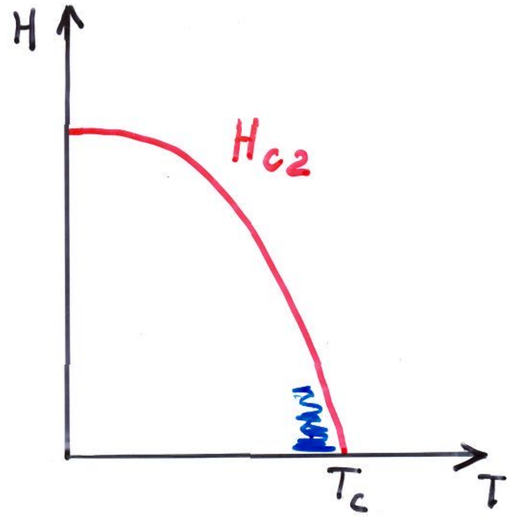
Electrodynamics of Larkin-Ovchinnikov-Fulde-Ferrell superconducting state

V. Mineev[1,2] & M. Houzet[1]

(1) Commissariat à l'Energie Atomique
Grenoble, France

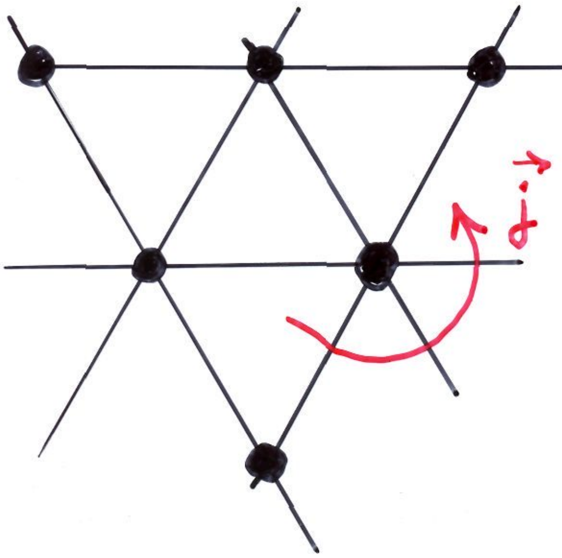
(2) Landau Institute for Theoretical Physics
Chernogolovka, Russia

$$\mathcal{F}_s = \mathcal{F}_n + \alpha|\Delta|^2 + \frac{\beta}{2}|\Delta|^4 + \gamma\left|\left(-i\hbar\vec{\nabla} + \frac{2e}{c}\vec{A}\right)\Delta\right|^2 + \frac{h^2}{8\pi}$$



