Tolya Larkin's contribution to the Phase Transitions theory

Larkin Conference,
Chernogolovka, June 24-28, 2007

Scale

Spontaneous mass generation (Higgs mechanism)

Phase Transitions

Paraconductivity

Coexistence of superconductivity and ferromagnetism (LOFF)

Disordered systems

Mesoscopics

Vortex matter

Deterministic chaos in conductors and superconductors

Collective dynamics of superconductors (Lagrangian formalism)

Gauge fields in strongly correlated electron systems

Great worker

25 articles with more than 100 citations

15 articles with more than 200 citations

12 articles with more than 300 citations

8 articles with more than 400 citations

6 articles with more than 500 citations

How Tolya worked

In vegetable garden

At home and in the office

Great teacher

Direct students

Yu.N. Ovchinnikov
S.A. Pikin
L.G. Aslamazov
D.E. Khmelnitskii
K.B. Efetov
V.M. Filyov
P.B. Wiegmann
K.A. Matveev
L.B. Ioffe
V.B. Geshkenbein
M.G. Vavilov
V.M. Galitsky

Young coworkers

V.I. Mel'nikov
A.M. Finkelstein
M.V. Feigelman
A.A. Varlamov
V.M. Vinokur
M.A. Skvortsov
I.S. Beloborodov
I.L. Aleiner

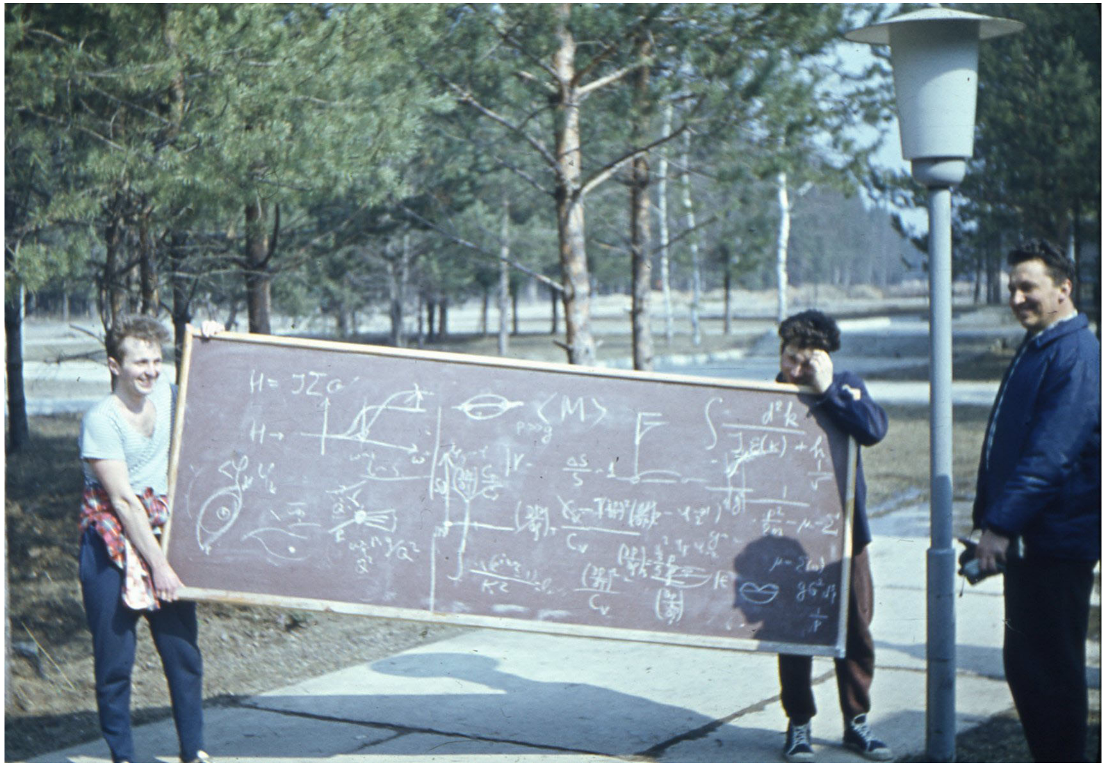
Coworkers

V.G. Vaks
I.E. Dzyaloshinski
L.P. Gor'kov
J. Suk
B.L. Altshuler
A.G. Aronov
P.A. Lee
S. Hikami
Y. Nagaoka
E. Abrahams
L.I. Glazman
A. Kamenev
G. Blatter

