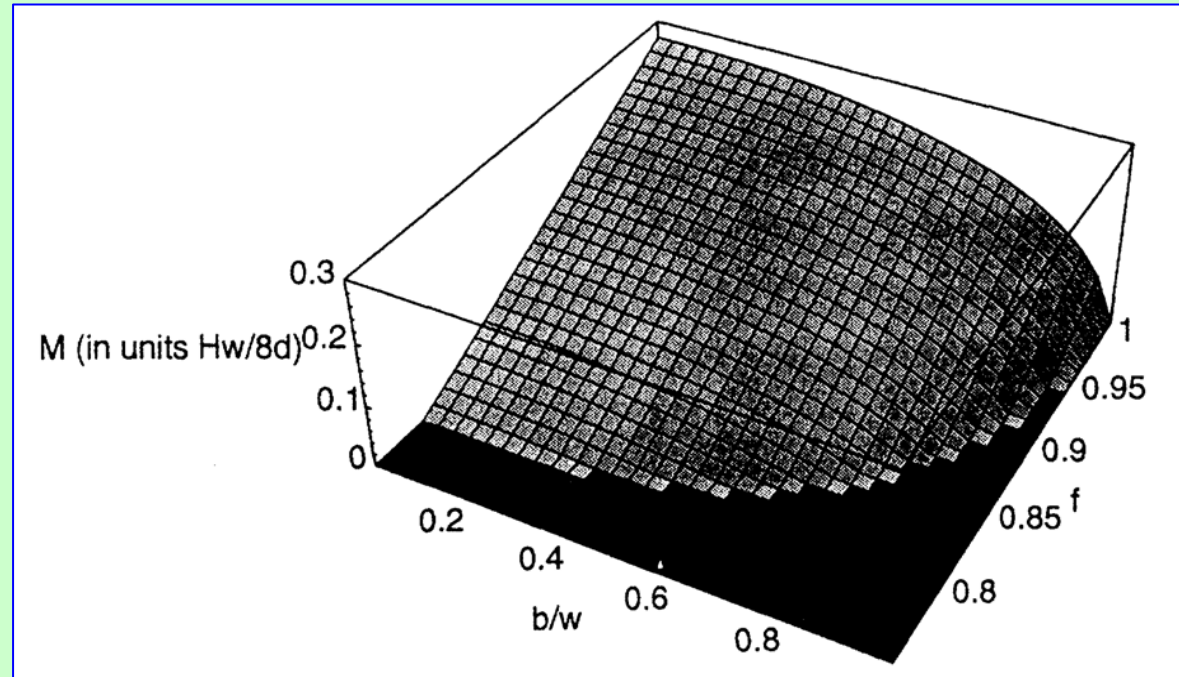
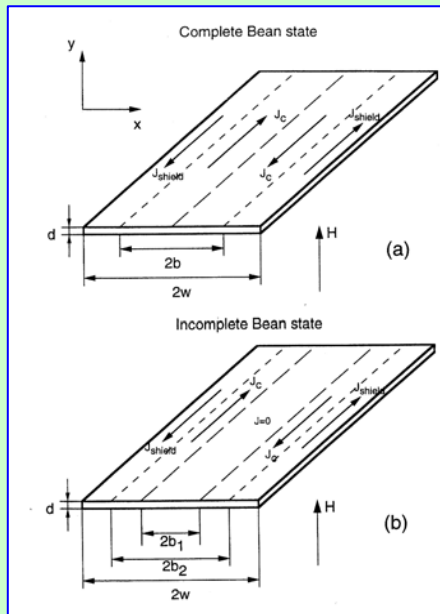
Motivation:

Paramagnetic Meissner effect in polycrystalline HTSC:

Spontaneous supercurrents due to random $0-\pi$ frustrations?

Problem: PME in Nb, P. Kostic *et al.*, [Phys. Rev. B 53, 791 \(1996\)](#)

Alternative mechanism based on flux compression

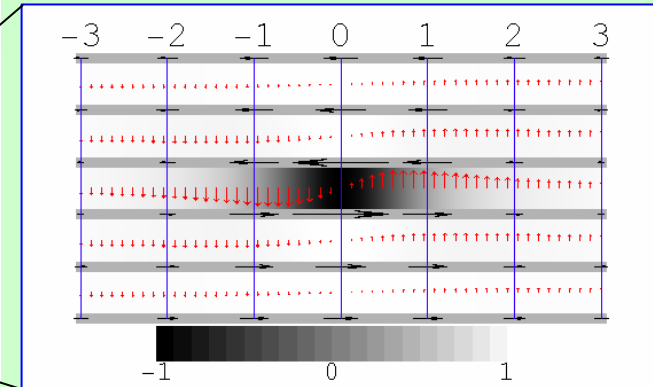
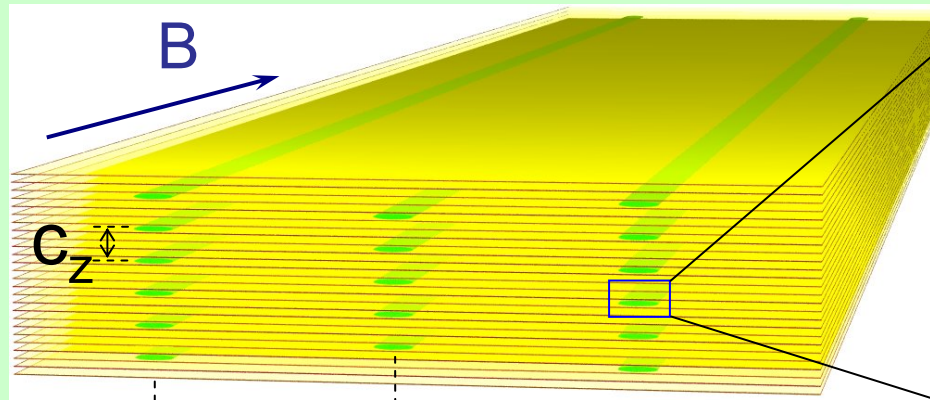


Lattice of Josephson vortices

$\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$
(BSCCO)

Small fields: Dilute lattice

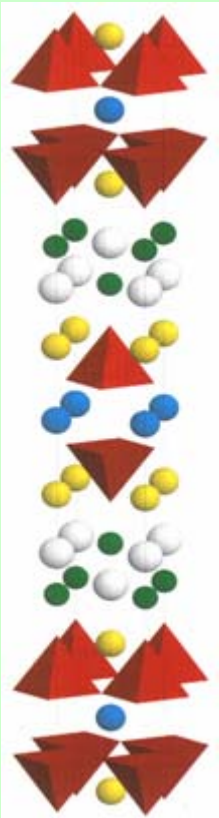
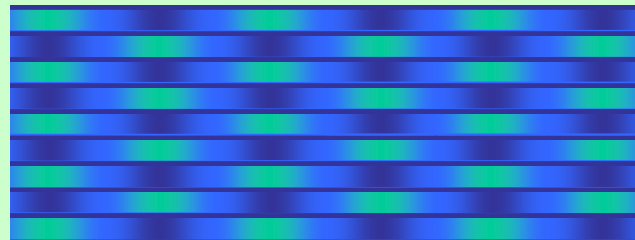
Core size $\lambda_J = \gamma s$



Bulaevskii 1973, Clem & Coffey 1991, AEK 1993

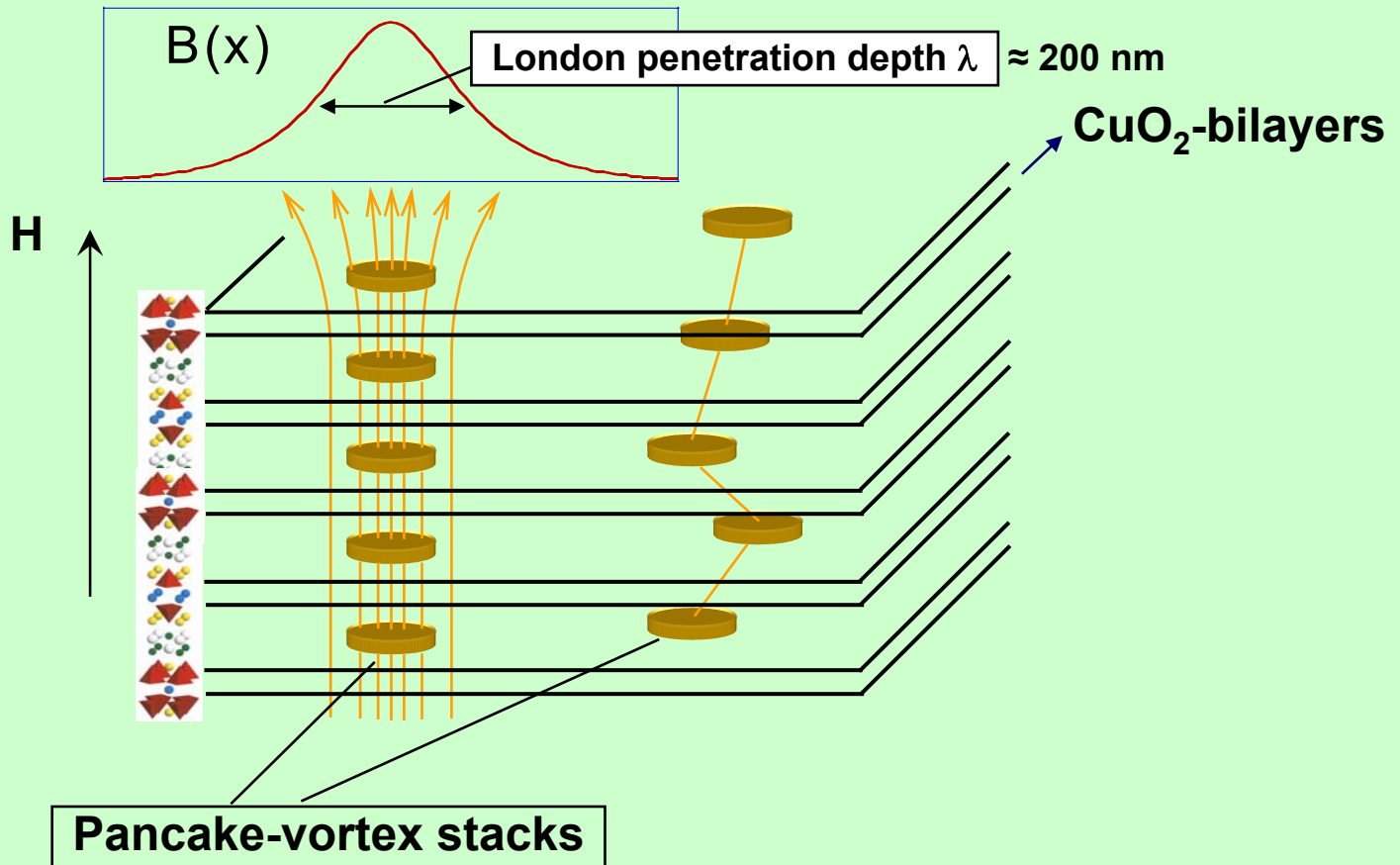
High fields $B_x > B_{cr} = \Phi_0/2\pi s\lambda_J$ (~ 0.5 T) : Dense lattice

