High Tc superconductivity in doped Mott insulators

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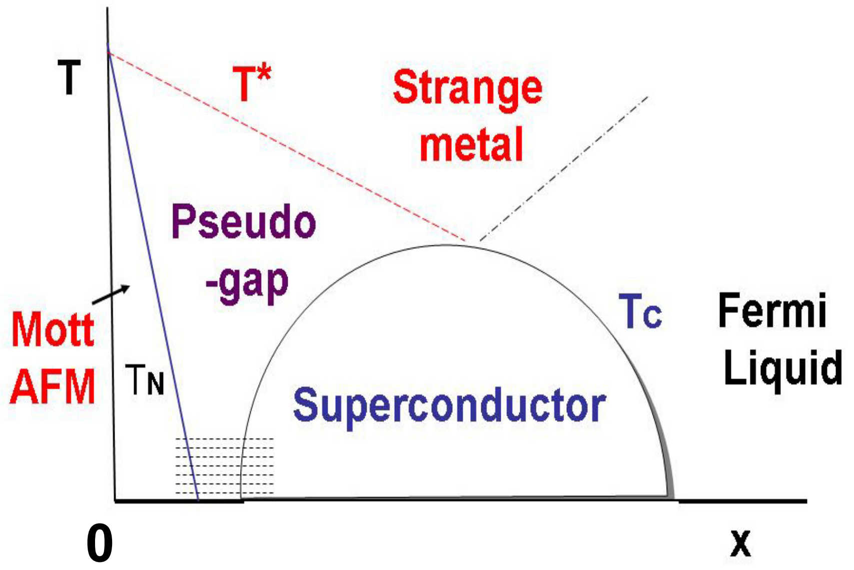
Outline:

- Introduction
- Sum Rules & p-h asymmetry
- Variational Theory of SC State
- Low energy excitations
- Superfluid density
- Disorder Effects

Failure of three central paradigms of Condensed Matter Physics

(1) Band theory fails for $x = 0$ **parent insulator**



(2) Landau's Fermi liquid theory fails for **strange metal** and **pseudogap regimes**

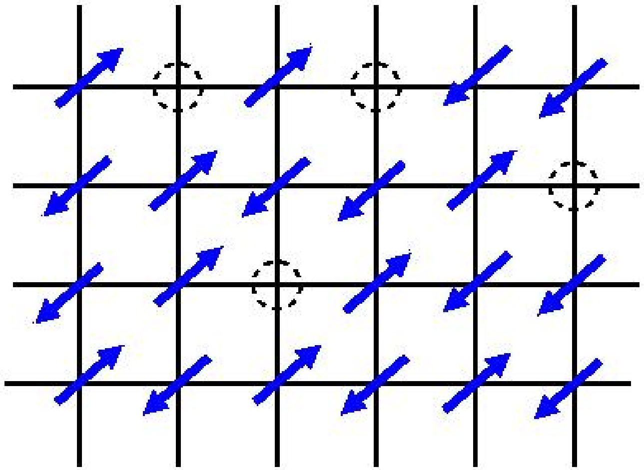
(3) BCS theory fails for **Unconventional SC** particularly for $x \ll 1$

Competing orders:
Antiferromagnetism;
Charge ordering; Spin glass
Circulating currents; DDW (?)

Hidden **Quantum Critical Point** under the dome(?)

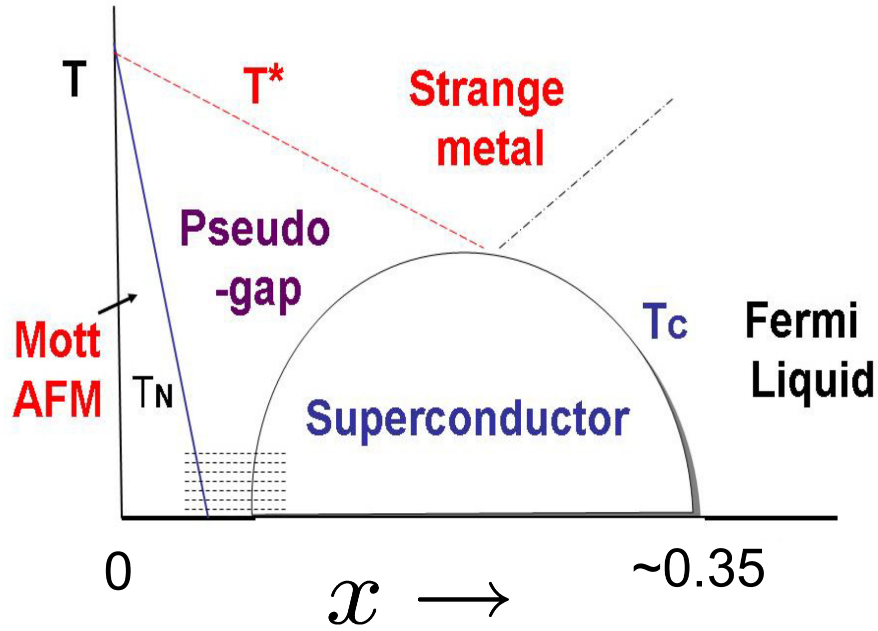
Key question:

holes in a 2-d S=1/2 Mott insulator



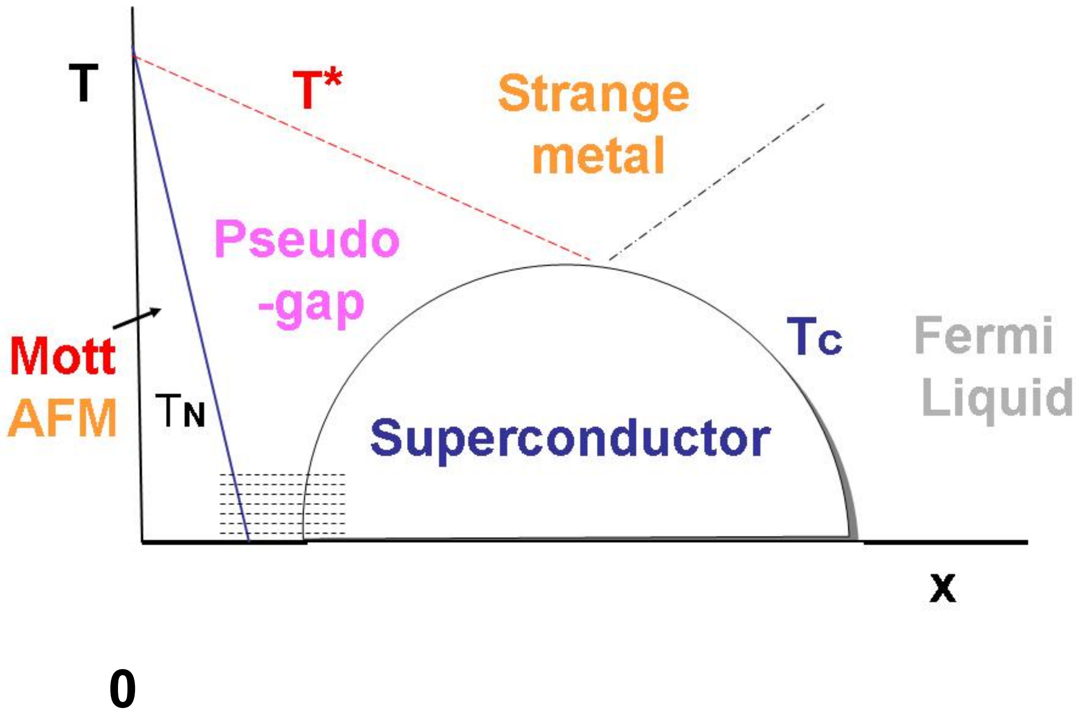
?

$\Rightarrow$



hole concentration

$x \ll 1$



will focus here on:
T=0 SC state and low-lying excitations

In collaboration with:

A. Paramekanti, Toronto

N. Trivedi, Ohio State

A. Paramekanti, MR & N. Trivedi,
PRL 87, 217002 (2001); PRB 69, 144509 (2004);
PRB 70, 054504 (2004); PRB 71, 069505 (2005).

R. SenSarma, Ohio State

R. Sensarma, MR & N. Trivedi,
PRL **98**, 027004 (2007) and unpublished.

F.C. Zhang, Hong Kong

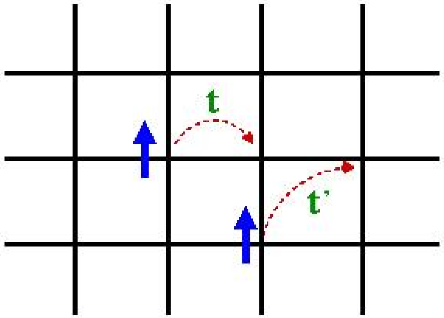
MR, R. Sensarma, N. Trivedi & F.C. Zhang,
PRL **95**, 137001 (2005).

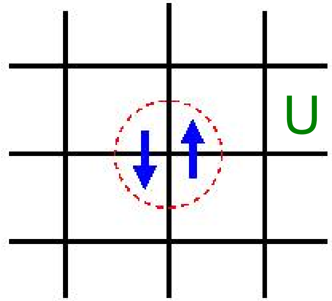
P.W. Anderson, Princeton

P.W. Anderson, P.A. Lee, MR,
T. M. Rice, N. Trivedi & F.C. Zhang,
J. Phys. Cond. Mat. 16, R755 (2004).

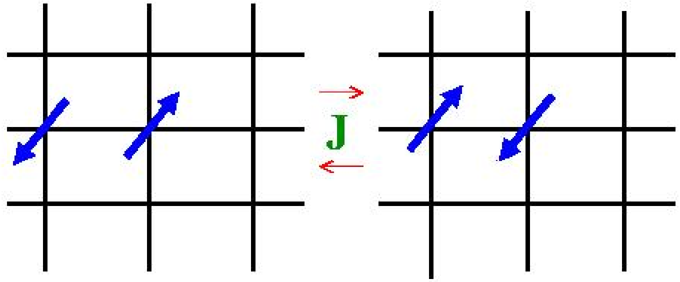
Hubbard model: **Minimal Model for CuO_2 planes**

$$\mathcal{H} = \sum_{\mathbf{k},\sigma} \epsilon(\mathbf{k}) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + U \sum_{\mathbf{r}} n_{\mathbf{r}\uparrow} n_{\mathbf{r}\downarrow}$$





$$\epsilon(\mathbf{k}) = -2t\left(\cos k_x + \cos k_y\right) + 4t' \cos k_x \cos k_y \qquad U \gg t, |t'|$$



$$J = 4t^2/U$$

$$J \leq |t'| \leq t \ll U$$ → 3.6 eV

- neutron
- Raman

~ 100 mev

300 meV

t' ~ - t/4

- photoemission
- band theory

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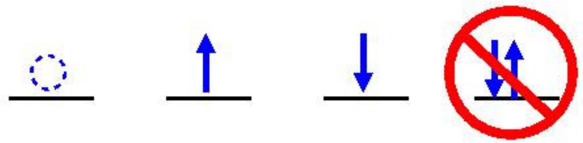
Qs: How does large U affect the low-energy properties of doped materials ?

