Two-eigenfunction correlation in multifractal metal and insulator

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Two-eigenfunction correlation function

$$C(E-E')=\frac{\sum_{n,m} V\int d^d r\left\langle \boxed{|\Psi_n(r)|^2|\Psi_m(r)|^2}\,\delta(E_n-E)\delta(E_m-E')\right\rangle}{\sum_{n,m}\left\langle \delta(E_n-E)\delta(E_m-E')\right\rangle}$$

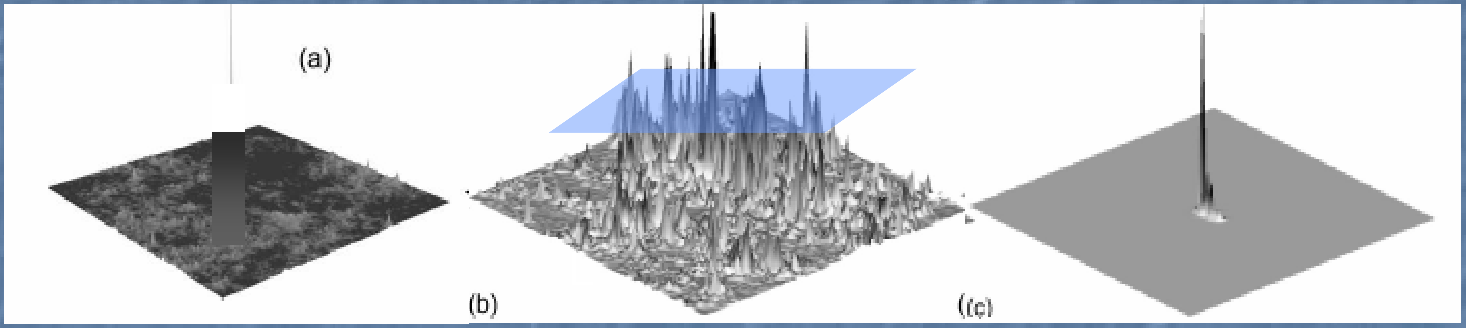
Eigenfunction overlap at an energy separation E-E'

Why to bother?

Matrix elements of local interactions, e.g. local attraction in superconductivity

$$J_{nm} = g\int d^d r \quad \Psi_n^2(r)\Psi_m^2(r)$$

Extended, localized and critical eigenstates

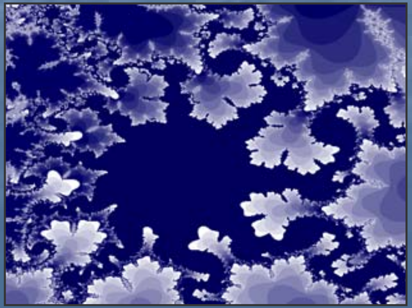


Extended states

$$\sum_r |\Psi_i(r)|^4 = \frac{1}{L^d} = \frac{1}{N}$$

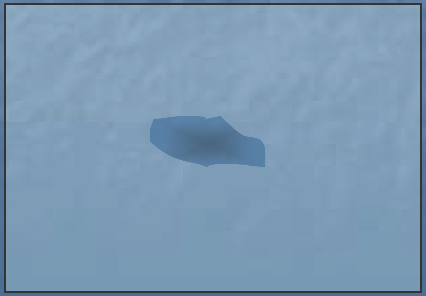
Critical multifractal states

$$\sum_r |\Psi_i(r)|^4 = \frac{1}{L^{d_2}} = \frac{1}{N^{d_2/d}}$$



Localized states

$$\sum_r |\Psi_i(r)|^4 = \frac{1}{\xi^d}$$



Ideal metal and insulator

$$V\int d^d r \left\langle \left|\Psi_n(r)\right|^2 \left|\Psi_m(r)\right|^2 \right\rangle$$

Metal:

$$V \quad V \quad \frac{1}{V} \quad \frac{1}{V} = 1$$

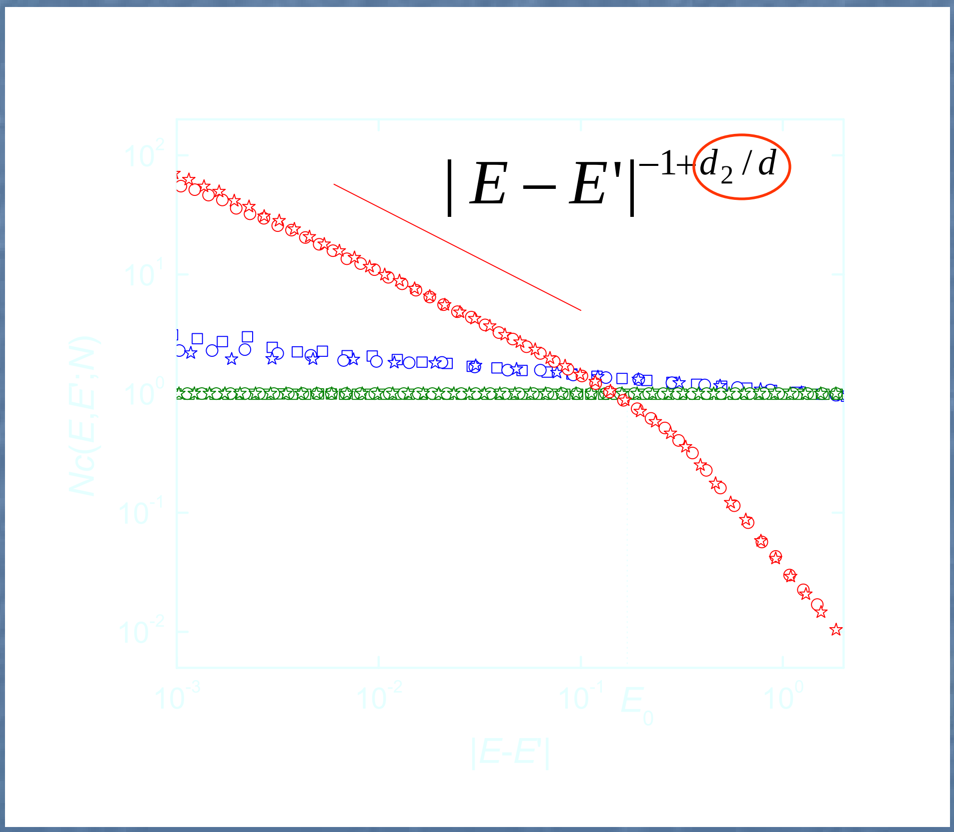
Small amplitude
100% overlap

Insulator:

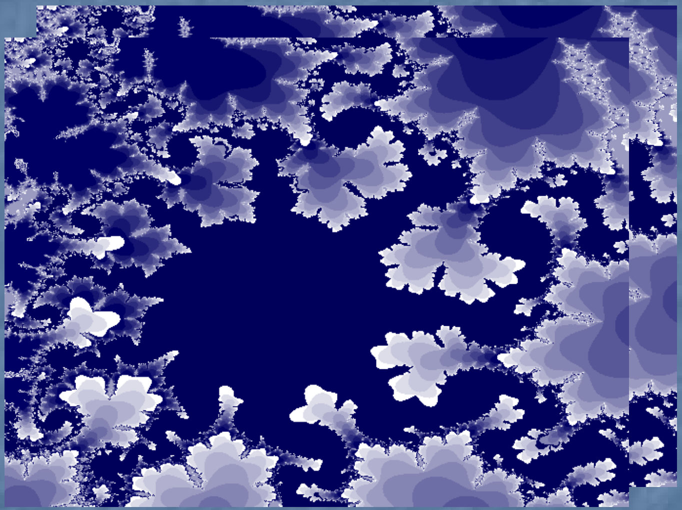
$$V \quad \xi^d \quad \frac{1}{\xi^d} \quad \frac{1}{\xi^d} \times \left(\frac{\xi^d}{V}\right) = 1$$

