Asymmetric Zero-Bias Anomaly for Strongly Interacting Electrons in One Dimension

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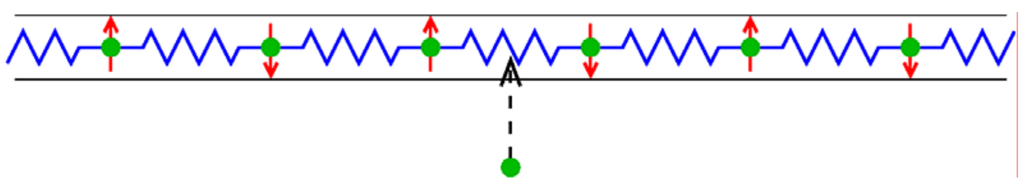

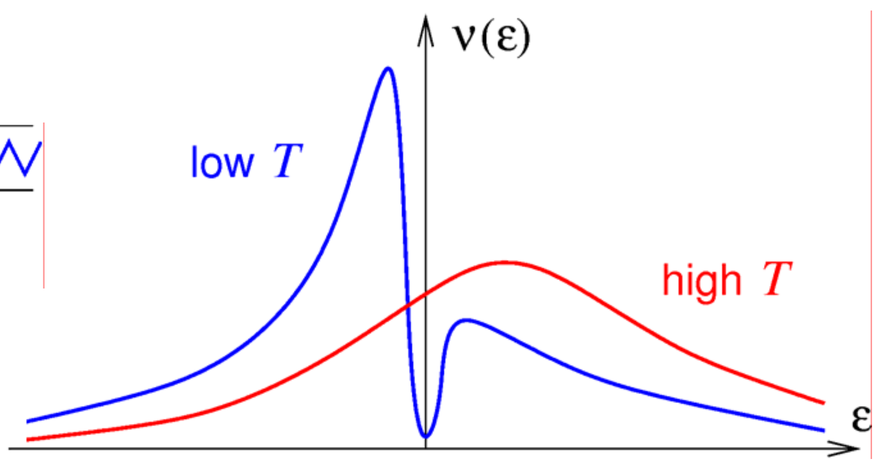

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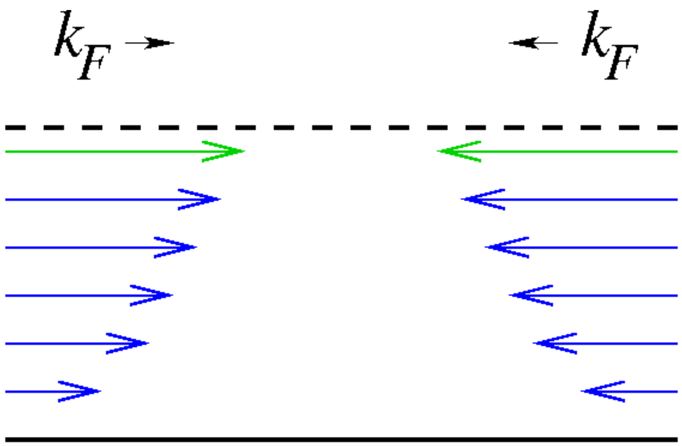
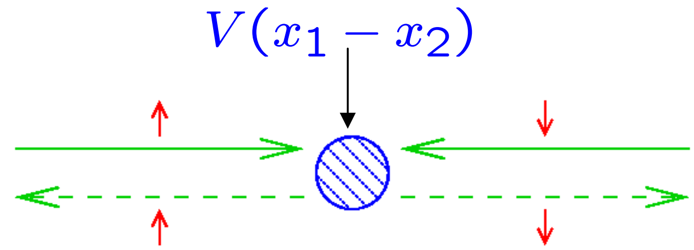
Interaction strength





Backscattering amplitude for two electrons at the Fermi surface $t \sim \frac{iV(2k_F)}{\hbar v_F}$

Strong interactions: $\frac{V(2k_F)}{\hbar v_F} \gg 1$

At strong backscattering electrons do not move freely, but are instead confined to finite regions of space:

Short-range interactions

Hubbard model:

$$H = -t\sum_i [c^\dagger_{i,\uparrow} c_{i+1,\uparrow} + c^\dagger_{i,\downarrow} c_{i+1,\downarrow} + \text{h.c.}] + U\sum_i n_{i,\uparrow} n_{i,\downarrow}.$$

holons $\quad U \gg t \quad$ spinons

Ogata & Shiba, 1990

$$H_c = -t\sum_i [c^\dagger_i c_{i+1} + \text{h.c.}].$$

$$H_s = J\sum_l \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}.$$

Exchange constant $\quad J = \frac{4t^2}{U} n_e \left(1 - \frac{\sin 2\pi n_e}{2\pi n_e}\right)$