- MR, SenSarma, Trivedi & Zhang, PRL **95**, 137001 (2005)
- Anderson & Ong, cond-mat/0405518; J. Phys. Chem . Sol. (2006)



$$P = \prod_i \left(1 - n_{i\uparrow} n_{i\downarrow}\right)$$

Particle-hole asymmetry in Spectral function:
Exact sum rules for Projected electrons at T=0

No assumptions about

- ground state
- broken symmetries
- translational invariance

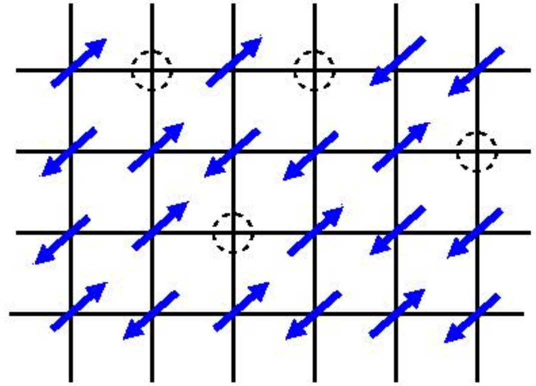
$$\mathrm{Im}G(\mathbf{r}, \mathbf{r}'; \omega)$$

Lehmann Representation
→**Energy-integrated**
Sum Rules

P-H asymmetry: Exact sum rules for local DOS

Local Hole doping

$$\langle n(\mathbf{r})\rangle = 1 - x(\mathbf{r})$$



Extracting electrons:

$$\int_{-\infty}^{0} d\omega N(\mathbf{r};\omega) = 1 - x(\mathbf{r})$$

(filled sites)

Adding electrons:

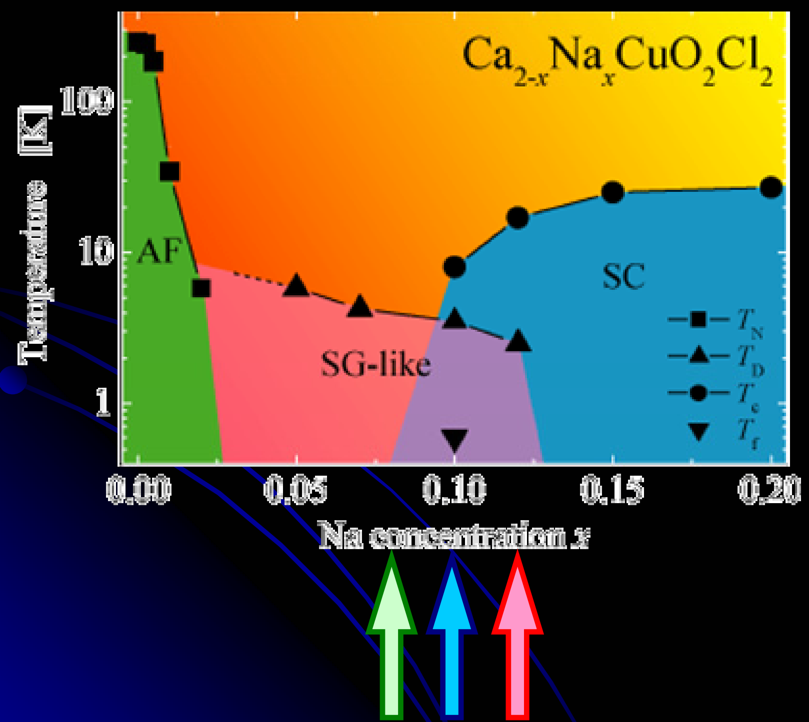
$$\int_{0}^{\Omega_L} d\omega N(\mathbf{r};\omega) = 2x(\mathbf{r}) + 2\left|\langle K(\mathbf{r})\rangle\right|/U$$

$$J < t \ll \Omega_L \ll U$$

(empty sites + ...)

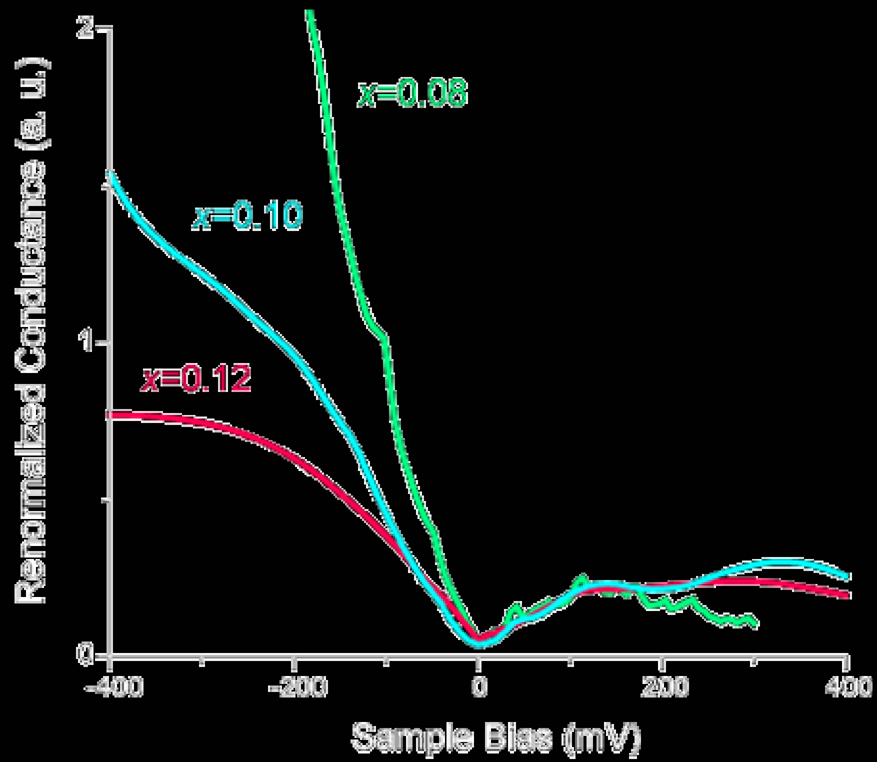
$$\left|\langle K(\mathbf{r})\rangle\right|/U \sim \mathcal{O}\left(xt/U\right)$$

Increasing p-h Asymmetry with Underdoping



K. Ohishi *et al.*, cond-mat/0412313

Averaged dI/dV spectra



T. Hanaguri *et al.*, Nature **430**, 1001 (2004)

Davis group (Cornell)

Using Sum Rules to estimate local density from STM data

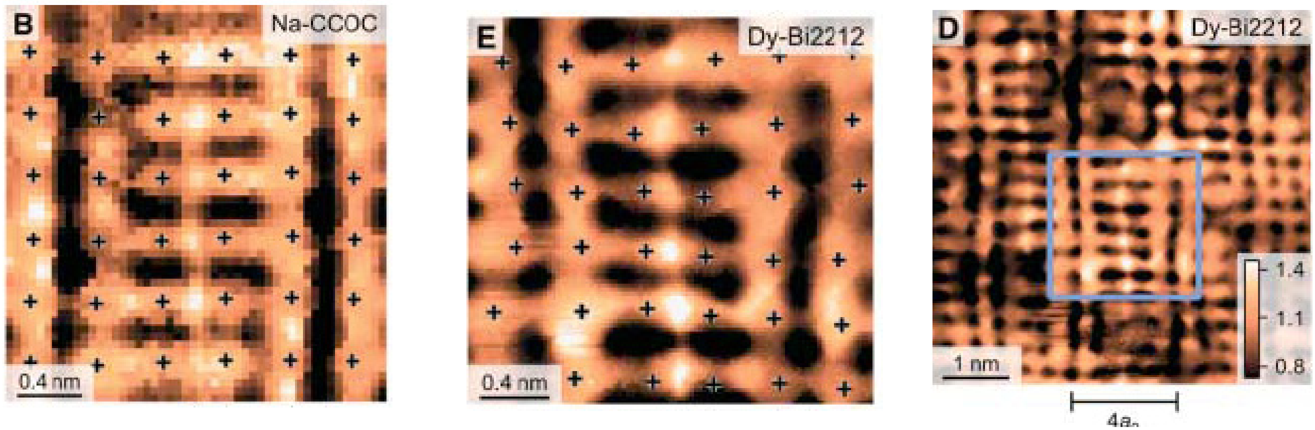
conductance $g(\mathbf{r}; eV) = M(\mathbf{r})N(\mathbf{r}; \omega = eV)$

Unknown matrix element M(r) cancels out in **ratio**

$$R(\mathbf{r}) \equiv \frac{\int_0^{\Omega_L} d\omega g(\mathbf{r};\omega)}{\int_{-\infty}^{0} d\omega g(\mathbf{r};\omega)} = \frac{2x(\mathbf{r})}{[1-x(\mathbf{r})]} + \mathcal{O}\left(\frac{xt}{U}\right)$$

Measured

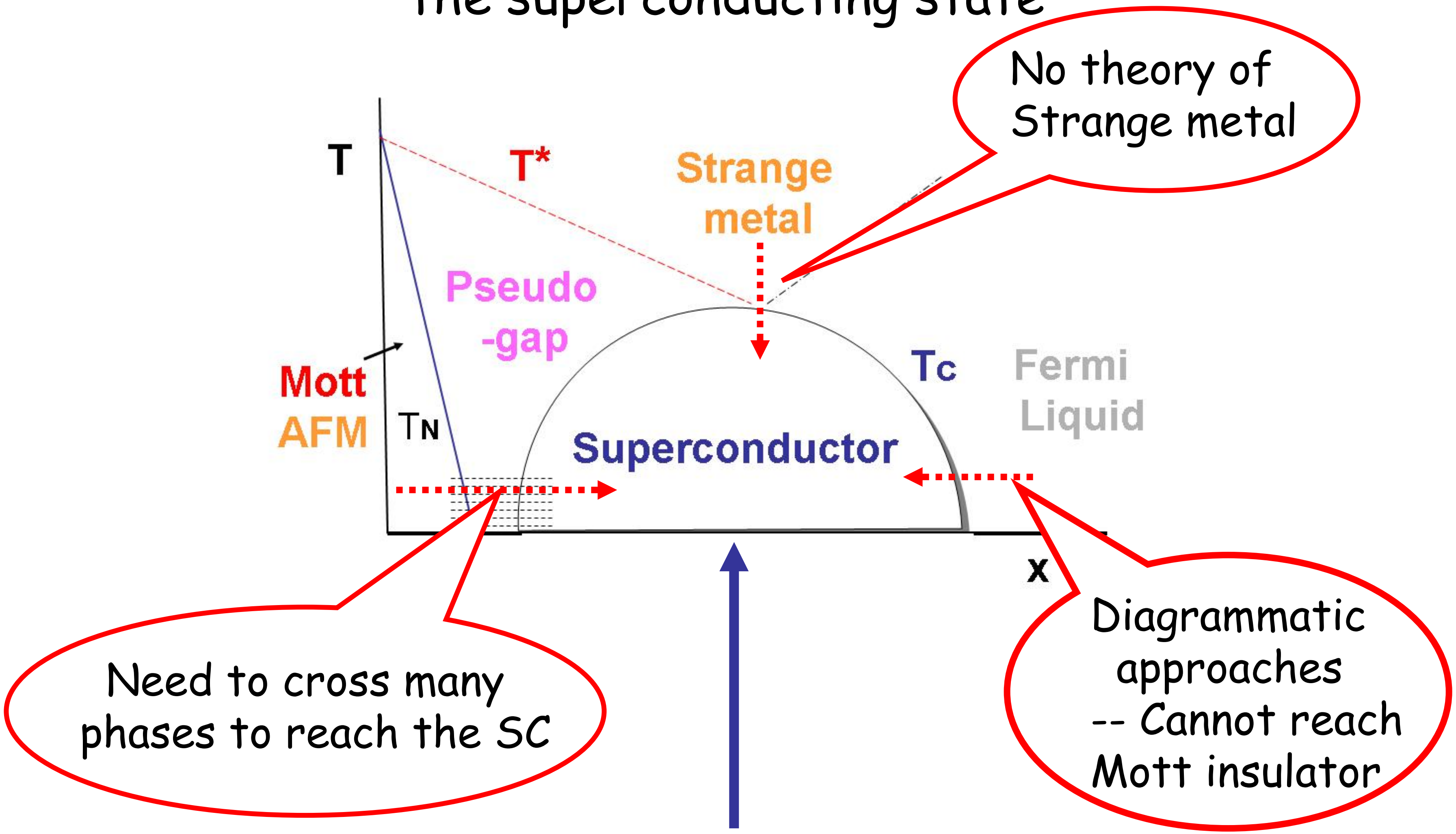
Determine x(**r**)



$R(\mathbf{r})$-maps

Y. Kohsaka et al, Science (2007)

Strategy for theoretical attack on the superconducting state



Use a variational approach to look directly at the T=0 SC state and low-lying excitations

