

Transport near strongly interacting quantum critical points

Subir Sachdev



Outline

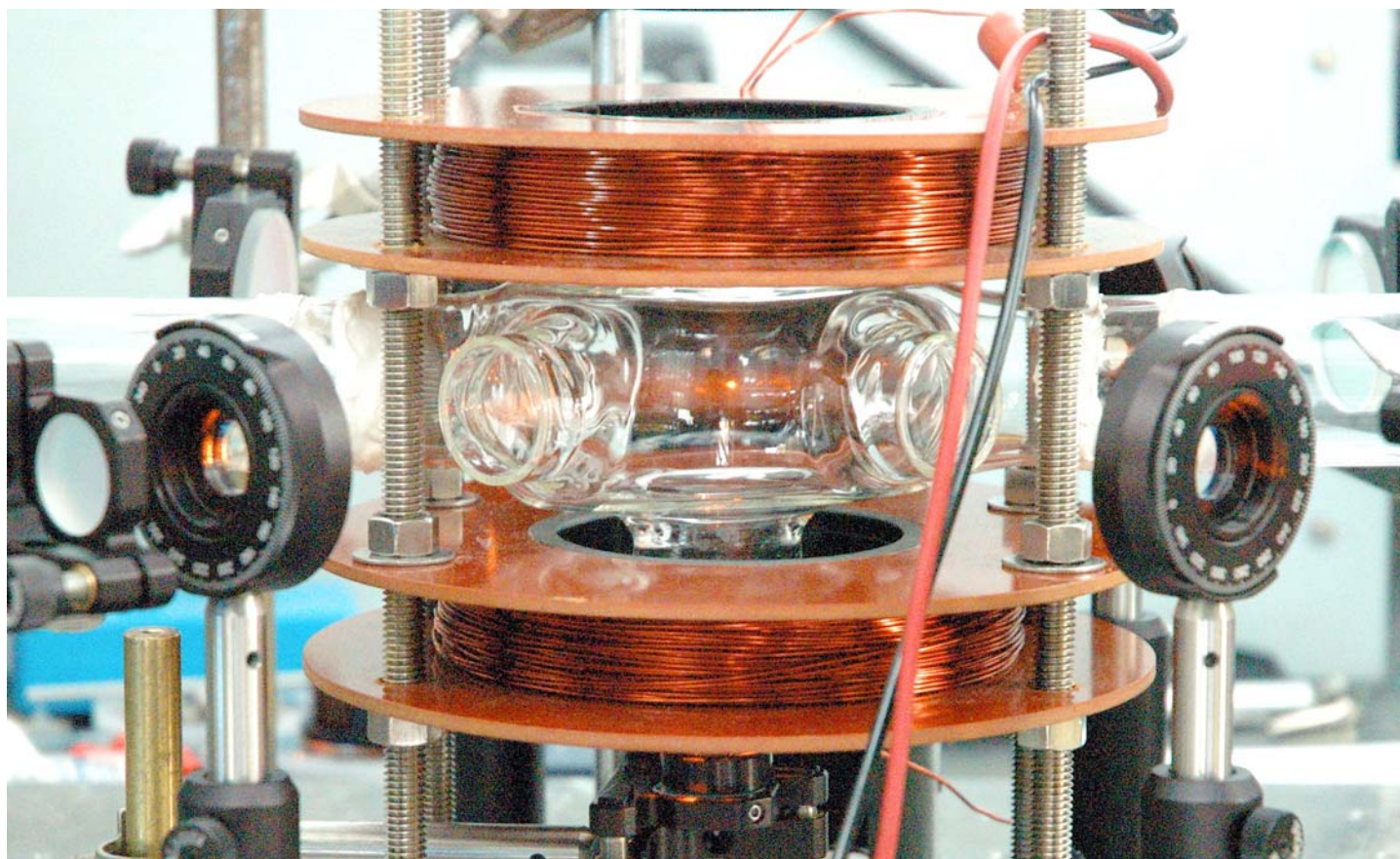
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1. The superfluid-insulator transition in the boson Hubbard model:
Hydrodynamic-collisionless crossover of a CFT
2. Exact solutions of CFTs in 1+1 dimensions
No hydrodynamics
3. Exact solution of a CFT in 2+1 dimensions - Yang-Mills theory
with N=8 supersymmetry:
Black holes in AdS₄
4. General hydrodynamic theory in the presence of a magnetic field,
chemical potential and impurities:
Nernst effect in the cuprate superconductors;
Dyonic black holes in AdS₄

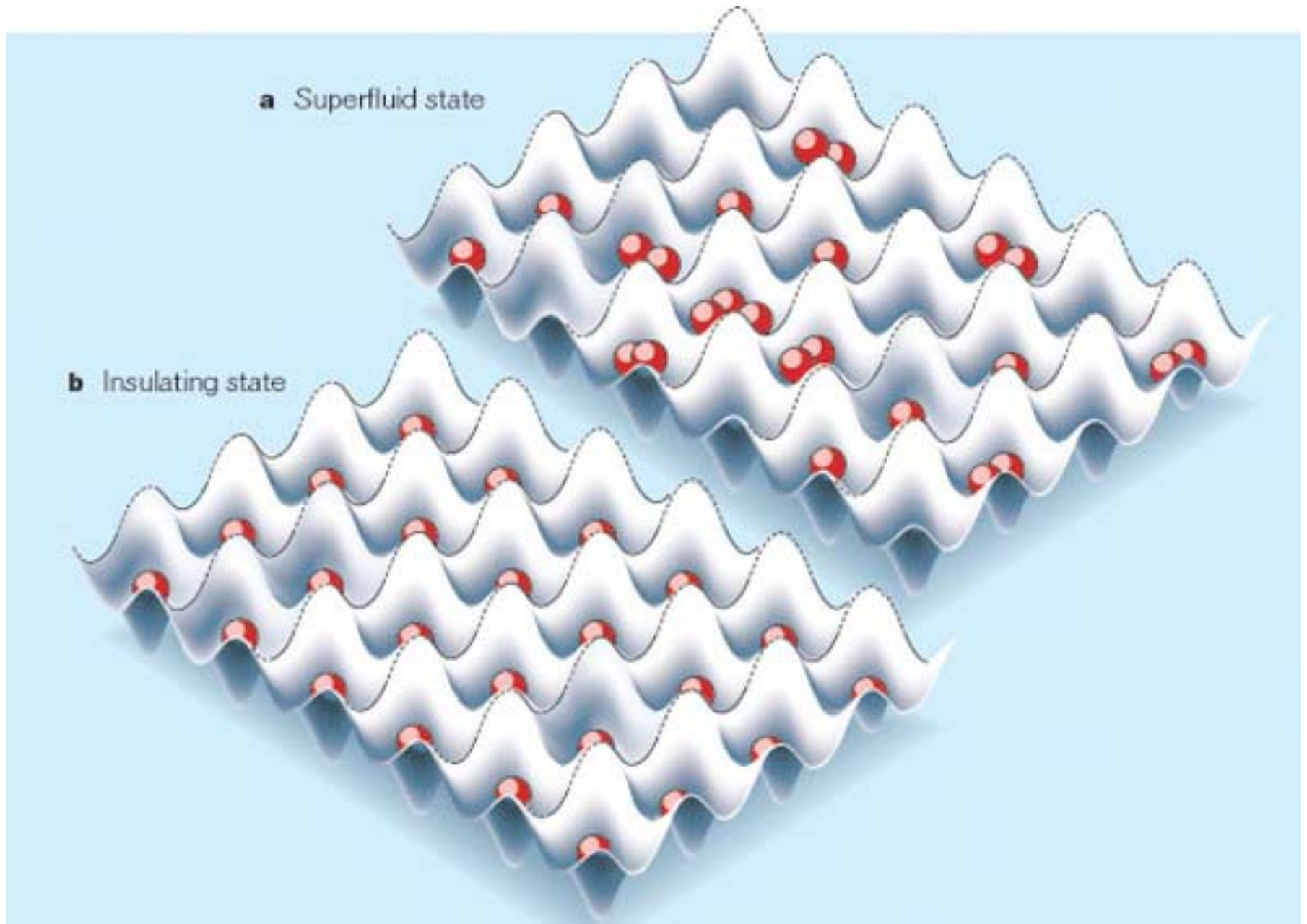
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Trap for ultracold ^{87}Rb atoms



M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Boson Hubbard model

Degrees of freedom: Bosons, b_j^\dagger , hopping between the sites, j , of a lattice, with short-range repulsive interactions.

$$H = -t \sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2} \sum_j n_j (n_j - 1) + \dots$$

$$n_j \equiv b_j^\dagger b_j$$

M.P.A. Fisher, P.B. Weichmann,
G. Grinstein, and D.S. Fisher
Phys. Rev. B **40**, 546 (1989).

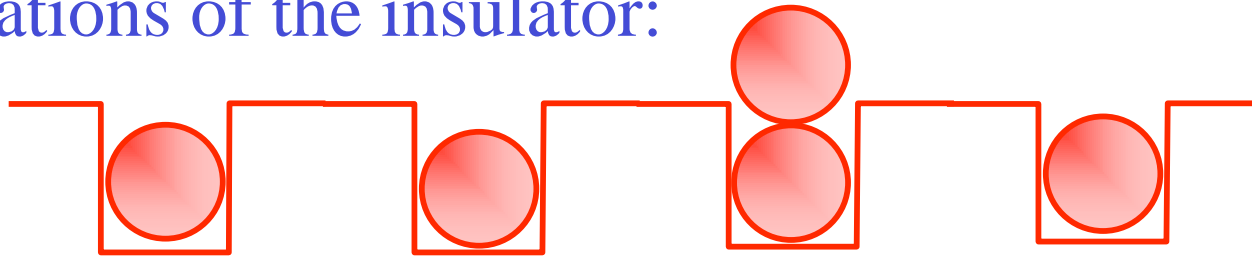
For small U/t , superfluid

For large U/t , insulator

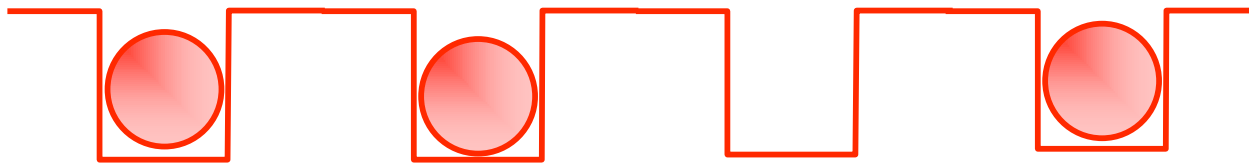
The insulator:



Excitations of the insulator:

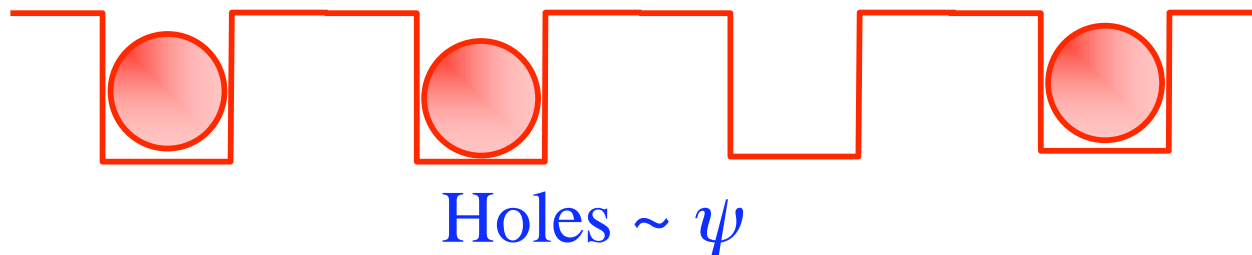
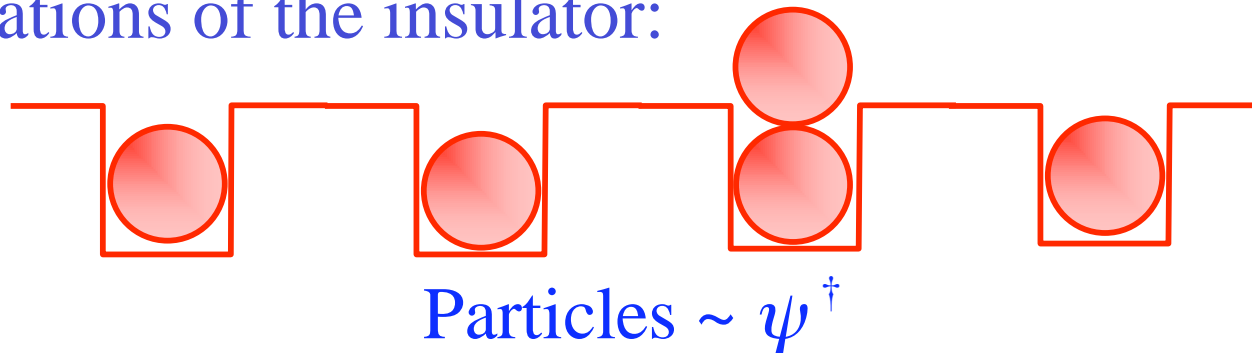


Particles $\sim \psi^\dagger$



Holes $\sim \psi$

Excitations of the insulator:



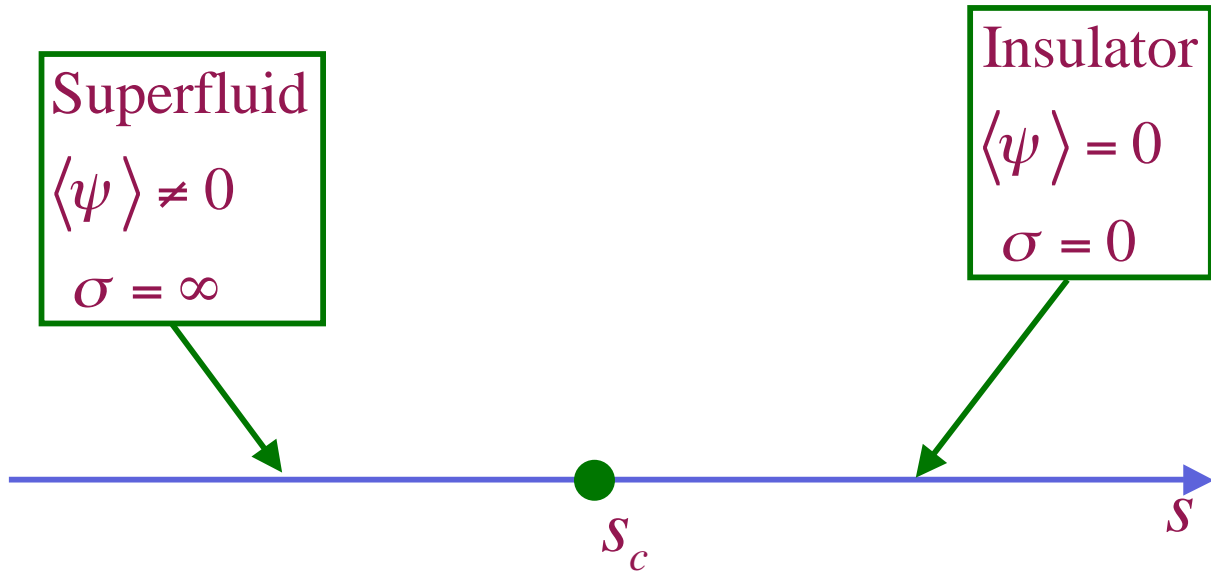
Density of particles = density of holes \Rightarrow

“relativistic” field theory for ψ :

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

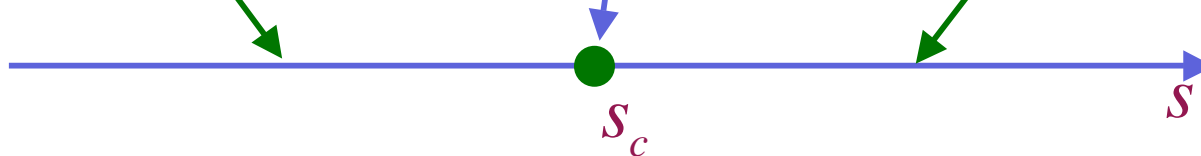
$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$



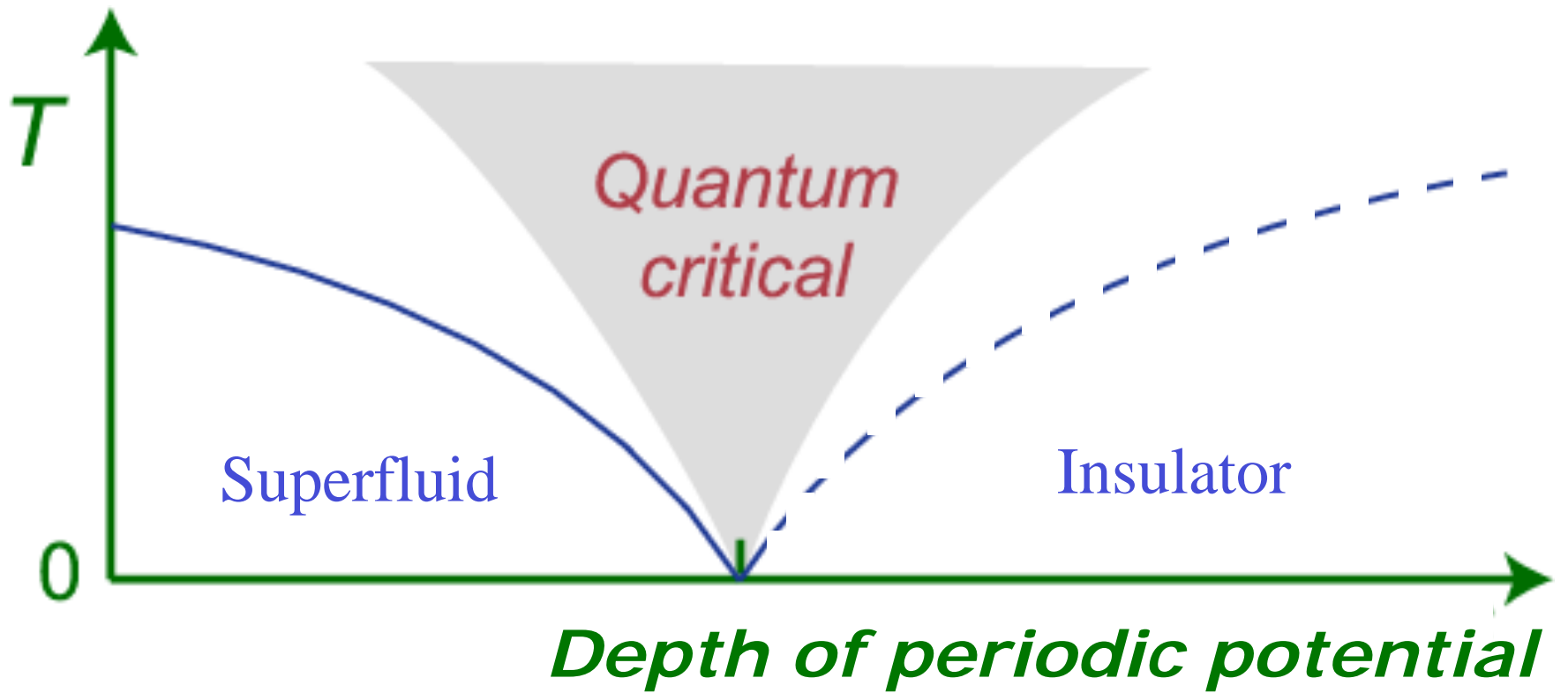
Conformal field theory:
Wilson-Fisher fixed point

Superfluid
 $\langle \psi \rangle \neq 0$
 $\sigma = \infty$

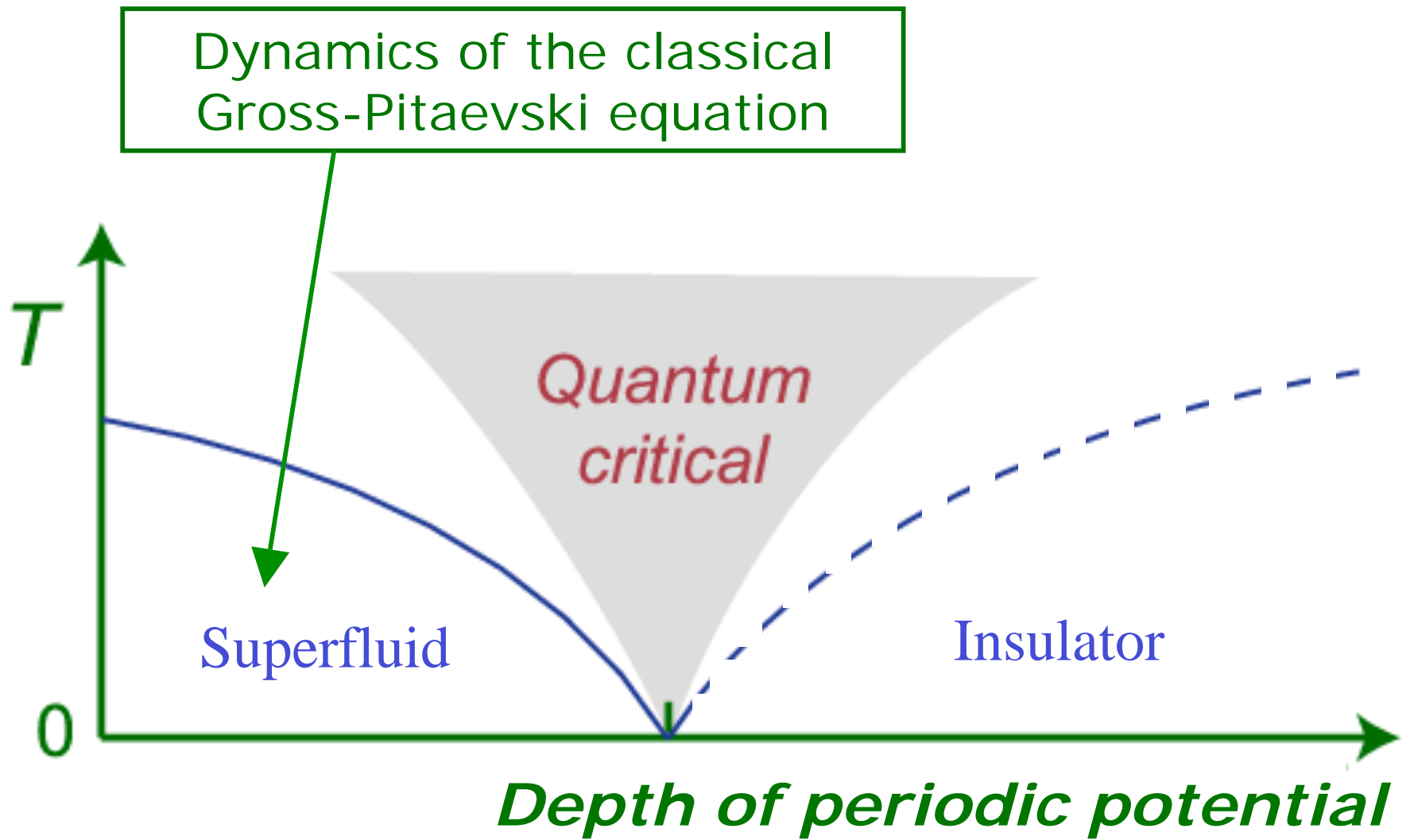
Insulator
 $\langle \psi \rangle = 0$
 $\sigma = 0$