Large amplitude
rare overlap

Critical enhancement of correlations

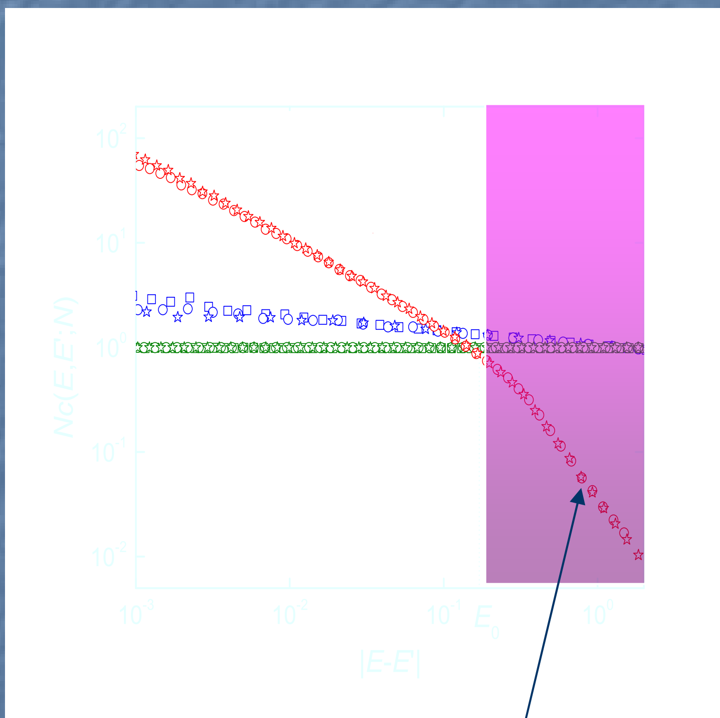


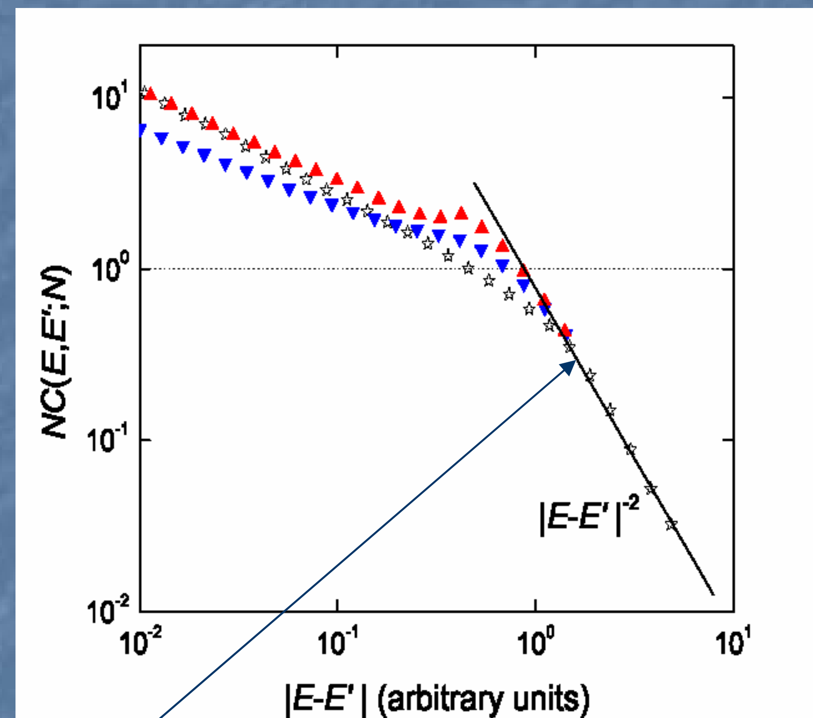
Amplitude higher than in a metal but almost full overlap