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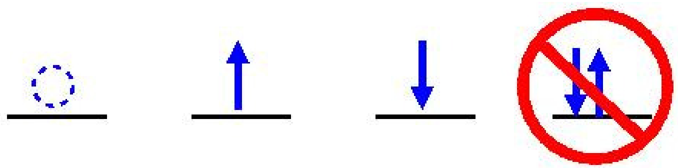
Variational Ground State Wavefunction

$$| \Psi_0 > = \exp(-iS)\, \mathbf{P}\, | dBCS >$$

| dBCS > = d-wave BCS state
variational parameters Δ and μ

$U = \infty$



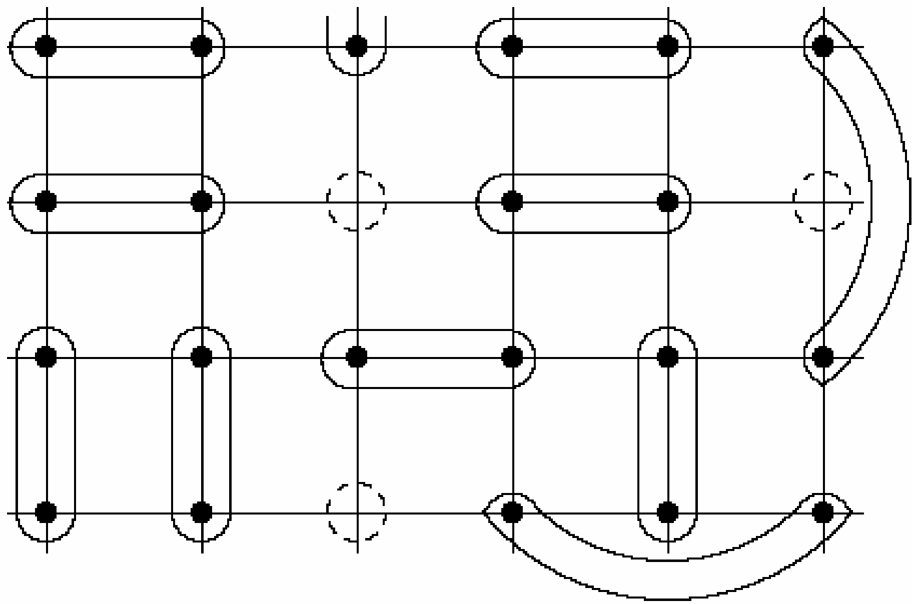
Gutzwiller Projection:

$$P = \prod_i \left(1 - n_{i\uparrow} n_{i\downarrow}\right)$$

exp(-iS) = Unitary transformation:
Hubbard → tJ + …   brings in the scale J

Kohn (`64); Gross, Joynt, Rice *(`87)*

$$|\Psi_0\rangle \equiv \mathcal{P}|BCS\rangle = \mathcal{P}[\sum_{\mathbf{r},\mathbf{r}'} \varphi(\mathbf{r}-\mathbf{r}')c^\dagger_{\mathbf{r}\uparrow}c^\dagger_{\mathbf{r}'\downarrow}]^{N/2}|0\rangle$$



Projected SC

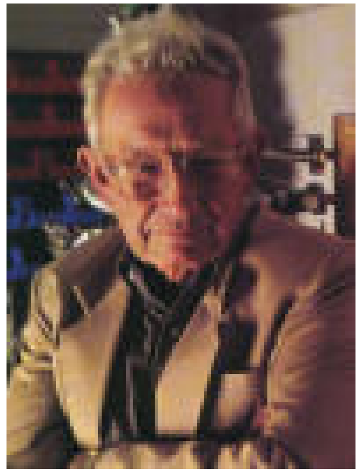
$\equiv$ a Resonating Valence Bond (RVB) liquid

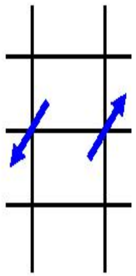
r  r′

$$= \frac{|\uparrow_{\mathrm{r}}\downarrow_{\mathrm{r}'}\rangle - |\downarrow_{\mathrm{r}}\uparrow_{\mathrm{r}'}\rangle}{\sqrt{2}} \quad \varphi(\mathbf{r}-\mathbf{r}')$$

$$\varphi(\mathbf{r}-\mathbf{r}') = \sum_k e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \left(\frac{v_{\mathbf{k}}}{u_{\mathbf{k}}}\right)$$

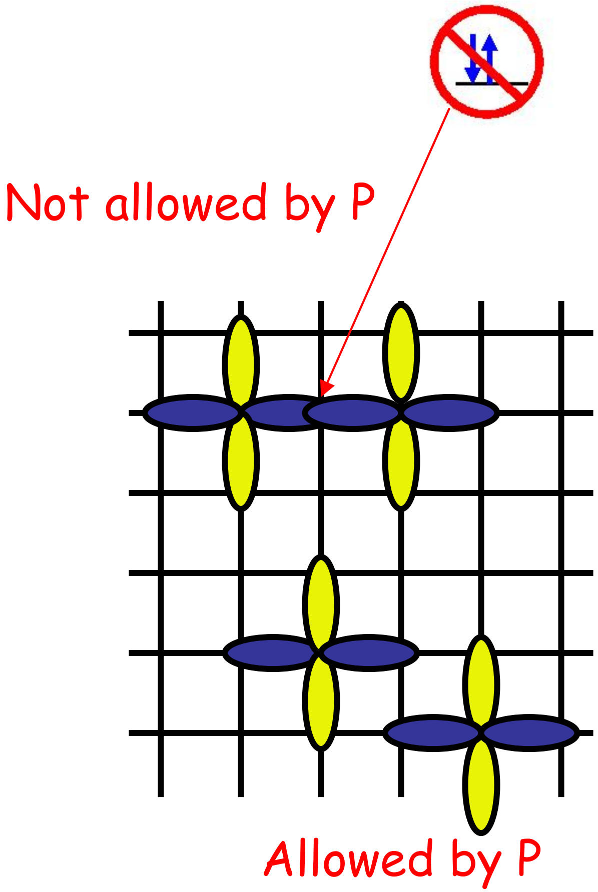
P.W. Anderson,
Science (1987)





S=0 pairing induced by superexchange $J = 4t^2/U$

Anderson (1987)
Baskaran et al (1987)
Kotliar(1988)
Zhang, Gros, Rice, and Shiba (1988)



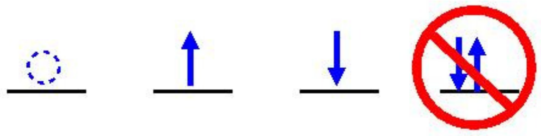
d-wave symmetry
→ Partners in a pair never at the same site

Projection P
→ eliminate all double occupancy

- What are the properties of the Projected SC P|dBCS>?

- How do these compare with those of the High Tc cuprates?

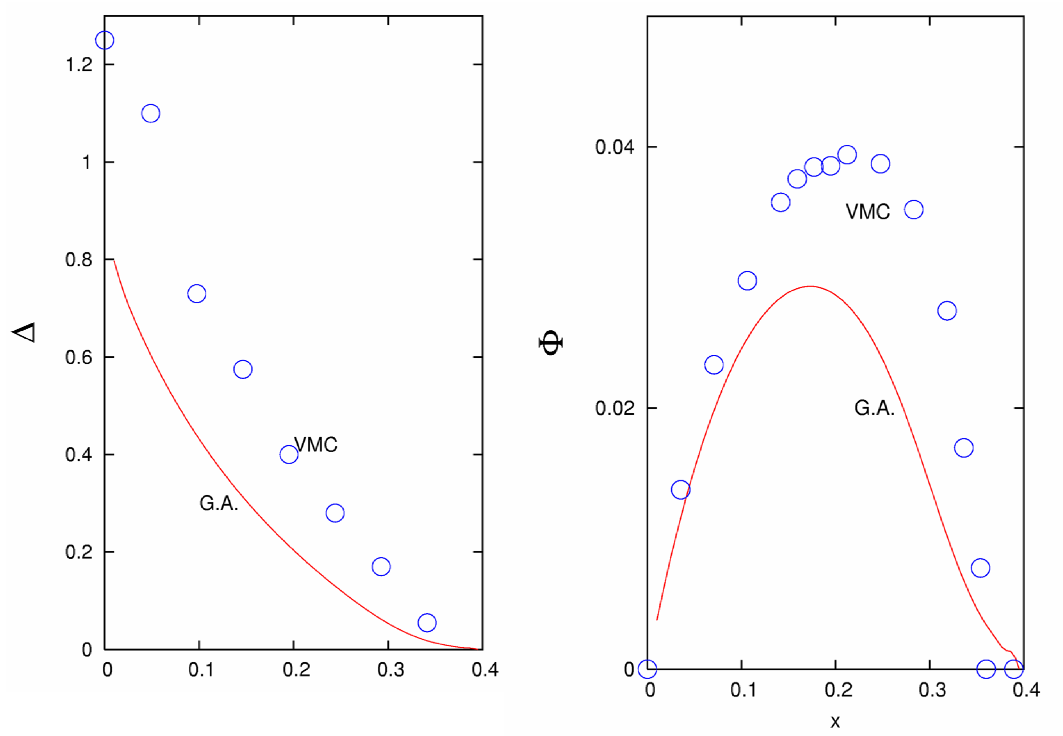
$$|\Psi_0\rangle = \mathcal{P}|BCS\rangle$$



variational Monte Carlo method: <u>only</u> known way to treat P *exactly*

Gutzwiller approximation:

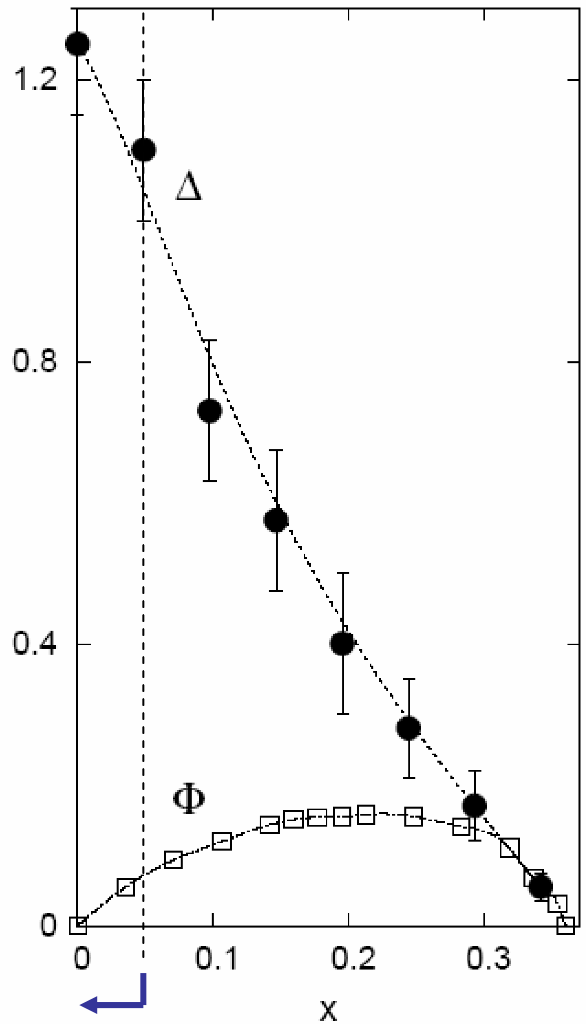
$$\langle \mathrm{K.E.}\rangle \simeq \frac{2x}{(1+x)}\langle \mathrm{K.E.}\rangle_0; \quad \langle s(i)s(j)\rangle \simeq \frac{4}{(1+x)^2}\,\langle s(i)s(j)\rangle_0$$



GA Calculation:
"renormalized"
mean field theory

→ analytical insights
→ excited states
→ dynamical correlations
→ disorder effects

$$\min \frac{\langle \Psi_0 | H | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle}$$



AFM

Pairing & Superconductivity

Pairing → variational Δ

d-wave SC order parameter Φ
→ from ODLRO $\langle c^\dagger c^\dagger c c \rangle$