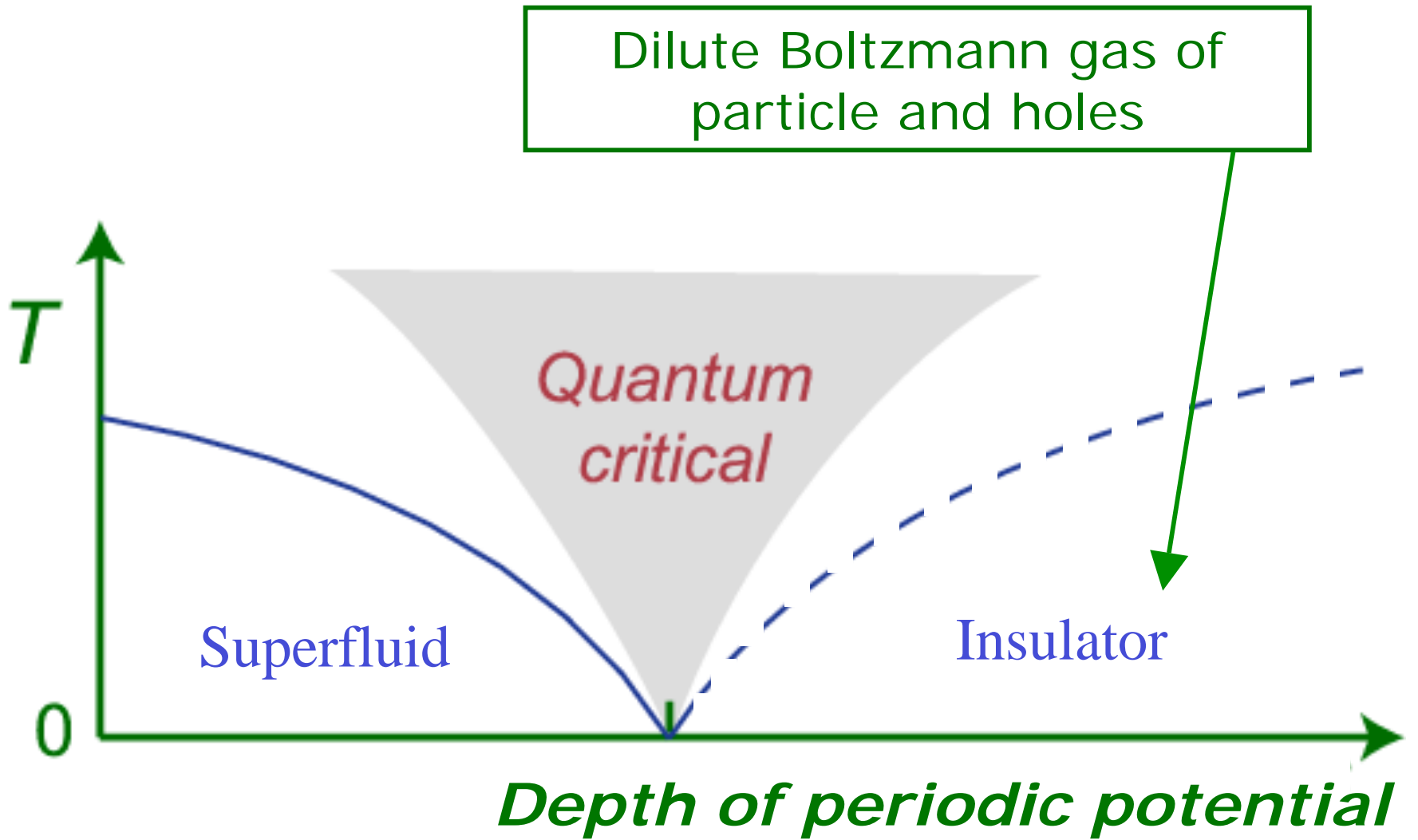
Non-zero temperature phase diagram



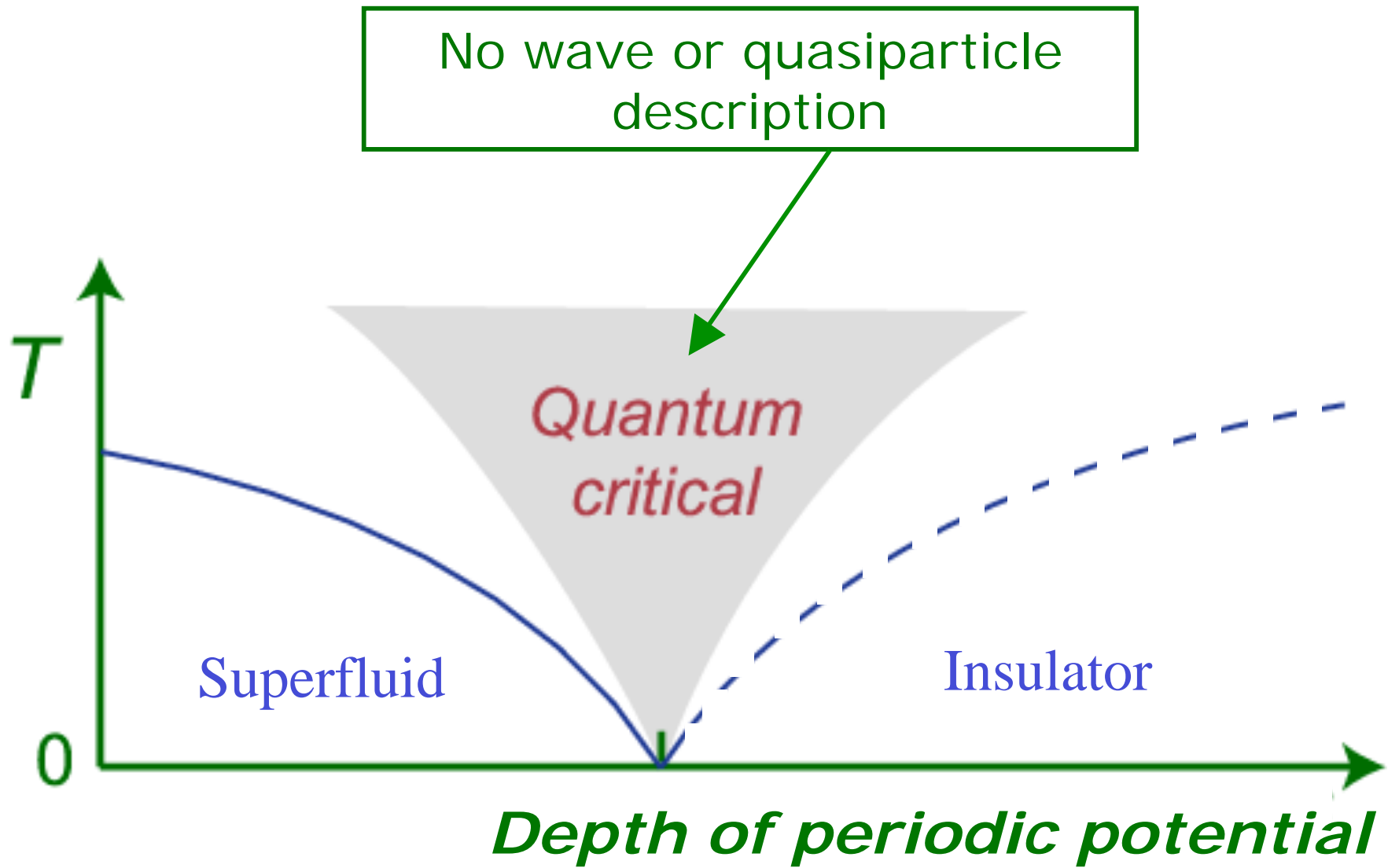
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Non-zero temperature phase diagram



Resistivity of Bi films

Conductivity σ

$$\sigma_{\text{Superconductor}} (T \rightarrow 0) = \infty$$

$$\sigma_{\text{Insulator}} (T \rightarrow 0) = 0$$

$$\sigma_{\text{Quantum critical point}} (T \rightarrow 0) \approx \frac{4e^2}{h}$$