$$\Delta_A = \sum_n C_n \exp(inqy)\, e^{-\frac{(x-x_n)^2}{2\xi^2}}$$

Abrikosov lattice

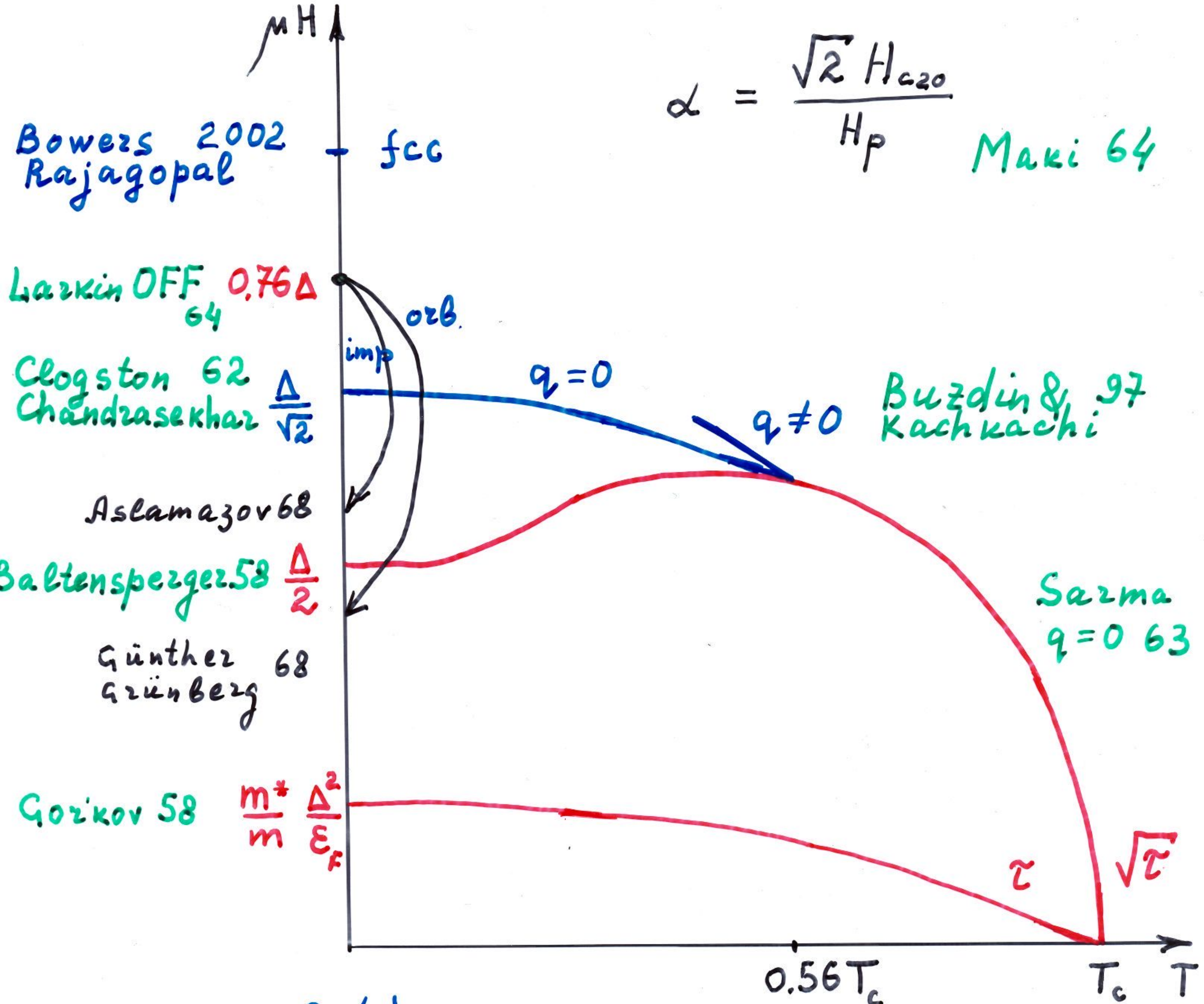


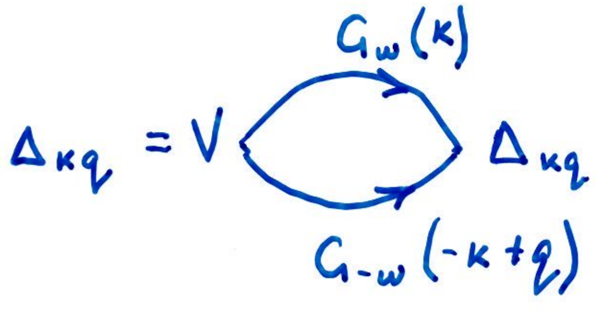
$$\varkappa = \frac{mc}{2e\hbar}\left(\frac{\beta}{2\pi}\right)^{1/2}$$

$$\vec{B} = \vec{H} - \frac{\vec{H}_{c2} - \vec{H}}{\beta_A\left(2\varkappa^2 - 1\right)}$$

$$\overline{\mathcal{F}} = \frac{B^2}{8\pi} - \frac{\left(B - H_{c2}\right)^2}{8\pi\left[\beta_A\left(2\varkappa^2 - 1\right) + 1\right]}$$

