Fluctuations in systems with superconducting islands

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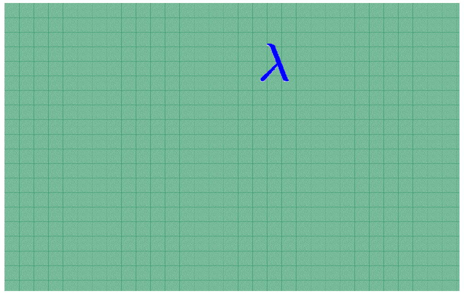
Outline

- Homogeneously disordered materials, grains, islands...
- **Preformed** superconducting islands in a metallic matrix
  - Phase transition
  - Dephasing
- **Fluctuation-induced** superconducting islands close to T_c due to mesoscopic fluctuations

Uniform materials, grains, islands

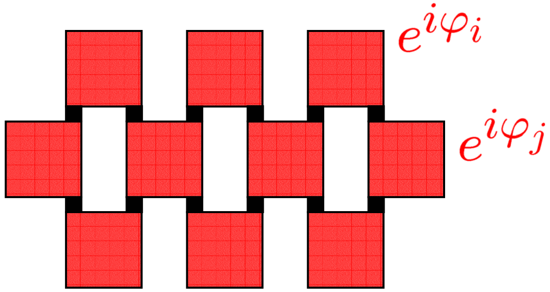
- **Homogeneously** disordered bulk materials

  Cooper-channel attraction λ

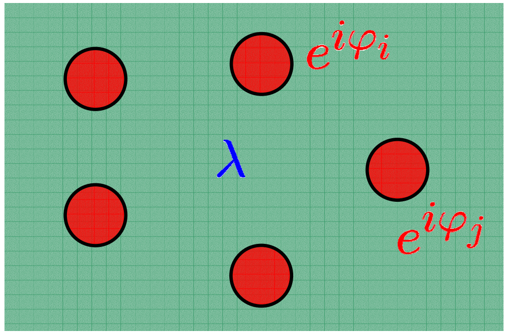


- **Granular** superconductors, Josephson junction arrays

  grains ($|\Delta|e^{i\varphi}$) + weak links



- Superconducting **islands** in a metallic matrix



Phases like in grains

Inter-island transport like in films

Two mechanism of T_c suppression

- **Fermionic** mechanism

  homogeneously disordered films

  mean-field BCS theory + Coulomb correction to $\lambda(E)$

  vanishing of $\lambda \longrightarrow$ vanishing of $|\Delta|$

- **Bosonic** mechanism

  granular superconductors,
  Josephson junction arrays

  superconducting grains ($|\Delta|e^{i\varphi}$) connected by weak links

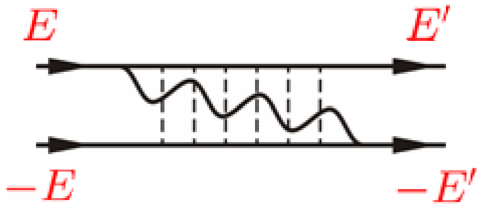
  fluctuations of φ suppress macroscopic phase coherence

Superconducting-metal transitions

Films: Coulomb suppression of T_c

Finkelstein (1987)