D. B. Haviland, Y. Liu, and A. M. Goldman,
Phys. Rev. Lett. **62**, 2180 (1989)

M. P. A. Fisher, *Phys. Rev. Lett.* **65**, 923 (1990)

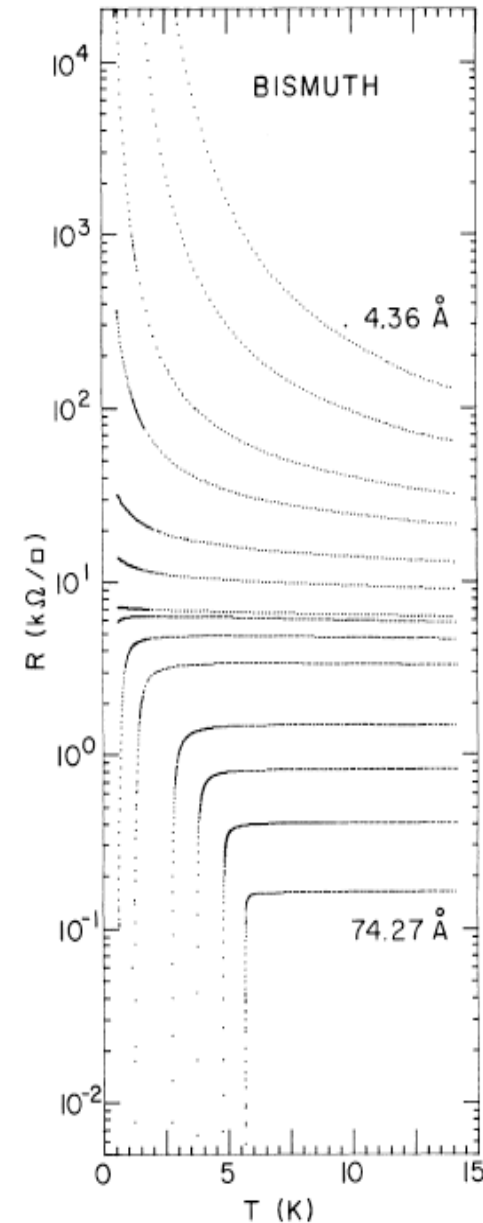
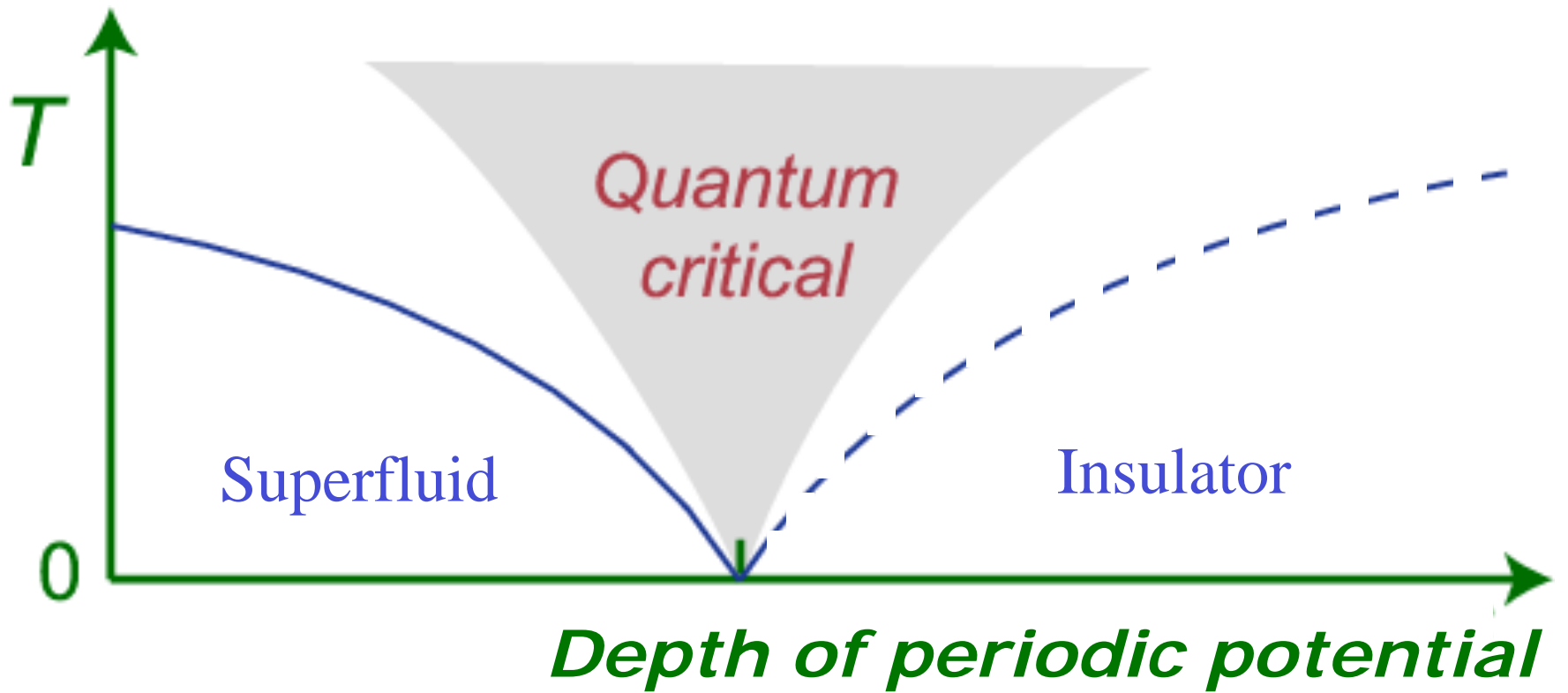
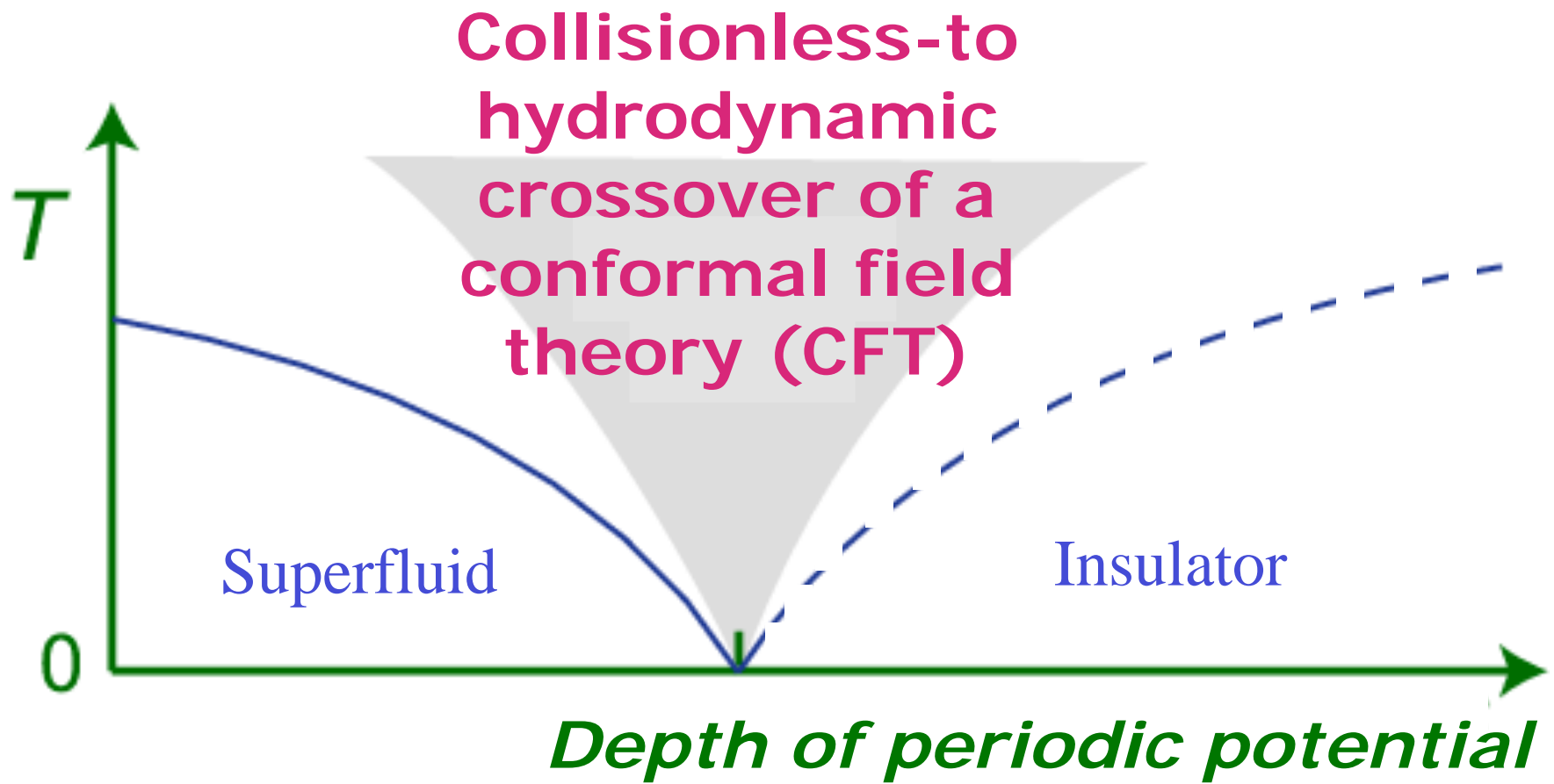


FIG. 1. Evolution of the temperature dependence of the sheet resistance $R(T)$ with thickness for a Bi film deposited onto Ge. Fewer than half of the traces actually acquired are shown. Film thicknesses shown range from 4.36 to 74.27 Å.

Non-zero temperature phase diagram



Non-zero temperature phase diagram



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Collisionless-to-hydrodynamic crossover of a CFT in 2+1 dimensions

Consider the retarded density-density correlation function

$$C(k, \omega) = \langle \rho(k, \omega) \rho(-k, -\omega) \rangle_{\text{ret}}$$

The characteristic collision time for excitations of the CFT is $\hbar/k_B T$. So, for $|\omega - k| \gg T$ we have the collisionless conformal behavior

$$C(k, \omega) = K \frac{k^2}{\sqrt{k^2 - \omega^2}}$$

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$$C(k, \omega) = K \frac{k^2}{\sqrt{k^2 - \omega^2}}$$

while for $\omega, k \ll T$, we have the hydrodynamic behavior

$$C(k, \omega) = \chi \frac{D_c k^2}{-i\omega + D_c k^2}.$$

So the high frequency conductivity $\sigma(\omega \gg T) = K$, and the hydrodynamic conductivity $\sigma(\omega \ll T) = D_c \chi$, and in general $K \neq D_c \chi$.

Hydrodynamics of a conformal field theory (CFT)

The scattering cross-section of the thermal excitations is universal and so transport coefficients are universally determined by $k_B T$

$$\text{Charge diffusion constant } D_c = \Theta \frac{c^2}{k_B T}$$

$$\text{Conductivity } \sigma_Q = D_c \chi = \Theta \frac{4e^2}{h}$$

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Exact solutions of CFTs in 1+1 dimensions

For all 1+1 dimensional CFTs, exact correlators of any conserved current at $T > 0$ can be obtained for all ω , k , and T by the conformal mapping of the plane to the cylinder. After analytic continuation in frequency to the retarded correlator we have, in general

$$C(k, \omega) = K \frac{k^2}{k^2 - \omega^2}$$

where K is a pure number - the central charge of a Kac-Moody algebra.

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There is no collisionless-hydrodynamic crossover for CFTs in 1+1 dimensions. Holomorphic factorization implies absence of collisions between left and right movers.

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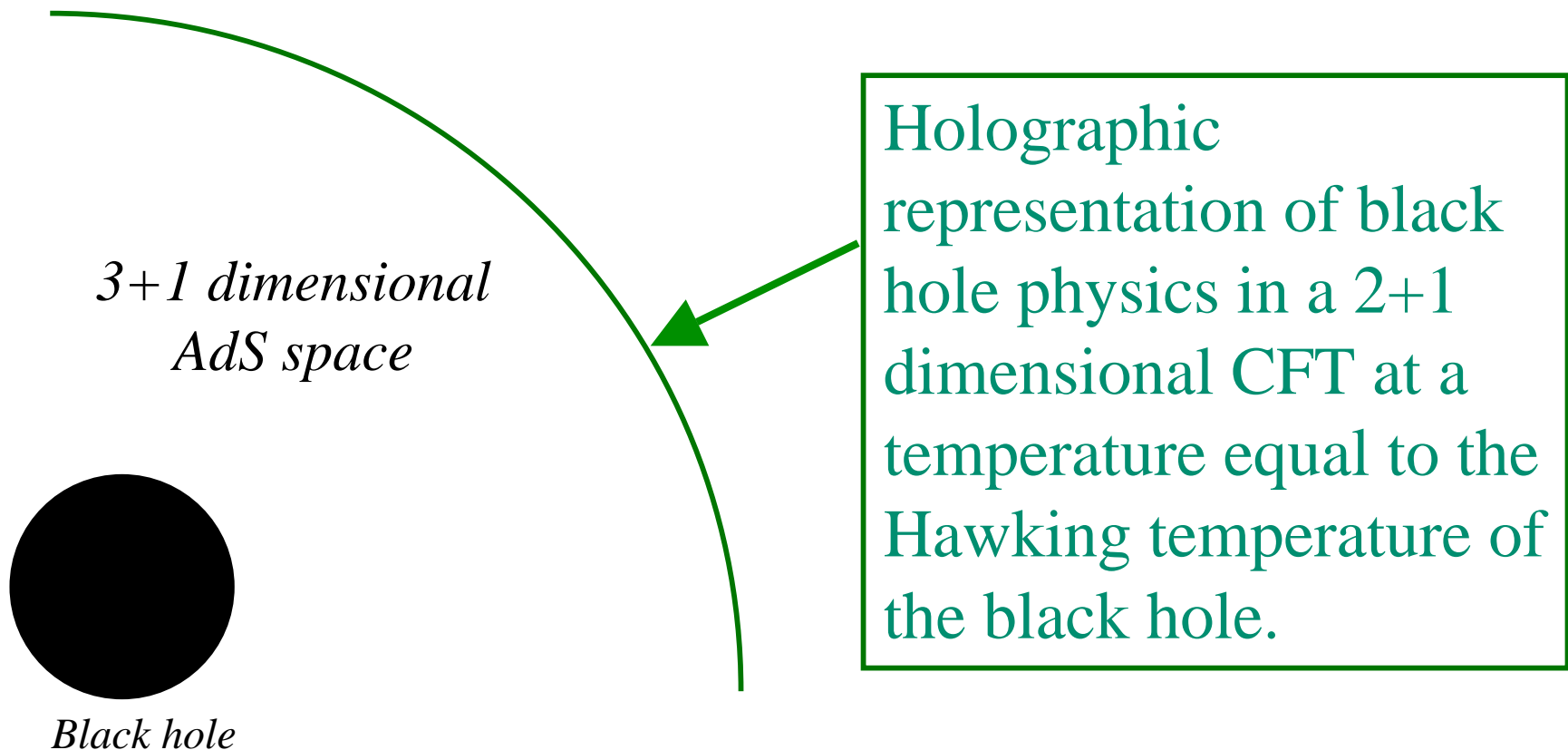
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Hydrodynamics of a conformal field theory (CFT)