E.Zel'dov
M. Konchykovsky
D. Majer
A.J. Millis
S.M. Girvin
C. Varma
E. Abrahams
O.S. Wagner
F.W.D. Hekking
A. Schmid
A. Barone
R.A. Klemm

Participation in discussions

VP, 1966	You are probably working on dynamic scaling
V.L. Berezinskii, 1970	You should work accurately with localized excitations
A.M. Polyakov 1975-1980	What are topological excitations?



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Works on Phase Transitions and Critical Phenomena

В.Г. Вакс и А.И. Ларкин. О фазовом переходе второго рода ЖЭТФ **49**, 975 (1965); On phase transition of the second order, Sov. Phys. JETP **22**, 678 (1966)

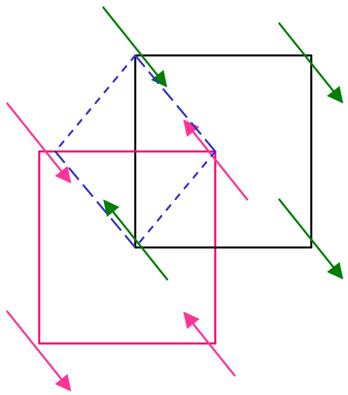
Universality hypothesis: the critical behavior depends only on the initial (broken) symmetry and the way of its violation
Microsopic details are irrelevant

Further development: L.P. Kadanoff and F.J. Wegner, Phys. Rev. B **4,** 3989 (1971)

Relevant perturbations with scaling dimensionality larger than dimensionality of space shift the transition point, but do not change critical exponents.

Perturbation of the marginal dimensionality Δ=d can lead to continuous dependence of other critical exponents.

Example: Ashkin-Teller or Baxter 8-vertex model



$$H = H_1 + H_2 + V$$

$$H_1 = \sum_{bonds} \varepsilon_b ; \quad \varepsilon_b = \sigma_1 \sigma_2$$

$$V = \sum_{b,\tilde{b}} \varepsilon_b \varepsilon_{\tilde{b}}$$

$$\Delta_\varepsilon = 1$$

$$\Delta_V = 2\Delta_\varepsilon = 2 = d$$

А.И. Ларкин и Д.Е. Хмельницкий. Фазовый переход в одноосном сегнетоэлектрике. ЖЭТФ **56**, 2087 (1969); Sov. Phys. JETP **29**, 1123 (1969)

First renormalization group calculation of phase transition

Precursor of 4-ε renormalization theory (Wilson,, Wilson and Fisher)

and the Ising model. Larkin and Khmelnitskii applied the field theoretic renormalization group of Gell-Mann and Low to critical phenomena in four dimensions and to the special case of uniaxial ferromagnets in three dimensions,[56] in both cases deriving logarithmic corrections to Landau's theory. Dyson formu-