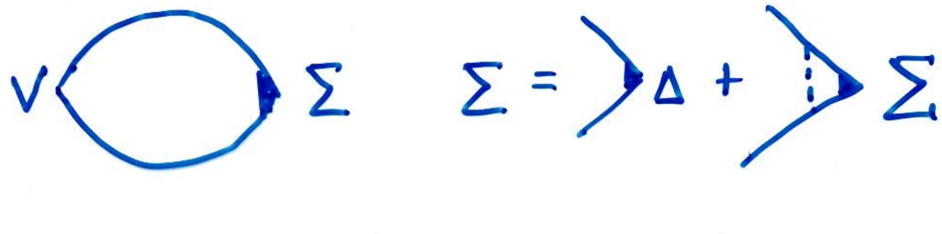
$$\beta_A = \frac{\overline{\Delta_A^4}}{\overline{\Delta_A^2}^{\,2}} = \begin{cases} 1.18 & \square \\ 1.16 & \triangle \end{cases}$$





$\vec{q} = 2e\vec{A}$ Gor'kov 58

$\omega \Rightarrow \omega + i\mu H$ Baltensperger 58



Aslamazov 68

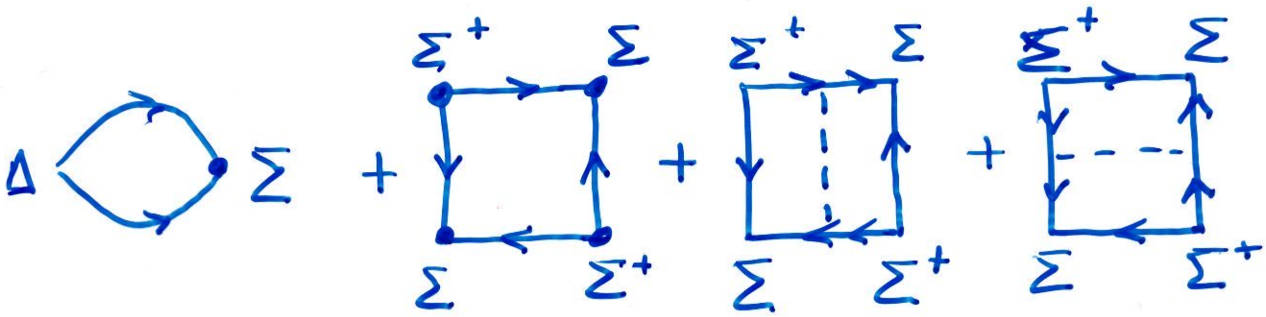
$$\mathcal{F}_s = \mathcal{F}_n + \frac{h^2}{8\pi} + \alpha|\Delta|^2 + \beta|\Delta|^4 + \gamma|\vec{\mathcal{D}}\Delta|^2 +$$

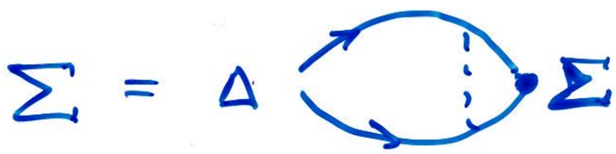
$$+ \delta\left[|\vec{\mathcal{D}}^2\Delta|^2 + (2eh)^2|\Delta|^2 - \frac{2e}{3}(\Delta^*\vec{\mathcal{D}}\Delta + c.c.)\,\mathrm{rot}\,\vec{h}\right]$$

$$+ \zeta|\Delta|^2|\vec{\mathcal{D}}\Delta|^2 - \eta\left[(\Delta^*)^2(\vec{\mathcal{D}}\Delta)^2 + c.c.\right] +$$

$$+ \varepsilon(h_z - B)|\Delta|^2$$

$\vec{\mathcal{D}} = -i\vec{\nabla} + 2e\vec{A}$





$$\overline{\mathcal{F}_s} = \mathcal{F}_n + \alpha_0(H - H_q)\,\overline{|\Delta|^2} + B_q\,\overline{|\Delta|^4}$$

$$H_q = H_0 + a q^2 + b q^4$$

$a < 0$

$q_0 > 0$

$B_{q_0} < 0 \quad - \quad \mathrm{I}$

$$\xi_0^2 \mathcal{D}_\perp^2 \sim \frac{\xi_0^2}{\lambda^2} = \frac{\xi_0^2\, 2eH}{\hbar c} \approx \frac{H}{H_{c2}} \approx \frac{1}{\alpha_M} \ll 1$$

$$\varkappa = \frac{\alpha_M}{\sqrt{k_F r_e \frac{m^*}{m}}} \qquad r_e = \frac{e^2}{mc^2} \sim 10^{-13}\,\mathrm{cm}$$