Two energy scales:

Holons, $\quad E_F \sim t$

Spinons, $\quad J \sim t^2/U$

$$\boxed{J \ll E_F}$$

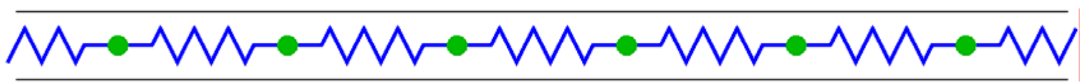
Long-range interactions

Quantum wires: $V(x) = \frac{e^2}{|x|}.$

At low electron density n, compare kinetic and Coulomb energies:

$$E_{\text{kin}} = \frac{\hbar^2 k_F^2}{2m} \propto n^2, \qquad E_{\text{Coul}} = \frac{e^2}{r} \propto n.$$

Coulomb energy dominates:



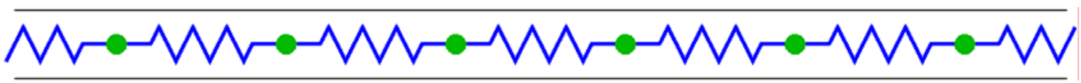
Electrons form a Wigner crystal and stay near their lattice sites

Density excitations are elastic waves in the crystal (plasmons)

$$\mathcal{H}_c = \frac{p^2}{2mn} + \frac{1}{2} nms^2 \left(\frac{du}{dx}\right)^2$$

$u(x)$ is displacement of the crystal, $p(x)$ is momentum density.

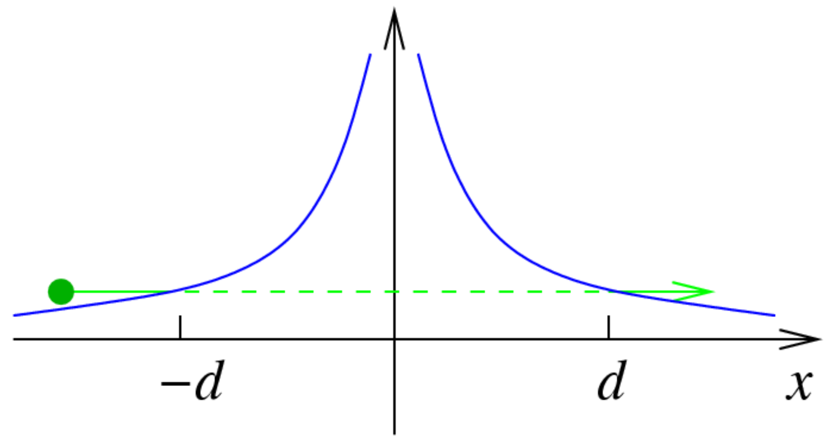
Spin coupling



To first approximation the spins do not interact

Weak exchange due to tunneling through the Coulomb barrier

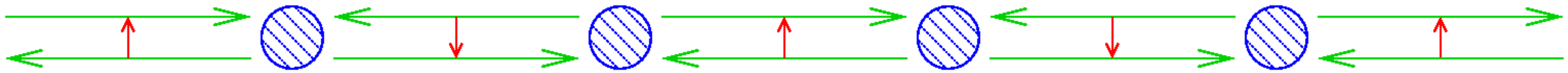
$$J \propto \exp\left(-\frac{\eta}{\sqrt{n a_B}}\right)$$



Antiferromagnetic spin chain: $$H_s = \sum_l J \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}$$

Exchange energy is very small: $$\boxed{J \ll E_F}$$

Spin-incoherent regime



At small J there is a broad range of energies between J and E_F

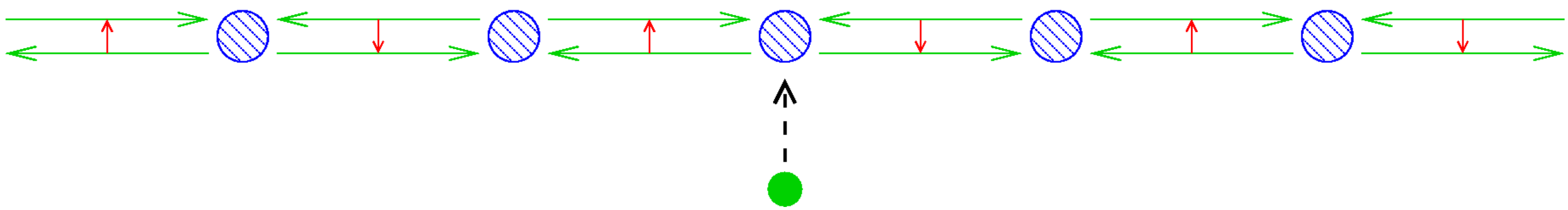
In the range of temperatures $J \ll T \ll E_F$ charge excitations (holons) form a degenerate Fermi system, while spins are completely random.