Bulaevskii & Clem, 1991



Pancake stacks

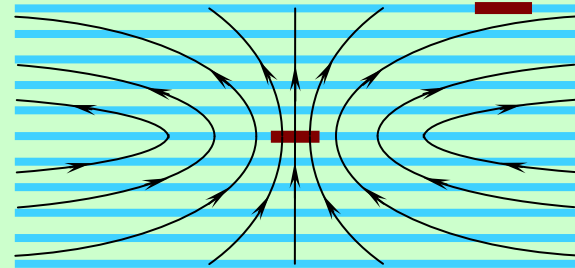
Efetov 1979, Artemenko&Kruglov 1990, Buzdin&Feinberg 1990, Clem 1991



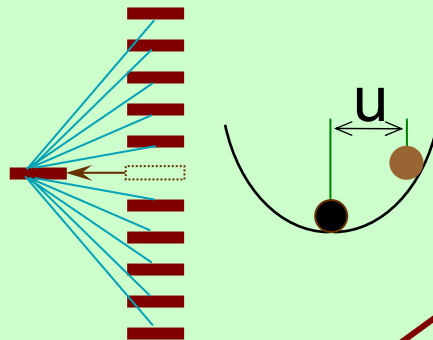
Magnetic and Josephson coupling

Magnetic coupling:

- nonlocal: range λ
- weaker than in-plane interaction by s/λ



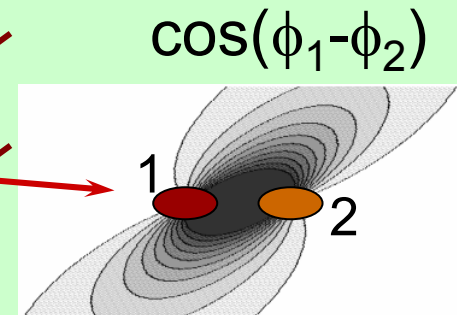
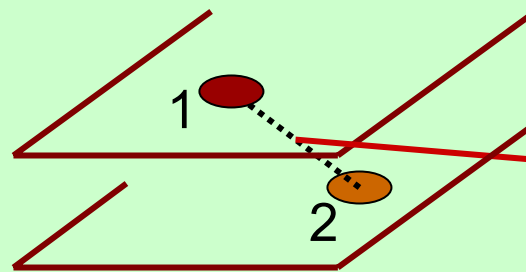
Magnetic cage:



Josephson coupling:

phase mismatch between neighboring layers

$$- E_J \cos(\phi_1 - \phi_2)$$



Relative strength:

Parameter $\alpha = \lambda/\gamma s$

BSCCO: $\gamma = 200 - 700$

$\lambda/s = 130 - 250$

$\alpha = 0.3-0.9$

Interaction between Jos. vortex and pancake stacks

Crossing configuration

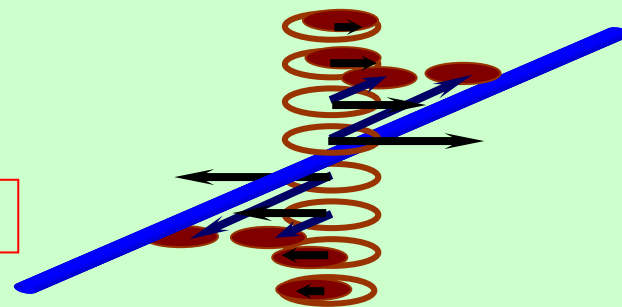
Phys.Rev.Lett. **83**, 187 (1999)

Deformation energy

$$E = \sum_n \left(-\frac{s\Phi_0}{c} j_n(0) u_n + \frac{K u_n^2}{2} \right)$$

Jos. Vortex current

Magnetic cage



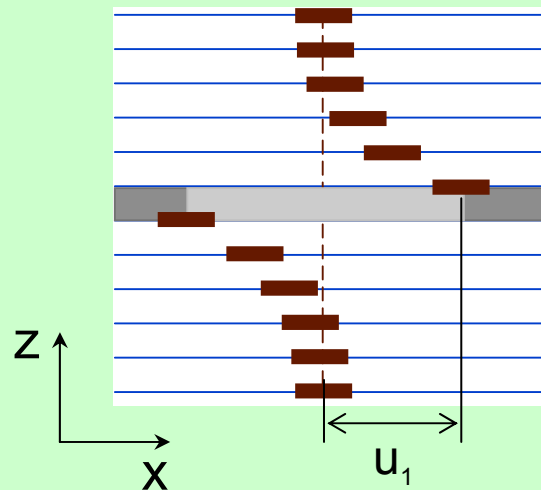
Crossing energy

$$E_x = -\frac{\Phi_0^2}{2\pi^2 \gamma^2 s \ln(3.5\gamma s / \lambda)} \quad \sim 200 \text{ K}$$

Maximum displacement:

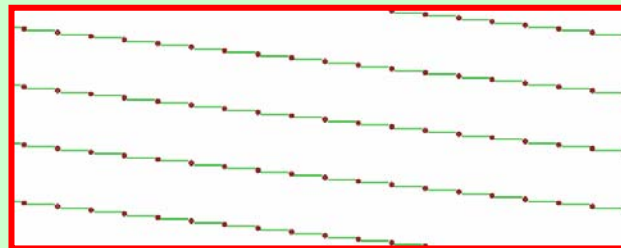
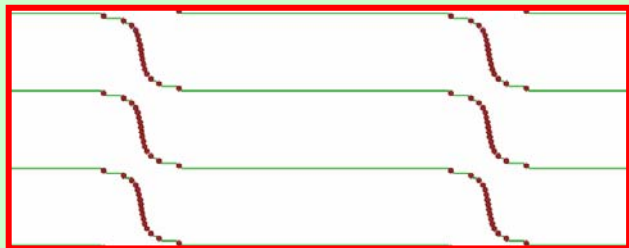
$$u_1 = \lambda^2 / \gamma s \quad \sim 100 \text{ nm}$$

Numerics: instability at $\alpha = \lambda / \gamma s \approx 0.69$



Penetration of c-axis field in presence of in-plane field

stacks vs kinks



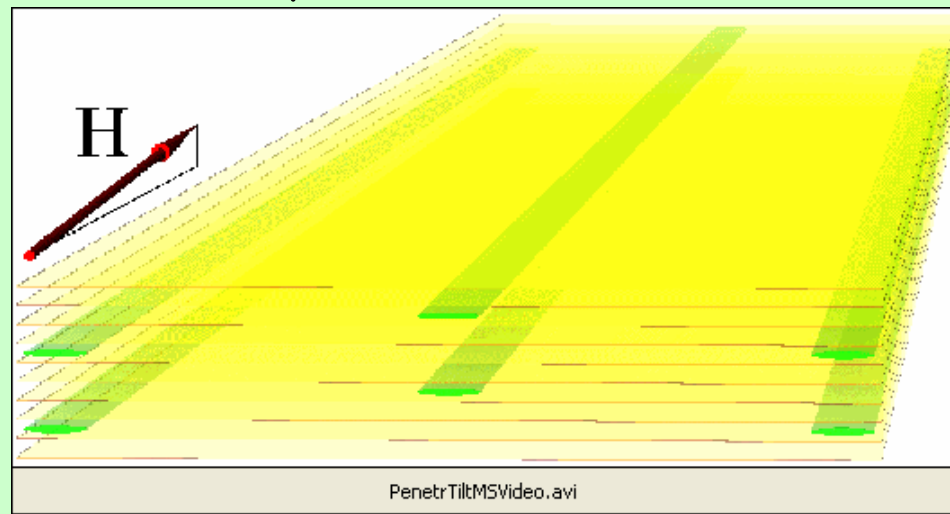
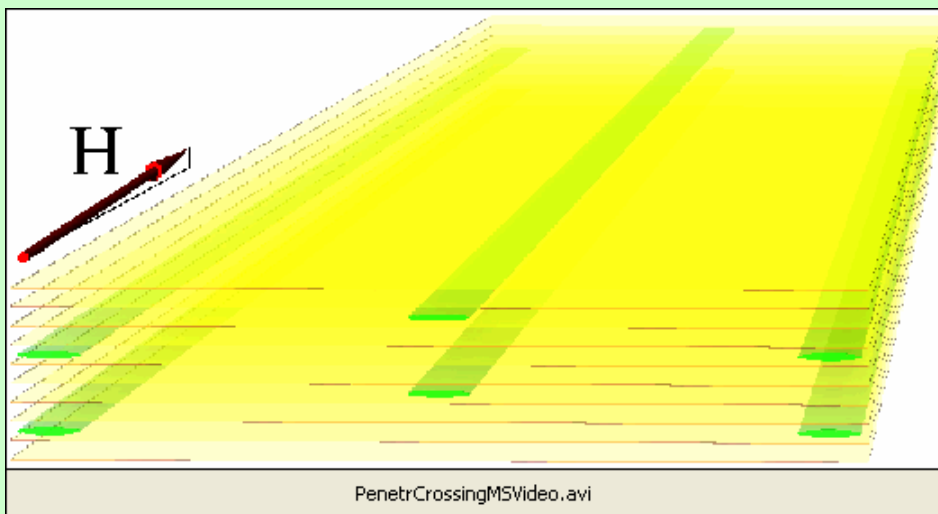
Bulaevskii *et al.* 1992; Huse 1992; AEK, 1999...

Feinberg *et al.* 1990; Ivlev *et al.* 1991; Bulaevskii *et al.* 1992 ...

$$\alpha = \lambda/\gamma s = 0.45-0.5$$

$$\alpha < 0.3$$

$$\alpha > 0.7, \gamma \gg 1$$



Visualization

Decorations: Bolle *et al.*, PRL, 1991
Grigorieva *et al.*, PRB, 1995
Tokunaga *et al.*, PR B, 2003
(Tamegai Group)

Magneto-optics:

Vlasko-Vlasov *et al.*, PR B 2002
Tokunaga *et al.*, PR B, 2002

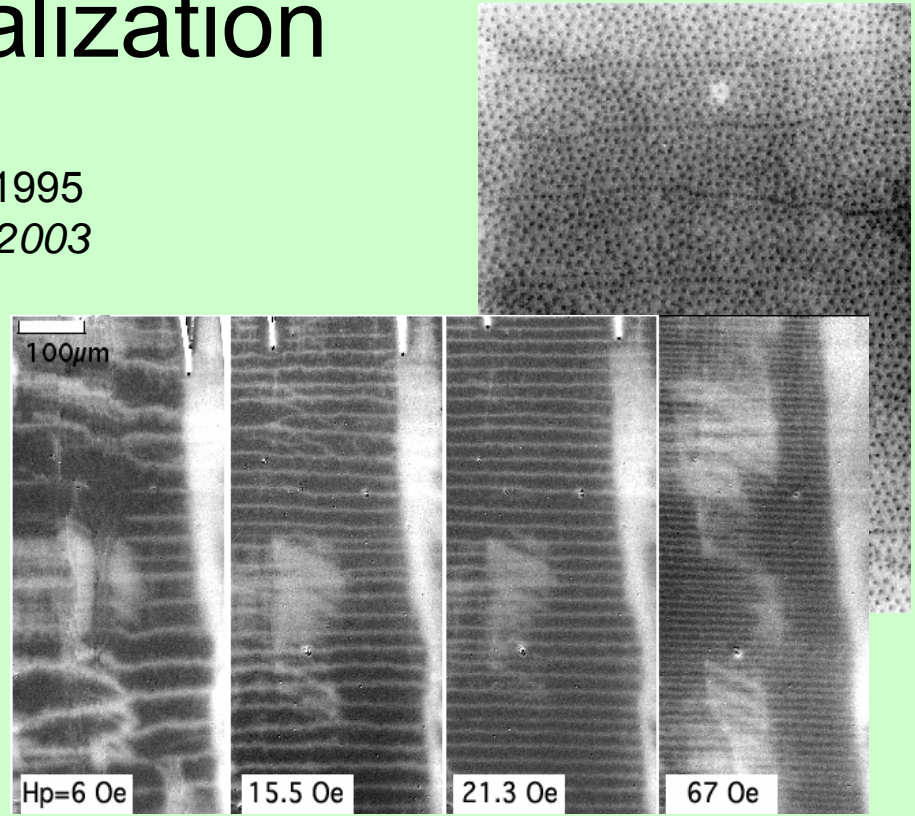
Also:

Lorentz microscopy:

Matsuda *et al.* (Tonomura group), *Science*, 2001

Scanning Hall Probe:

Grigorenko *et al.* (Bending group), *Nature*, 2001



Outline

- Vortex-chain states and transitions
- Josephson vortex inside pancake-vortex lattice
- Phase diagram in tilted fields at intermediate anisotropies

Lawrence-Doniach model in London limit

Regular phase $\phi_n(\mathbf{x}, y)$ + pancake displacements u_n (vortex phase $\phi_{v,n}$)

Energy (distances, $x, y < \lambda_c$, $z < \lambda_{ab} \rightarrow$ no screening):

$$E[\phi_n, u_n] = \sum_n \int d^2r \left[\frac{J}{2} (\nabla \phi_n)^2 + E_J \left(1 - \cos \left(\phi_{n+1} - \phi_n + \phi_{v,n+1} - \phi_{v,n} - \frac{2\pi s}{\Phi_0} B_x y \right) \right) \right] + \frac{1}{2} \sum_{n,m} U(u_n - u_m)$$

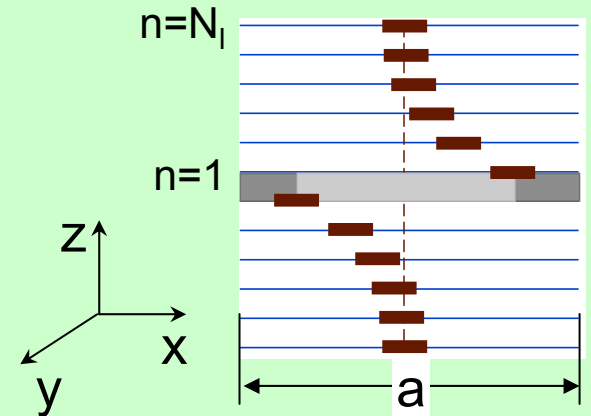
Full 3D model was used to study **isolated chain**

2D model (pancake vortex lattice, at $a < \gamma s$):

$$1. \phi_n(\mathbf{x}, y) = \bar{\phi}_n(y) + \tilde{\phi}_n(\mathbf{x}, y)$$

$$2. \tilde{\phi}_n(\mathbf{x}, y) \approx \tilde{\phi}_{v,n}(\mathbf{x}, y)$$

$$3. F_{J,loc}[\bar{\phi}_n, u_n] = E_J \int dx dy \left[\cos(\tilde{\phi}_{n+1} - \tilde{\phi}_n + \bar{\phi}_{n+1} - \bar{\phi}_n) - \cos(\bar{\phi}_{n+1} - \bar{\phi}_n) \right]$$



Local Josephson energy

Vortex Chains in tilted fields

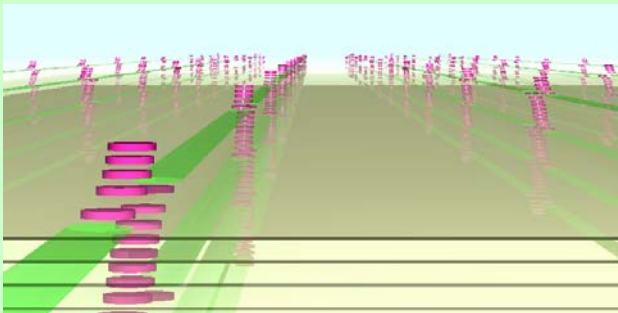
Phys. Rev. B **71**, 174507 (2005)

Dilute Josephson vortex lattice + very small c-axis field \rightarrow **Vortex Chains**

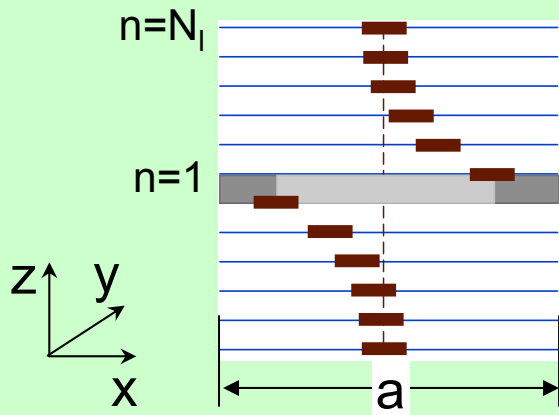
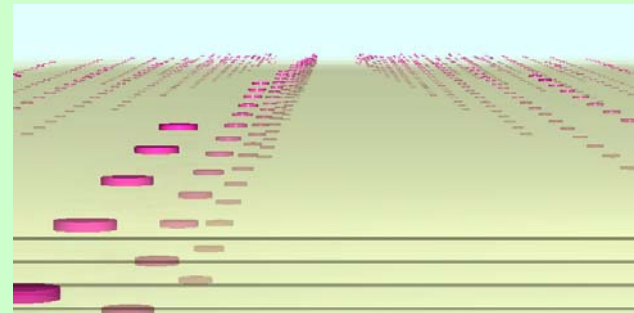
key parameter $\alpha = \lambda/\gamma s$

Really high anisotropy $\alpha < 0.3$ vs **not-so-high anisotropy** $\alpha > 0.7$

Crossing Chains



Tilted Chains



Competition between
Magnetic coupling and **interaction with JVs**
(likes straight stacks) (likes tilted stacks)

two periods: **a** and **$N = 2N_l$**

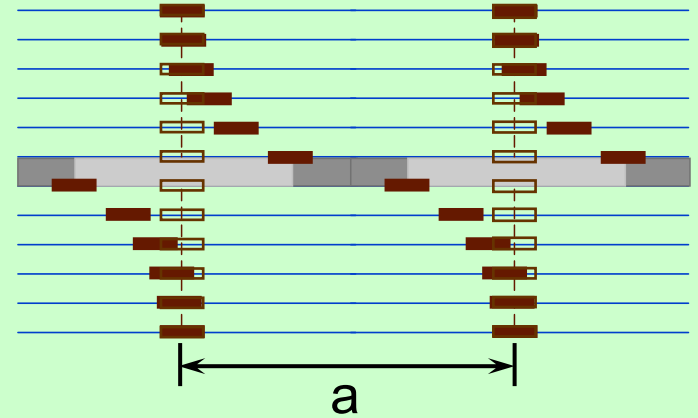
Attraction between deformed stacks

A. Buzdin and I. Baladié, PRL 2002

Interaction energy for $a < \gamma s$

$$U_{\text{int}}(a) = 2\varepsilon_0 \left[\underbrace{K_0(a/\lambda)}_{\propto \exp(-a/\lambda)} - \frac{\langle u^2 \rangle}{a^2} \right]$$

Minimum of $U_{\text{int}}(a)$ at $a_m = \lambda \ln \frac{C\lambda^2}{\langle u^2 \rangle}$

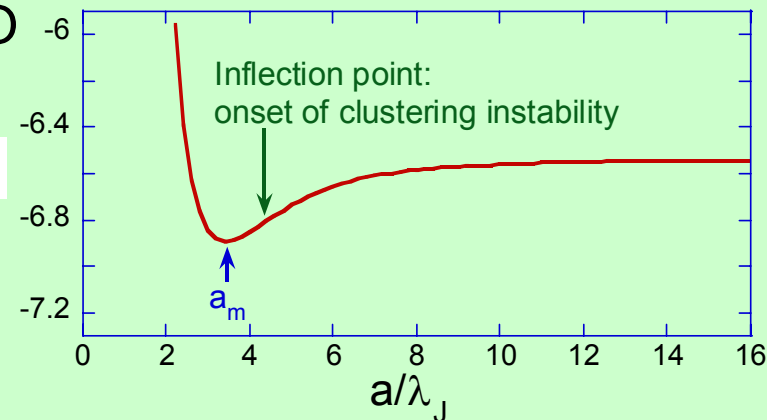


Chains with $a > a_m$ do not realize

Valid only if $a_m < \gamma s \rightarrow$ not practical for BSCCO

In general: $\min_a [\mathcal{U}(a)]$
 Energy per unit cell

\mathcal{U}



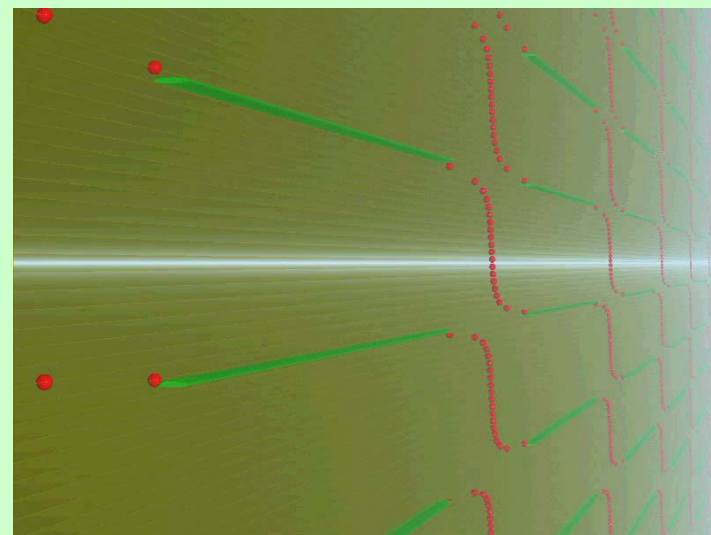
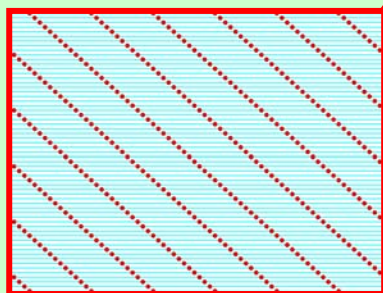
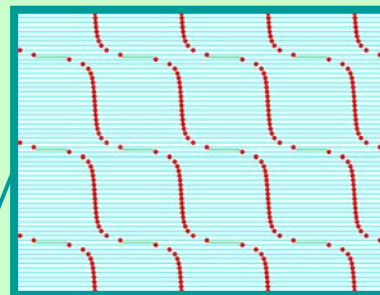
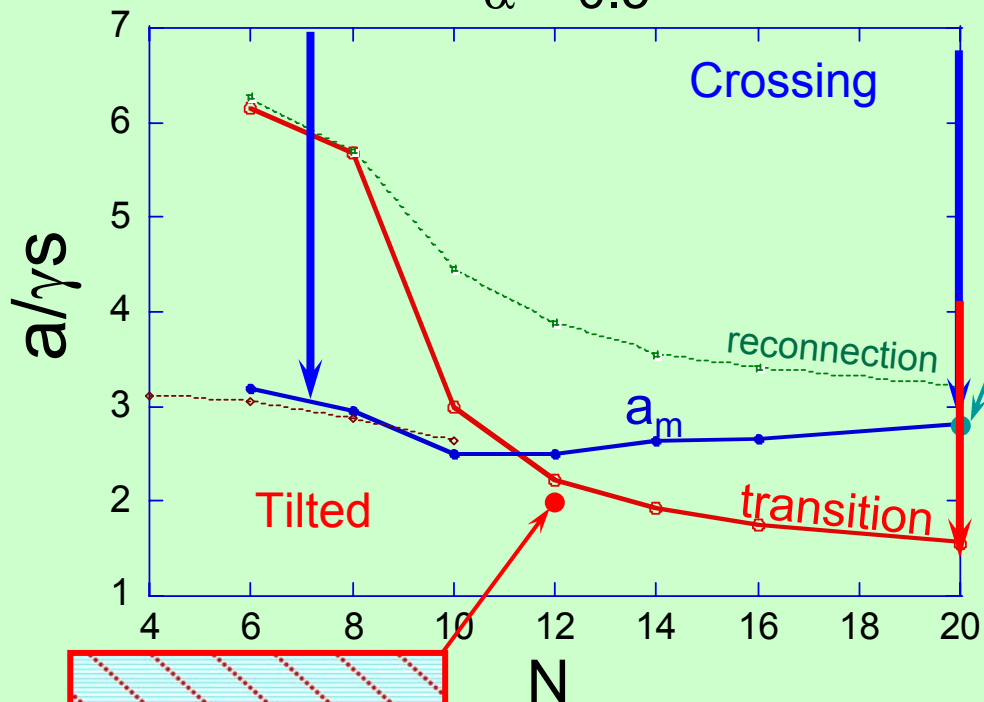
Also: attraction between **tilted stacks**