Strong Coulomb U
Φ**(x) ~ x as x** $\to 0$

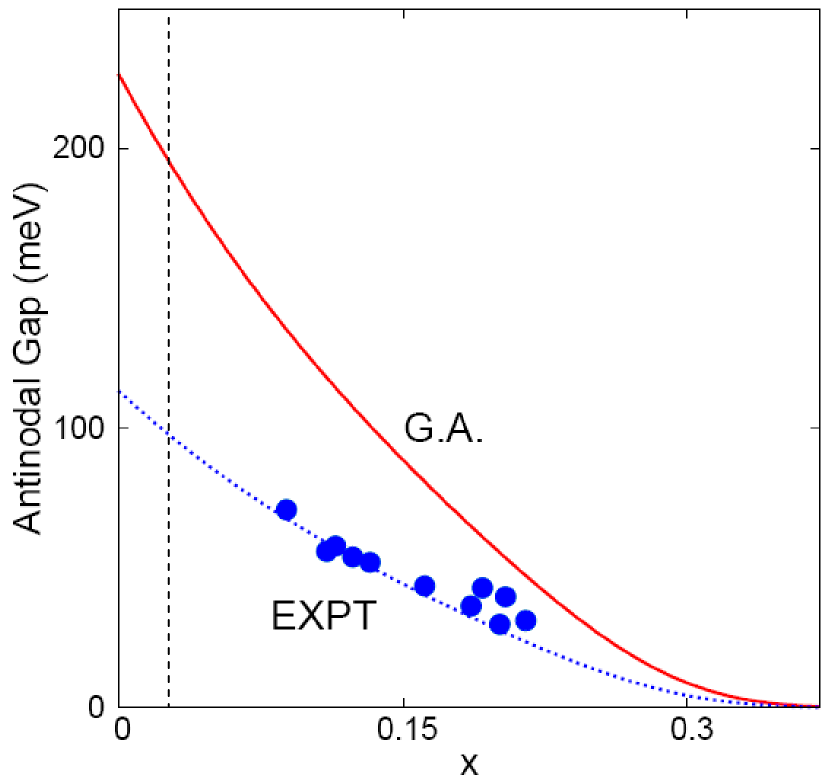
P → local, quantum
phase fluctuations

→ $\Delta \neq \Phi_{sc}$

Variational Estimate of SC Energy Gap

Variational QP state $|k\sigma\rangle = e^{-iS} P \gamma^{\dagger}_{k\sigma} |\mathrm{dBCS}\rangle$

QP excitation energy → SC gap



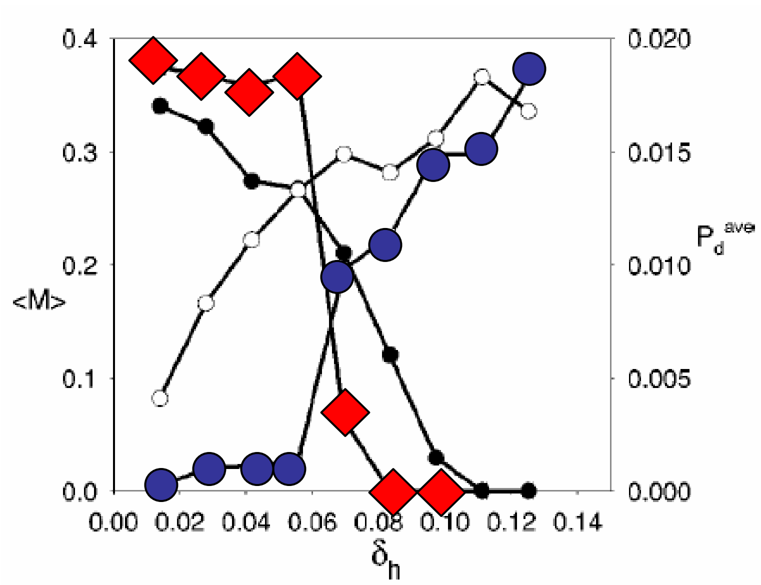
ARPES Expt:
Campuzano et al, PRL (1999)

SC Gap = Variational Δ

- ~ factor of 2 larger than experiment
- same x-dependence as experiment

Competition between SC and AFM as x → 0

Energetics: energies of different states differ by few % J → details of H are important



At x=0 AF magnetism wins;
RVB spin-liquid insulator
Energy/bond = - 0.3199 J
v/s
AFM Long range order
Energy/bond = - 0.3346J
Trivedi & Ceperley, (1989)

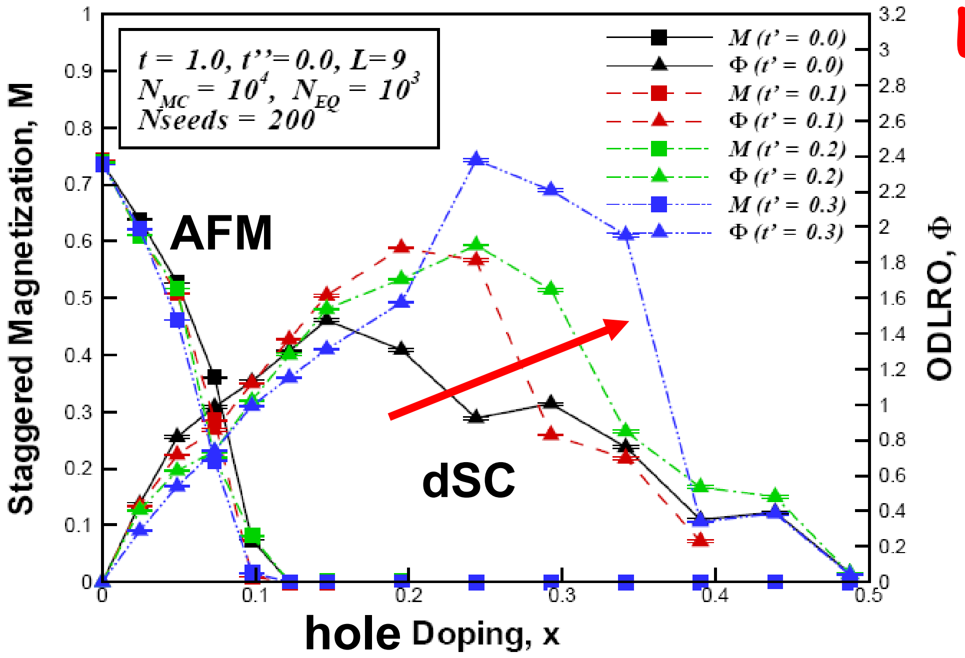
- ◆ AFM for x < 7%
- ● SC for x > 7%

J/ t =0.3,
t ' / t= - 0.3
t"/ t=0.2

Shih, Chen, Chou, & TK Lee, PRB **70**, 220502(R) (2004)

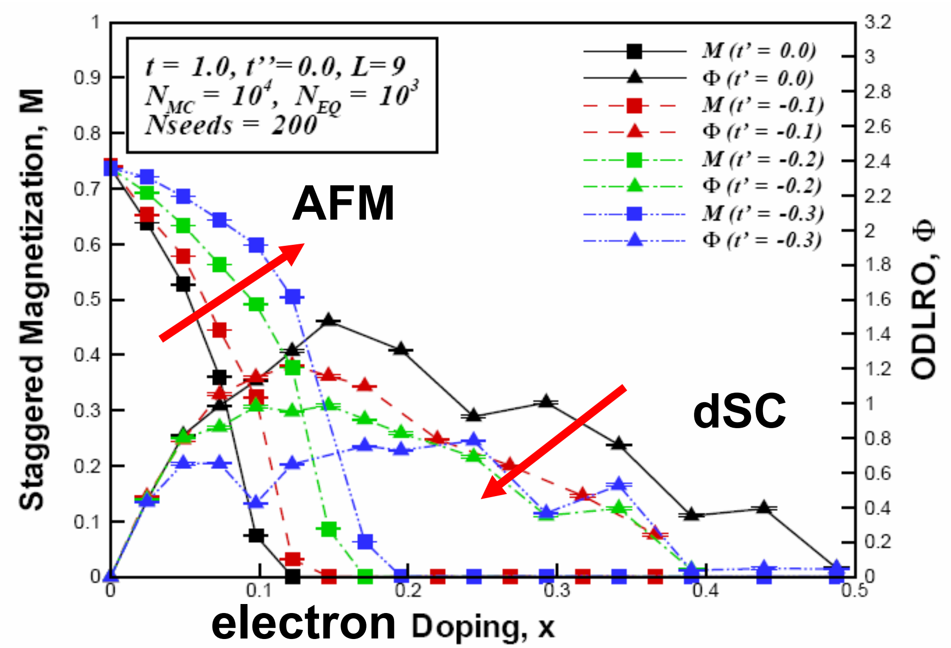
Electron-Hole Asymmetry In Phase Diagram

t,t′,J model



Hole Doping:

- AFM insensitive to t′
- SC grows with t′



Electron Doping:

- AFM grows with t′
- SC suppressed with t′

S.Pathak, V.Shenoy, N.Trivedi & MR

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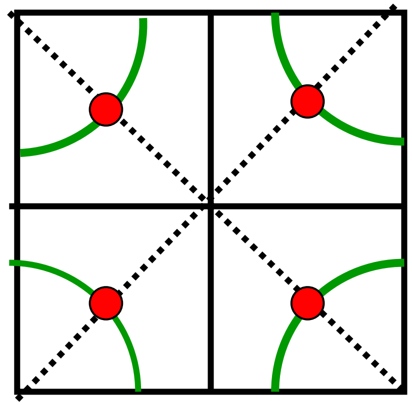
- Variational Theory of SC State
- Low energy excitations:
  - * nodal quasiparticles
  - * "underlying Fermi surface"
- Superfluid density
- Disorder Effects

Low Energy Excitations in SC state



Sharp Nodal Quasiparticles

- existence
- $k_F(x)$
- coherent spectral weight $Z(x)$
- dispersion $v_F(x)$

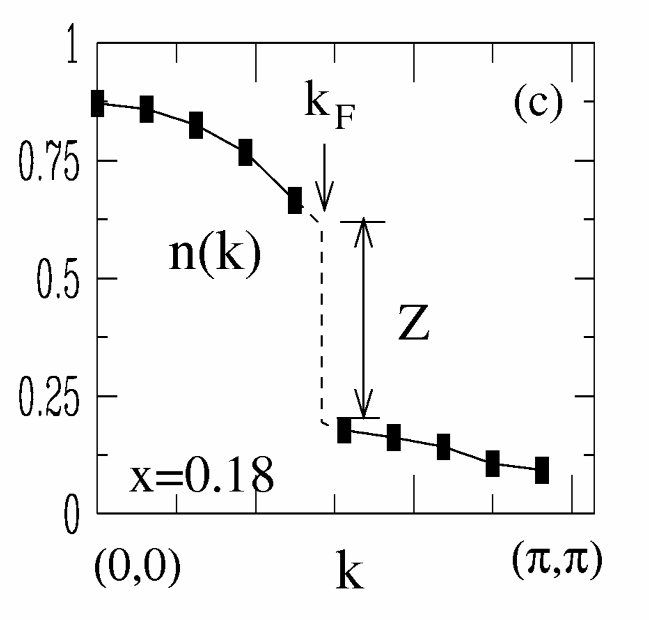
Moments of spectral function = equal-time correlators

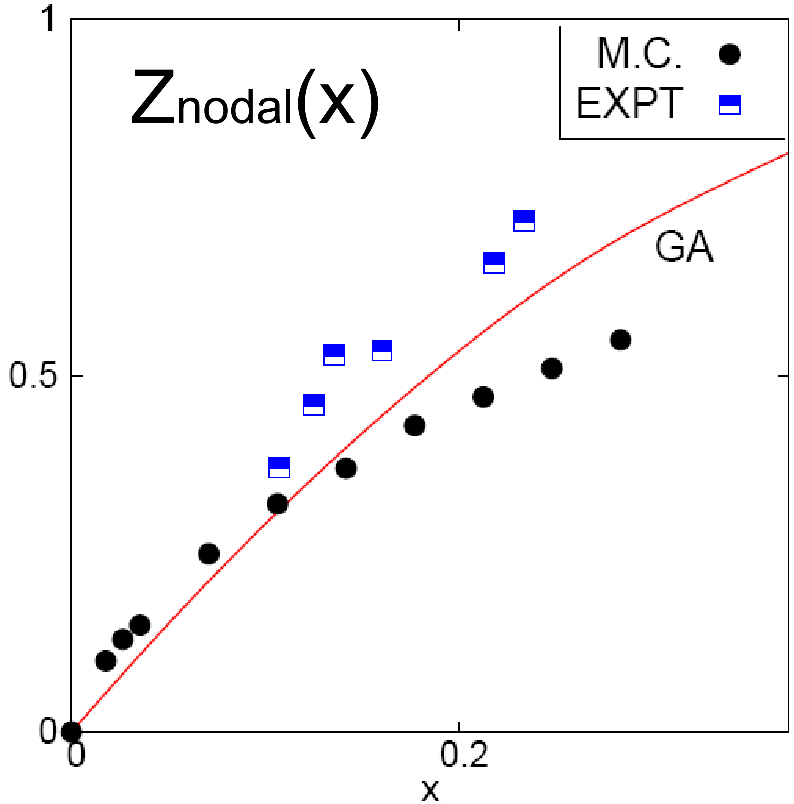
$$M_\ell(\mathbf{k}) = \int_{-\infty}^{0} d\omega\, \omega^\ell A(\mathbf{k},\omega)$$