States far away in energy are strongly correlated

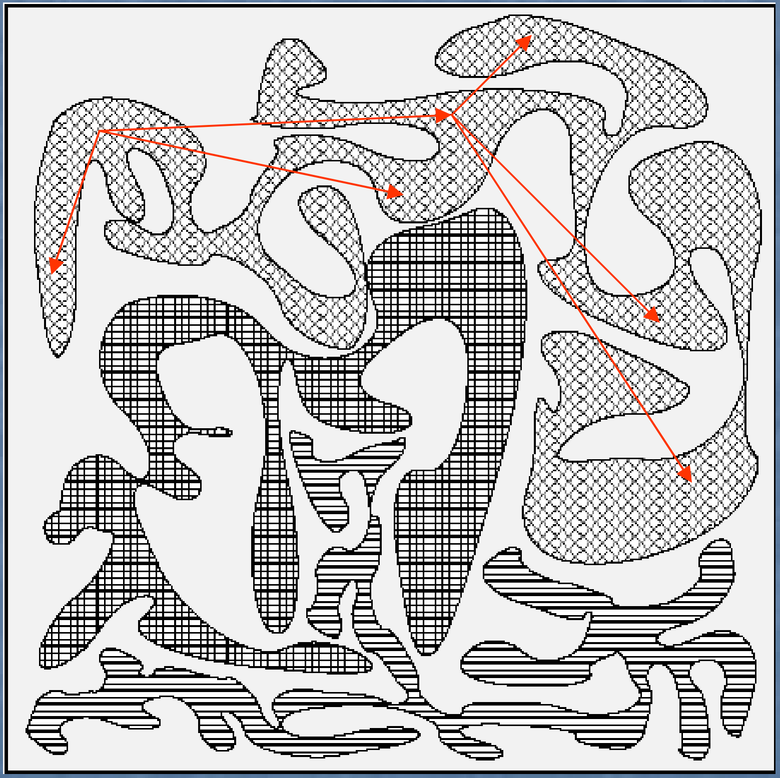
Self-avoiding of eigenfunctions





Overlap is smaller than for uncorrelated eigenfunctions

Stratification of space



Each shell consists of resonance sites for which $|E-E'|<V$

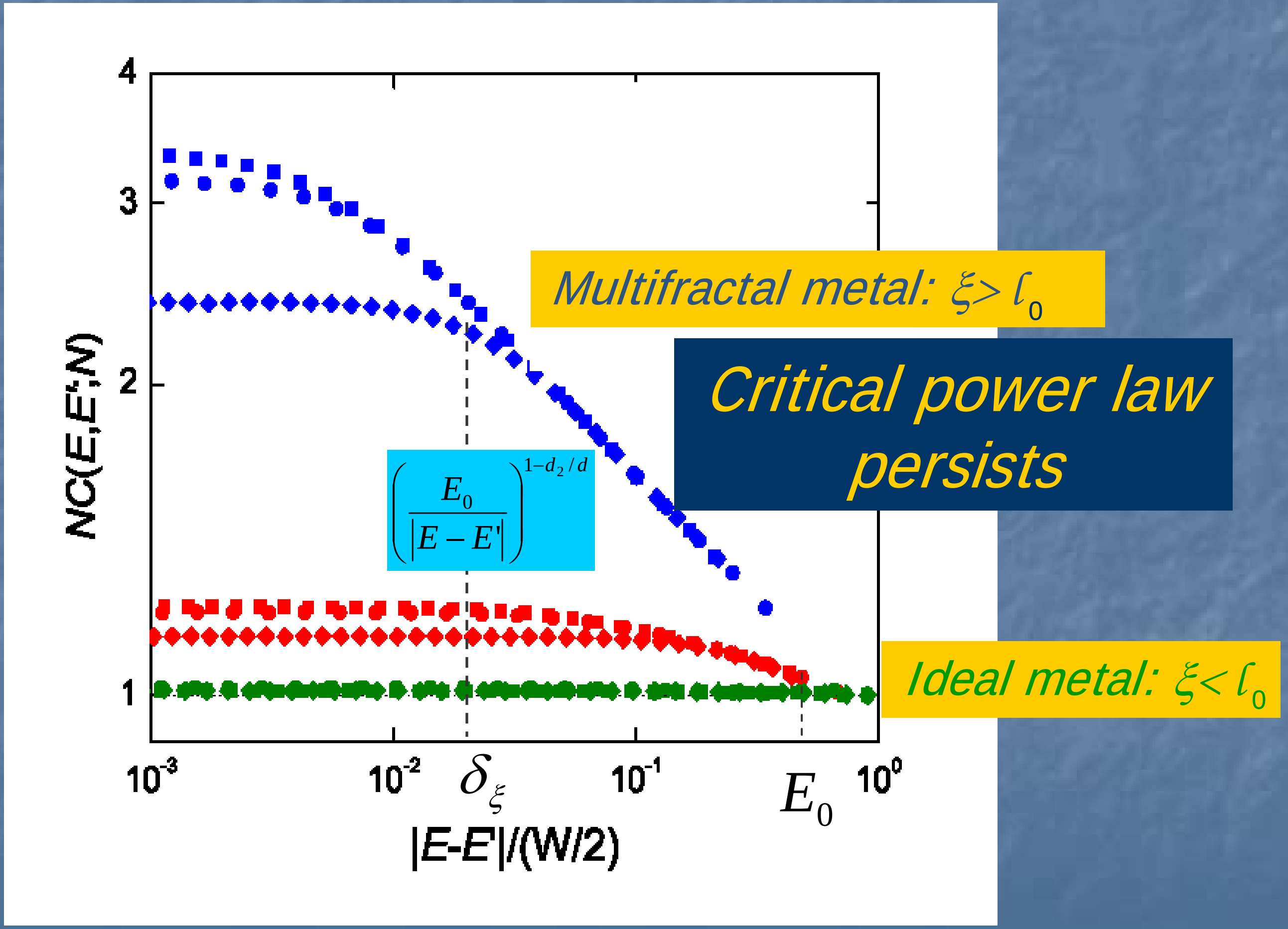
For $W = (\delta E_n) > V$ there are more than one shell which avoid each other in space

Intra-shell states overlap almost like in metal: enhancement of $C(\omega)$ at ω < bandwidth $=E_0$

Inter-shell states avoid each other: $C(\omega)$ rapidly decreases for $\omega > E_0$.