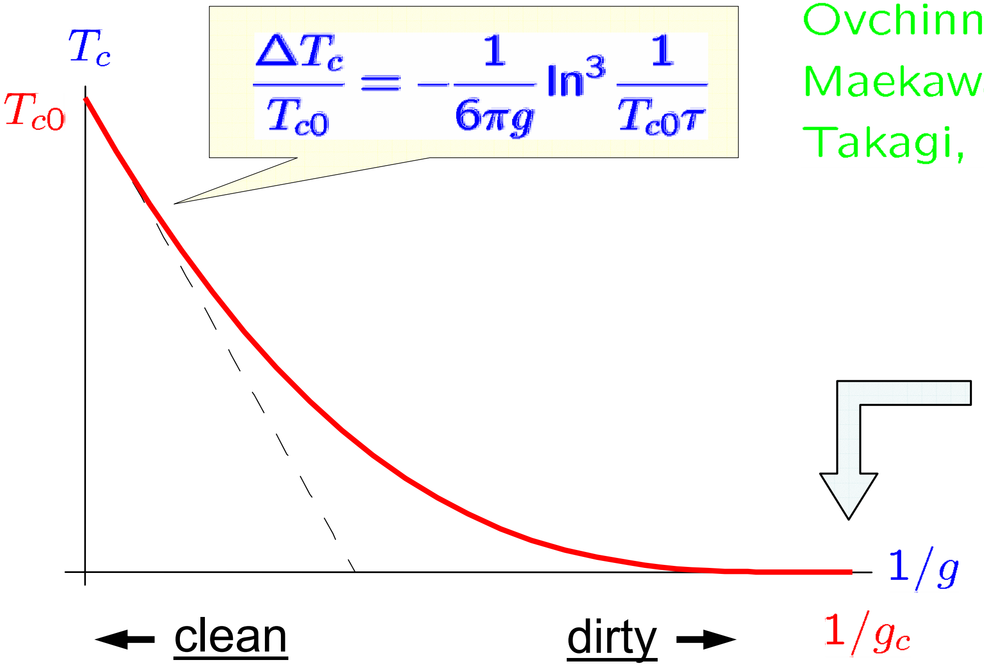
Coulomb correction to the Cooper channel at $E < \tau^{-1}$:



RG equation for $\lambda(E)$:

$$\frac{\partial\lambda}{\partial\zeta} = -\lambda^2 + \frac{1}{2\pi g}$$

$$\zeta = \ln\frac{1}{E\tau}, \qquad g = \frac{h}{e^2 R_\square} \gg 1$$



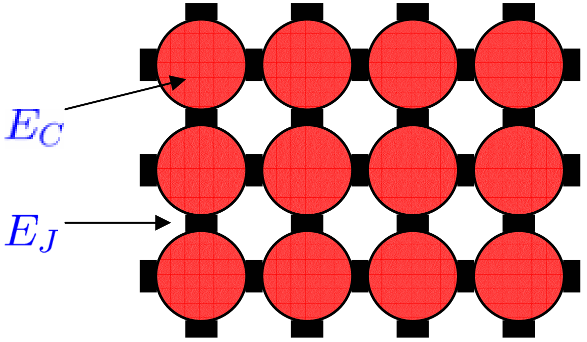
$$\frac{\Delta T_c}{T_{c0}} = -\frac{1}{6\pi g}\ln^3\frac{1}{T_{c0}\tau}$$

Ovchinnikov (1973)
Maekawa, Fukuyama (1982)
Takagi, Kuroda (1982)