New behavior is expected for

1. Conductance [KM, 2004]
2. Tunneling density of states
   [Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]

Tunneling density of states

Consider the problem of electron tunneling into a strongly interacting 1D system:



In the simplest case of the Hubbard model the density of states

$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{1/2}} \frac{1}{\sqrt{\ln(E_F/|\varepsilon|)}}, \qquad J \ll T \ll |\varepsilon| \ll E_F.$$

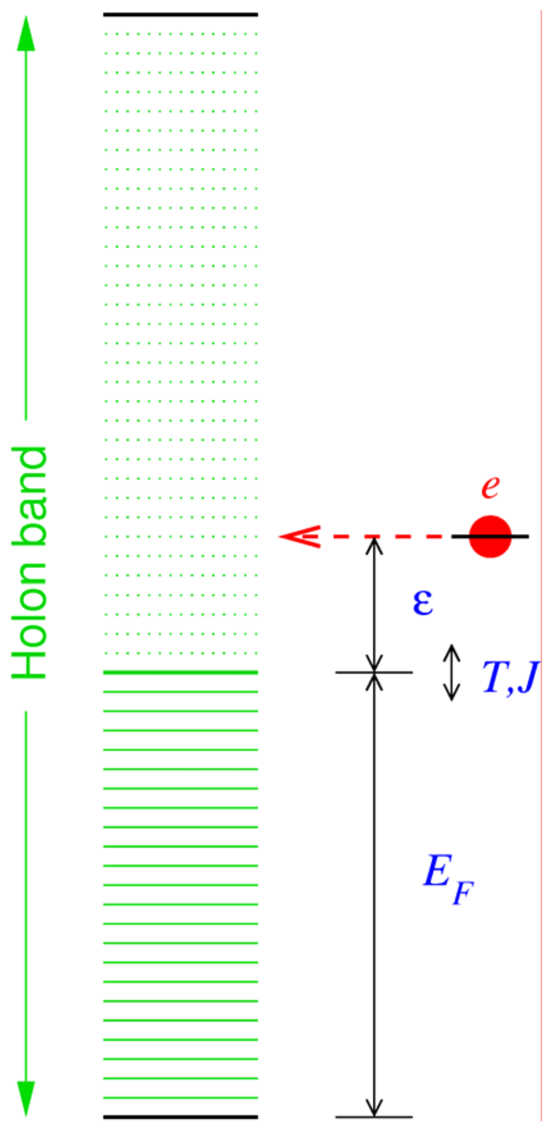
[Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]

On the other hand,

$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{3/8}}, \qquad T = 0, \; J \ll |\varepsilon| \ll E_F.$$

[Penc, Mila & Shiba, 1995]

Questions



1. In the Hubbard model at $U \gg t$ the electrons essentially become non-interacting spinless fermions. Why is there an enhancement of the density of states at low energies?

2. Why does the density of states at energy ε depend on the temperature when $T \ll \varepsilon$?

Luttinger liquid & Bosonization

$$H = \frac{\hbar v}{2\pi} \int dx \left[K (\partial_x \theta)^2 + K^{-1} (\partial_x \phi)^2 \right].$$

ϕ and θ are bosonic fields; $\phi(x)$ is the displacement of the system at point x from equilibrium, $\partial_x \theta$ is momentum density

Luttinger-liquid parameter K depends on the interaction strength. For repulsive interactions $K < 1$.

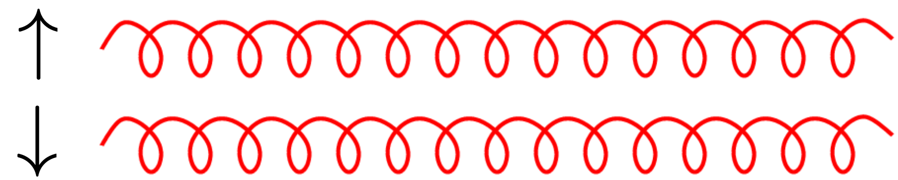
Electron creation operator:

$$\psi(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{i[k_F x + \phi(x)] - i\theta(x)}$$

Bosonization for electrons with spin

Two pairs of bosonic fields:

$\phi_\uparrow, \theta_\uparrow$ and $\phi_\downarrow, \theta_\downarrow$



Charge and spin modes: $\phi_{c,s} = \dfrac{\phi_\uparrow \pm \phi_\downarrow}{\sqrt{2}}$ and $\theta_{c,s} = \dfrac{\theta_\uparrow \pm \theta_\downarrow}{\sqrt{2}}$

Hamiltonian: $H = H_c + H_s$

$$H_c = \frac{\hbar v_c}{2\pi} \int \left[K_c(\partial_x\theta_c)^2 + K_c^{-1}(\partial_x\phi_c)^2\right] dx$$

$T \ll v_c k_F \sim E_F$

$$\begin{aligned} H_s &= \frac{\hbar v_s}{2\pi} \int \left[K_s(\partial_x\theta_s)^2 + K_s^{-1}(\partial_x\phi_s)^2\right] dx \\ &+ \frac{2g_{1\perp}}{(2\pi\alpha)^2} \int \cos[\sqrt{8}\phi_s(x)]dx \end{aligned}$$

$T \ll v_s k_F \sim J$

Intermediate energies

At $J \sim T \ll E_F$ the charge excitations are still bosonic:

$$H_c = \frac{\hbar v_c}{2\pi} \int dx \left[K(\partial_x \theta)^2 + K^{-1}(\partial_x \phi)^2 \right].$$

1. For short range repulsion (e.g., Hubbard) one finds this by bosonizing the non-interacting fermions. Then $K = 1$.
2. In the Wigner crystal case, this is the phonon Hamiltonian.

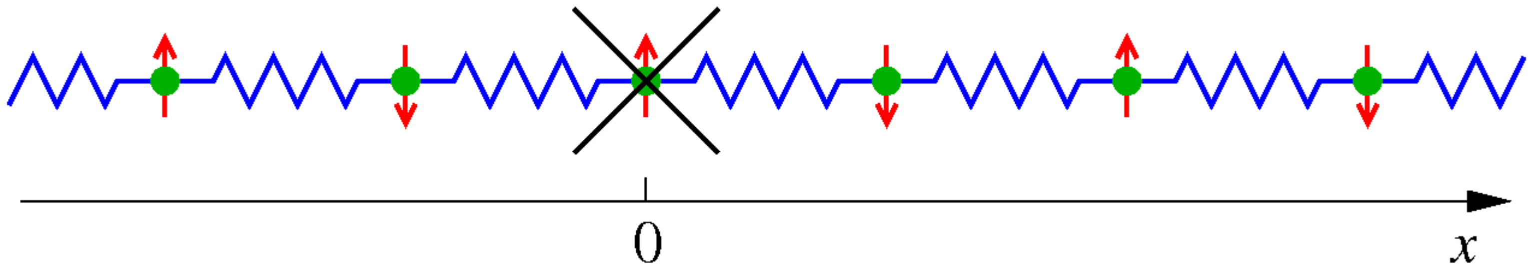
Spin excitations are described by the Heisenberg model

$$H_s = \sum_l J \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}$$

It can be bosonized, but only if $T \ll J$.

$$\boxed{\text{How to make a fermion out of bosons and spins?}}$$

Electron annihilation operator (1)



1. We destroy a spinless fermion and
2. Remove a site from the spin chain

$$\psi_{\uparrow}(0) = \psi_h(0) Z_{0,\uparrow}$$

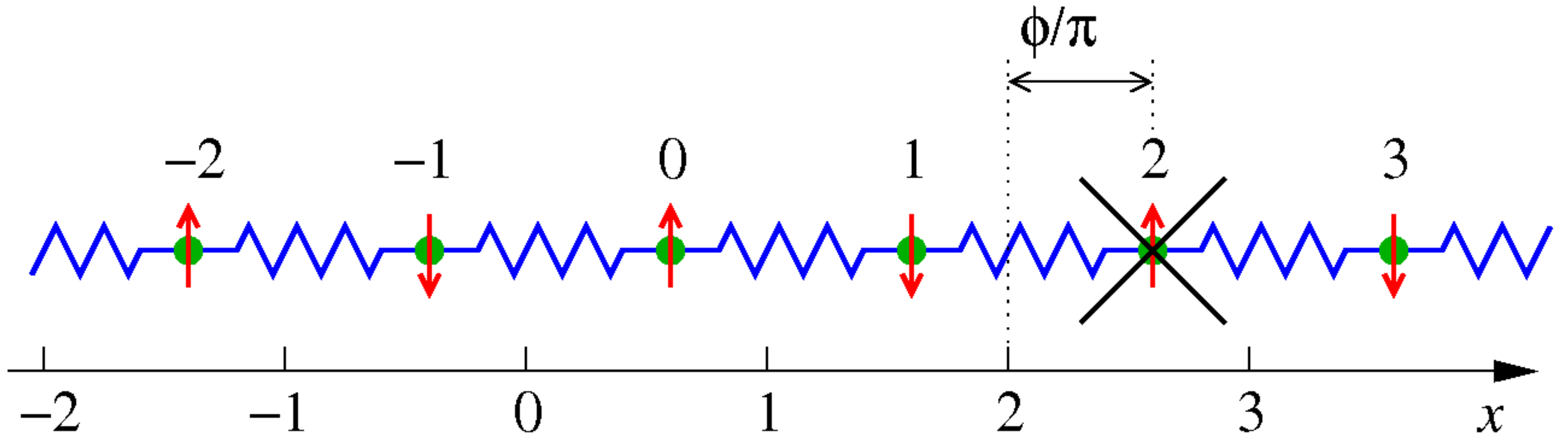
[Penc, Mila & Shiba, 1995]