Singularities in

$M_\ell(\mathbf{k}) \Leftrightarrow$ **gapless Quasiparticles**

Nodal QP Spectral Weight Z





Loss of coherence with underdoping

— GA $Z = \frac{2x}{(1+x)} + \mathcal{O}(xt/U)$

MR, Sensarma et al., PRL (2005)

● MC: Paramekanti. MR, Trivedi, PRL(2001)

▬ Expt: Johnson *et al.*, PRL (2001)
ARPES Bi2212

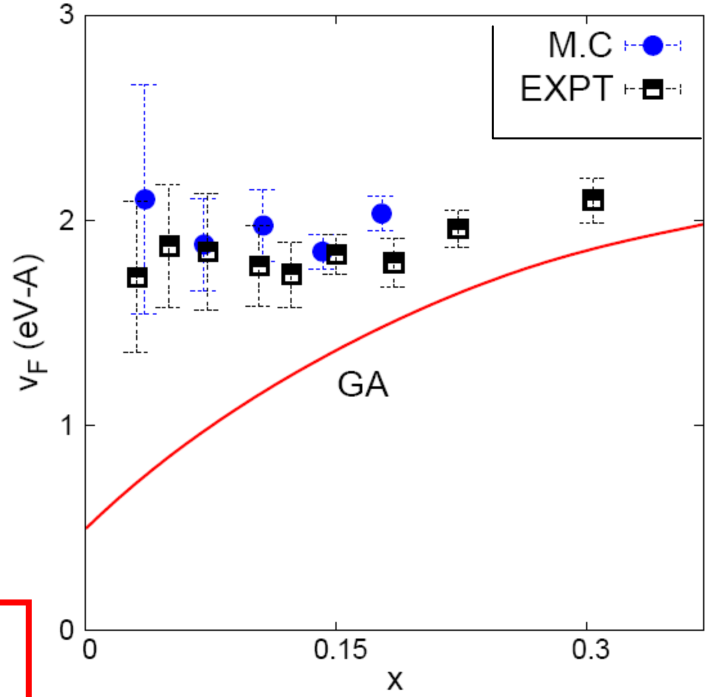
Dispersion of Nodal Quasiparticles

Nodal V_F or m* independent of x

as $x \to 0$

$$Z \sim x \Rightarrow |\partial\Sigma'/\partial\omega| \sim 1/x$$

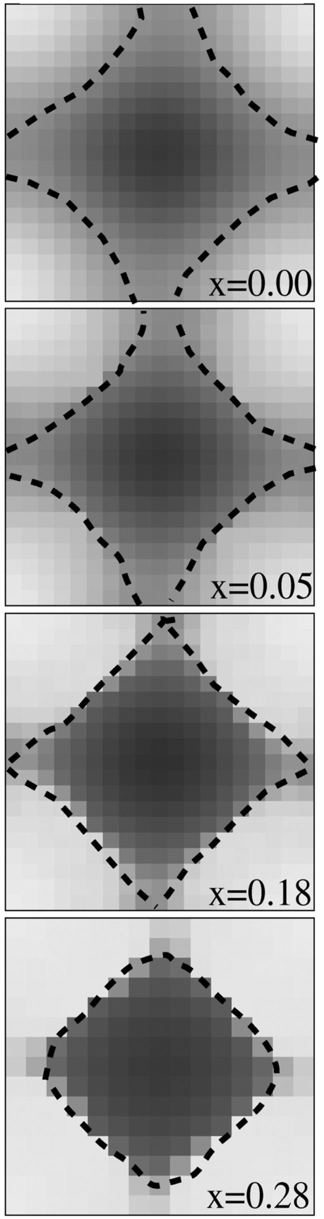
$$v_F = \text{const.} \Rightarrow |\partial\Sigma'/\partial k| \sim Ja/x$$



Σ' has **singular 1/x** dependence on both ω <u>and</u> **k** along zone diagonal

- *MC*: Paramekanti. MR, Trivedi, PRL(2001)
- ARPES Expt: Zhou et al, Nature (2003)
- *GA*: Sensarma et al, PRL (2005)

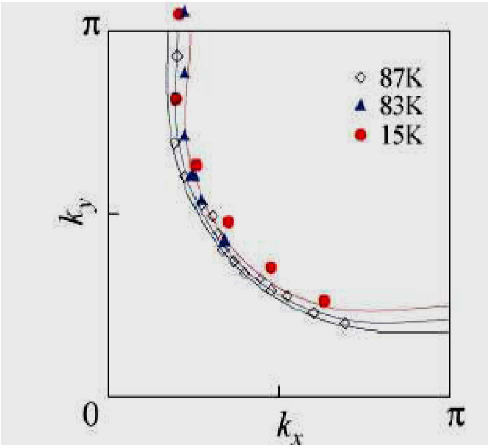
n(k) from MC



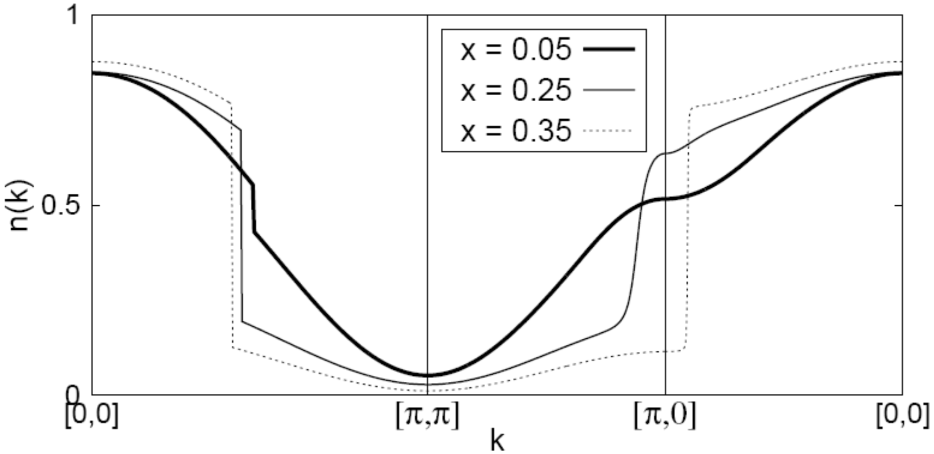
x-dependence of the Momentum distribution

---- n(k) = 1/2

"FS" topology change at x ~ 0.2; depends sensitively on t'/t



Expt: H. Ding, *et.al* PRL (1997)



GA n(K)

Qs: Is there a way to determine the "underlying FS" that is gapped out in a SC state at T = 0?

R. Sensarma, MR & N. Trivedi, PRL **98**, 027004 (2007)

C. Gros, B. Edegger, V. Muthukumar & P.W. Anderson, PNAS (2006)

BCS answer: $$\mathrm{G}_{11}^{\mathrm{bcs}}(\mathbf{k},\omega) = \frac{\omega - \xi_{\mathbf{k}}}{\omega^2 + E_{\mathbf{k}}^2}$$ $$\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$$

$$\mathrm{G}_{11}^{\mathrm{bcs}}(\mathbf{k},0) = 0 \;\Rightarrow\; \xi_{\mathbf{k}} = 0$$ (but … see below!)

Is SC state "FS" given by $G(\mathbf{k},0) = 0$?

↑ (1,1)-component of Nambu G

cf. Fermi surface in Landau's FLT $$G(\mathbf{k},0) = \infty$$

<u>Unlike</u> the normal Landau FL, the SC state "FS"…

$G(\mathbf{k},0)=0$ does <u>not</u> enclose n electrons

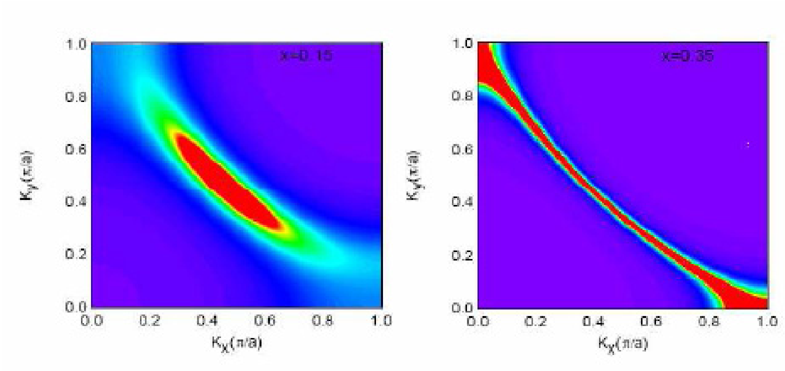
No Luttinger Sum Rule for state with ODLRO

$$\Phi[\hat{\mathrm{G}}] \text{ with } \delta\Phi/\delta\hat{\mathrm{G}}=\hat{\Sigma}$$

$$\int d\omega \sum_{\mathbf{k}} \mathrm{Tr}\{\tau_3\hat{\mathrm{G}}\}=(n_\uparrow+n_\downarrow)$$

"Number is not conserved in SC state"

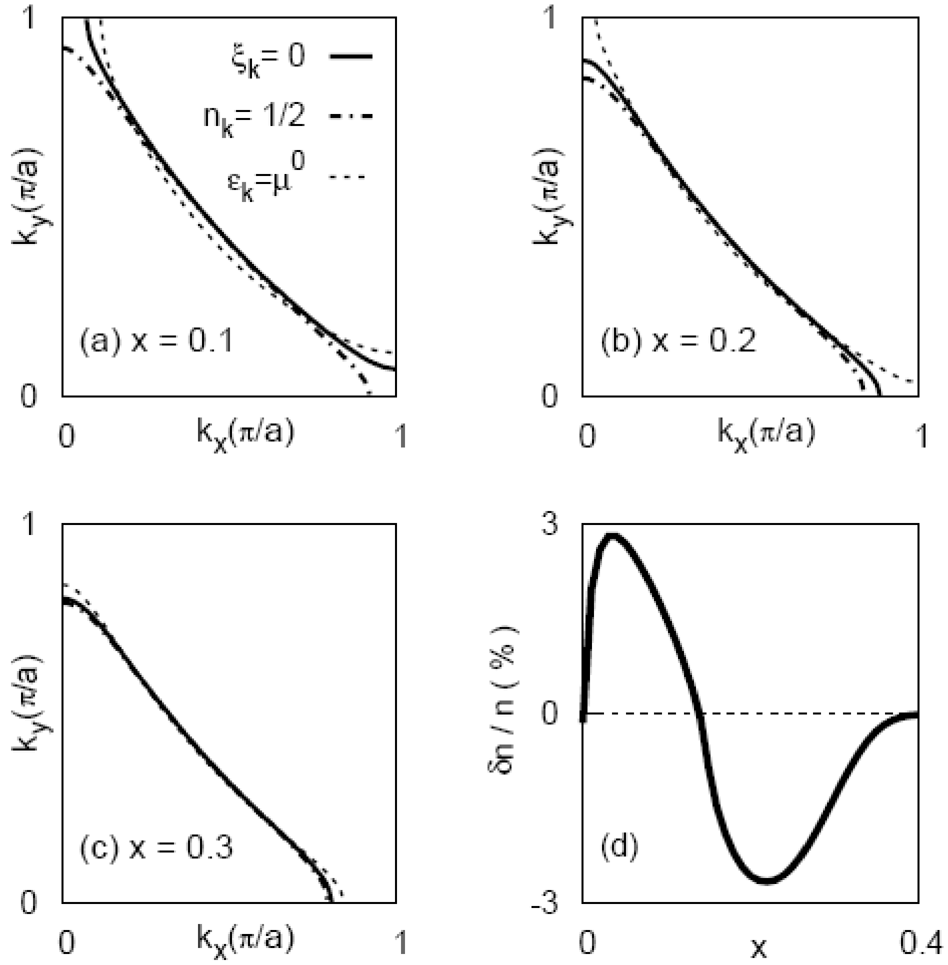
[Dzyloshinskii, PRB(2003)]



- Angle-Resolved Photoemission expts.

-- measure locus of zero-energy ARPES intensity
-- "minimum gap locus"
which "<u>looks like</u> a Fermi surface". What does this mean?

