Asymmetric Zero-Bias Anomaly for Strongly Interacting Electrons in One Dimension

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Interaction strength

\[ t \sim \frac{iV(2k_F)}{\hbar v_F} \]

Backscattering amplitude for two electrons at the Fermi surface

Strong interactions: \( \frac{V(2k_F)}{\hbar v_F} \gg 1 \)

At strong backscattering electrons do not move freely, but are instead confined to finite regions of space:
Short-range interactions

Hubbard model:

\[ H = -t \sum_i \left[ c_{i,\uparrow}^\dagger c_{i+1,\uparrow} + c_{i,\downarrow}^\dagger c_{i+1,\downarrow} + \text{h.c.} \right] + U \sum_i n_{i,\uparrow} n_{i,\downarrow}. \]

\[ H_c = -t \sum_i \left[ c_{i}^\dagger c_{i+1} + \text{h.c.} \right]. \]

Exchange constant

\[ J = \frac{4t^2}{U} n_e \left( 1 - \frac{\sin 2\pi n_e}{2\pi n_e} \right) \]

Two energy scales:

Holons, \( E_F \sim t \)

Spinons, \( J \sim t^2 / U \)

Ogata & Shiba, 1990

\[ H_s = J \sum_l S_l \cdot S_{l+1}. \]
Long-range interactions

Quantum wires: \( V(x) = \frac{e^2}{|x|} \).

At low electron density \( n \), compare kinetic and Coulomb energies:

\[
E_{\text{Kin}} = \frac{\hbar^2 k_F^2}{2m} \propto n^2, \quad E_{\text{Coul}} = \frac{e^2}{r} \propto n.
\]

Coulomb energy dominates:

Electrons form a Wigner crystal and stay near their lattice sites. Density excitations are elastic waves in the crystal (plasmons)

\[
\mathcal{H}_c = \frac{p^2}{2mn} + \frac{1}{2} nms^2 \left( \frac{du}{dx} \right)^2
\]

\( u(x) \) is displacement of the crystal, \( p(x) \) is momentum density.
Spin coupling

Weak exchange due to tunneling through the Coulomb barrier

\[ J \propto \exp \left( -\frac{\eta}{\sqrt{na_B}} \right) \]

Antiferromagnetic spin chain:

\[ H_s = \sum_l J s_l \cdot s_{l+1} \]

Exchange energy is very small:

\[ J \ll E_F \]
At small $J$ there is a broad range of energies between $J$ and $E_F$.

In the range of temperatures $J \ll T \ll E_F$ charge excitations (holons) form a degenerate Fermi system, while spins are completely random.

New behavior is expected for

2. Tunneling density of states
   [Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]
Tunneling density of states

Consider the problem of electron tunneling into a strongly interacting 1D system:

\[ \nu(\varepsilon) \propto \begin{cases} \frac{1}{\sqrt{|\varepsilon|}} \frac{1}{\ln(E_F/|\varepsilon|)}, & T \ll J \ll |\varepsilon| \ll E_F. \\ \frac{1}{|\varepsilon|^{3/8}}, & T = 0, \quad J \ll |\varepsilon| \ll E_F. \end{cases} \]

[Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]

On the other hand,

[Penc, Mila & Shiba, 1995]
Questions

1. In the Hubbard model at $U \gg t$ the electrons essentially become non-interacting spinless fermions. Why is there an enhancement of the density of states at low energies?

2. Why does the density of states at energy $\varepsilon$ depend on the temperature when $T \ll \varepsilon$?
Luttinger liquid & Bosonization

\[
H = \frac{\hbar v}{2\pi} \int dx \left[ K (\partial_x \theta)^2 + K^{-1} (\partial_x \phi)^2 \right].
\]

\( \phi \) and \( \theta \) are bosonic fields; \( \phi(x) \) is the displacement of the system at point \( x \) from equilibrium, \( \partial_x \theta \) is momentum density. Luttinger-liquid parameter \( K \) depends on the interaction strength. For repulsive interactions \( K < 1 \).