Buzdin and Simonov, JETP Lett. (1990); Grishin, Martynovich, and Yampolskii, JETP (1990).

1st scenario, $0.4 < \alpha < 0.55$

Locked \rightarrow Crossing Chain \rightarrow Tilted Chain

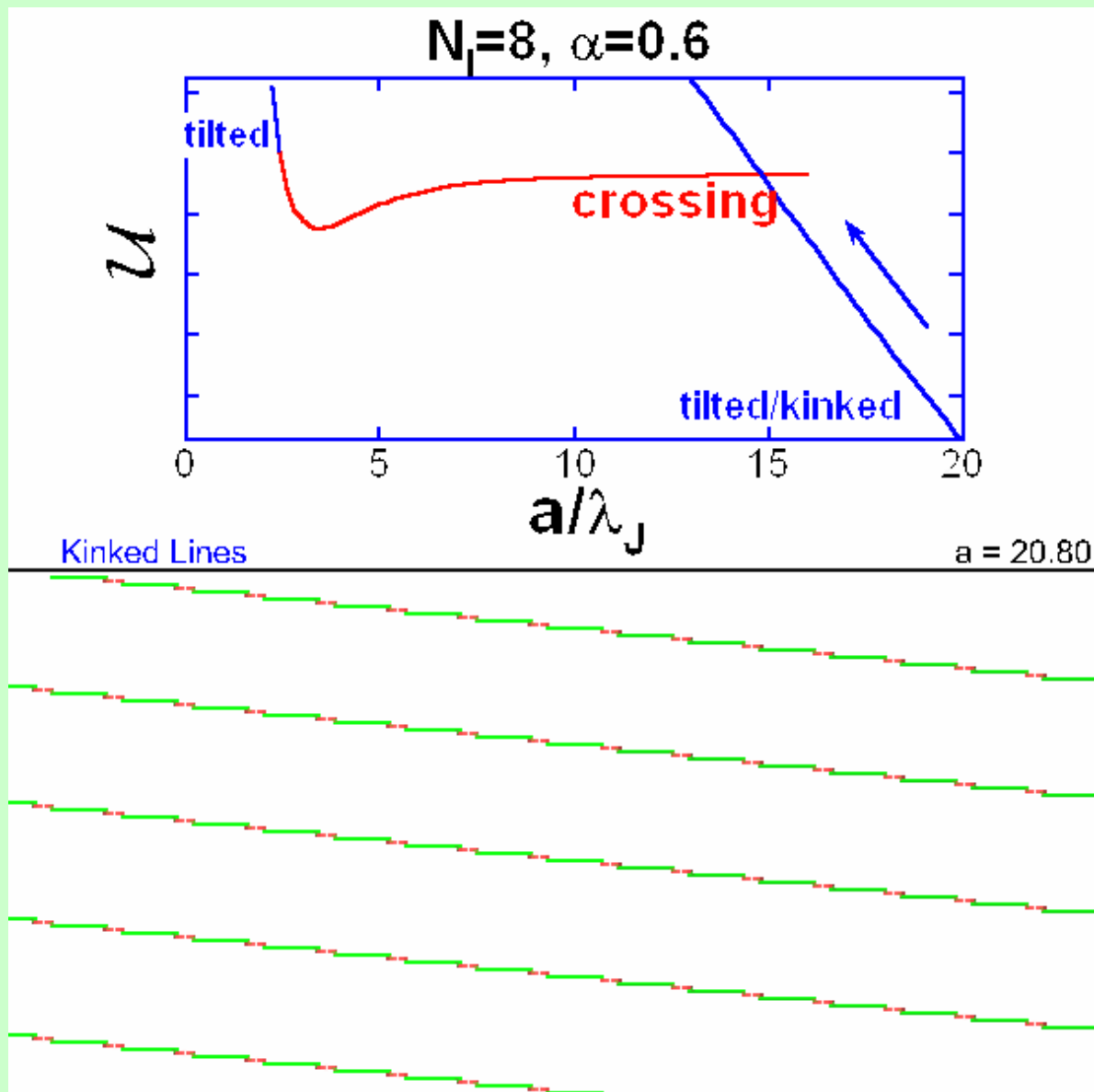
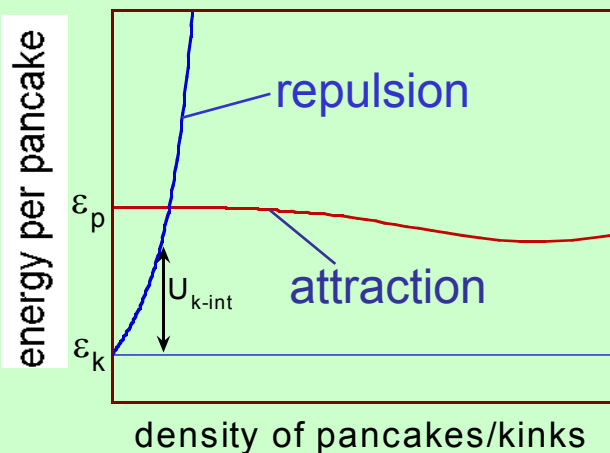
$\alpha = 0.5$



2nd scenario $0.55 < \alpha < 0.65$

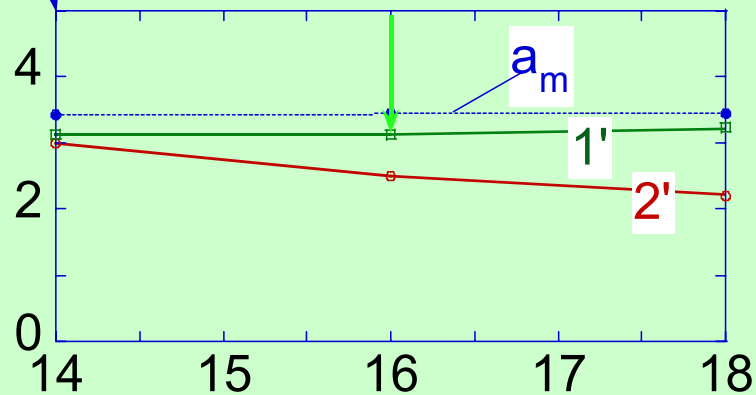
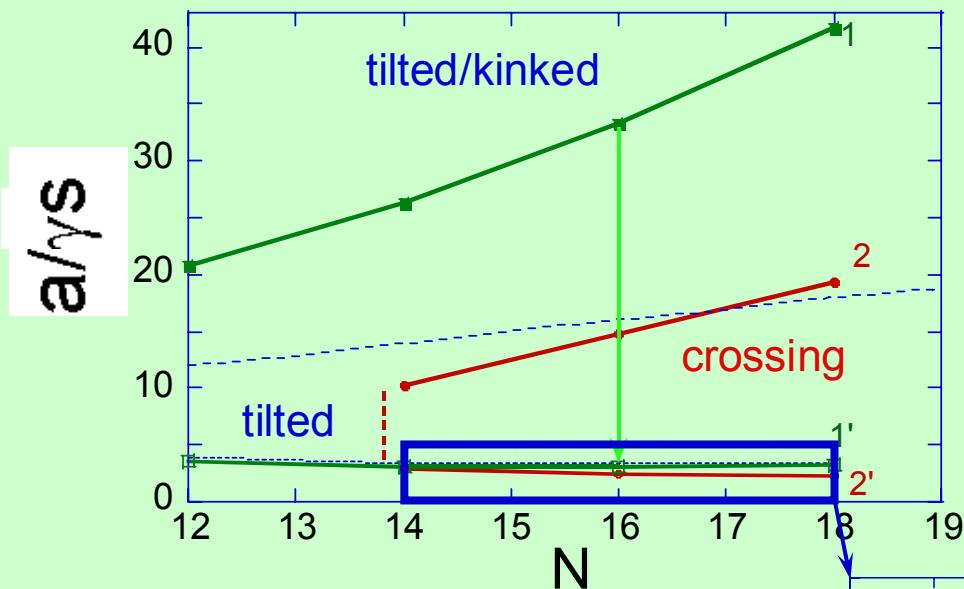
Locked \rightarrow Kinked \rightarrow Crossing Chain \rightarrow Tilted Chain

1. kink penetration
2. 1st order transition due to competing interaction energies



Typical phase diagram for $0.5 < \alpha < 0.65$

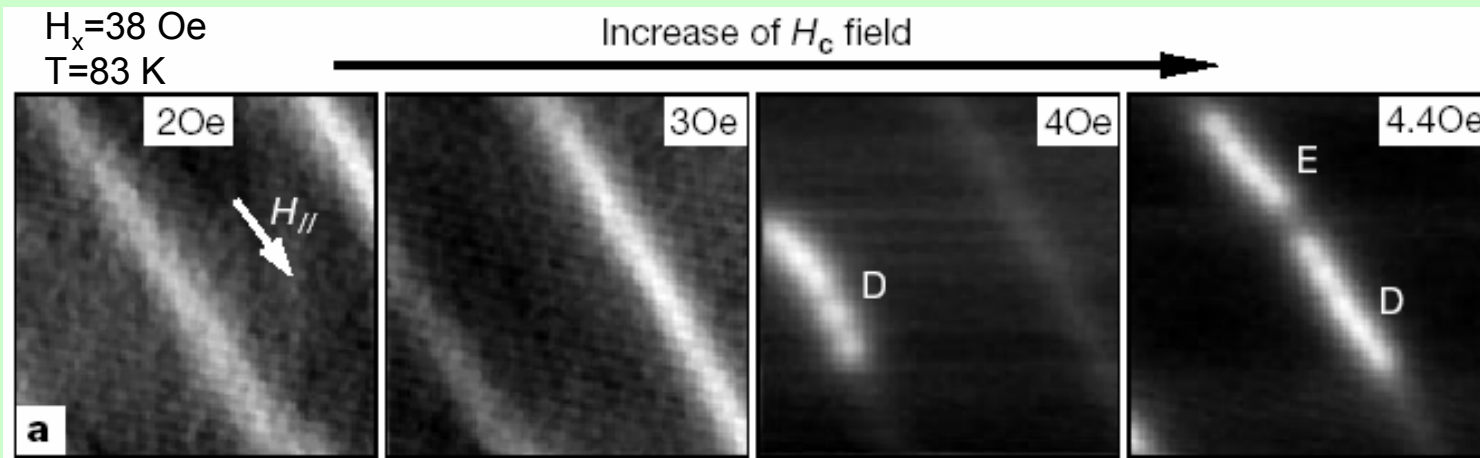
$\alpha = 0.6$



Experimental observation of transition

A. Grigorenko *et. al.* (S. Bending Group),
Nature, 2001

Transition to tilted chains
M. Dodgson, Phys. Rev. B 2002



“... as H_c is increased further, ‘crystallites’ of well resolved PV stacks, **with ten times higher flux density**, nucleate and grow ($H_c = 40$ e and 4.4 Oe).”