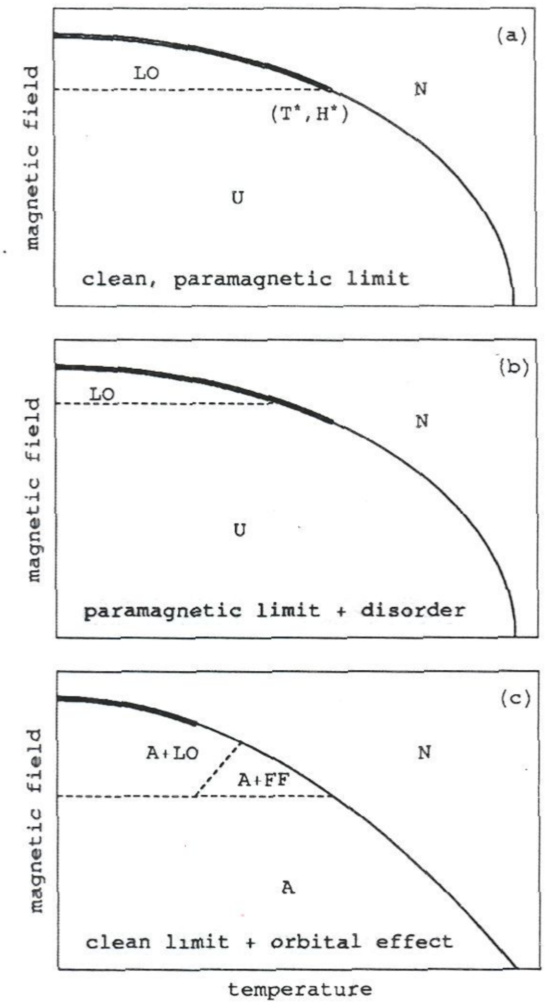


FIG. 1. Qualitative superconducting phase diagram for a three-dimensional *s*-wave superconductor in the presence of a strong paramagnetic effect. The clean, purely paramagnetic case is shown in (a), the purely paramagnetic case with disorder is shown in (b), and the clean case, with some orbital effect, is shown in (c). Possible phases are the normal state (N), the conventional superconducting state, uniform state (U) in the absence or orbital effect, Abrikosov vortex lattice state (A) in the presence of the orbital effect, and the FFLO modulated state with exponential modulation (FF) or sinusoidal modulation (LO) along the applied field, eventually with the Abrikosov vortex lattice (A+FF or A+LO) if the orbital effect is present. Thin lines correspond to second-order transitions, thick lines correspond to first-order transitions, and dashed lines correspond to transitions between different superconducting states, and they have not been calculated in the present work.

INTERPLAY OF PARAMAGNETIC, ORBITAL, AND...