Excerpt from Wilson's Nobel presentation

Why the dipolar interaction in 3d leads to effective 4d theory

Ising symmetry, transition point:

$$G^{-1} = Jq^2 + g\cos^2\theta = Jq^2 + gu^2$$

Exchange interaction

Demagnetizing factor

$$d^3x = 2\pi x^2 dx d\cos\theta = 2\pi x^2 dx du$$

Specific heat: $C \sim |\ln t|^{1/3}; t = \dfrac{T - T_c}{T_c}$

Further developments

A. Aharoni, 1973 – *derivation by Wilson renormgroup in 4-ε*

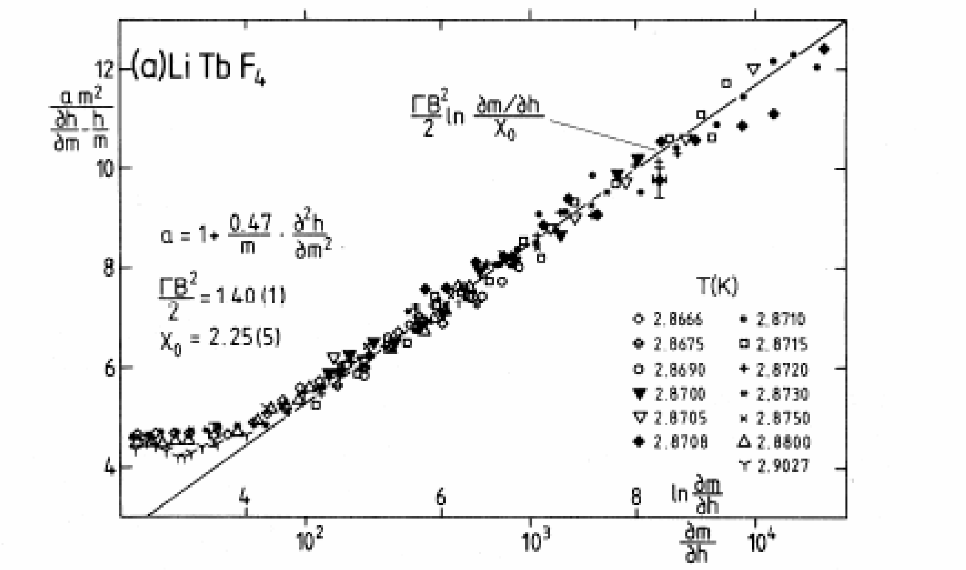
B. Cowley, 1976, R. Folk, H. Iro and F. Schwabl – *piezoelectricity decreases the marginal dimensionality and suppresses logarithmic corrections*

A. Aharoni and P.C. Hohenberg: universal relations 1976

$$\xi^2 \xi_{||} C_p t^2 / k_B = \frac{3}{32\pi} |\ln(t)|.$$

$$R = t^2 C_p \chi T_C / M^2 = \frac{1}{3} \quad (t<0)$$

Experimental verifications



R. Frowein and J. Kötzler,
Phys. Rev. B **25**, 3292 (1982)

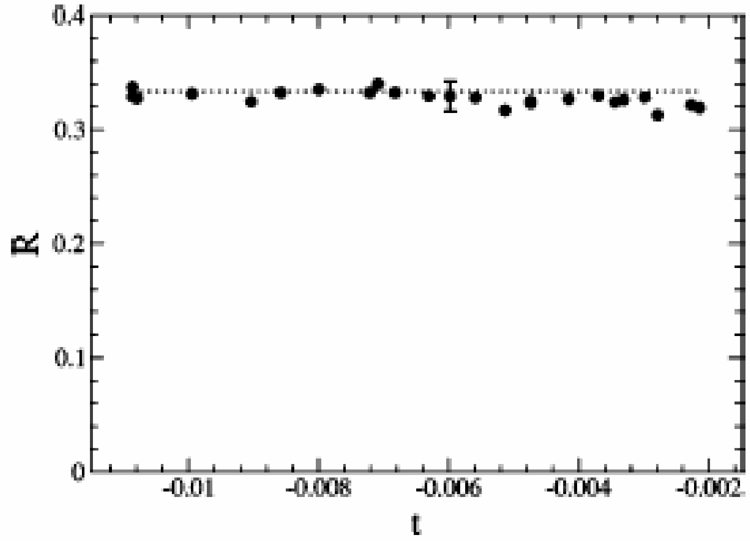


FIG. 5. The quantity $R = t^2 C_P \chi T_C / M^2$. The dashed line is the RGT predicted value of 1/3.