Vortex Chains: practical application

Physics Today:
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VORTICES IN BSCCO
IMAGE COURTESY OF
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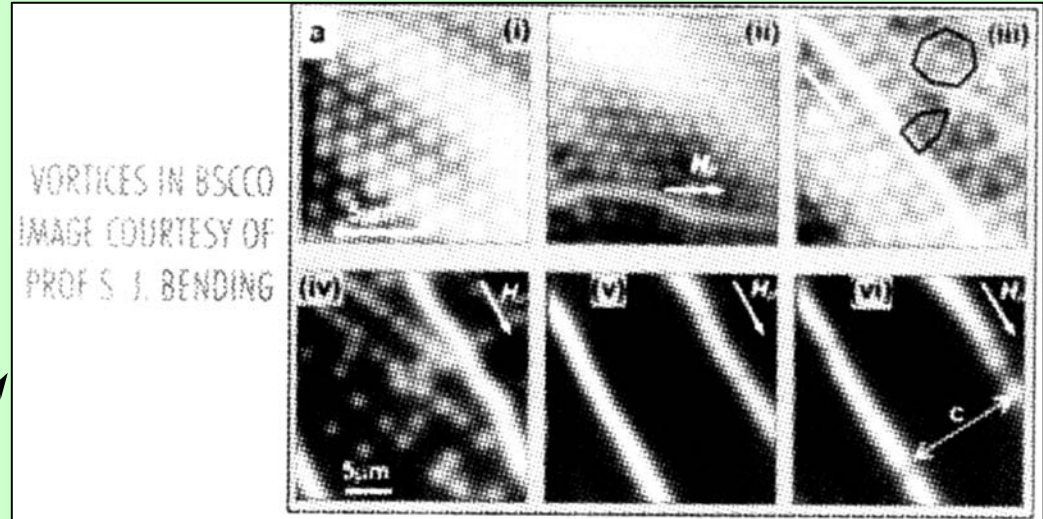
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In-plane vortex inside dense pancake lattice

$$B_z = 10 - 100 \text{ G}$$

Crossover Josephson vortex \rightarrow soliton

Physical Review B **68**, 094520 (2003)

Renormalization of Josephson vortex by pancakes

$$\alpha = \lambda/\gamma s \ll 1$$

Typical field

$$B_\lambda = \frac{\Phi_0}{4\pi\lambda^2} \ln \frac{a}{r_w} \sim 30 - 70 \text{ G}$$

$$\alpha = 0.2$$

Vortex energy

$$\varepsilon_J = \sqrt{\frac{B_\lambda}{B_z + B_\lambda}} \varepsilon_{J0}$$

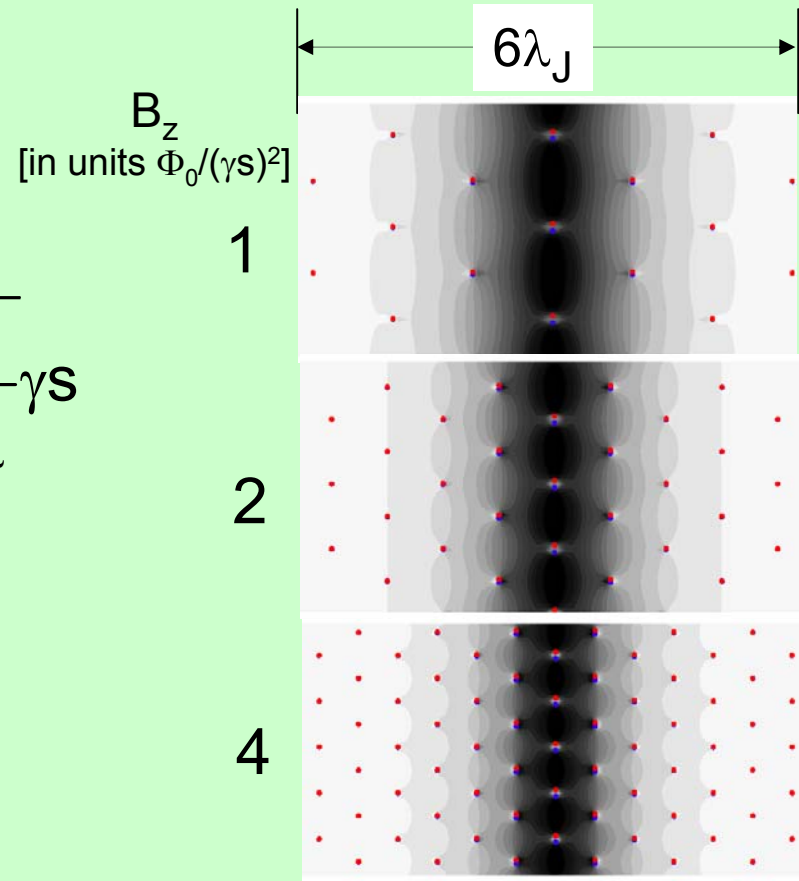
Core size

$$\lambda_J = \sqrt{\frac{B_\lambda}{B_z + B_\lambda}} \gamma s$$

Dense Lattice, $B_z > B_\lambda$:

Maximum displacement $u_1/a \approx 0.5 \alpha$

Number of rows in the core $\approx 1/\alpha$



Solitonlike cores at $\alpha > 0.5$

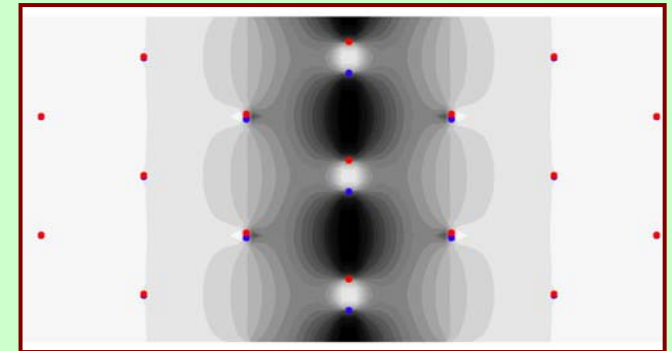
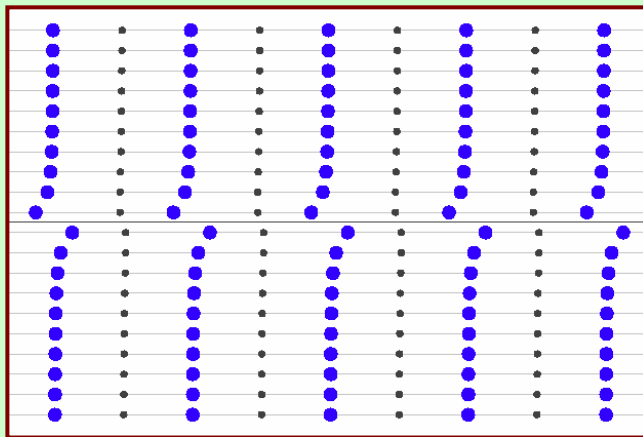
$$B = \Phi_0 / (\gamma s)^2$$

Displacements
in the central row

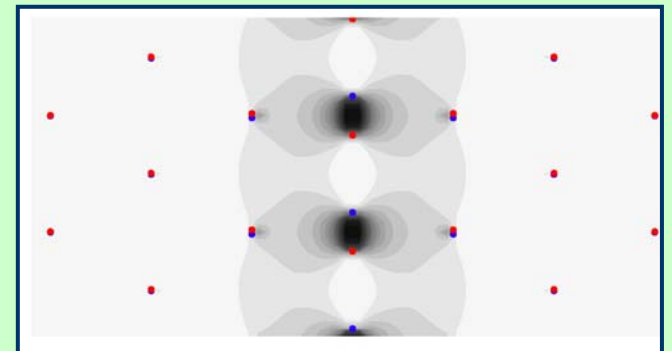
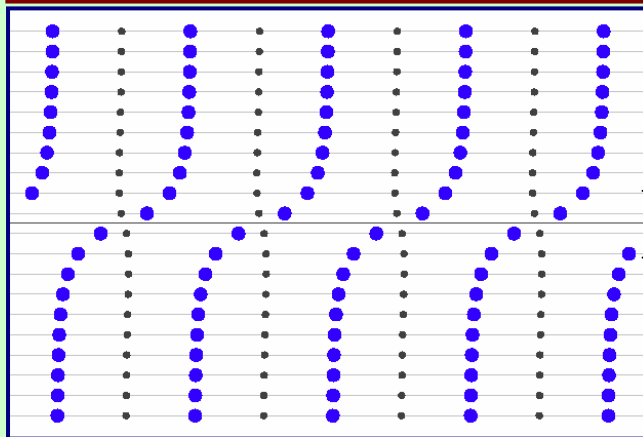
Phase difference
between two central layers