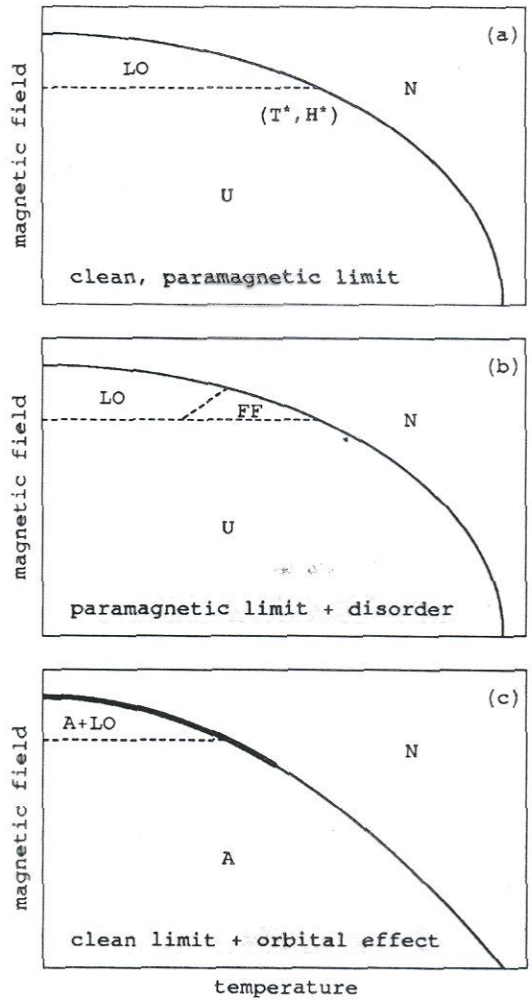
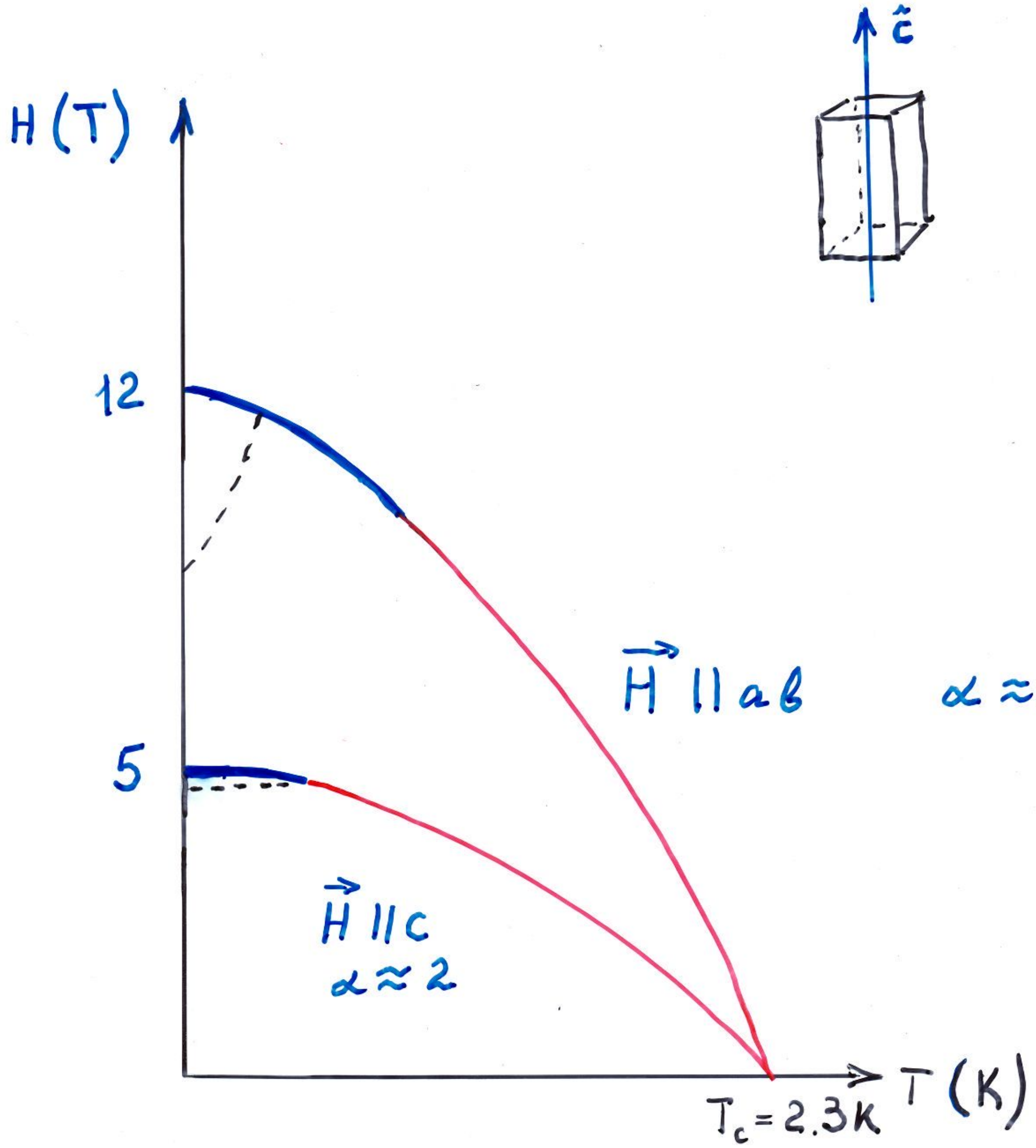


FIG. 3. Qualitative superconducting phase diagram for the quasi-two-dimensional *d*-wave superconductor in the presence of a strong paramagnetic effect (see also the legend of Fig. 1).