Testing renormalization group theory at the critical dimension in $LiHoF_4$

James Nikkel and Brett Ellman
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Unexpected applications

J. Phys. A: Math. Gen. **15** (1982) L317–L320. Printed in Great Britain

LETTER TO THE EDITOR

New method of analysing self-avoiding walks in four dimensions

S Havlin and D Ben-Avraham

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The solution for the problem of self-avoiding walks (SAW) on a four-dimensional lattice is known through the analogy with the $n=0$ limit of the ferromagnet vector model (Larkin and Khmelnitskii 1969, Brézin *et al* 1976, de Gennes 1979). Consider for example the theoretical prediction for the correlation length ξ of this model

$$\xi \sim t^{-1/2}|\ln t|^{1/8} \qquad (d=4) \tag{1}$$

where t is the reduced temperature $t=(T-T_c)/T_c$. In the SAW problem, t is analogous to $1/N_0$ (N_0 being the total number of steps) and ξ is analogous to the end-to-end distance $\langle R^2_{N_0}\rangle^{1/2}$ (de Gennes 1979), so that

$$\langle R^2_{N_0}\rangle^{1/2} \sim N_0^{1/2}(\ln N_0)^{1/8} \qquad (d=4). \tag{2}$$

А.И. Ларкин, С.И. Пикин. Фазовые переходы первого рода, близкие ко второму. ЖЭТФ **56**, 1664 (1969);
Sov. Phys. JETP **29**, 891 (1969)

Main idea: quadratic magnetostriction turns the magnetic Curie point into a weakly first order phase transition.

$$H_{\mathrm{int}} = q\int \varphi^2(\mathbf{x})\, u_{\alpha\alpha}(\mathbf{x})\, d^3x$$

Unexpected sophistication: only the shear deformation are responsible for this effect

$$H_{ind} = -\frac{2q^2\mu}{VK(3K+4\mu)}\left[\int \varphi^2(\mathbf{x})\, d^3x\right]^2$$

Further developments

Anisotropic elasticity: D.E. Khmelnitskii and V.L. Shneerson, 1975
D.J. Bergmann and B.I. Halperin, 1976
I.F. Lyuksyutov, 1977

Multi-charge renormalization group, complicated phase diagram

Linear striction: A.P. Levanyuk, A.A. Sobyanin, 1970

$$H_{\text{int}} = q\int \varphi(\mathbf{x})u_{xy}d^3x$$

Suppression of fluctuations, mean field exponents in a very
Close vicinity of Curie point

Beyond perturbation theory: A. Aharony 1973, Y. Imry, 1974

First order transition at small pressure, second order transition at large pressure; tricritical point between.

Beyond perturbation theory: S.N. Gadekar and T.V. Ramakrishnan, 1980

Summation of ring diagrams. Conclusion: always first order

Unexpected development: B.J. Schulz, B. Dünweg, K. Binder, M. Müller, 2005

Suppression of capillary waves broadening of interface in binary alloys due to elastic interactions

В.Г. Вакс, А.И. Ларкин, С.А. Пикин, Спиновые волны и корреляции в ферромагнетиках. ЖЭТФ 53, 1083 (1967); Spin waves and correlations in ferromagnets. Sov. Phys. JETP **26**, 674 (1968)

Renormalization of long-scale thermodynamic properties due to spin wave fluctuations

$$\chi \sim h^{-1/2}$$

Further developments

A.Z. Patashinskii, V.P., 1973. Generalization to any Goldstone mode, Principle of modulus conservation

S. Coleman, K.G. Wilson, 1973 Non-linear renormalization in field theory

E.I. Kats, V.P., Anomalous longitudinal scattering in nematics, 1975

W. Zwerger, 2004, Anomalous size dependence of fluctuations in systems with broken continuous symmetry

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