Operator $Z_{0,\uparrow}$ removes site 0 if it has spin $\uparrow$, and destroys the state otherwise.

[Sorella & Parola, 1992]

This expression does not explicitly account for the fact that phonons shift the spin chain!

Electron annihilation operator (2)



The shift of electrons by density waves is conveniently accounted for in bosonization approach:

$$\psi_\uparrow(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{\pm i[k_F x + \phi(x)] - i\theta(x)} Z_{l,\uparrow}\Big|_{l=\frac{k_F x + \phi(x)}{\pi}}$$

Electron annihilation operator (3)

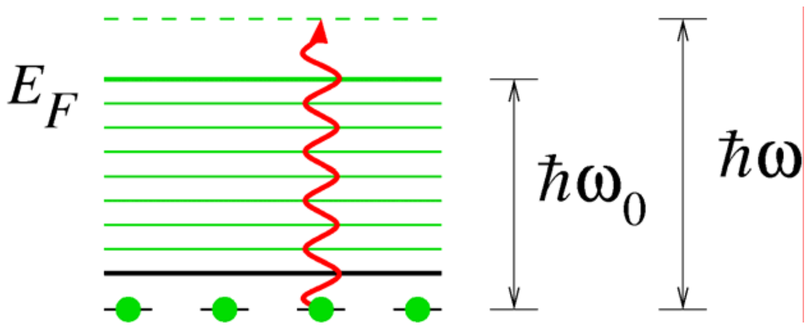
Let us perform Fourier transform on Z-operator:

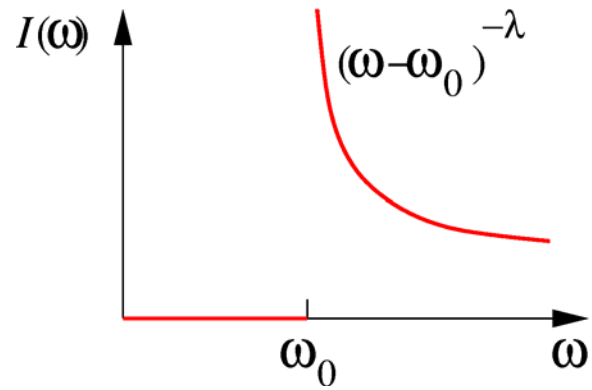
$$Z_{l,\uparrow} = \int_{-\pi}^{\pi} \frac{dq}{2\pi} z_{\uparrow}(q)\, e^{iql}$$

$$\psi_{\uparrow}(x) = \int_{-\pi}^{\pi} \frac{dq}{2\pi}\, z_{\uparrow}(q) \frac{e^{ik_F(1+\frac{q}{\pi})x}}{\sqrt{2\pi\alpha}}\, e^{i(1+\frac{q}{\pi})\phi(x) - i\theta(x)}$$

1. At $q = 0$ the green factor is the destruction operator for a spinless fermion (holon).
2. At $q \neq 0$ it represents an operator that removes a fermion and adds a phase shift $\delta =$ -q to the boundary conditions. C.f. x-ray absorption edge problem [Shotte & Shotte, 1969]