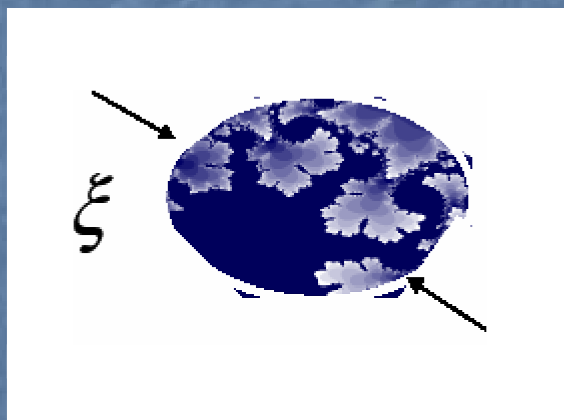
Two-eigenfunction correlation in 3D Anderson model (metal)

New length scale ℓ_0, new energy scale $E_0 = 1/\nu\, \ell_0^3$

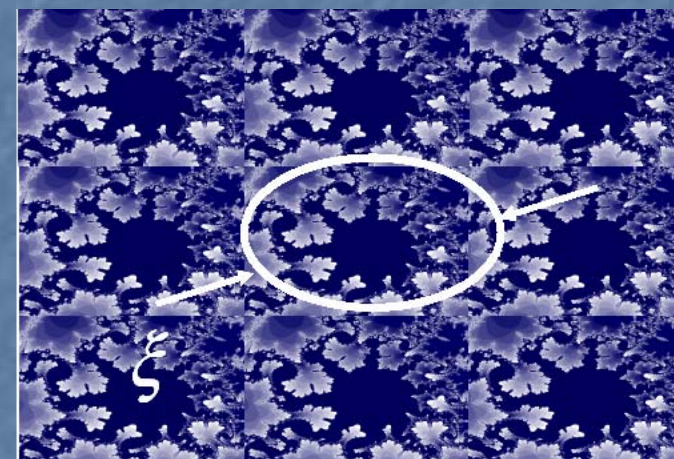


Multifractal metal and insulator

Fractal texture persists in the metal and insulator



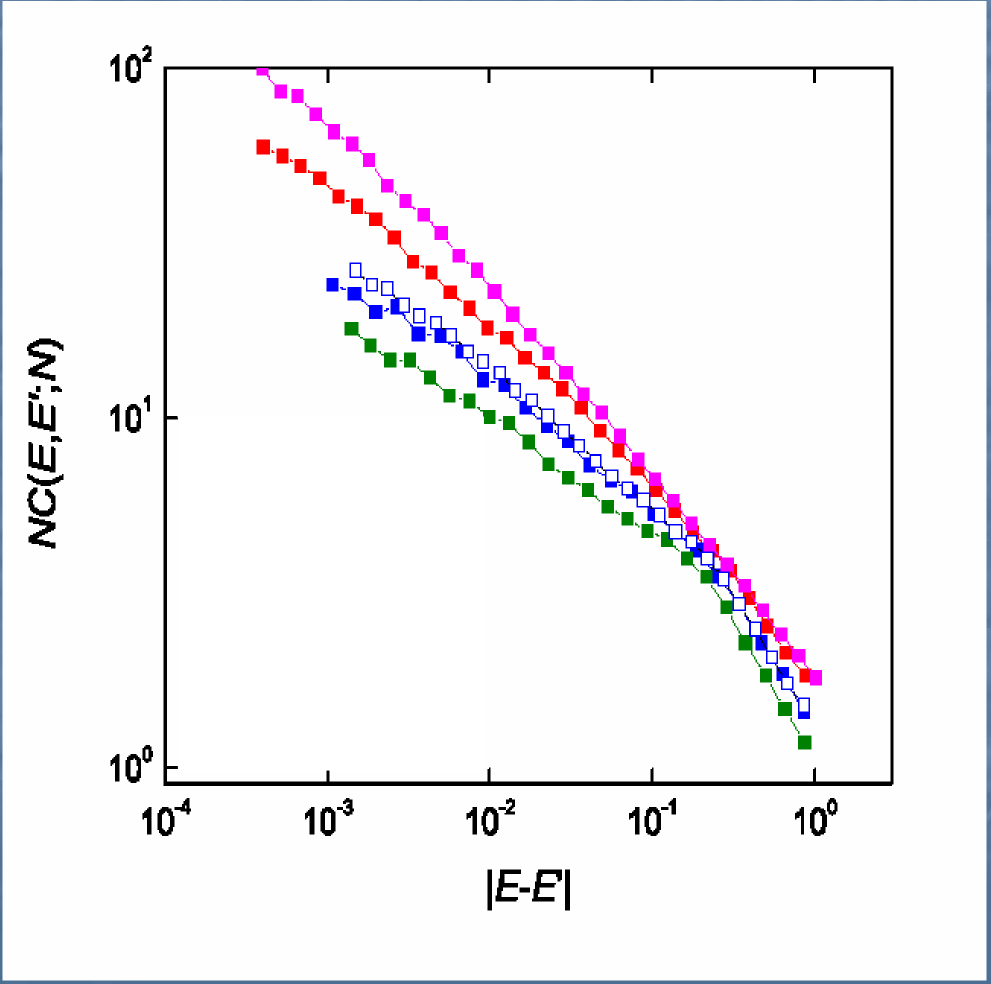