$\lambda/\gamma s$

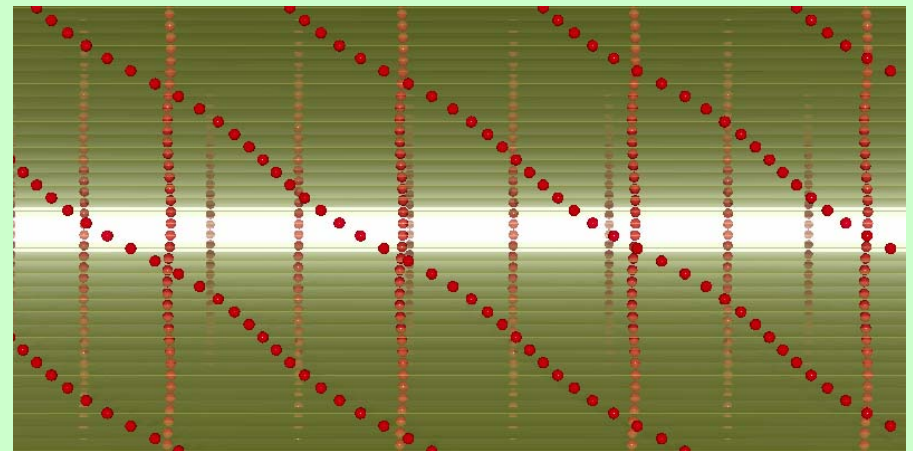
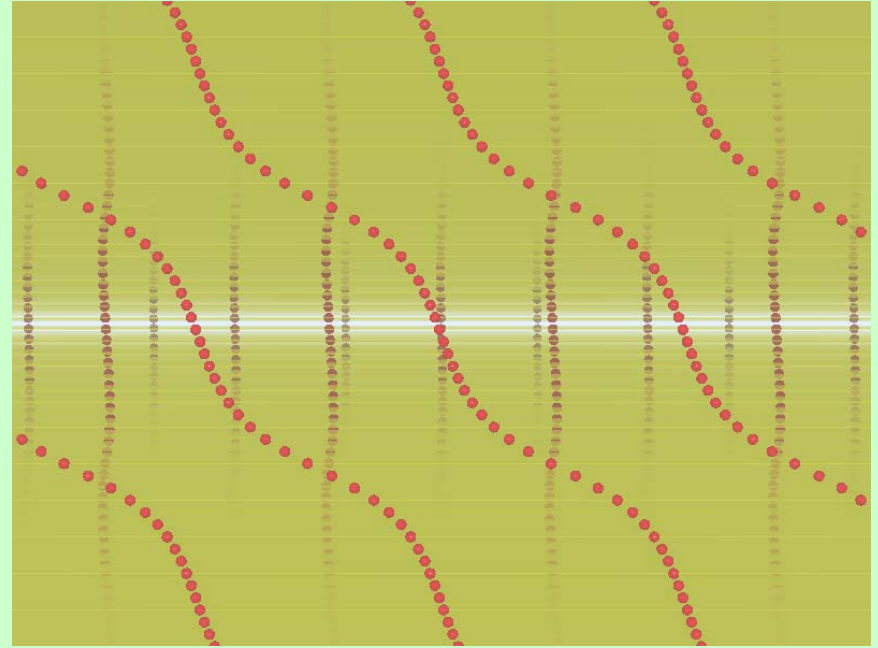
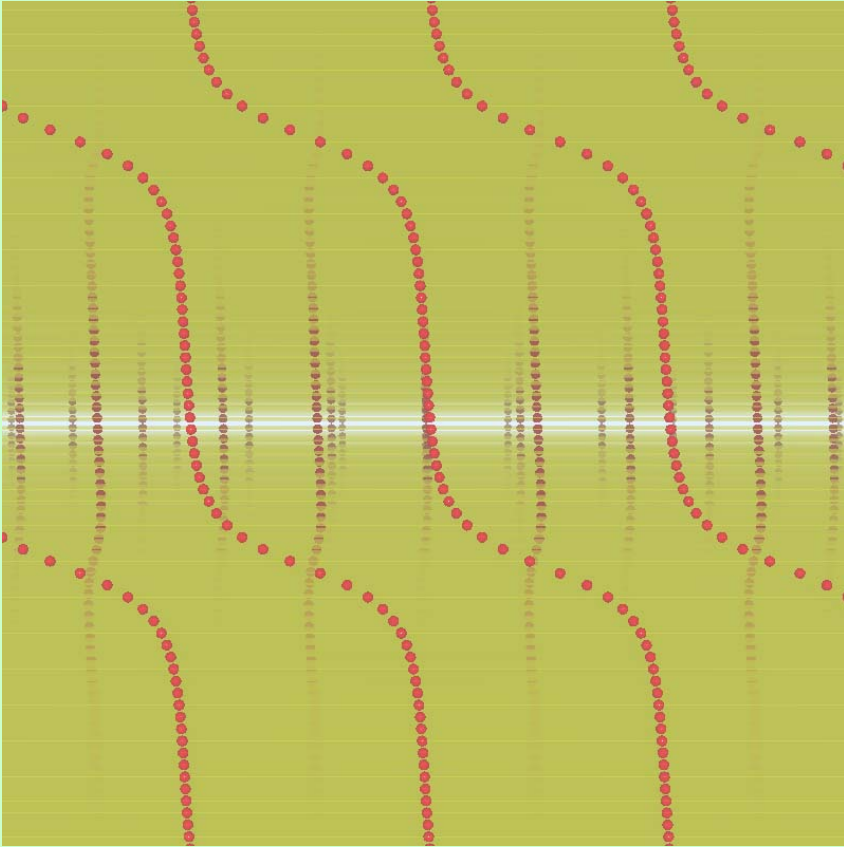
0.3

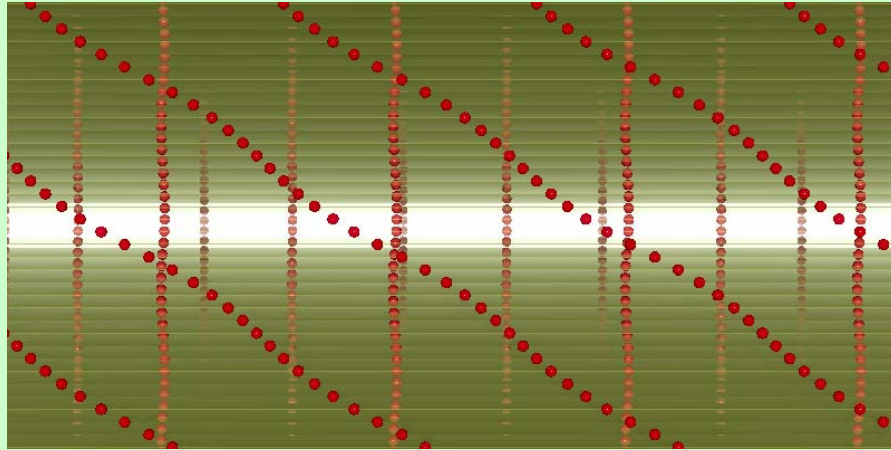


0.5



Soliton lattice with increasing B_x



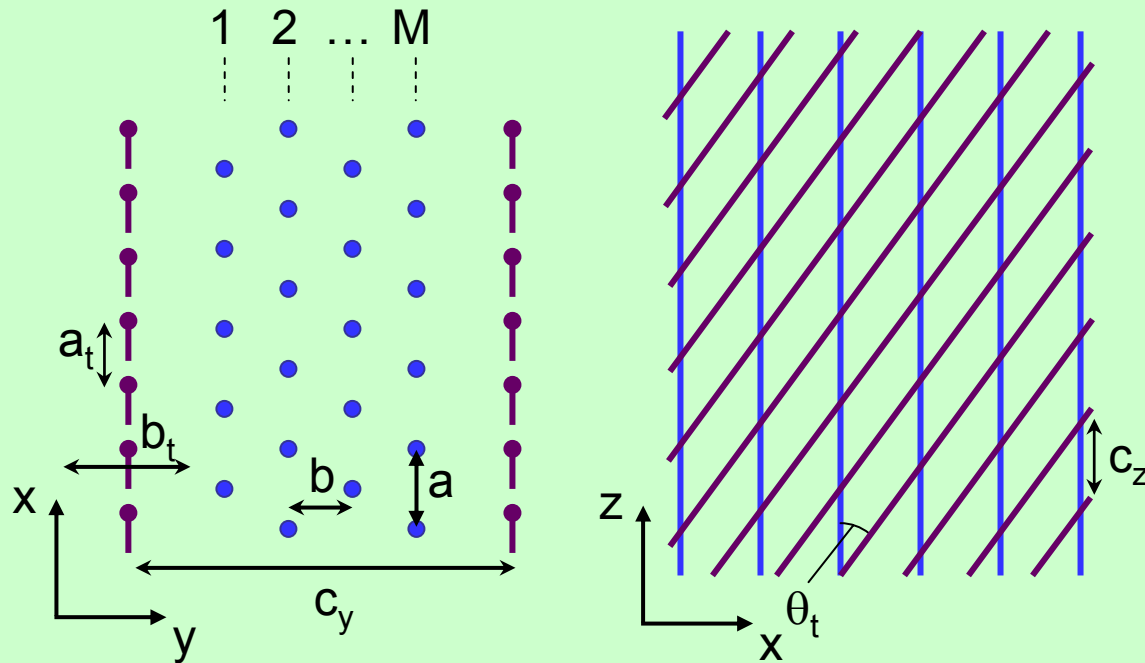


Composite Lattices and Tilted Lattice at large tilt angles

Phase diagram at moderate α

Composite Lattices

Daemen et al. 1993, Sudbø et al. 1993, Sardella 1997



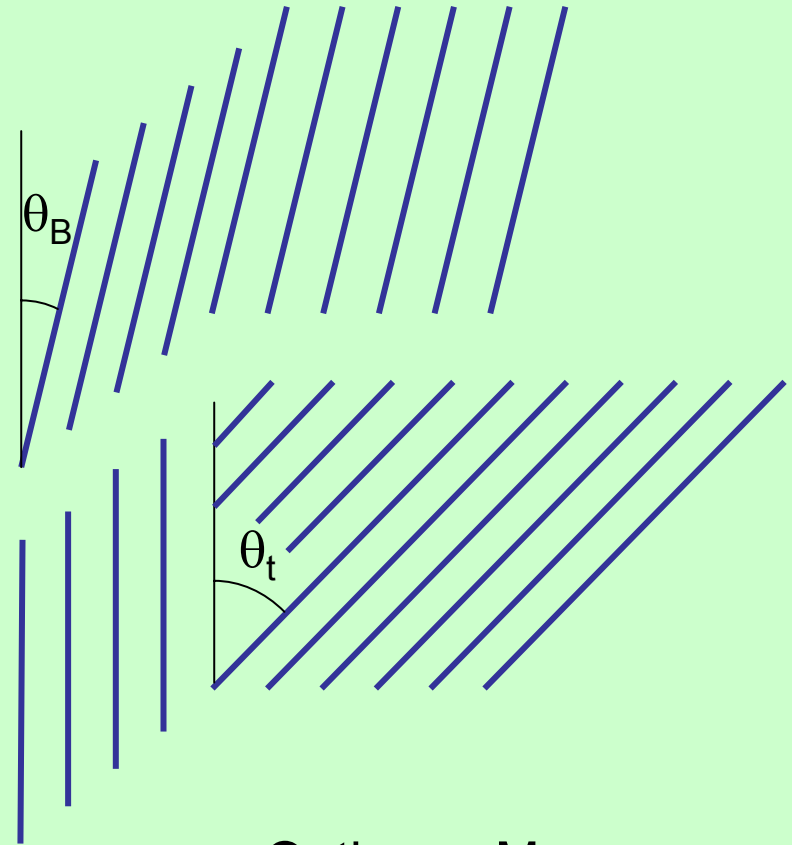
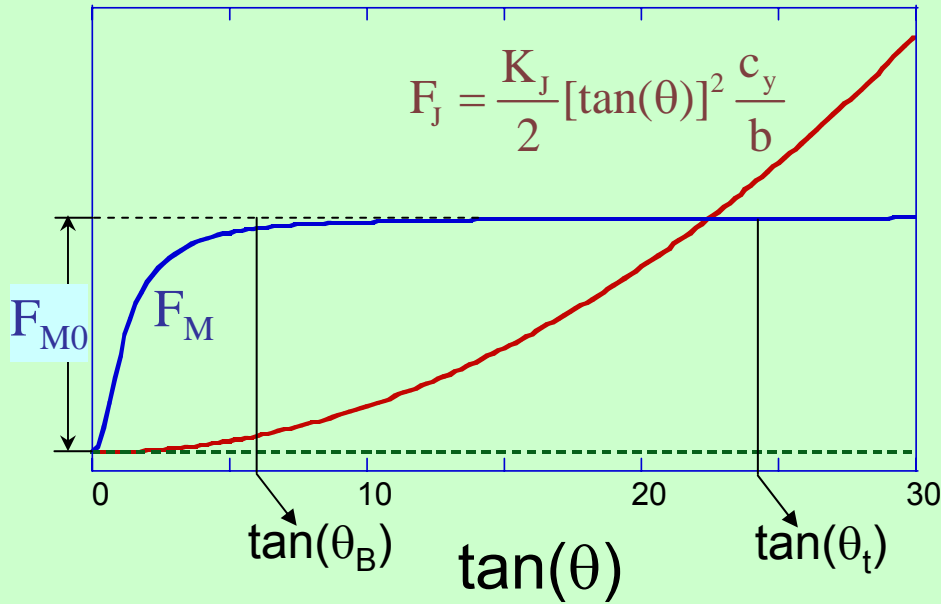
- Dense-lattice case $a < \lambda$
- Tilt angle of field $1 \ll \tan \Theta \ll \gamma$

For fixed field: two independent parameters: M and $r = a/a_t$

$$E = \min_{M,r} [E(M,r)]$$

Why composite lattice may be better?