X-ray absorption edge problem





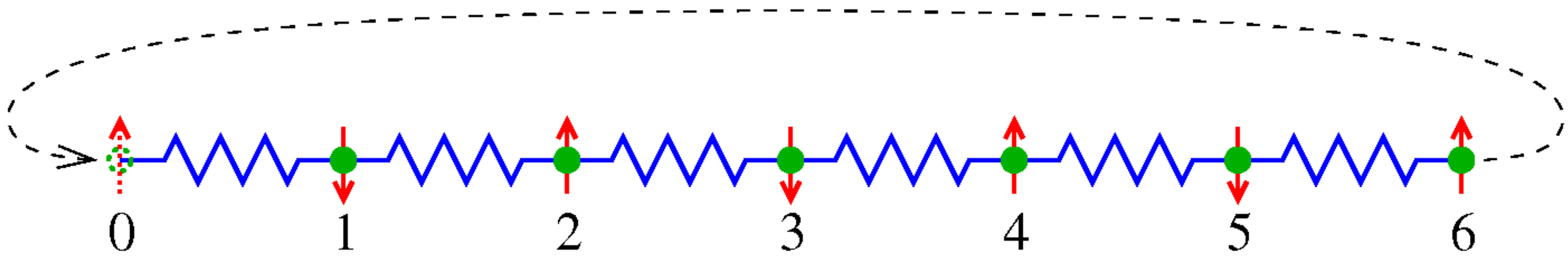
When X-rays excite electrons above the Fermi level, the remaining core hole scatters all the electrons in the Fermi sea. The resulting absorption intensity shows a power-law singularity.

Bosonization solution [Shotte & Shotte, 1969]

$$\psi \propto e^{i[\phi-\theta]} \quad \Longrightarrow \quad \psi \propto e^{i[(1-\delta/\pi)\phi-\theta]}$$

δ is the scattering phase shift of the core-hole potential

Periodic boundary conditions



Move the whole system to the right by system size L

Total momentum is quantized $PL = 2\pi m$

$$PL = \sum_{i=1}^{N} p_i L + qN = \sum_{i=1}^{N} (p_i L + q) = \sum_{i=1}^{N} 2\pi m_i$$

As a holon goes around the circle, it acquires a phase factor

$$e^{ip_i L} = e^{-iq}$$

[Penc, Mila & Shiba, 1995]

Tunneling density of states

Fermion operators $\Rightarrow$ Green's functions $\Rightarrow$ Density of states

$$\nu_\sigma^{\pm}(\varepsilon) = \nu_0 \int_{-\pi}^{\pi} \frac{dq}{2\pi} \frac{c_\sigma^{\pm}(q)}{\Gamma(\lambda(q)+1)} \left(\frac{|\varepsilon|}{E_F}\right)^{\lambda(q)} \qquad J, T \ll |\varepsilon| \ll E_F$$

At $q \neq 0$ we find a power-law singularity with the exponent

$$\lambda(q) = \frac{1}{2}\left[\left(1+\frac{q}{\pi}\right)^2 K + \frac{1}{K}\right] - 1$$

Short-range interactions (Hubbard): non-interacting holons, $K=1$

Properties of the spin chain enter through the equal-time correlators

$$\begin{aligned} c_\sigma^{+}(q) &= \sum_l \langle Z_{l,\sigma} Z_{0,\sigma}^{\dagger}\rangle\, e^{-iql}, \\ c_\sigma^{-}(q) &= \sum_l \langle Z_{0,\sigma}^{\dagger} Z_{l,\sigma}\rangle\, e^{-iql} \end{aligned}$$

Density of states for the Hubbard model at $U \gg t$

In the limit $|\varepsilon|/E_F \to 0$ the true asymptotic behavior of the density of states is

$$J, T \ll |\varepsilon| \ll E_F$$

$$\nu_\sigma^{\pm}(\varepsilon) = \frac{\nu_0}{2\sqrt{2}}\, c_\sigma^{\pm}(\pi) \left(\frac{E_F}{|\varepsilon|}\right)^{1/2} \frac{1}{\sqrt{\ln(E_F/|\varepsilon|)}}$$