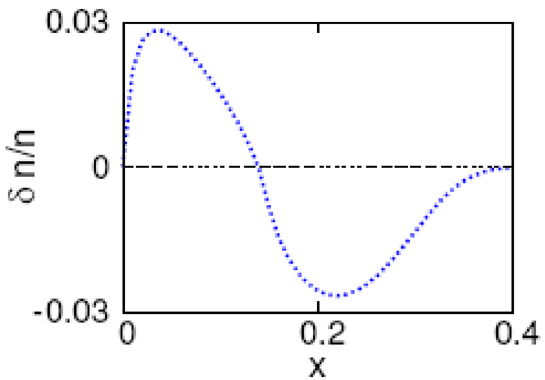
Various SC state "FS" definitions lead to similar contours, but all of them violate the Luttinger count



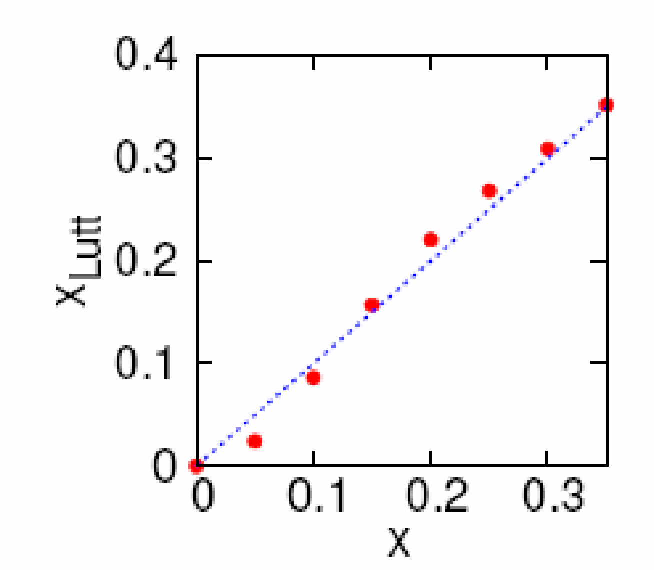
Size of violation

$$\left|\frac{\delta n}{n}\right| \sim \left(\Delta / E_f\right)^2$$

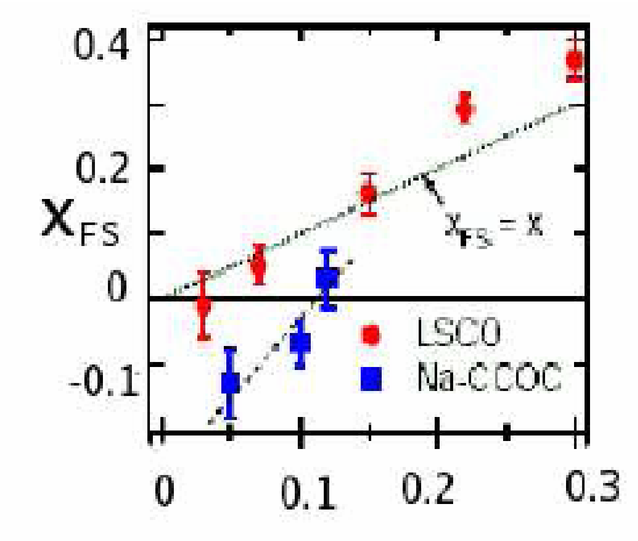
Sign of violation related to "FS" topology



Experiments:
Consistent with prediction,
but not definitive yet
* determination of x;
* k-resolution



Theory:
Sensarma, MR & Trivedi
PRL **98**, 027004 (2007)



ARPES experiment:
T. Yoshida et. al,
PRB **74**, 224510 (2006)

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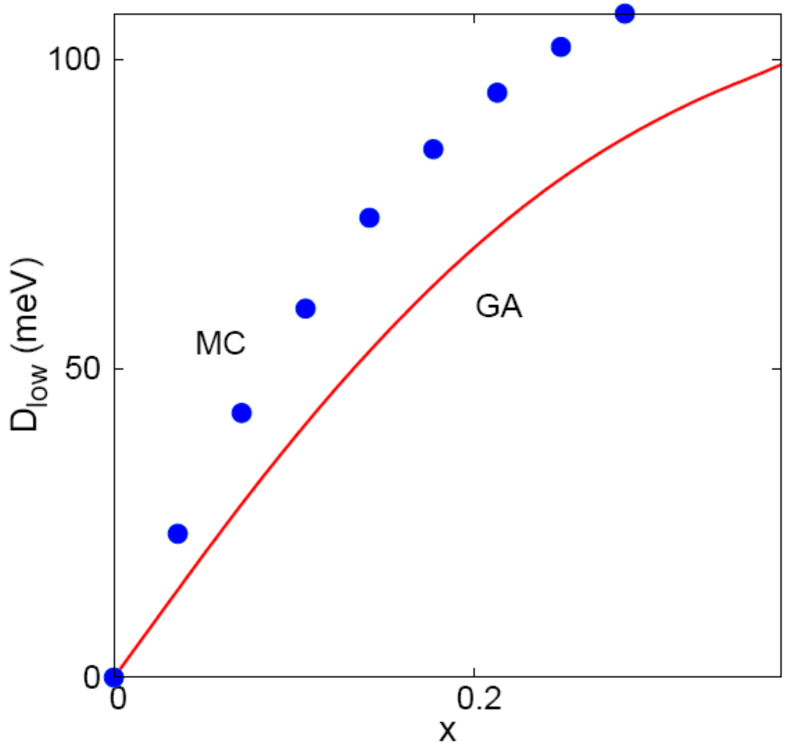
Optical Spectral Weight or Drude Weight

$$D_{\text{low}} = \frac{2}{\pi}\int_0^{\Omega_c} d\omega \ \text{Re}\sigma(\omega)$$

$$J < t \ll \Omega_c \ll U$$

$$\omega_p^{*2} = \frac{4\pi e^2}{d} D_{\text{low}}$$

Values & trend in agreement with optics:
Orenstein, et al., PRB (1990)
Cooper, et al., PRB (1993)



Upper Bound on Superfluid Density

$$\rho_s \leq D_{\text{low}}$$

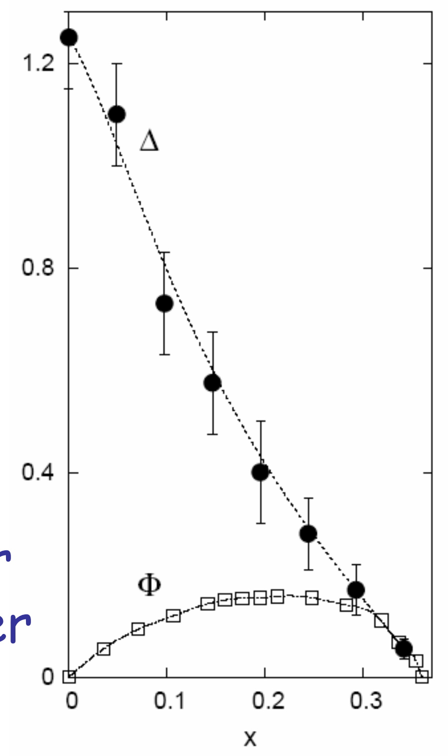
$$\Rightarrow \rho_s \to 0$$

as $x \to 0$

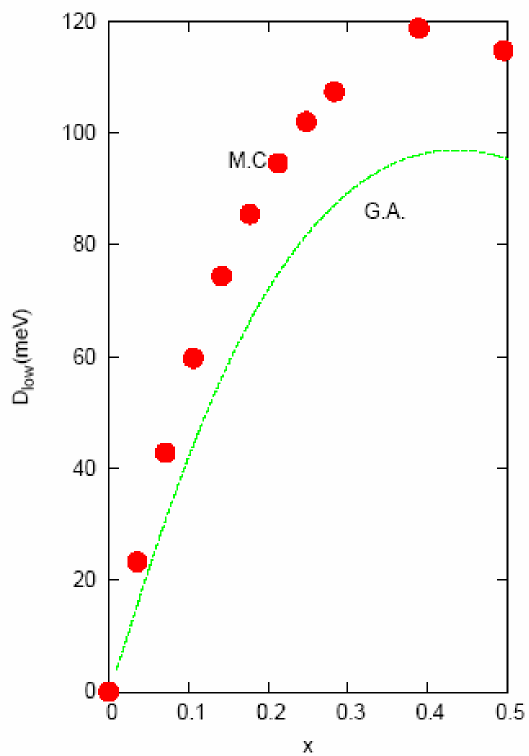
Implications of SC state results for Pseudogap:

Energy Gap

SC Order parameter



Superfluid density:



For large x, BCS-like: $\Delta \ll \rho_s$

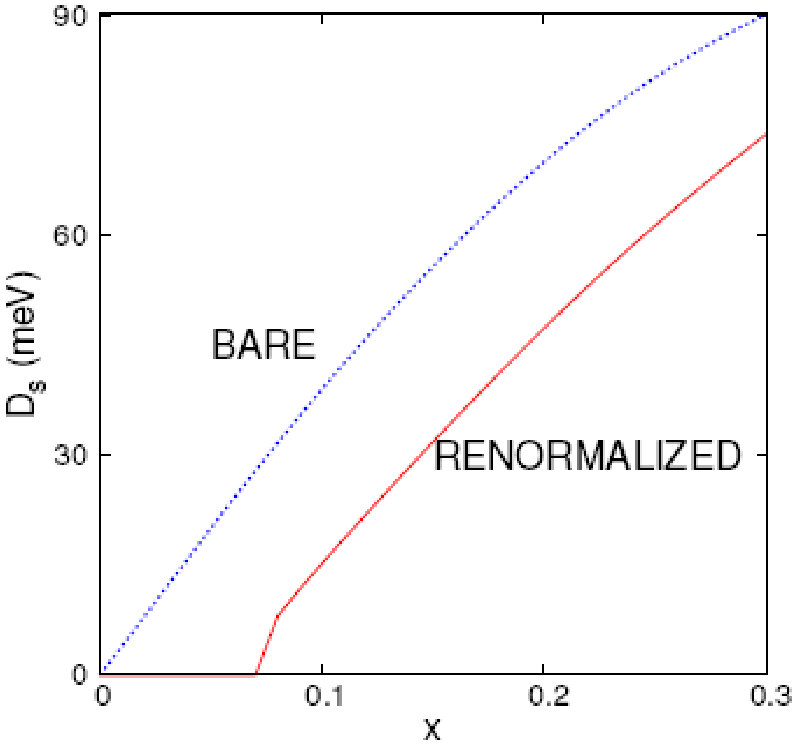
For small x, non-BCS: $\rho_s \ll \Delta$

$T_C \sim \min\{\rho_s, \Delta\}$

Emery & Kivelson, Nature (1995)

Pairing Pseudogap: MR *et al*, PRL (1992); Trivedi & MR, PRL (1995)

Effect of Long-Wavelength Quantum Phase Fluctuations:



T= 0 Self-Consistent Harmonic Approximation

Paramekanti, MR, Ramakrishnan & Mandal PRB (2000)

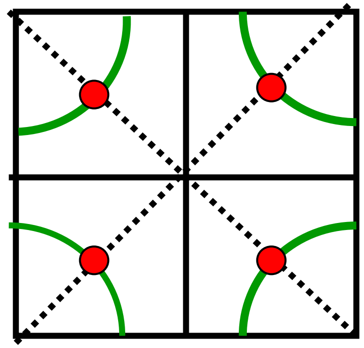
$$\rho_s = \rho_s^0 \exp\left(-\langle \delta\theta^2 \rangle / 2\right)$$