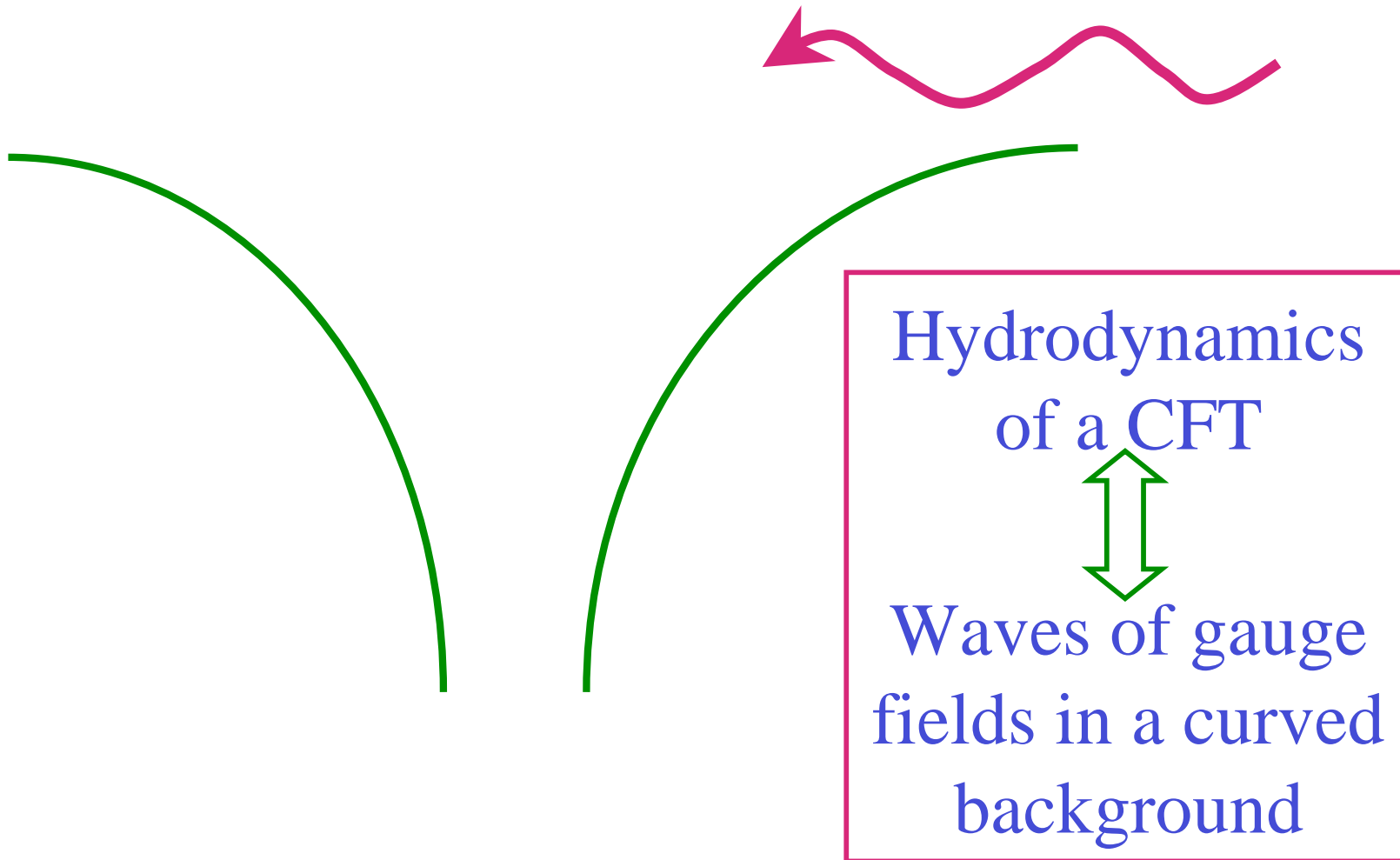
The AdS/CFT correspondence (Maldacena, Polyakov) relates the hydrodynamics of CFTs to the quantum gravity theory of the horizon of a black hole in Anti-de Sitter space.

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Hydrodynamics of a conformal field theory (CFT)

For the (unique) CFT with a $SU(N)$ gauge field and 16 supercharges, we know the exact diffusion constant associated with a global $SO(8)$ symmetry:

$$\text{Spin diffusion constant} \quad D_c = \frac{3}{4\pi} \frac{c^2}{k_B T}$$

$$\text{Spin conductivity} \quad \sigma_Q = \frac{N^{3/2}}{3\sqrt{2}\pi}$$

Collisionless-to-hydrodynamic crossover of solvable SYM₃

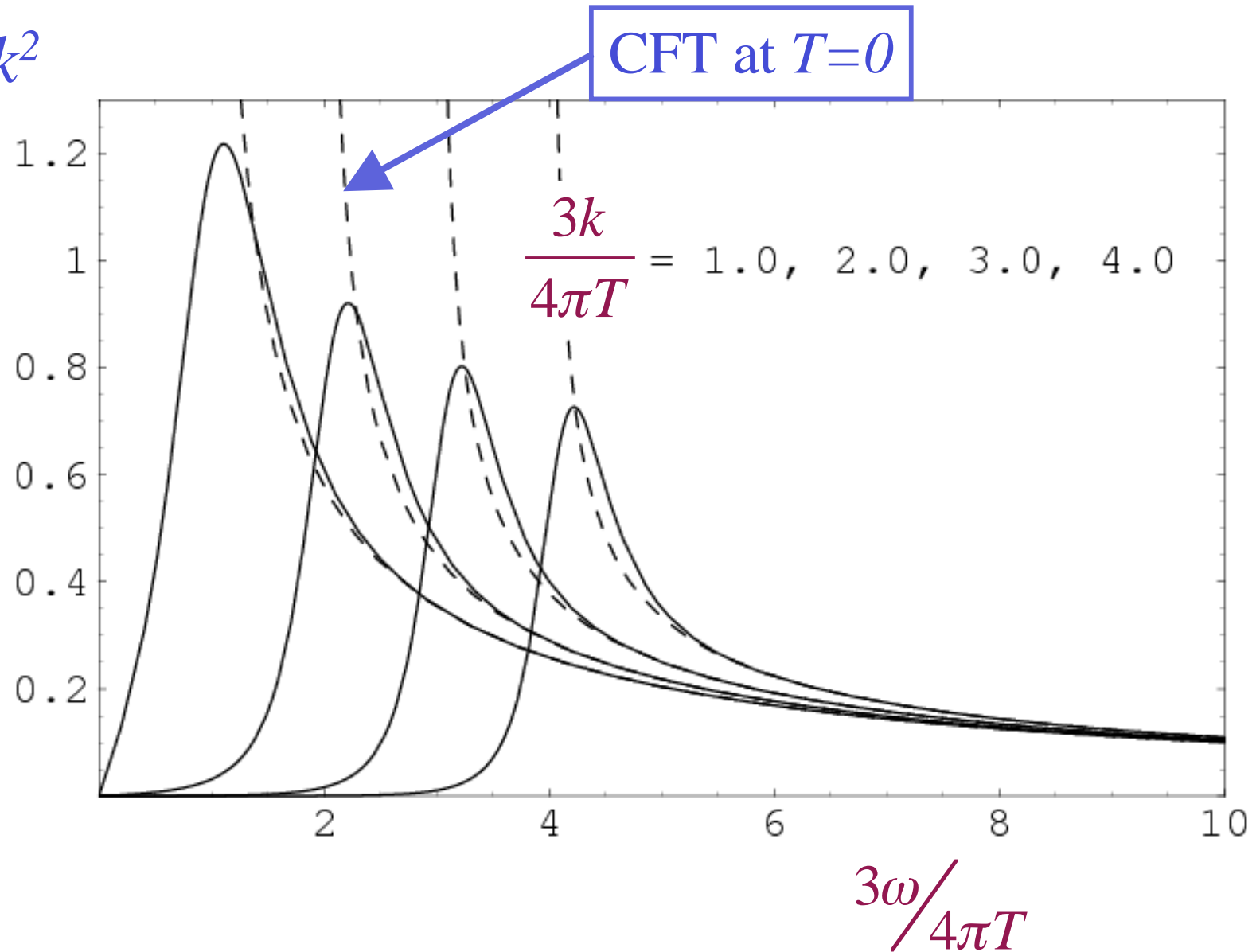
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for $|\omega - k| \gg T$ we have the collisionless conformal behavior

$$C(k, \omega) = K \frac{k^2}{\sqrt{k^2 - \omega^2}}$$

$\text{Im}C/k^2$



P. Kovtun, C. Herzog, S. Sachdev, and D.T. Son, Phys. Rev. D **75**, 085020 (2007)

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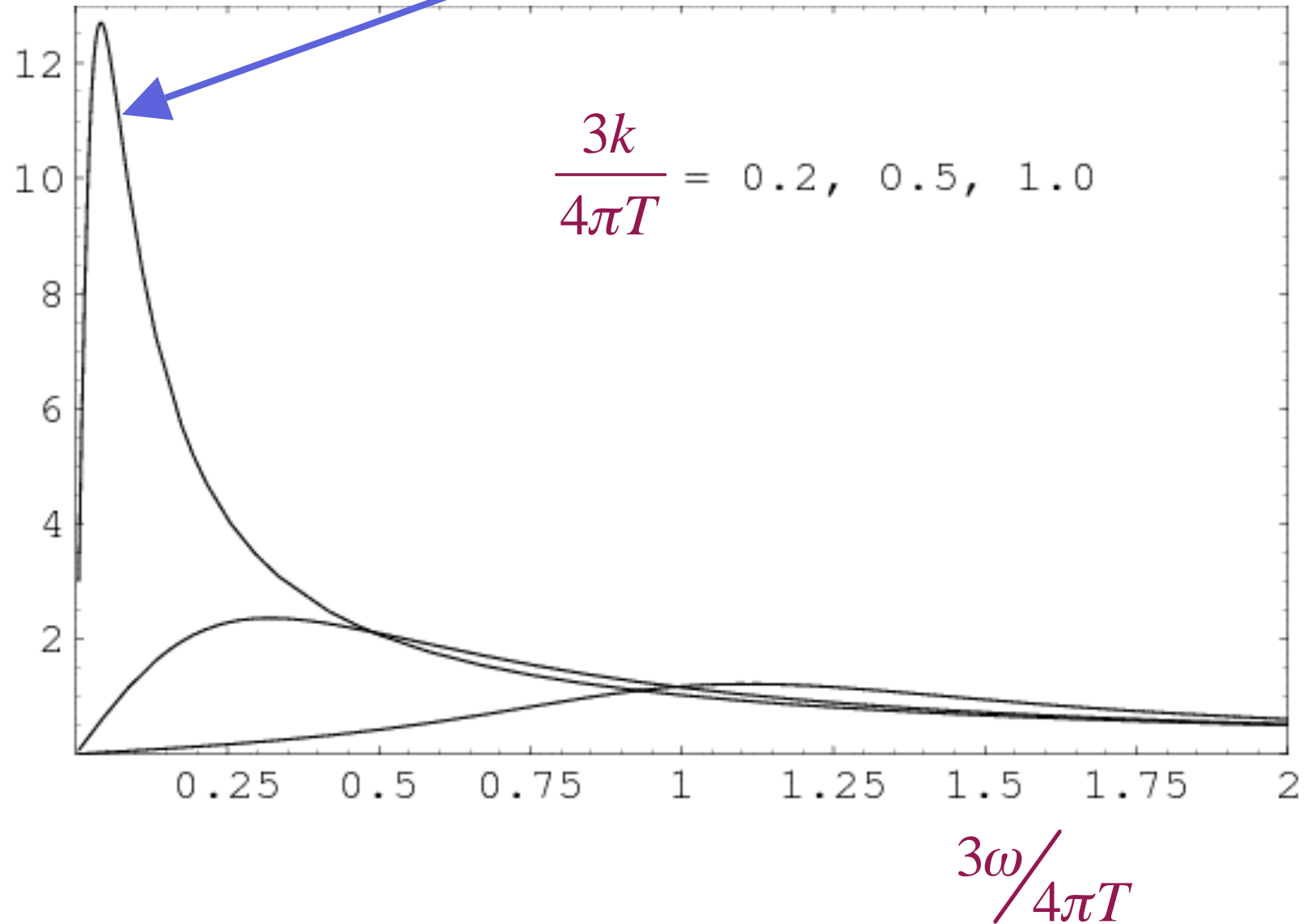
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diffusion peak