$$g_c = \frac{1}{2\pi}\ln^2\frac{1}{T_{c0}\tau}$$

Grains: Superconductive transition

From local to global coherence

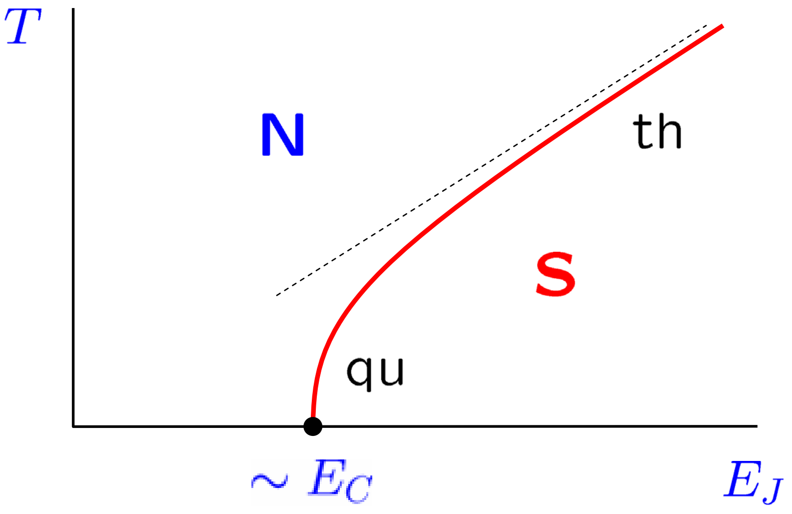


$$S = \sum_i \int_0^\beta d\tau \frac{\dot{\varphi}_i^2}{E_C} + \sum_{\langle i,j \rangle} \int_0^\beta d\tau E_J \cos(\varphi_i - \varphi_j)$$

MF transition: $\mathcal{J} \cdot \int_0^\beta d\tau \, C(\tau) = 1$

$\mathcal{J} = \sum_j (E_J)_{ij}$

$C(\tau) = \langle e^{i\varphi(\tau) - i\varphi(0)} \rangle$



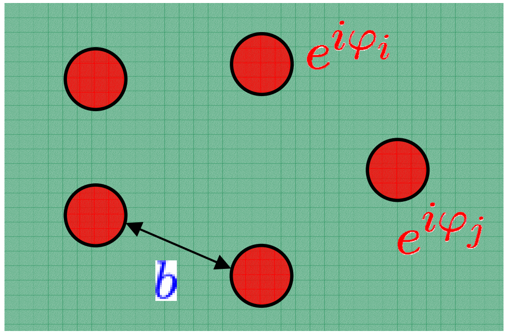
(Efetov 1980)

Columb blockade regime:

$$C(\tau) = \exp(-E_C \tau)$$

$$\int_0^\infty C(\tau) \, d\tau = \frac{1}{E_C}$$

Islands on a film: what's the difference?



- **Josephson proximity coupling:**

$$E_J(r) \propto \frac{1}{r^2} e^{-r/L_T}, \quad L_T = \sqrt{D/2\pi T}$$

short-range at $T > D/b^2$

long-range at $T < D/b^2$

- **Single-island phase dynamics:**

$$S = T \sum_n \frac{\omega_n^2}{E_C} |\varphi_n|^2 + T \sum_n G_A |\omega_n| (e^{i\varphi})_n (e^{-i\varphi})_{-n}$$

Andreev conductance of the island

$$G_A \int\int d\tau\, d\tau' \frac{\cos[\varphi(\tau) - \varphi(\tau')]}{(\tau - \tau')^2}$$

Ambegaokar, Eckern, Schön

$G_A > 1$ kills the Coulomb blockade

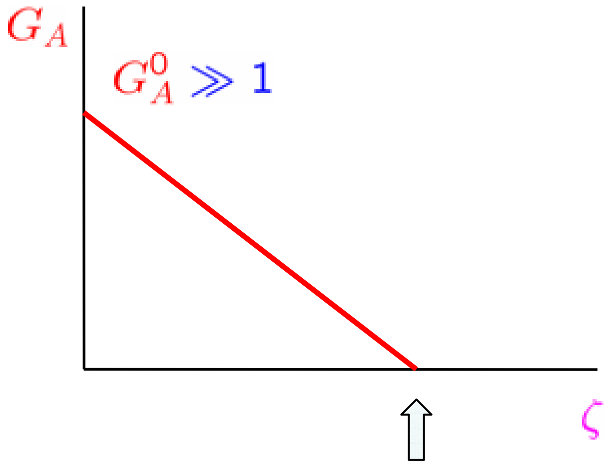
$\Downarrow$

What about the QPT? Always superconductive?