Multifractal insulator



Multifractal metal

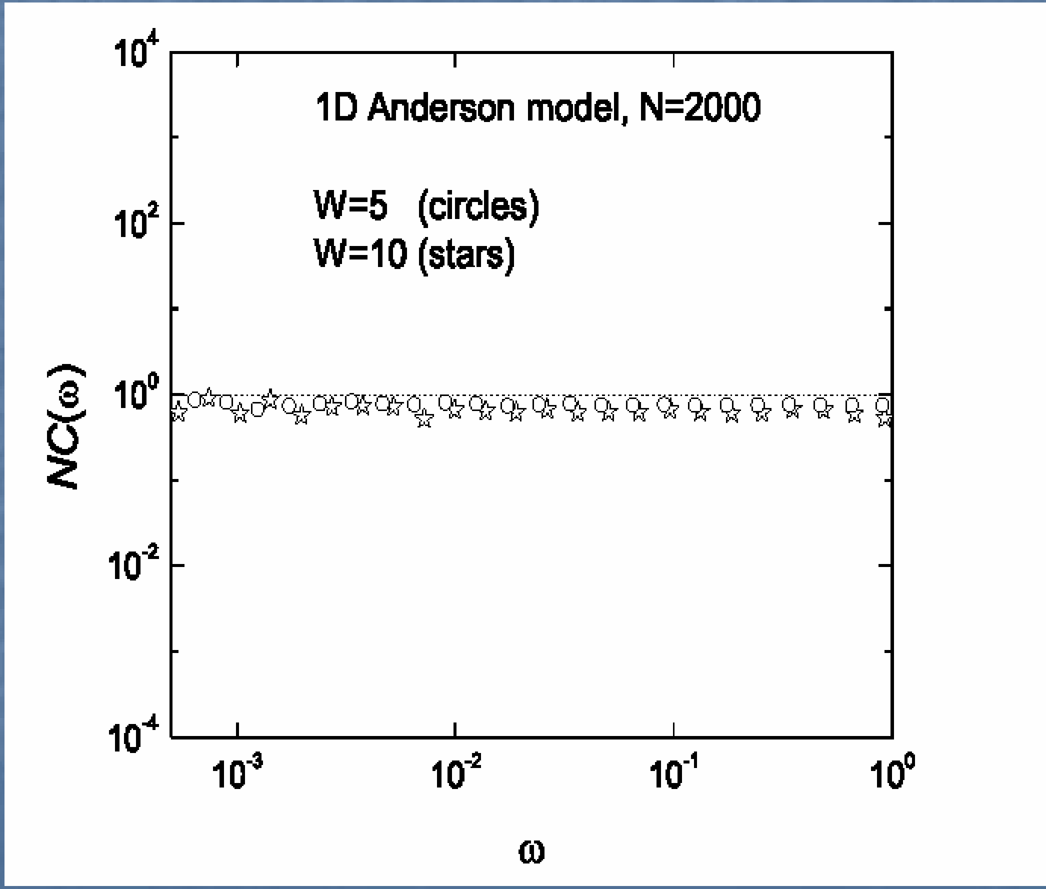
Localization/correlation length ξ is much larger than the minimal length scale for fractality l_0

Two-eigenfunction correlation in 3D Anderson model (insulator)



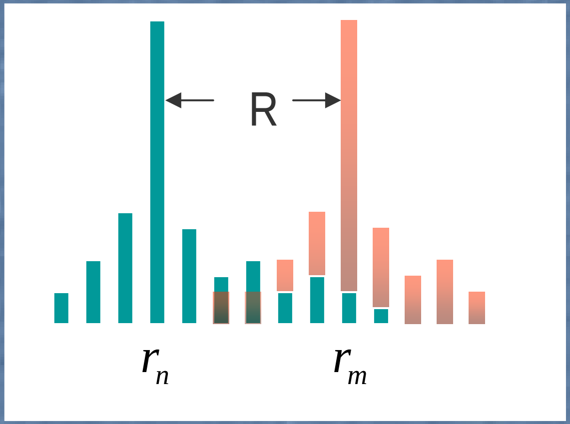
No ideal insulator even for very strong disorder!