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For experimental applications, we must move away from the ideal CFT

$$\text{e.g. } \mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 - g |\psi|^2 + \frac{u}{2} |\psi|^4 \right]$$

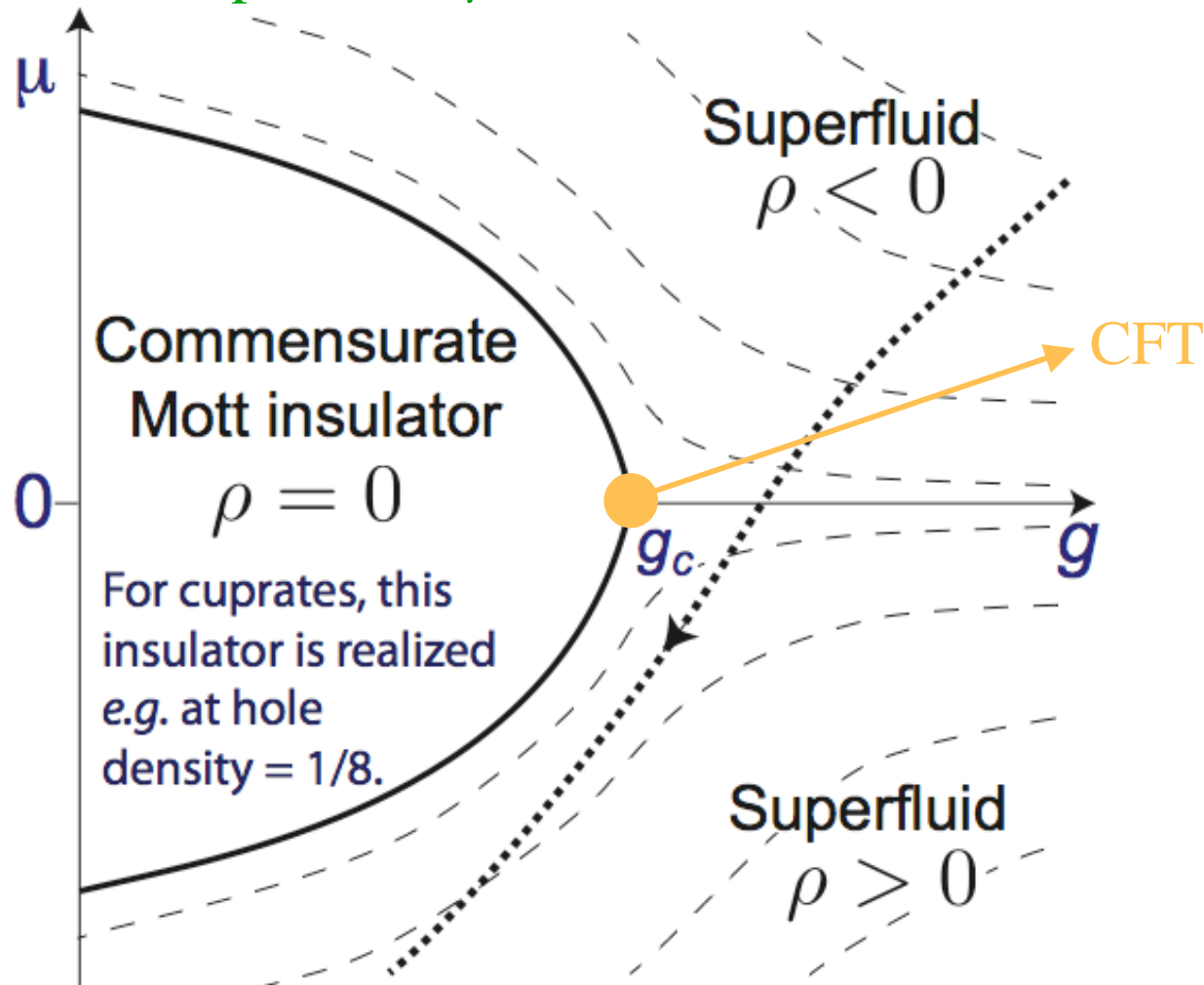
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- A chemical potential μ

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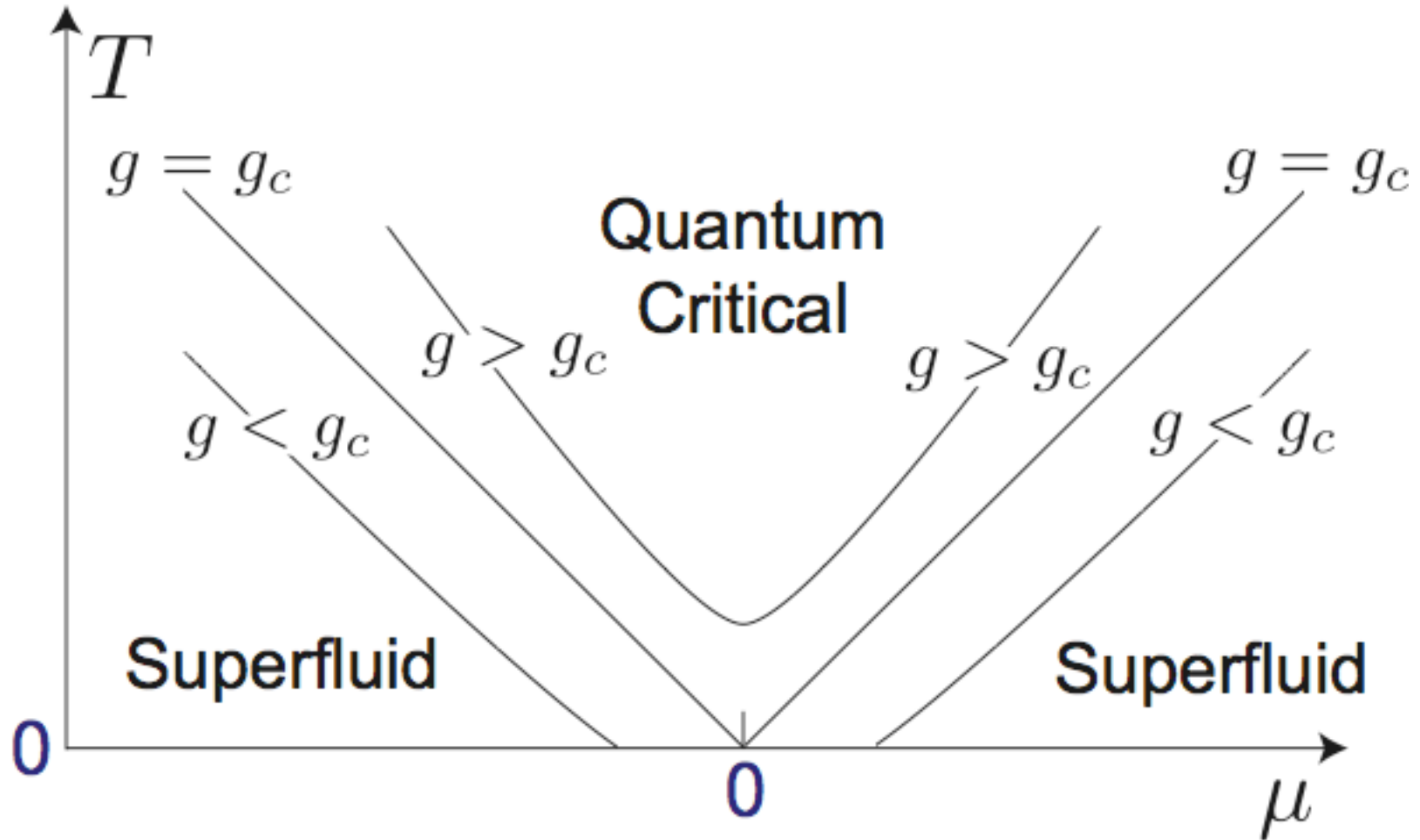
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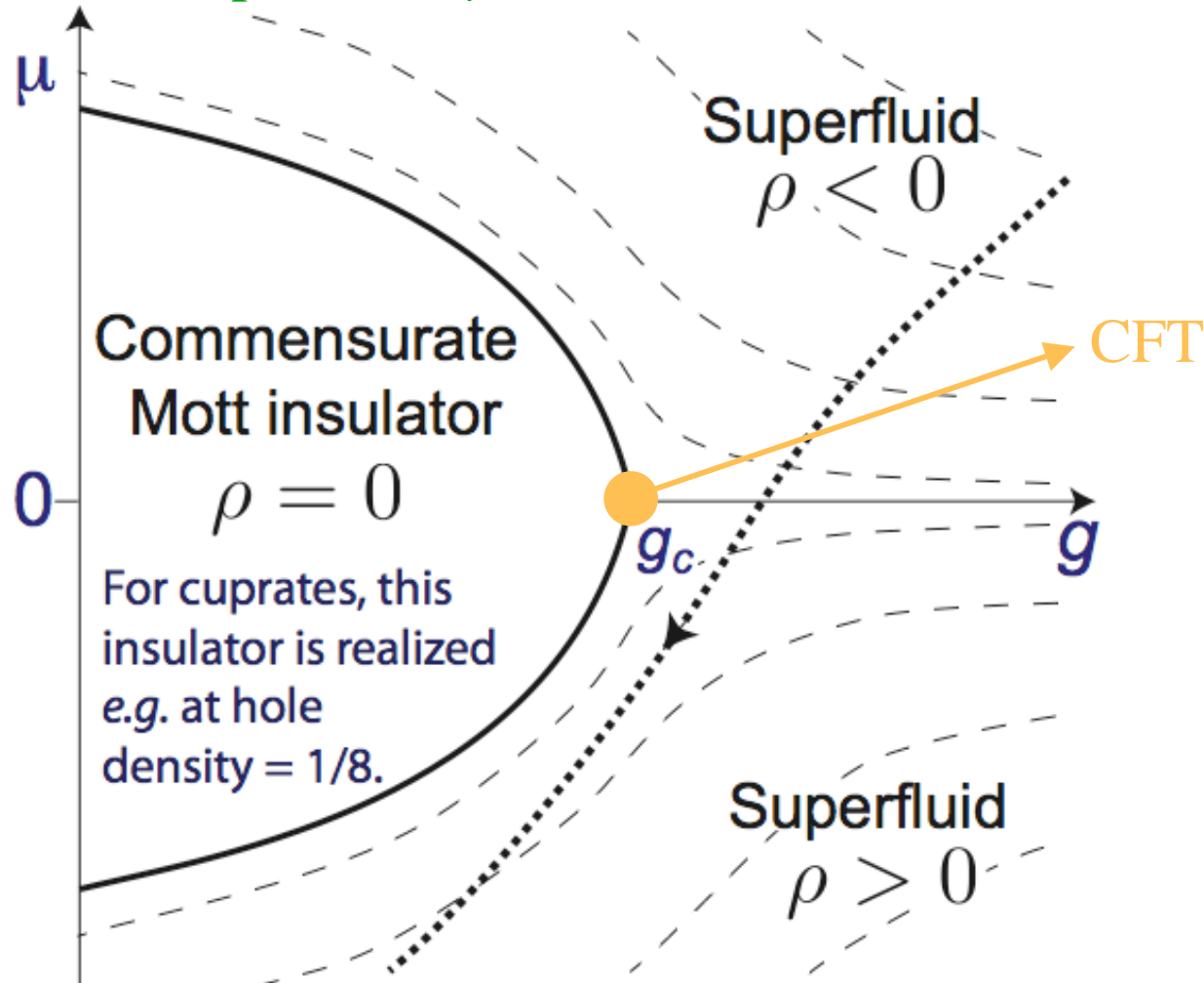
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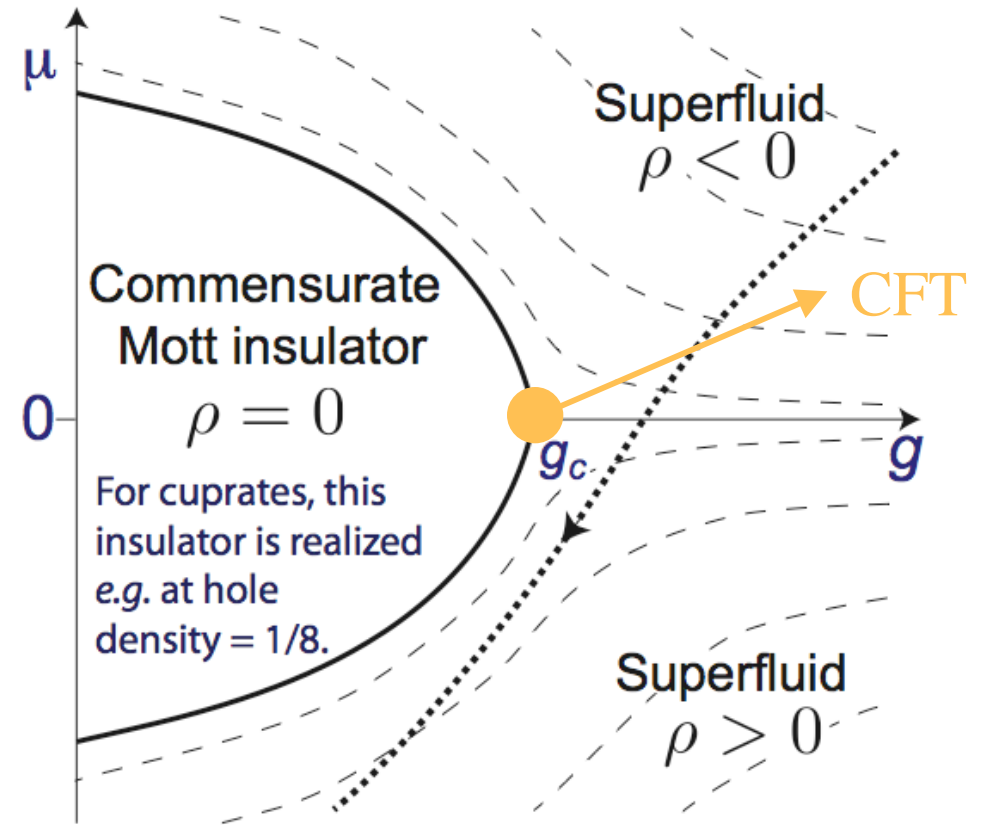
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For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field B