T-dependence of Superfluid density $(T \ll T_c)$

$$\rho_s(T) = \rho_s(0) - \frac{2\ln 2}{\pi}\ \alpha^2 \left(\frac{v_F}{v_2}\right) T + \ldots$$

$$\lambda^{-2}(T) = 4\pi e^2 D_s(T)/(\hbar^2 c^2 d)$$

$\rho_s(0,x)$ ← doping dependence near Mott insulator



T-dependence from **nodal QPs**

Nodal QP dispersion

$$E_{\mathbf{k}} \approx \sqrt{(v_F \delta k_\perp)^2 + (v_2 \delta k_{||})^2}$$

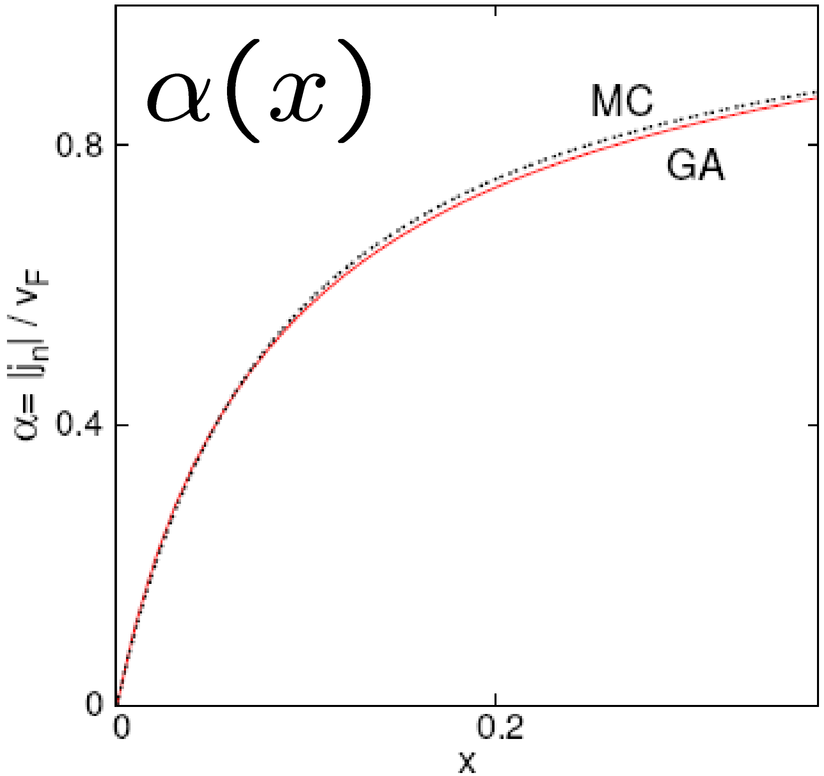
$$j = \alpha e v_F$$

← Current carried by nodal QPs backflow corrections

$$J_{nqp} = \langle qp|\hat{J}|qp\rangle$$
$$= \alpha e v_F$$

Current carried by a Nodal Quasiparticle

$\alpha =$ "Effective charge" of QP



tJ Monte Carlo
Nave, Ivanov & P.A.Lee, PRB (2005)

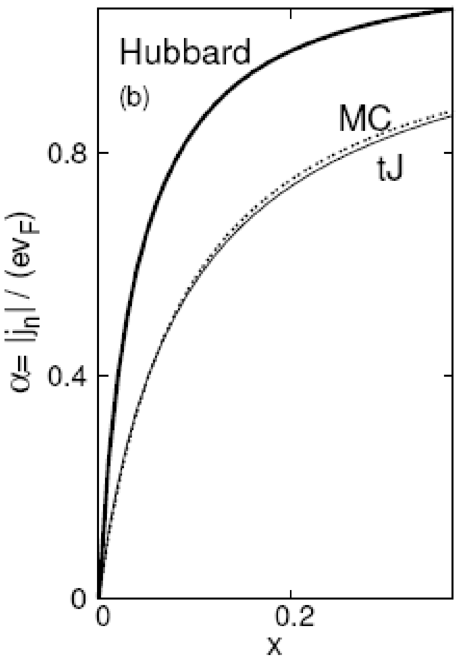
our Gutzwiller Approximation result for tJ model:

$$J_{nqp} = eZv_F^0$$

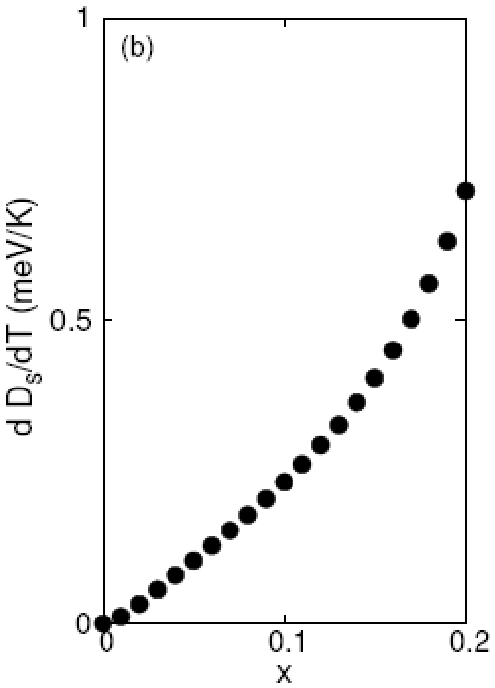
$$\alpha \approx g_t/(g_t + 1/6)$$
$$= x/(1.08x + 0.08)$$

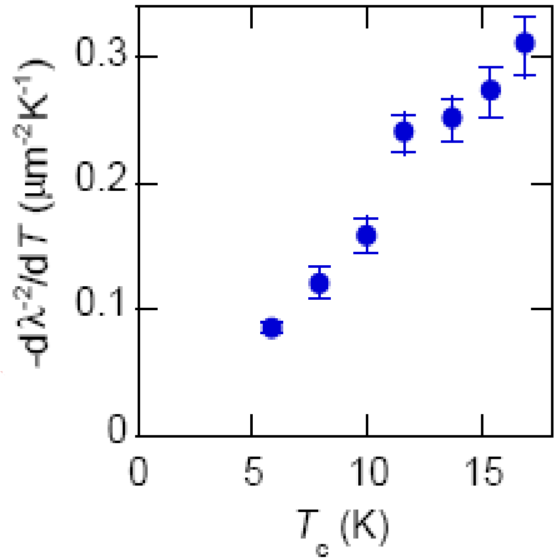
Slope of SF density:

$$\alpha(x)$$



$$d\rho_s/dT$$





UBC YBCO data
Broun et al,
cond-mat/0509223

Outline:

- Introduction
- Sum Rules & p-h asymmetry
- Variational Theory of SC State
- Low energy excitations
- Superfluid density
- Disorder Effects

Why are HTSC's so robust against disorder?

* (most) impurities lie off CuO_2 planes

* the coherence length is very short
→ Spatially inhomogeneous response
  not captured by standard A-G theory

* Disorder effects are suppressed
  in strongly correlated SCs

Arti Garg, N. Trivedi & MR, cond-mat/0609666

Inhomogeneous response to impurities:
-- Bogoliubov-deGennes (BdG) theory

Correlation effects
-- inhomogeneous Gutzwiller approx.(GA)

e.g., KE renormalization: $g_t(\mathbf{r},\mathbf{r}') = g_t(\mathbf{r})g_t(\mathbf{r}')$
where $g_t(\mathbf{r}) = [2x(\mathbf{r})/(1+x(\mathbf{r}))]^{1/2}$

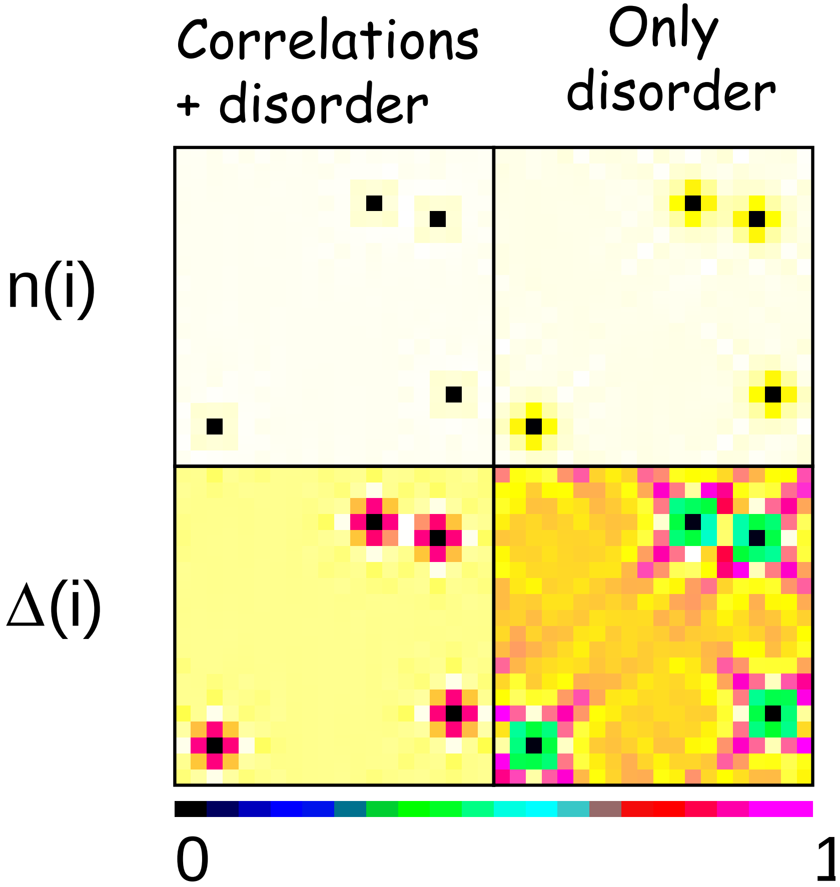
Compare results with and without correlations:

For correlated system:
- fewer low-energy excitations
- robust nodal QPs - "V" in DOS protected
- nodes in k-space protected

Response to weak (Born) impurities:

Local density

Local d-wave Pairing amplitude



$\langle n \rangle = 0.8$

$n_{imp} = 0.01$

$V_{imp} = t$

Short healing length in correlated system

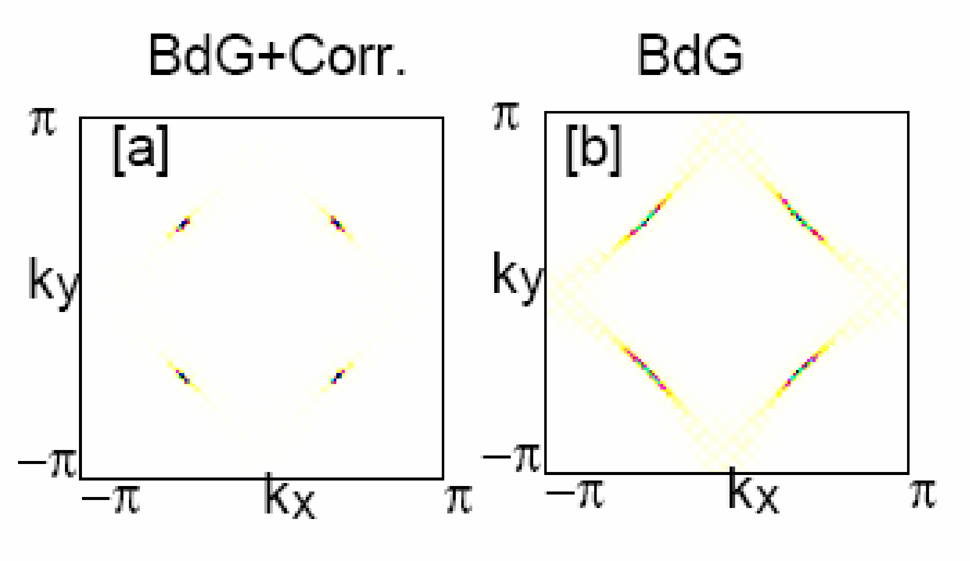
Impurity potential is renormalized by interactions

Longer healing Length in calculation which ignores correlations

Where do the low energy excitations live?