Compare with the earlier results

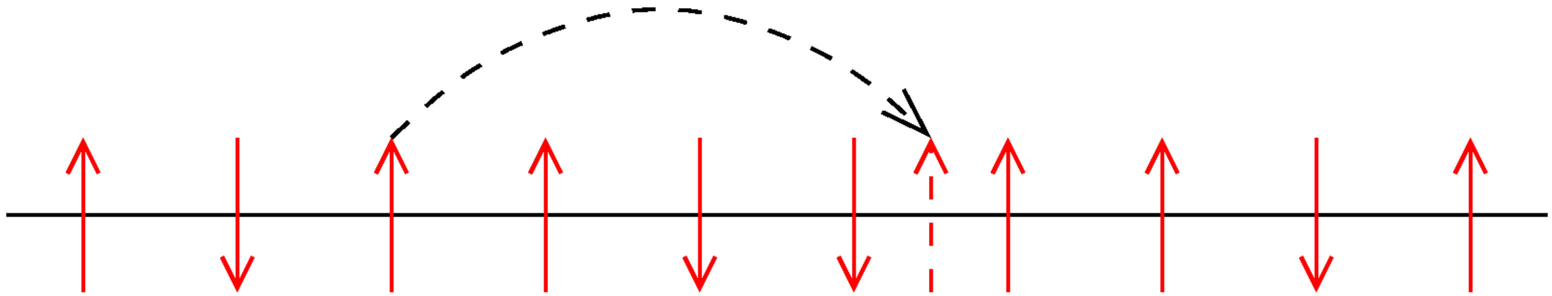
$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{1/2}} \frac{1}{\sqrt{\ln(E_F/|\varepsilon|)}}, \qquad J \ll T \ll |\varepsilon| \ll E_F.$$

[Cheianov & Zvonarev, 2004; Fiete & Balents, 2005]

$$\nu(\varepsilon) \propto \frac{1}{|\varepsilon|^{3/8}}, \qquad T = 0, \;\; J \ll |\varepsilon| \ll E_F.$$

[Penc, Mila & Shiba, 1995]

High temperature



All l spins between the initial and final positions must be aligned

$$\langle Z_{l,\uparrow} Z^\dagger_{0,\uparrow} \rangle = \frac{1}{2^{|l|}}, \quad \langle Z^\dagger_{0,\uparrow} Z_{l,\uparrow} \rangle = \frac{1}{2^{|l|+1}}$$

$$c^+_\uparrow(\pi) = \frac{1}{3}, \quad c^-_\uparrow(\pi) = \frac{1}{6}$$

Factor of 2: one can always add spin ↑, but the probability of removing one is ½.

Density of states:

$$(1) \quad \nu(\varepsilon) \propto 1/\sqrt{|\varepsilon|}$$

$$(2) \quad \nu(+\varepsilon) = 2\nu(-\varepsilon)$$

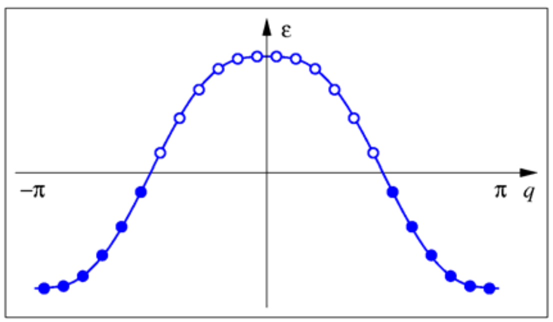
Zero temperature

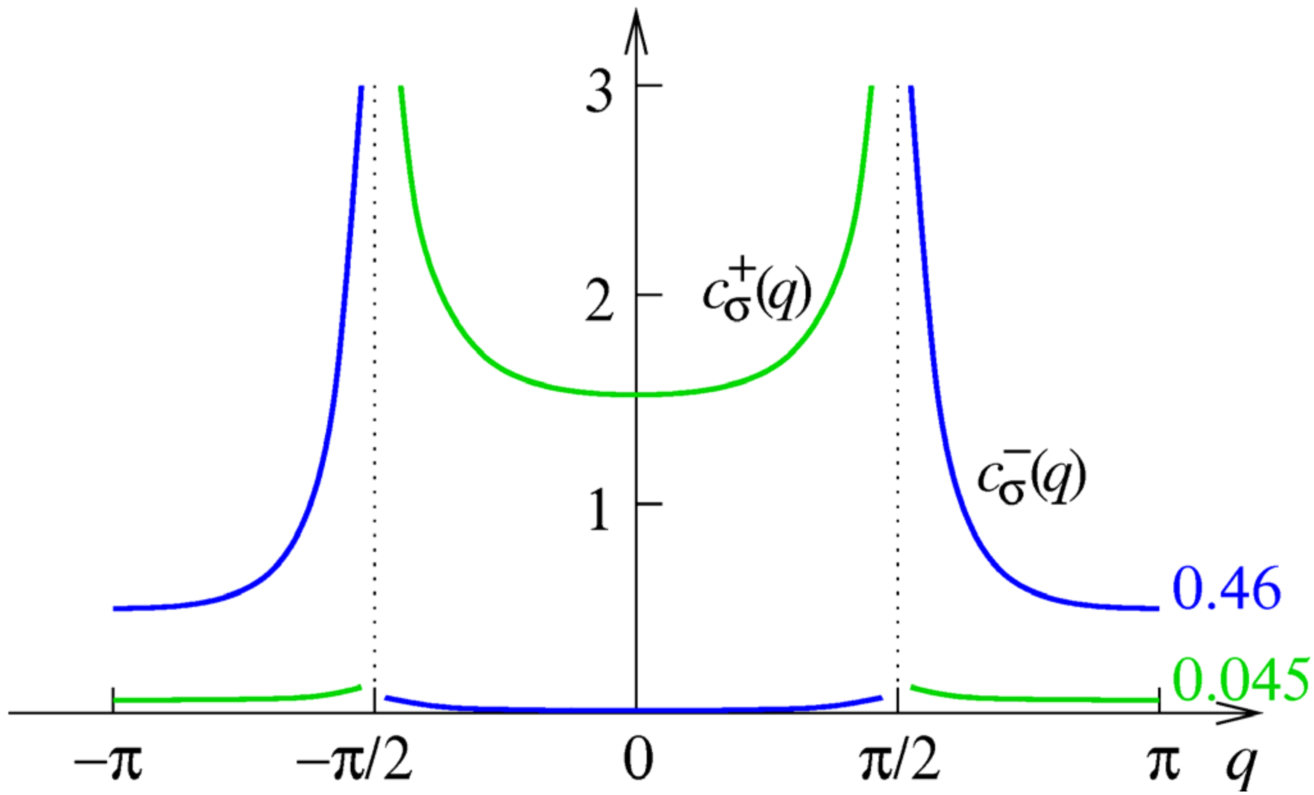
Numerical results for

$c_\sigma^+(q)$ and $c_\sigma^-(q)$

[Penc, Mila & Shiba, 1995]

$c_\sigma^+(\pi) \approx 0.1 c_\sigma^-(\pi)$





Density of states:

$$(1) \quad \nu(\varepsilon) \propto 1/\sqrt{|\varepsilon|}$$

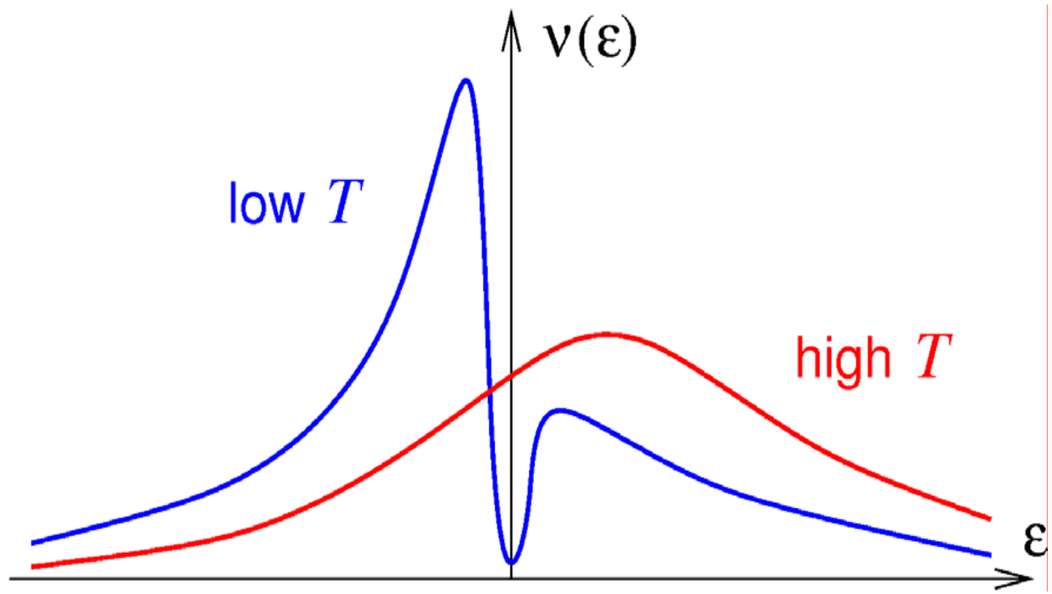
$$(2) \quad \nu(+\varepsilon) \approx 0.1 \nu(-\varepsilon)$$

Subleading contribution at $\epsilon > 0$

$$\tilde{\nu}^+(\varepsilon) \propto \frac{1}{\varepsilon^{3/8}}$$

Summary

- The tunneling density of states is asymmetric around the Fermi level;
- The asymmetry changes sign when temperature is tuned between $T \gg J$ and $T \ll J$ regimes;



- For short-range interactions, the nature of the peak in the density of states is similar that in the x-ray absorption edge problem.