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 - g|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B$$

In the hydrodynamic regime, $\hbar\omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu\nu}$,

$$F^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu\nu}$, the stress energy tensor,
- J^μ , the current,
- ρ , the local number density,
- ε , the local energy density,
- P , the local pressure,
- u^μ , the local velocity, and
- σ_Q , a universal conductivity, which is the **single transport co-efficient**.

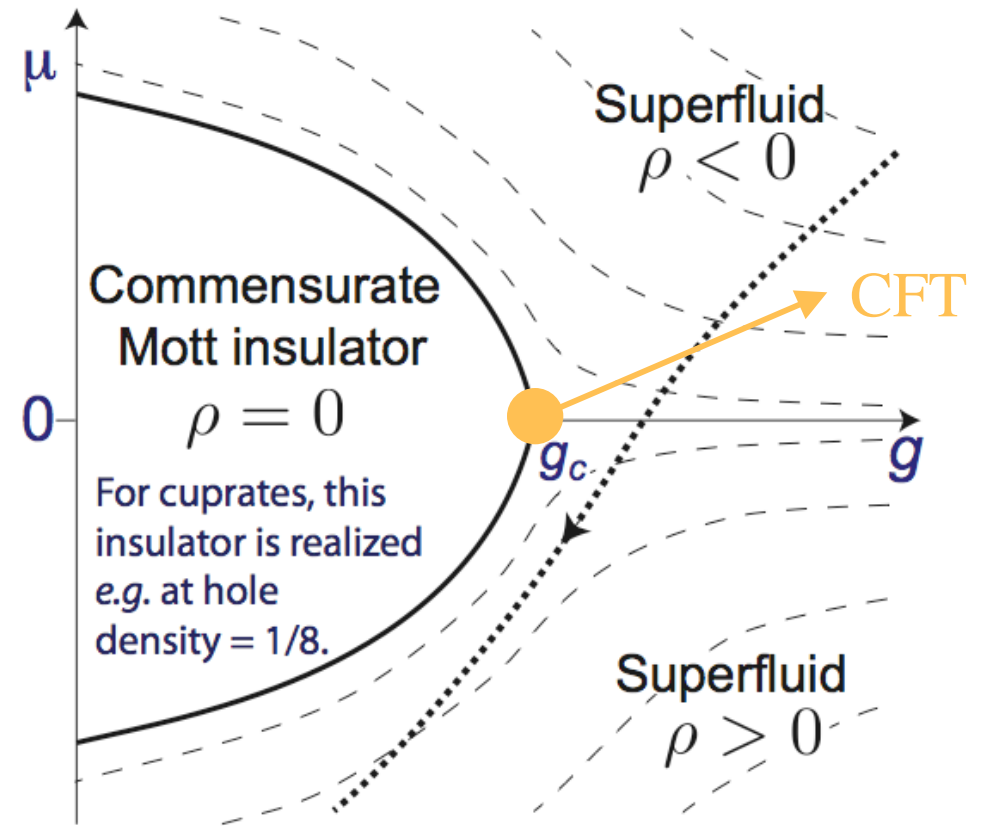
The dependence of ε , P , σ_Q on T and v follows from simple scaling arguments

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\begin{aligned}\partial_\mu J^\mu &= 0 \\ \partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\ T^{\mu\nu} &= (\varepsilon + P)u^\mu u^\nu + P g^{\mu\nu} \\ J^\mu &= \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[(-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]\end{aligned}$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field B



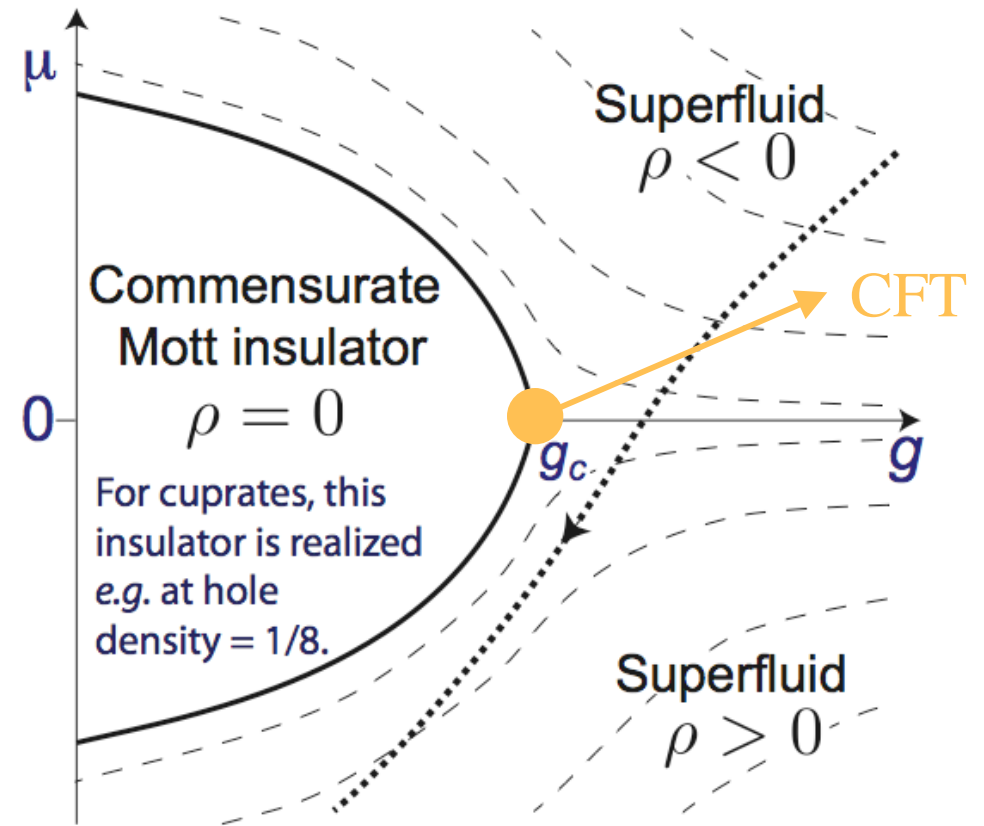
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$$\nabla \times \vec{A} = B$$

For experimental applications, we must move away from the ideal CFT

- A chemical potential μ
- A magnetic field B
- An impurity scattering rate $1/\tau_{\text{imp}}$ (its T dependence follows from scaling arguments)



e.g.

$$\mathcal{S} = \int d^2r d\tau \left[|(\partial_\tau - \mu)\psi|^2 + v^2 |(\vec{\nabla} - i\vec{A})\psi|^2 - g|\psi|^2 + V(r)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\nabla \times \vec{A} = B \quad , \quad \overline{V(r)} = 0 \quad , \quad \overline{V(r)V(r')} = V_{\text{imp}}^2 \delta^2(r - r')$$

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

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From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

Longitudinal conductivity

$$\sigma_{xx} = \sigma_Q \left[\frac{(\omega + i/\tau_{\text{imp}})(\omega + i\gamma + i\omega_c^2/\gamma + i/\tau_{\text{imp}})}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] .$$

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Hall conductivity

$$\begin{aligned} \sigma_{xy} &= -\frac{2e\rho c}{B} \left[\frac{\gamma^2 + \omega_c^2 - 2i\gamma\omega + 2\gamma/\tau_{\text{imp}}}{(\omega + i\gamma + i/\tau_{\text{imp}})^2 - \omega_c^2} \right] \\ &= B \left[\sigma_Q \frac{4e\rho v^2}{(\varepsilon + P)(1/\tau_{\text{imp}} - i\omega)} + \frac{8e^3 \rho^3 v^4}{(\varepsilon + P)^2 (1/\tau_{\text{imp}} - i\omega)^2} \right] \\ &\quad \text{as } B \rightarrow 0 \end{aligned}$$

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 \left[\frac{(\omega_c^2/\gamma)(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B} \right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \end{aligned}$$

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Thermal conductivity

$$\begin{aligned} \kappa_{xx} &= \sigma_Q \left(\frac{k_B^2 T}{4e^2} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right)^2 \boxed{\rightarrow 1 \text{ as } B \rightarrow 0} \\ &= \frac{1}{\sigma_Q} k_B^2 T \left(\frac{c(\varepsilon + P)}{k_B T B} \right)^2 \left[\frac{\gamma(\omega_c^2/\gamma + 1/\tau_{\text{imp}})}{(\omega_c^2/\gamma + 1/\tau_{\text{imp}})^2 + \omega_c^2} \right] \end{aligned}$$