Energy per tilted row



Optimum M

$$M + 1 \approx \left(\frac{F_{M0}}{K_J \tan^2 \theta_B} \right)^{1/3}$$

Energy per row

$$F(M) = \frac{1}{M+1} \left((M+1)^3 \frac{K_J}{2} \tan^2 \theta_B + F_{M0} \right)$$

Estimates

Number of straight rows M in between tilted rows

$$M + 1 = \left(\frac{0.24 \Phi_0 \gamma^2 B_z}{4\pi\lambda^2 B_x^2} \right)^{1/3} \gg 1$$

Energy

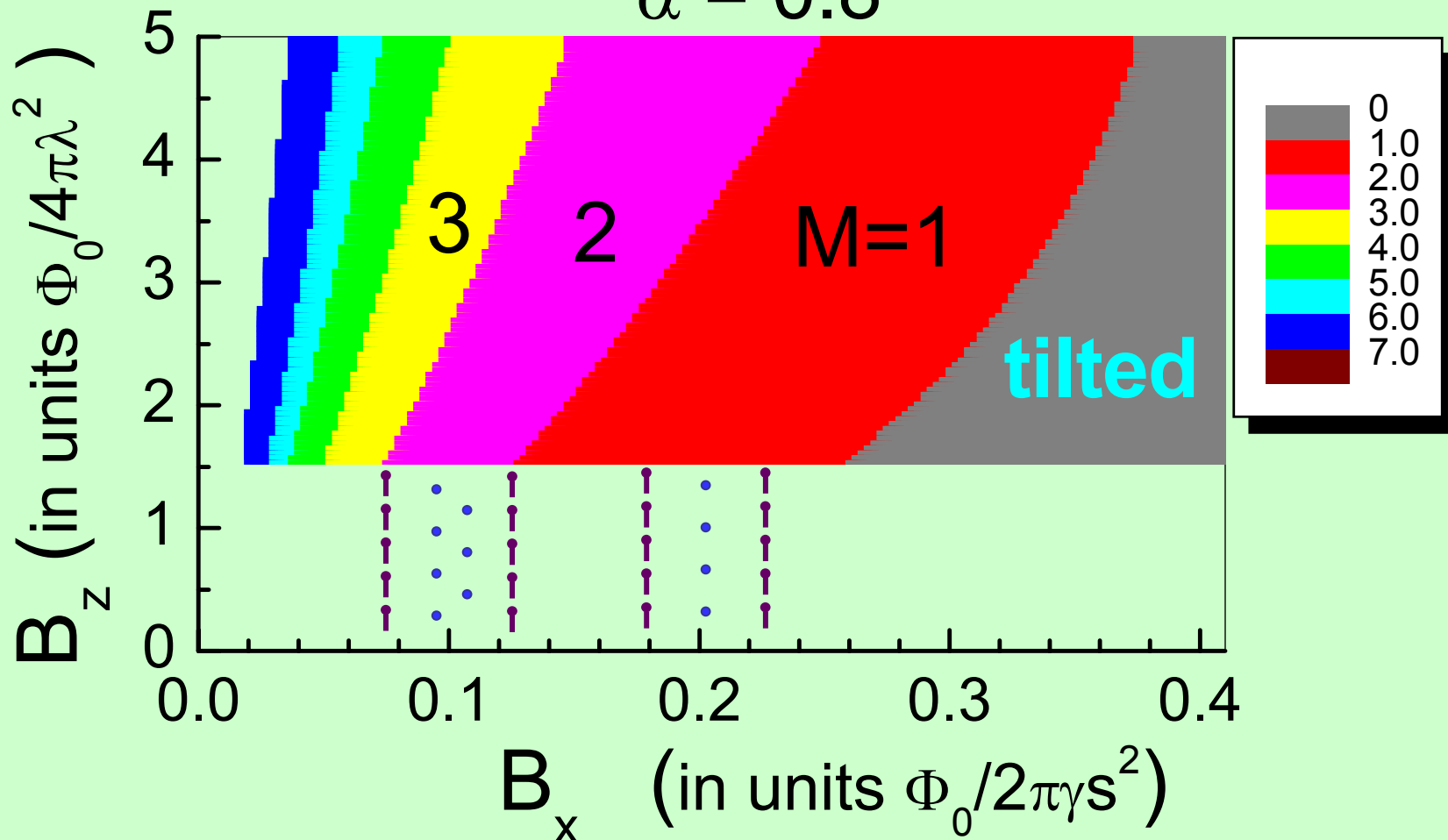
$$F(B_x, B_z) - F(0, B_z) \approx \frac{\epsilon_0}{2\pi\lambda^2} \left[\frac{B_x}{\gamma B_z} \right]^{2/3} \left[\frac{2\pi\lambda^2 B_z}{\Phi_0} \right]^{1/3}$$

Transition composite \rightarrow tilted lattice $E_{M=1} = E_{\text{tilt}}$

$$B_{x,t}(B_z) \approx \gamma \sqrt{\frac{B_z \Phi_0}{4\pi\lambda^2}}$$

Phase diagram using analytical energies

$\alpha = 0.8$



Numerical exploration of lattice states

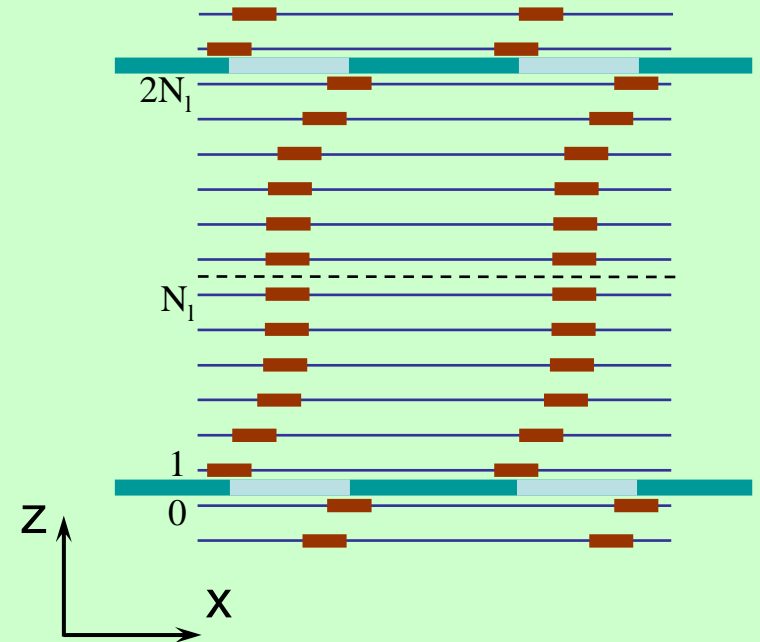
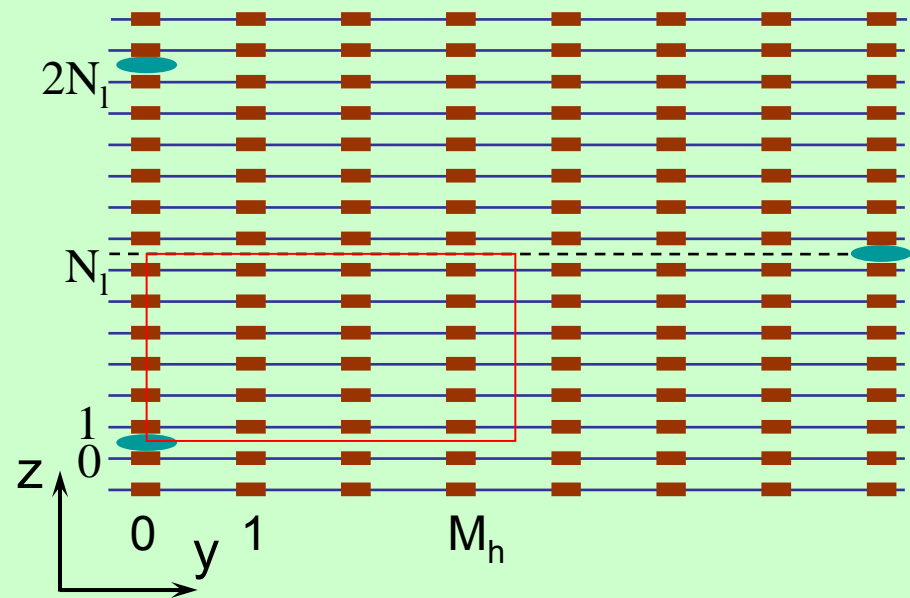
Numerical minimization of energy of unit cell with respect to

- row displacements $u_{n,l}$
- phase $\phi_n(y)$

$$E[\phi, u] = E_{\text{phase}}[\phi] + E_M[u] + E_{\text{shear}}[u] + E_J[\phi, u]$$