Electron creation operator:

\[
\psi(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{i[k_F x + \phi(x)]} - i\theta(x)
\]
Bosonization for electrons with spin

Two pairs of bosonic fields:

\[ \phi_{\uparrow}, \theta_{\uparrow} \text{ and } \phi_{\downarrow}, \theta_{\downarrow} \]

Charge and spin modes:

\[ \phi_{c,s} = \frac{\phi_{\uparrow} \pm \phi_{\downarrow}}{\sqrt{2}} \text{ and } \theta_{c,s} = \frac{\theta_{\uparrow} \pm \theta_{\downarrow}}{\sqrt{2}} \]

Hamiltonian:

\[ H = H_C + H_S \]

\[ H_C = \frac{\hbar v_c}{2\pi} \int \left[ K_c (\partial_x \theta_c)^2 + K_c^{-1} (\partial_x \phi_c)^2 \right] dx \quad T \ll vck_F \sim E_F \]

\[ H_S = \frac{\hbar v_s}{2\pi} \int \left[ K_s (\partial_x \theta_s)^2 + K_s^{-1} (\partial_x \phi_s)^2 \right] dx + \frac{2g_{1\perp}}{(2\pi\alpha)^2} \int \cos[\sqrt{8}\phi_s(x)] dx \quad T \ll vsk_F \sim J \]
Intermediate energies

At $J \sim T \ll E_F$ the charge excitations are still bosonic:

$$H_c = \frac{\hbar v_c}{2\pi} \int dx \left[ K(\partial_x \theta)^2 + K^{-1}(\partial_x \phi)^2 \right].$$

1. For short range repulsion (e.g., Hubbard) one finds this by bosonizing the non-interacting fermions. Then $K = 1$.

2. In the Wigner crystal case, this is the phonon Hamiltonian.

Spin excitations are described by the Heisenberg model

$$H_S = \sum_l J S_l \cdot S_{l+1}$$

It can be bosonized, but only if $T \ll J$.

How to make a fermion out of bosons and spins?
Electron annihilation operator (1)

1. We destroy a spinless fermion and
2. Remove a site from the spin chain

\[ \psi_{\uparrow}(0) = \psi_{h}(0)Z_{0,\uparrow} \]

Operator \( Z_{0,\uparrow} \) removes site 0 if it has spin \( \uparrow \), and destroys the state otherwise.

This expression does not explicitly account for the fact that phonons shift the spin chain!
The shift of electrons by density waves is conveniently accounted for in bosonization approach:

\[
\psi_\uparrow(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{\pm i[k_F x + \phi(x)] - i\theta(x)} Z_{l,\uparrow} l = \frac{k_F x + \phi(x)}{\pi} \quad Z_{l,\uparrow} = e^{i\phi/\pi}
\]
Electron annihilation operator (3)

Let us perform Fourier transform on $Z$-operator:

$$Z_{l,\uparrow} = \int_{-\pi}^{\pi} \frac{dq}{2\pi} z_{\uparrow}(q) e^{iql}$$

$$\psi_{\uparrow}(x) = \int_{-\pi}^{\pi} \frac{dq}{2\pi} z_{\uparrow}(q) \frac{e^{ik_F(1+\frac{q}{\pi})x}}{\sqrt{2\pi\alpha}} e^{i(1+\frac{q}{\pi})\phi(x) - i\theta(x)}$$

1. At $q = 0$ the green factor is the destruction operator for a spinless fermion (holon).

2. At $q \neq 0$ it represents an operator that removes a fermion and adds a phase shift $\delta = -q$ to the boundary conditions. C.f. x-ray absorption edge problem [Shotte & Shotte, 1969]
X-ray absorption edge problem

When X-rays excite electrons above the Fermi level, the remaining core hole scatters all the electrons in the Fermi sea. The resulting absorption intensity shows a power-law singularity.

\[ \psi \propto e^{i\left(1 - \frac{\delta}{\pi}\right)\phi - \theta} \]

\( \delta \) is the scattering phase shift of the core-hole potential.
Periodic boundary conditions