Weak Coulomb blockade

$$S = T\sum_n \frac{\omega_n^2}{E_C}|\varphi_n|^2 + T\sum_n G_A|\omega_n|(e^{i\varphi})_n(e^{-i\varphi})_{-n}$$

at $\omega < G_A^0 E_C$: $\boxed{\frac{dG_A}{d\zeta} = -8}$ $\zeta = \ln \frac{G_A^0 E_C}{\omega}$



$$\zeta_c = \ln \frac{G_A^0 E_C}{\tilde{E}_C} = \frac{G_A^0}{8}$$

$C(\tau) = \langle e^{i\varphi(\tau) - i\varphi(0)} \rangle$ decays

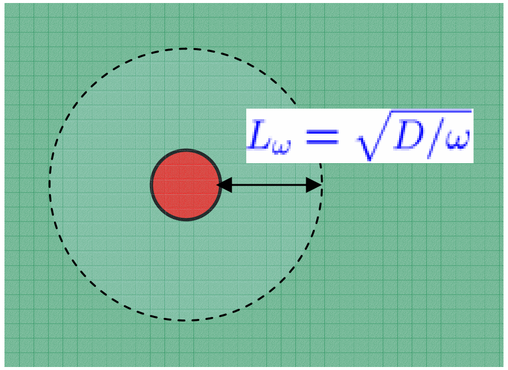
at the time scale $\tau \sim 1/\tilde{E}_C$

$\boxed{\text{QPT at } \tilde{E}_C \sim \mathcal{J}}$

Effective charging energy: $\boxed{\tilde{E}_C \sim E_C (G_A^0)^4 e^{-G_A^0/8}}$

Beloborodov, Andreev, Larkin (2003)

Andreev conductance + multicharge action



Depends on the ratio G_T/G_D:

$$G_A(\omega) = \begin{cases} G_T^2/G_D(\omega), & G_T \ll G_D \\ G_D(\omega), & G_T \gg G_D \end{cases}$$

2D

$$1/G_D(\omega) = \frac{1}{4\pi g} \ln \frac{D}{d^2\omega}$$

At ω such that $G_T \sim G_D(\omega)$
there is still no Coulomb blocking

Multicharge proximity action: $S_{AES} = G_A \operatorname{Tr}(Q_S\Lambda)^2 \Rightarrow \sum_m \gamma_m \operatorname{Tr}(Q_S\Lambda)^{2m}$

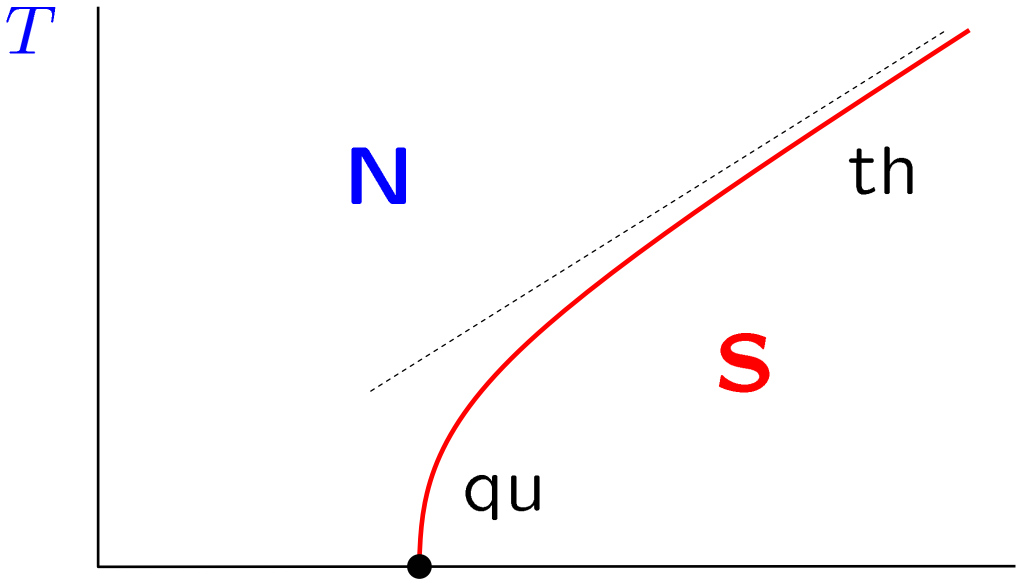
Renormalization of γ's due to

- phase fluctuations
- ω-dependence of $G_D(\omega)$
- Cooper-channel renormalization

$$\tilde{E}_C \sim e^{-3.1\sqrt{g}}$$

Feigel'man, Larkin, M.S. (2001)