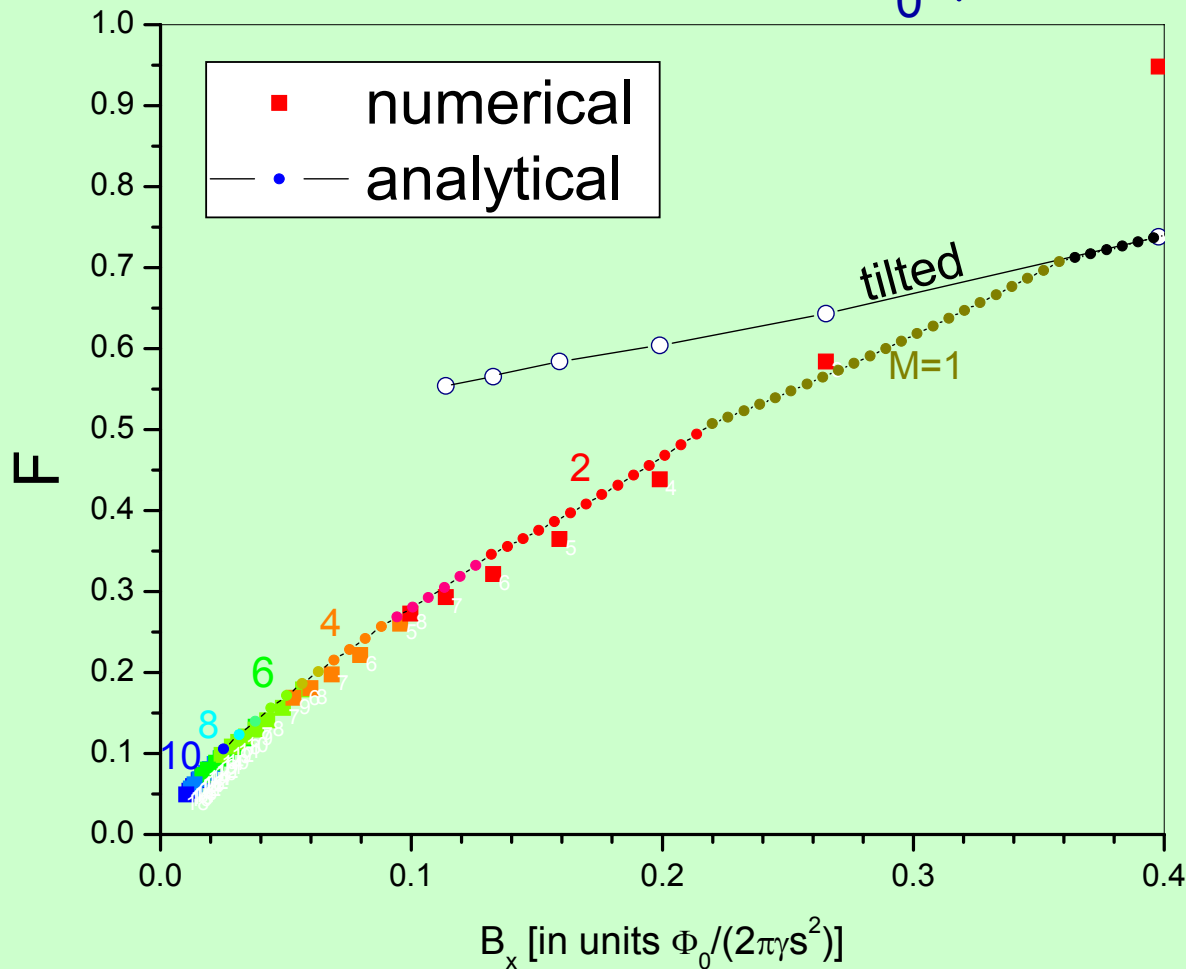
Long-range+local

Cell $2N_l \times (2M_h + 1)$

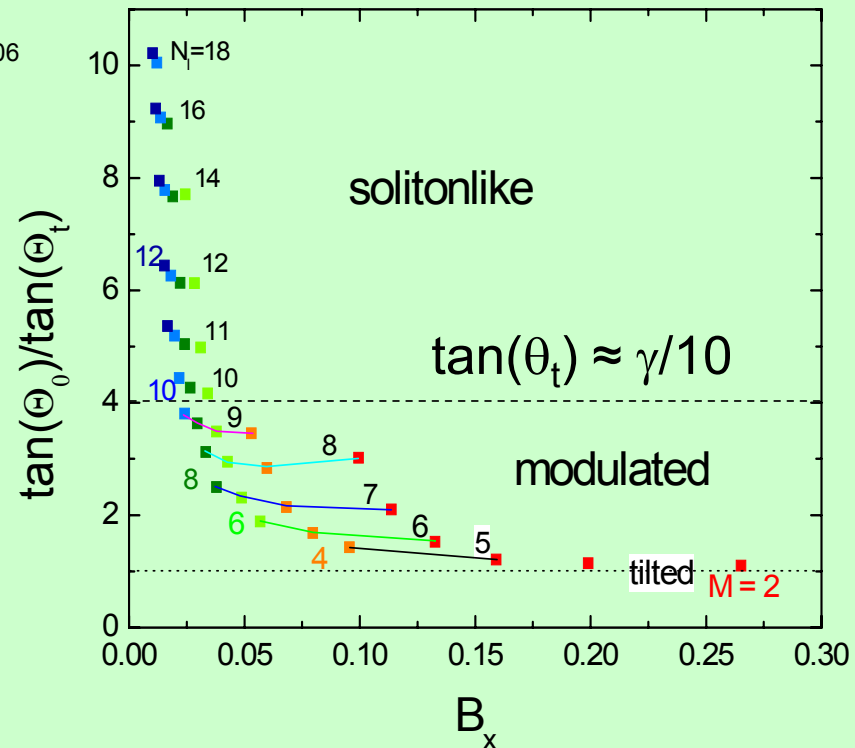
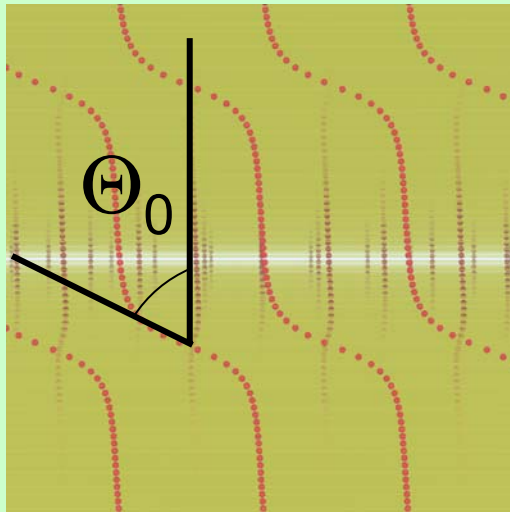
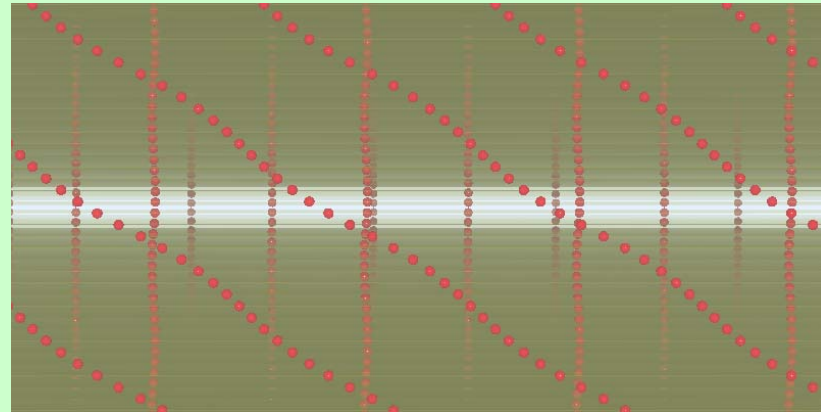
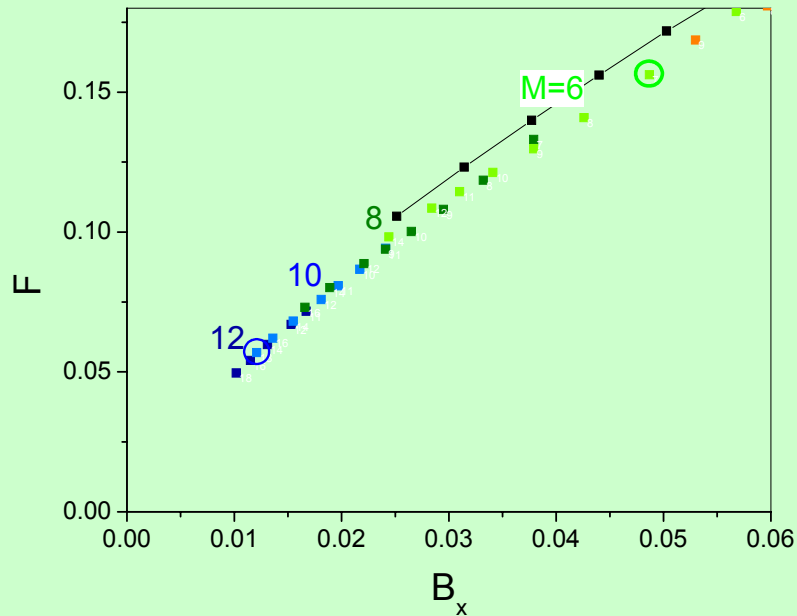


Comparison of analytical and numerical results

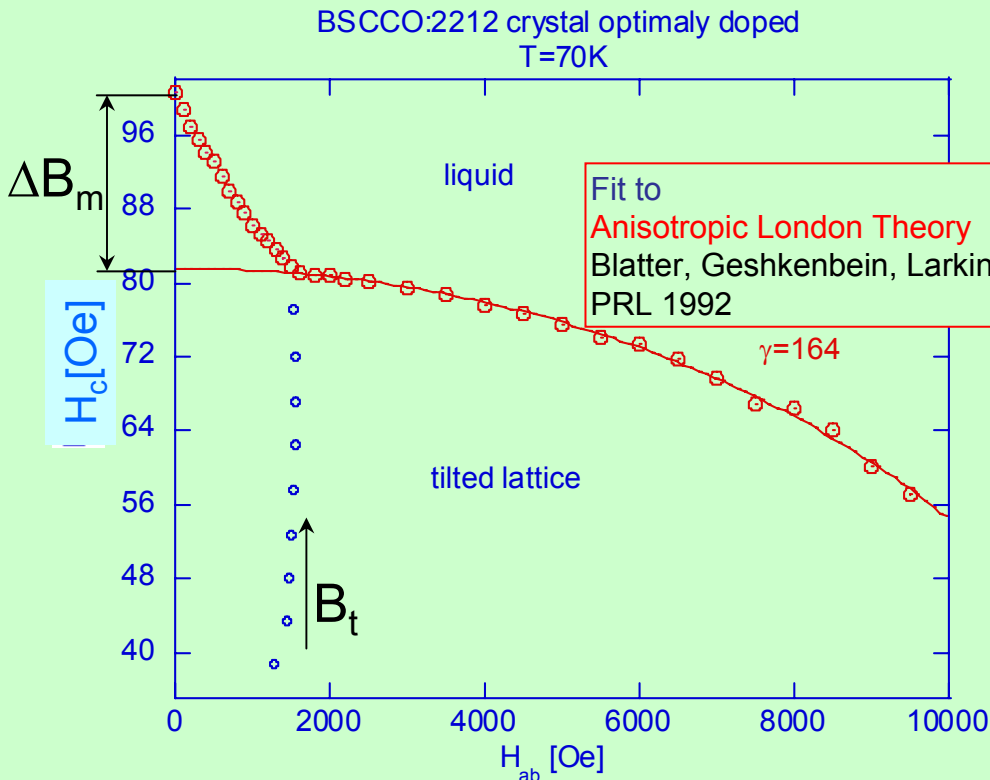
$$\alpha = 0.8, B=0.5\Phi_0/\gamma s^2$$



Crossover from tilted to soliton rows



BSCCO phase diagram



M. Konczykowski *et al.*, PRL 2006

Konczykowski *et al.*, 2000

Mircović *et al.*, 2001

Tokunaga *et al.*, 2002

Parameters small ?
 $\lambda_{ab} = 300\text{nm}$ and $\gamma = 200$ ($\alpha = 0.96$)
 give
 $B_t \approx 1.7 \text{ kG}$ at $B_z = 80\text{G}$

role of fluctuations ?

$$\Delta B_m = \frac{E_M}{\Delta M} \approx 30 \text{ G}$$

magnetic-coupling shift

$$E_M \approx -\frac{9\Phi_0^2}{256\pi^5\lambda^4} \exp\left(-\frac{8\pi^2 c_L^2}{3}\right)$$

Summary

- Chain transitions
 - Transition crossing \rightarrow tilted at moderate B_z (1-3 G)
 - Intermediate states
 - Transition [kinked lines] \rightarrow [crossing chain] at very small B_z (0.1-0.3 G)
 - Finite anisotropy range $0.5 < \alpha < 0.65$
 - Density jump
- Crossover Josephson vortex-soliton at $\alpha \sim 0.5$
- Composite-lattice phase diagram for $\alpha \sim 1$
 - Solitons \rightarrow Composite \rightarrow Tilted