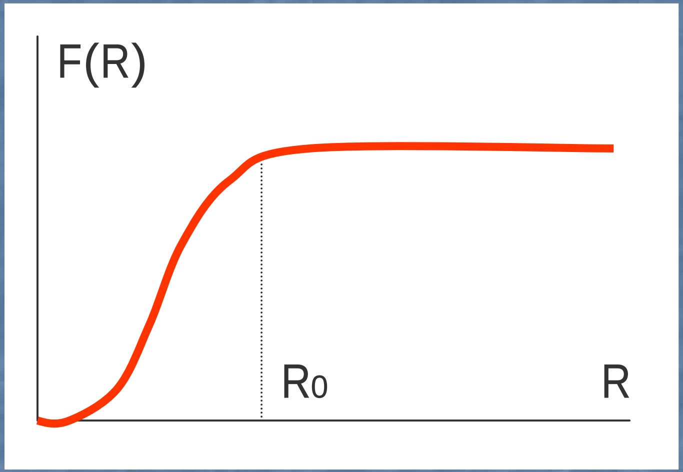
Two-eigenfunction correlation in 1D Anderson model (insulator)



Ideal insulator for sufficiently strong disorder

Repulsion of centers of localization



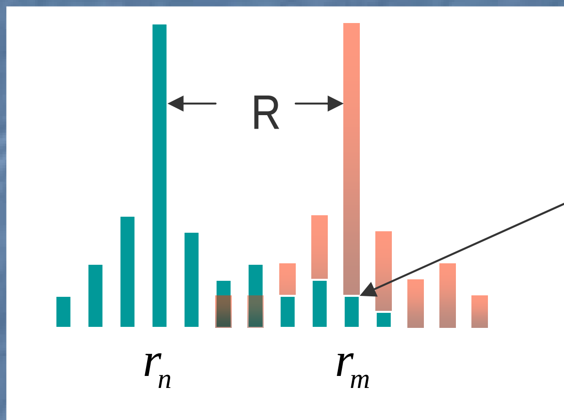


Resonance repulsion of centers of localization

$$R_0 = 2\xi \ln\left(\frac{\delta_\xi}{\omega}\right)$$

$$\omega = \left|E - E'\right| << \delta_\xi$$

Resonance enhancement of overlap



$$|\Psi_n(r_m)|^2 \approx \frac{|H_{nm}|^2}{(E_n - E_m)^2} \sim \left(\frac{\delta_\xi}{\omega}\right)^2 \exp\left[-\frac{R}{\xi}\right]$$

Enhancement of overlap at $\delta_\xi >> \omega$

$$NC(\omega) \sim \left(\frac{\delta_\xi}{\omega}\right)^2 \int_{R_0}^{\infty} dR \;\; R^{d-1} \;\; \exp\left[-\frac{R}{\xi}\right]$$

$$R_0 = 2\xi \ln\left(\frac{\delta_\xi}{\omega}\right) >> \xi$$

$$NC(\omega) \propto \ln^{d-1}\left(\frac{\delta_\xi}{\omega}\right)$$

At d=1 repulsion of centers of localization and the resonance enhancement of overlap compensate each other

$$NC(\omega) = 1$$

At d>1 resonance enhancement prevails

$$NC(\omega) \propto \ln^{d-1}\left(\frac{\delta_\xi}{\omega}\right) >> 1$$

Averaged matrix elements of interaction are enhanced

Random-matrix theory for 3D multifractal insulator

$$\left\langle \left|H_{nm}\right|^2 \right\rangle = \frac{1}{\left(1+\frac{|n-m|^2}{b^2}\right)} \exp\left[-\left(\frac{|n-m|}{B}\right)^{1/3}\right]$$