Islands: Superconductive transition



$n_c \sim d^{-2}e^{-3.1\sqrt{g}}$

$n_i = 1/b^2 \quad (\propto \mathcal{J})$

islands concentration

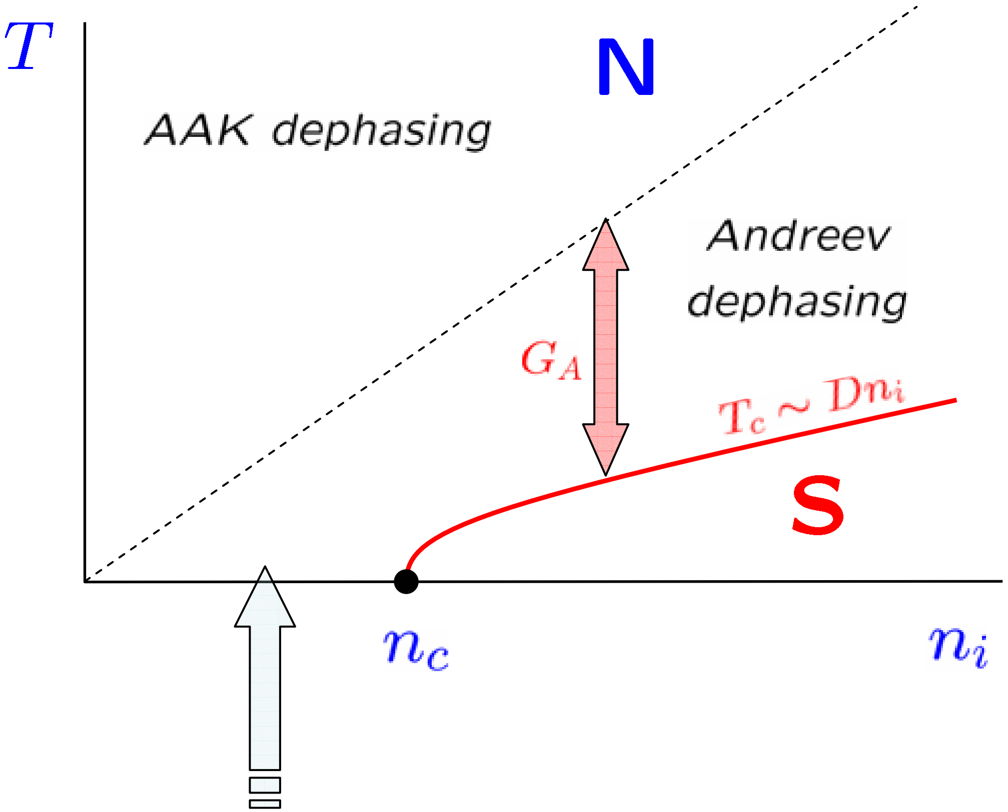
At fixed n_i, QPT takes place at $R_\Box \approx \frac{h}{e^2}\frac{2}{\ln^2(b/d)}$

inter-island distance *island size*

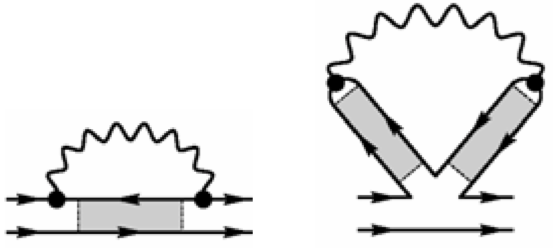
Dephasing due to Andreev reflection

Dephasing due to Andreev scattering off the islands

Larkin, Feigel'man, M.S. (2004)