From these relations, we obtained results for the transport co-efficients, expressed in terms of a “cyclotron” frequency and damping:

$$\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \sigma_Q \frac{B^2 v^2}{c^2(\varepsilon + P)}$$

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Nernst signal

$$e_N = \left(\frac{k_B}{2e} \right) \left(\frac{\varepsilon + P}{k_B T \rho} \right) \left[\frac{\omega_c / \tau_{\text{imp}}}{(\omega_c^2 / \gamma + 1 / \tau_{\text{imp}})^2 + \omega_c^2} \right]$$
$$\frac{k_B}{2e} = 43.086 \mu\text{V/K}$$

Outline

Transport near strongly interacting quantum critical points

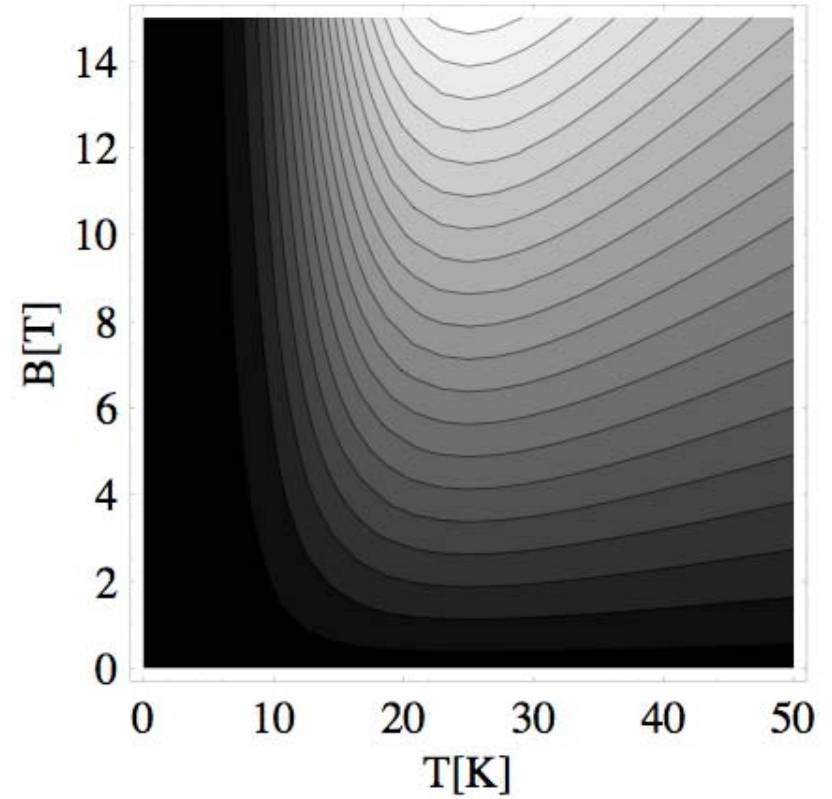
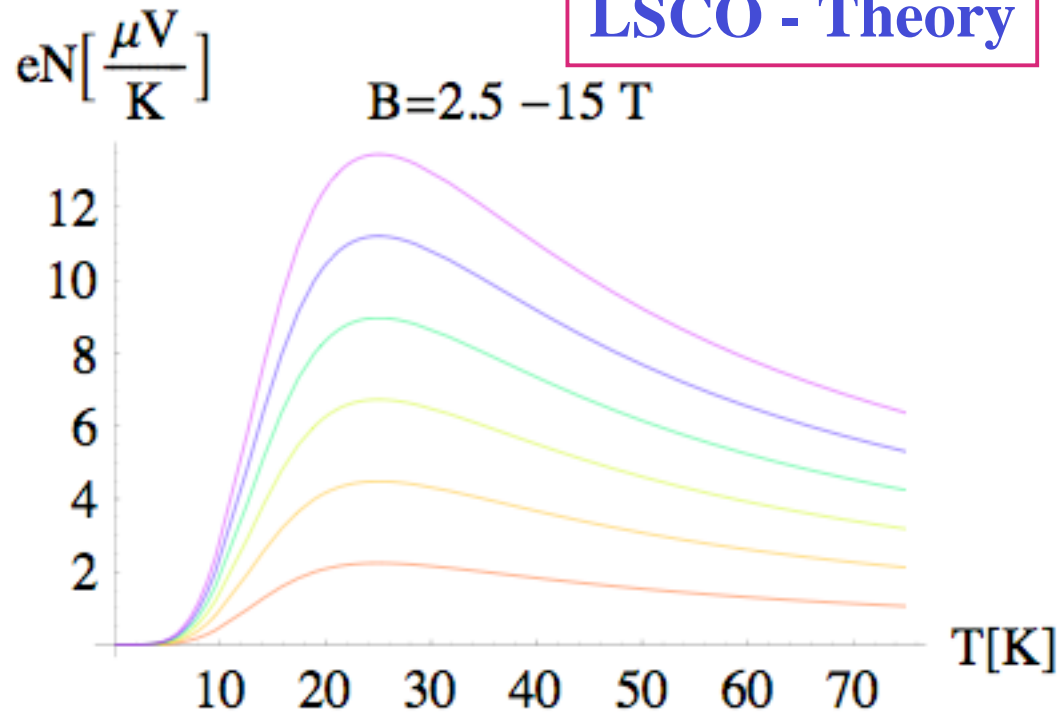
1. The superfluid-insulator transition in the boson Hubbard model:
Hydrodynamic-collisionless crossover of a CFT
2. Exact solutions of CFTs in 1+1 dimensions
No hydrodynamics
3. Exact solution of a CFT in 2+1 dimensions - Yang-Mills theory
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Black holes in AdS₄
4. General hydrodynamic theory in the presence of a magnetic field,
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LSCO - Theory



Only input parameters

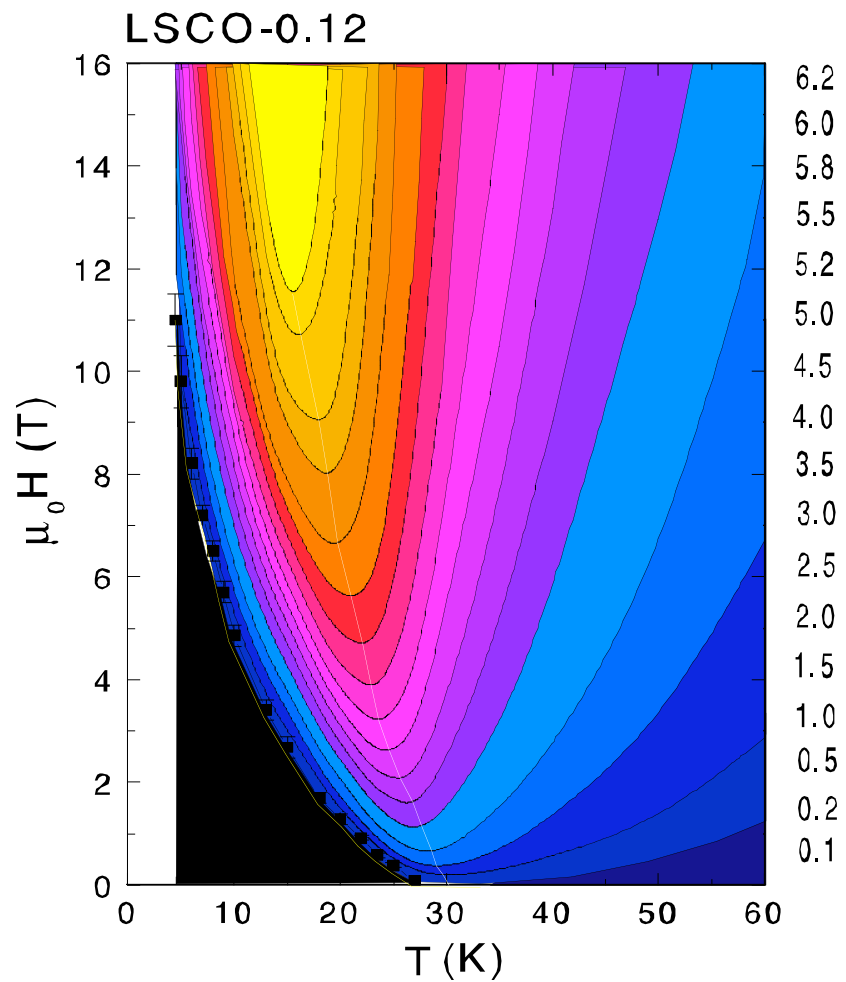
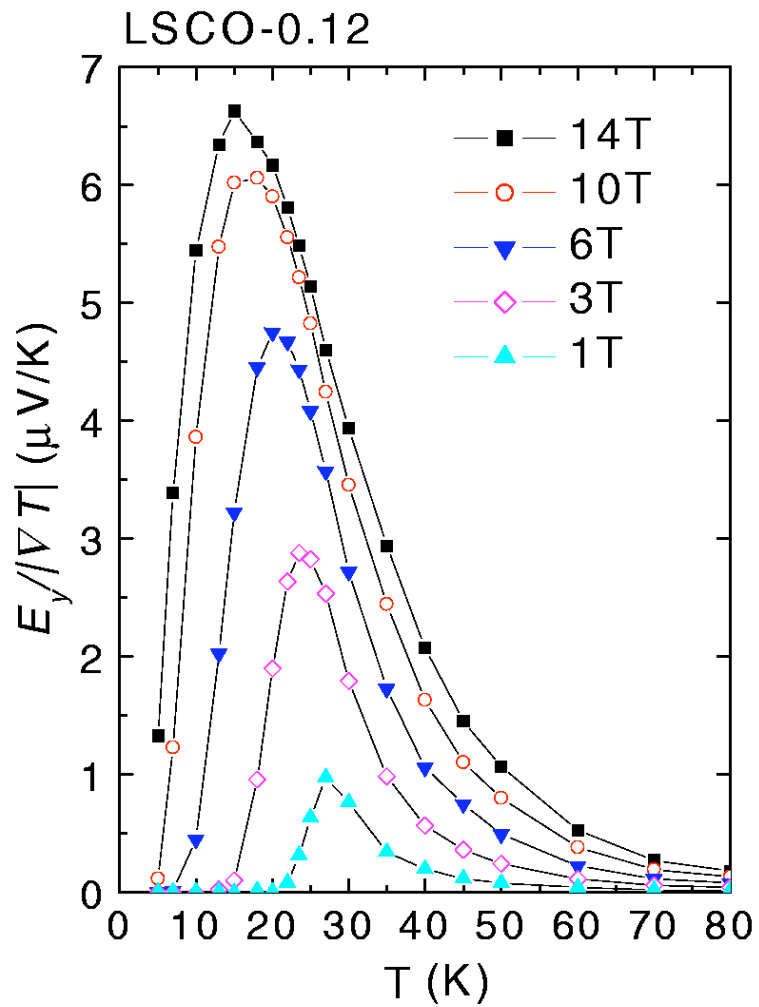
$$\hbar v = 15 \text{ meV } \text{\AA}$$

$$\tau_{\text{imp}} = 6.25 \cdot 10^{-12} \text{ s}$$

Output

$$\omega_c = 180 \text{ MHz} \cdot \frac{B}{1 \text{ T}} \left(\frac{25 \text{ K}}{T} \right)^3$$

LSCO - Experiments



N. P. Ong *et al.*

Outline

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To the solvable supersymmetric, Yang-Mills theory CFT, we add

- A chemical potential μ
- A magnetic field B

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

The exact results are found to be in *precise* accord with *all* hydrodynamic results presented earlier

Conclusions

- General theory of transport in a weakly disordered ``vortex liquid'' state.
- “Relativistic” magnetohydrodynamics offers an efficient approach to disentangling momentum and charge transport
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Simplest model reproduces many trends of the Nernst measurements in cuprates.