Houzet & Mineev 2006

$$\Delta(x,y,z) = \Delta_A(x,y)\, f(z)$$

$$f(z) = \begin{cases} 1 & \text{Abrikosov} \\ e^{iqz} & \text{Fulde-Ferrell} \\ \sqrt{2}\sin qz & \text{Larkin-Ovchinnikov} \end{cases}$$



$CeCoIn_5$

Clean limit

V. Mitrović et al 2006

R. Movshovich ....

Y. Matsuda ....

K. Izawa ....

F. Steglich ....

$$\beta_A = \frac{\overline{\Delta_A^4}}{\overline{\Delta_A^2}^{\,2}}$$

$$\underline{\min \beta_A} = \beta_{A\Delta} = 1.16$$

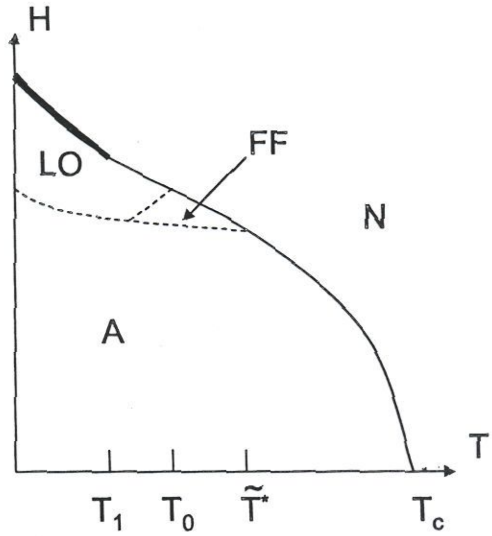


FIG. 1: Schematic phase diagram of a clean three-dimensional superconductor with large Maki parameter. Thin (thick) line is for second (first) order transition. Transitions shown with dashed lines are not discussed in the present work.

$$\overline{\mathcal{F}} = \frac{B^2}{8\pi} - \frac{(B - H_{c2}(T))^2}{8\pi\left[\beta_A\left(2\varkappa_{eff}^2 - C\right) + 1\right]}$$

$$\varkappa_{eff} = \varkappa_{eff}(T, H)$$

$$\varkappa_{eff}^2(\tilde{T}^*, \tilde{H}^*) \approx \frac{\varkappa^2}{\alpha_M^3} \sim \frac{1}{K_F l_e}\frac{m}{m^*}\frac{1}{\alpha_M} \gtrsim \frac{C}{2}$$

$$C = \begin{cases} 1 & A \\ 1 & FF \\ \frac{3}{2} + \frac{2q^2\lambda^2}{1 + 4q^2\lambda^2} & LO \end{cases}$$

$$\lambda^2 = \frac{\hbar c}{2eH}$$

$$B = H - \frac{H_{c2} - H}{\beta_A\left(2\varkappa_{eff}^2 - C\right)}$$

Currents

$$\vec{j}_s = \vec{j}_{kin} + \vec{j}_z$$

$$\vec{j}_{kin} = -2e\gamma\left[\Delta\left(\vec{\mathcal{D}}\Delta\right)^* + c.c.\right] - \quad = -2e\gamma \,\mathrm{rot}|\Delta|^2\hat{z}$$

$$- 4e\delta\left[\vec{\mathcal{D}}^2\Delta\left(\vec{\mathcal{D}}\Delta\right)^* + c.c.\right] +$$

$$+ \frac{2e\delta}{3}\,\mathrm{rot}\,\mathrm{rot}\left[\Delta\left(\vec{\mathcal{D}}\Delta\right)^* + c.c.\right] +$$

$$+ 8e^2\delta\vec{B}\times\vec{\nabla}|\Delta|^2$$

$$\vec{j}_z = -\frac{\delta\,\varepsilon\,(h_z - B)|\Delta|^2}{\delta\vec{A}} = -\varepsilon\,\mathrm{rot}|\Delta|^2\hat{z}$$