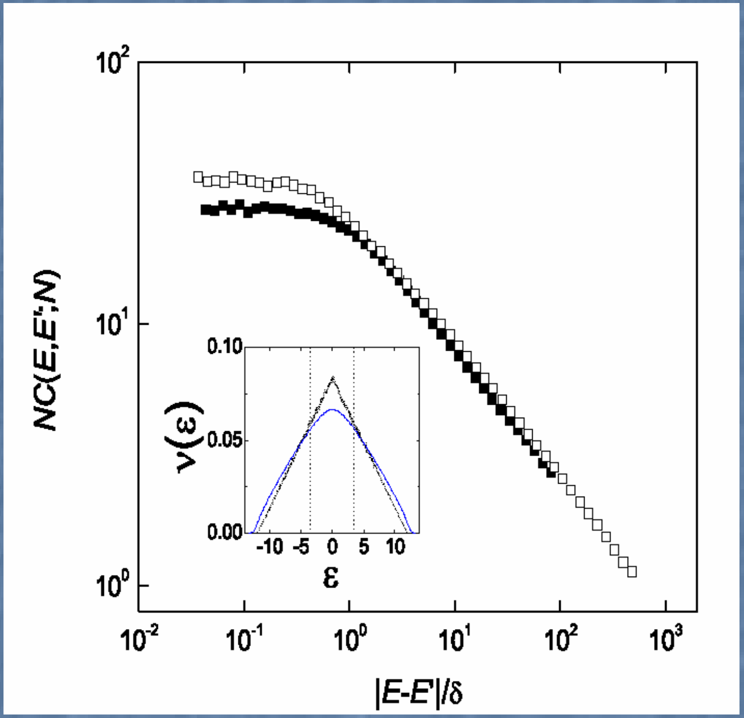
criticality

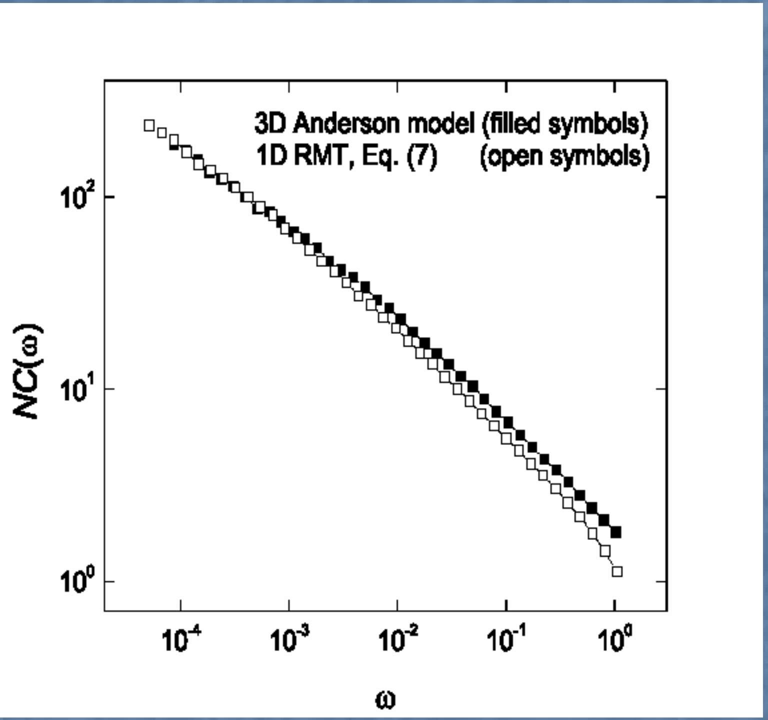
dimensionality of space

b controls fractality

B controls localization radius

RMT vs. 3D Anderson model





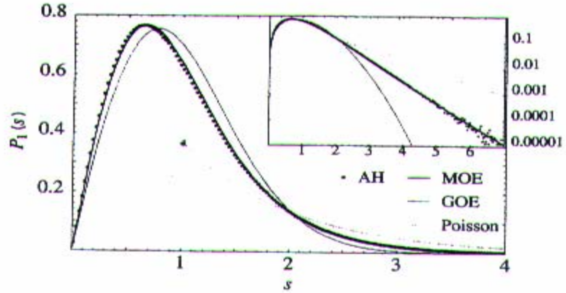
Mobility edge:
B=infinity, b=0.38

Strong insulator:
B=5, b=0.38

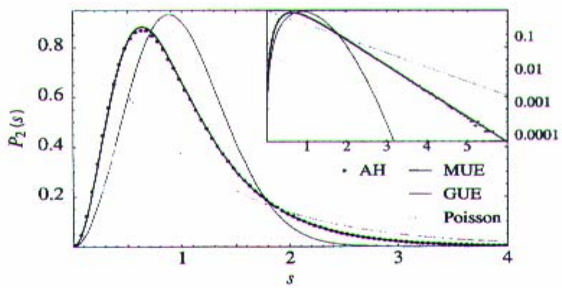
Conclusion

- Fraclal texture of eigenfunctions persists in metal and insulator (multifractal metal and insulator).
- Critical power-law enhancement of eigenfunction correlations.
- Log enhancement of eigenfunction overlap in deep insulator for D=2,3. No enhancement for D=1.
- Stratification of space.
- Random matrix model for the 3D multifractal insulator

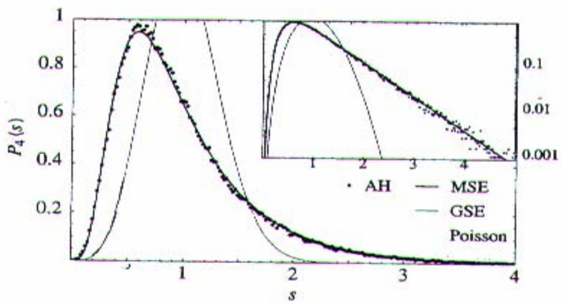
Spectral statistics



Potential disorder
b=0.38



Potential disorder
+ magnetic field
b=0.16



Potential disorder
+ spin-orbit inter.
b=0.07