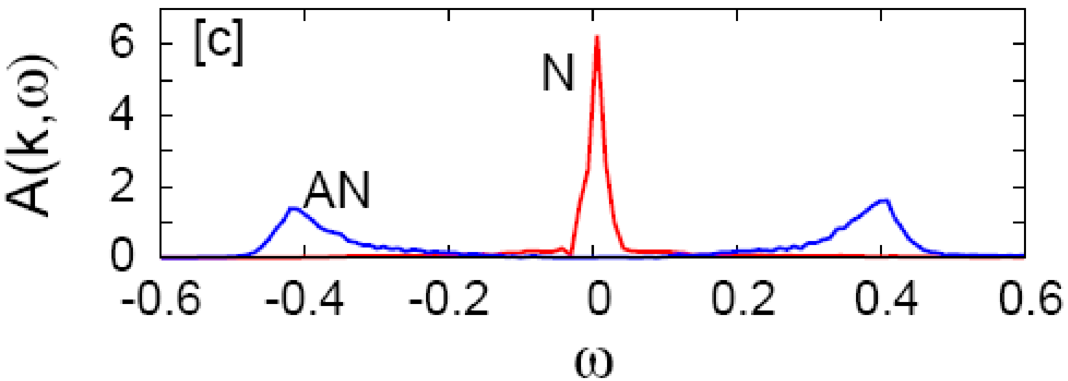
$$A(\mathbf{k},\omega) = \mathsf{FT}_{\mathbf{r}\to\mathbf{k}} \langle \mathrm{Im}\, G(\mathbf{r},\mathbf{R},\omega)\rangle_{\mathbf{R}}$$

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2; \quad \mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$$



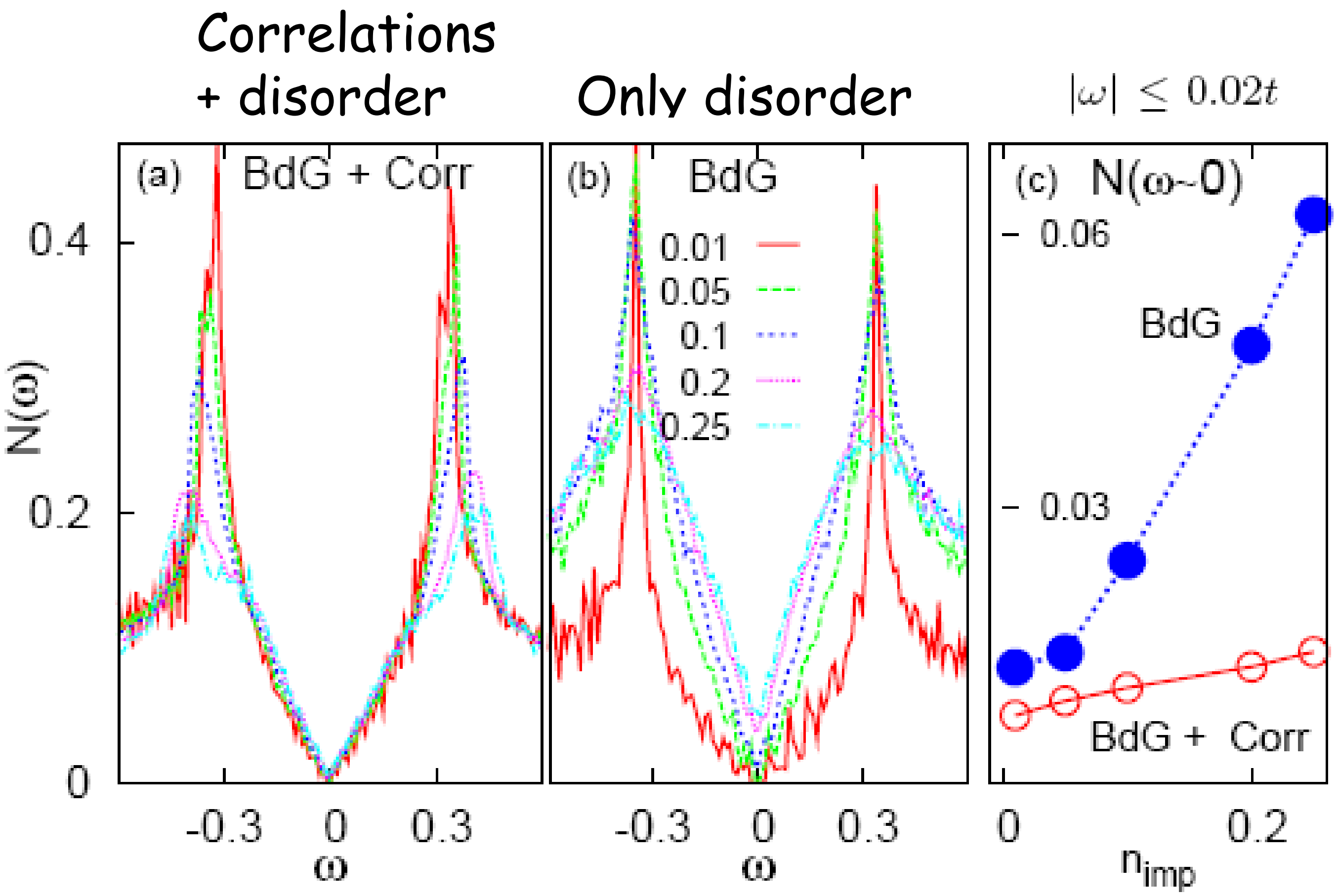
$A(\mathbf{k}, |\omega| \leq 0.02)$

Nodes are protected



Nodal QPs much less affected by Disorder than the Antinodal QPs

Spatially averaged DOS

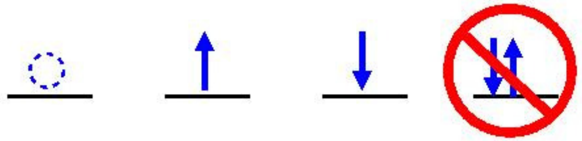


In the correlated system

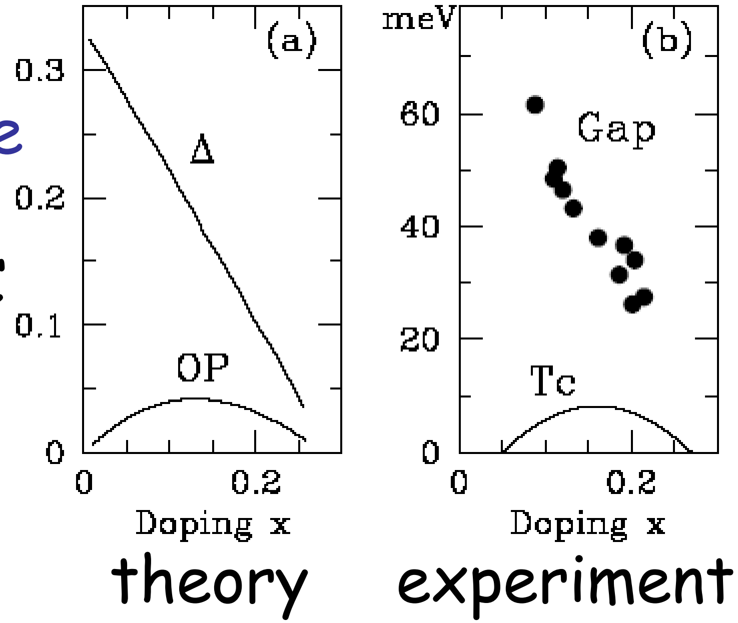
- low energy 'V' from nodal QPs
  very weakly affected by disorder
- disorder induces fewer low-energy excitations

Summary: SC in doped Mott insulators

- p-h asymmetry in STM & ARPES
- local x(r) from sum rule ratio



- SC "dome" with optimal doping
- energy gap and SC order have qualitatively different x-dependence



- Evolution from large x BCS-like SC to small x SC near Mott insulator
- nodal QPs: $k_F(x)$, $Z(x)$, $V_F(x)$
- underlying "Fermi surface"
- optical spectral weight and superfluid density $\rho_s(x;T)$

- disorder effects suppressed in presence of strong correlations

The end

Canonical Transformation **exp(iS)**

transforms Hubbard to tJ model

(plus three-site terms)

$$\mathcal{K}_0 = -\sum_{\mathbf{r},\mathbf{r}',\sigma} t_{\mathbf{r}\mathbf{r}'}[n_{\mathbf{r}\bar{\sigma}} c^\dagger_{\mathbf{r}\sigma} c_{\mathbf{r}'\sigma} n_{\mathbf{r}'\bar{\sigma}} + h_{\mathbf{r}\bar{\sigma}} c^\dagger_{\mathbf{r}\sigma} c_{\mathbf{r}'\sigma} h_{\mathbf{r}'\bar{\sigma}}]$$

$$\mathcal{K}_{+1} = -\sum_{\mathbf{r},\mathbf{r}',\sigma} t_{\mathbf{r}\mathbf{r}'} n_{\mathbf{r}\bar{\sigma}} c^\dagger_{\mathbf{r}\sigma} c_{\mathbf{r}'\sigma} h_{\mathbf{r}'\bar{\sigma}}$$

$$\mathcal{K}_{-1} = -\sum_{\mathbf{r},\mathbf{r}',\sigma} t_{\mathbf{r}\mathbf{r}'} h_{\mathbf{r}\bar{\sigma}} c^\dagger_{\mathbf{r}\sigma} c_{\mathbf{r}'\sigma} n_{\mathbf{r}'\bar{\sigma}}$$

$$h_{\mathbf{r}\sigma} = (1 - n_{\mathbf{r}\sigma}) \qquad \bar{\sigma} = -\sigma$$

$$iS \equiv iS^{[1]} + iS^{[2]}$$

$$= \frac{1}{U}(\mathcal{K}_{+1} - \mathcal{K}_{-1}) + \frac{1}{U^2}([\mathcal{K}_{+1}, \mathcal{K}_0] + [\mathcal{K}_{-1}, \mathcal{K}_0])$$

Kohn (1964); Gros, Joynt, Rice(1987); MacDonald, Girvin, Yoshioka (1988)

Gutzwiller Approximation:

Gutzwiller; Brinkman-Rice
Vollhardt; Zhang *et al.*

approximation scheme to analytically
evaluate matrix elements in Projected states

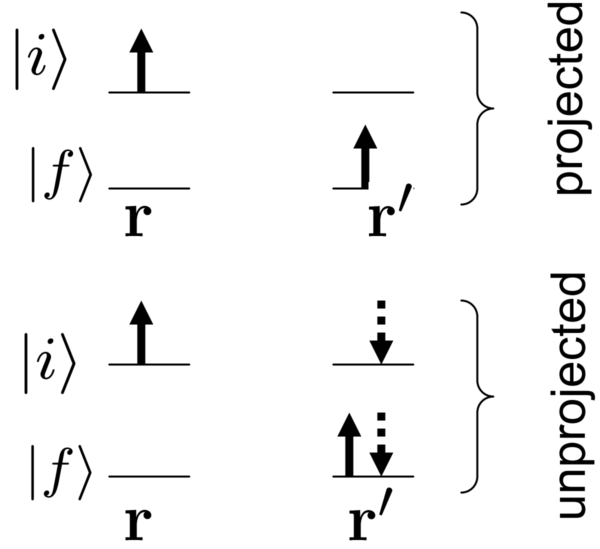
$$\langle \Phi_0 | \mathcal{P} Q \mathcal{P} | \Phi_0 \rangle \simeq g_Q(x) \langle \Phi_0 | Q | \Phi_0 \rangle$$

examples:

$$\langle c^\dagger_{i\sigma} c_{j\sigma} \rangle \simeq \frac{2x}{(1+x)} \langle c^\dagger_{i\sigma} c_{j\sigma} \rangle_0 \qquad \langle s_i s_j \rangle \simeq \frac{4}{(1+x)^2} \langle s_i s_j \rangle_0$$

$$g_t = \frac{\langle c^\dagger_{i\sigma} c_{j\sigma} \rangle}{\langle c^\dagger_{i\sigma} c_{j\sigma} \rangle_0}$$

$$g_t = \left[\frac{n_{\mathbf{r}'\uparrow}(1-n_{\mathbf{r}}) n_{\mathbf{r}\uparrow}(1-n_{\mathbf{r}'})}{n_{\mathbf{r}'\uparrow}(1-n_{\mathbf{r}\uparrow}) n_{\mathbf{r}\uparrow}(1-n_{\mathbf{r}'\uparrow})} \right]^{\frac{1}{2}} = \frac{2x}{1+x}$$



Singularities of Spectral Moments & Gapless QPs

$$A(\mathbf{k},\omega) = -\mathrm{Im}G(\mathbf{k},\omega+i0^+)/\pi$$

$$= \sum_m [|\langle m|c^\dagger_{\mathbf{k}\sigma}|0\rangle|^2\delta(\omega+\omega_0-\omega_m) + |\langle m|c_{\mathbf{k}\sigma}|0\rangle|^2\delta(\omega-\omega_0+\omega_m)]$$

$$M_\ell(\mathbf{k}) \equiv \int_{-\infty}^{0} d\omega\,\omega^\ell A(\mathbf{k},\omega)$$

$$M_0(\mathbf{k}) = \int_{-\infty}^{0} d\omega A(\mathbf{k},\omega) = \sum_m |\langle m|c_{\mathbf{k}\sigma}|0\rangle|^2 = n(\mathbf{k})$$

Jump discontinuity: k_F & Z

$$M_1(\mathbf{k}) = \int_{-\infty}^{0} d\omega\,\omega A(\mathbf{k},\omega) = \sum_m (\omega_0-\omega_m)|\langle m|c_{\mathbf{k}\sigma}|0\rangle|^2$$

$$= \langle c^\dagger_{\mathbf{k}\sigma}[c_{\mathbf{k}\sigma},\mathcal{H}-\mu\mathcal{N}]\rangle$$

Slope discontinuity: v_F