diamagnetic

$$\left|\frac{\vec{j}_{kin}}{\vec{j}_z}\right|_{\tilde{T}^*} \approx \frac{1}{\alpha_M^2} \ll 1$$

$$\gamma = \frac{\pi N_0 v_F^2}{12}\cdot 2T\,\mathrm{Re}\sum_{n\geq 0}\frac{1}{(\omega + i\mu B)^3}$$

$$\varepsilon = -\pi N_0\mu\, 2T\,\mathrm{Im}\sum\frac{1}{(\omega + i\mu B)^2}$$

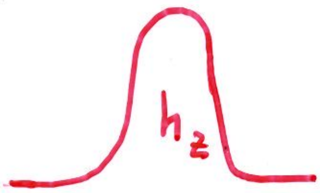
Fields

$$\vec{h}(\vec{r}) = \vec{B} + \vec{h}_1(\vec{r})$$

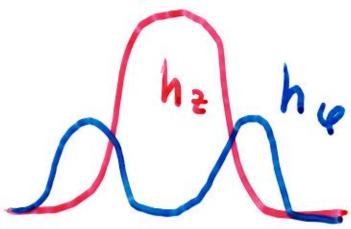
$$\overline{\vec{h}_1(\vec{r})} = 0$$

Maxwell $\begin{cases} \mathrm{rot}\, \vec{h}_1 = -4\pi\varepsilon\, \mathrm{rot}\, |\Delta|^2 \hat{z} - \begin{cases} 0 & A \\ 0 & LO \\ \frac{16\pi e \delta q}{3} \vec{\nabla}^2 |\Delta|^2 \hat{z} & FF \end{cases} \\ \mathrm{div}\, \vec{h}_1 = 0 \end{cases}$

A $\quad \vec{h}_1 = -4\pi\varepsilon \left(|\Delta|^2 - \overline{|\Delta|^2} \right) \hat{z}$

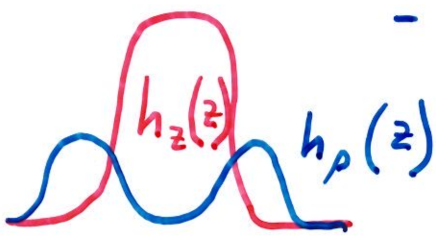


FF $\quad \vec{h}_1 = -4\pi\varepsilon \left(|\Delta|^2 - \overline{|\Delta|^2} \right) \hat{z} - \frac{16\pi e \delta q}{3} \hat{z} \times \vec{\nabla} |\Delta|^2$

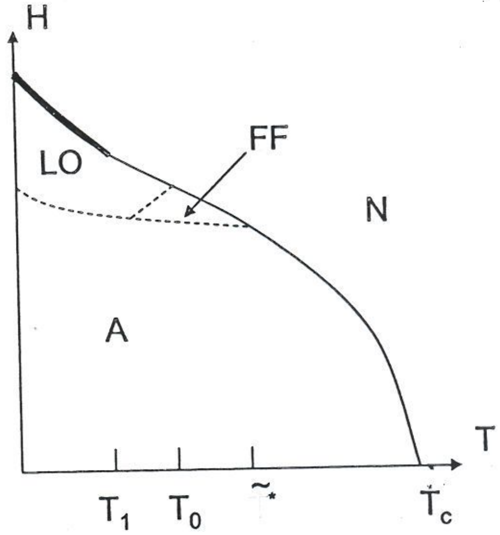


LO $\quad \vec{h}_1 = -4\pi\varepsilon \left(|\Delta|^2 - \overline{|\Delta|^2} \right) z - 8\pi q \vec{\nabla} \chi(x,y) \sin 2qz$

$$-\nabla_\perp^2 \chi + 2q^2 \chi = \varepsilon |\Delta_A|^2$$



Resumé



Houzet & Mineev
cond-mat/0703104

1. Vortex lattice in FF and LO states in an isotropic s-wave superconductor is always triangular.

2. The effect of appearance of space oscillating transverse field component has model independent character and will also be present in anisotropic materials with different type of superconducting pairing.

3. The effect can be revealed by measurements of spin flips in polarized neutron scattering and by relaxation rate in μSR with muon spins polarized along the external field.