Move the whole system to the right by system size $L$

Total momentum is quantized

$$PL = 2\pi m$$

$$PL = \sum_{i=1}^{N} p_i L + qN = \sum_{i=1}^{N} (p_i L + q) = \sum_{i=1}^{N} 2\pi m_i$$

As a holon goes around the circle, it acquires a phase factor

$$e^{ip_i L} = e^{-iq}$$

[Penc, Mila & Shiba, 1995]
Tunneling density of states

Fermion operators $\rightarrow$ Green’s functions $\rightarrow$ Density of states

\[
\nu_\sigma^\pm(\varepsilon) = \nu_0 \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{c_\sigma^\pm(q)}{\Gamma(\lambda(q) + 1)} \left( \frac{\vert\varepsilon\vert}{E_F} \right)^{\lambda(q)} \quad J, T \ll \vert\varepsilon\vert \ll E_F
\]

At $q \neq 0$ we find a power-law singularity with the exponent

\[
\lambda(q) = \frac{1}{2} \left[ \left(1 + \frac{q}{\pi}\right)^2 K + \frac{1}{K} \right] - 1
\]

Short-range interactions (Hubbard): non-interacting holons, $K=1$

Properties of the spin chain enter through the equal-time correlators

\[
c_\sigma^+(q) = \sum_l \langle Z_{l,\sigma} Z^\dagger_{0,\sigma} \rangle e^{-iql},
\]

\[
c_\sigma^-(q) = \sum_l \langle Z^\dagger_{0,\sigma} Z_{l,\sigma} \rangle e^{-iql}
\]
Density of states for the Hubbard model at $U \gg t$

In the limit $|\varepsilon|/E_F \to 0$ the true asymptotic behavior of the density of states is

$$\nu_{\sigma}^{\pm}(\varepsilon) = \frac{\nu_0}{2\sqrt{2}} c_{\sigma}^{\pm}(\pi) \left( \frac{E_F}{|\varepsilon|} \right)^{1/2} \frac{1}{\sqrt{\ln(E_F/|\varepsilon|)}}$$

Compare with the earlier results

$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{1/2}} \sqrt{\ln(E_F/|\varepsilon|)}, \quad J \ll T \ll |\varepsilon| \ll E_F.$$  
[Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]

$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{3/8}}, \quad T = 0, \quad J \ll |\varepsilon| \ll E_F.$$  
[Penc, Mila & Shiba, 1995]
High temperature

All $l$ spins between the initial and final positions must be aligned

\[ \langle Z_{l,↑} Z_{0,↑}^\dagger \rangle = \frac{1}{2|l|}, \quad \langle Z_{0,↑}^\dagger Z_{l,↑} \rangle = \frac{1}{2|l|+1} \]

\[ c_{↑}(\pi) = \frac{1}{3}, \quad c_{↑}(\pi) = \frac{1}{6} \]

Factor of 2: one can always add spin $\uparrow$, but the probability of removing one is $\frac{1}{2}$.

Density of states:

(1) \[ \nu(\varepsilon) \propto \frac{1}{\sqrt{|\varepsilon|}} \]

(2) \[ \nu(\pm \varepsilon) = 2\nu(-\varepsilon) \]
Zero temperature

Numerical results for $c_\sigma^+(q)$ and $c_\sigma^-(q)$

$[\text{Penc, Mila & Shiba, 1995}]$

$c_\sigma^+(\pi) \approx 0.1 c_\sigma^-(\pi)$

Density of states:

1. $\nu(\varepsilon) \propto 1/\sqrt{|\varepsilon|}$
2. $\nu(+\varepsilon) \approx 0.1 \nu(-\varepsilon)$

Subleading contribution at $\varepsilon > 0$

$\tilde{\nu}^+(\varepsilon) \propto \frac{1}{\varepsilon^{3/8}}$
Summary

- The tunneling density of states is asymmetric around the Fermi level;
- The asymmetry changes sign when temperature is tuned between $T \gg J$ and $T \ll J$ regimes;
- For short-range interactions, the nature of the peak in the density of states is similar that in the x-ray absorption edge problem.