$$\frac{1}{\tau_\varphi^{\mathrm{A}}} \propto T \qquad (T \ll \tilde{E}_C)$$



Dephasing by Andreev scattering

$$\frac{1}{\tau_\varphi^{\mathrm{A}}} \sim \frac{T_c}{g} G_A(T) \qquad (T \gg T_c)$$

AAK dephasing rate in 2D

$$\frac{1}{\tau_\varphi^{(0)}} \sim \frac{T}{g} \ln g$$

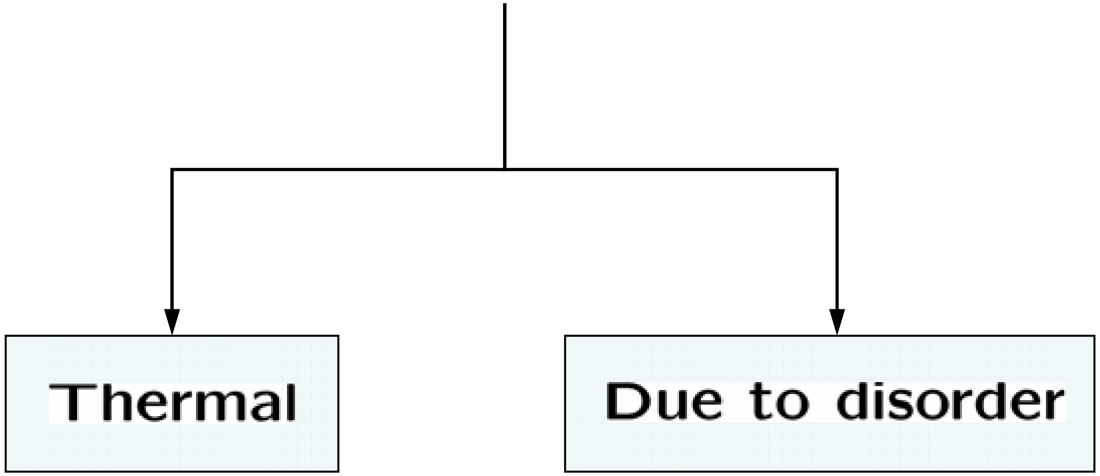
Dephasing in homogeneous films

$$\frac{1}{\tau_\varphi^{(fl)}} \sim \frac{T}{g} \frac{T}{T - T_c}$$

Abrahams, Wolfle, et al. (1985)

Droplets near the superconducting transition

Fluctuations near T_c



Thermal

Due to disorder

Thermal fluctuations: disorder and dimensionality

Ginzburg number

		clean $(l > \xi_0)$	dirty $(l < \xi_0)$
	3D	$\frac{1}{(k_F\xi_0)^4}$	$\frac{1}{k_F^4\xi_0 l^3}$
film of thickness d:	2D	$\frac{1}{k_F^2\xi_0 d}$	$\frac{1}{k_F^2 l d}$
wire of area S:	1D	$\frac{1}{(k_F^2 S)^{2/3}}$	$\frac{(\xi_0/l)^{1/3}}{(k_F^2 S)^{2/3}}$

$$\frac{1}{k_F^2 l d} \rightarrow \frac{1}{g}$$

$$g = \frac{h}{e^2 R_\square} \gg 1$$

dimensionless
sheet conductance

$$R_Q = 25\,\mathrm{k\Omega}$$

Fluctuations due to disorder

Linearized gap equation: $\Delta(\mathbf{r}) = \lambda \int K(\mathbf{r}, \mathbf{r}')\, \Delta(\mathbf{r}')\, d\mathbf{r}'$

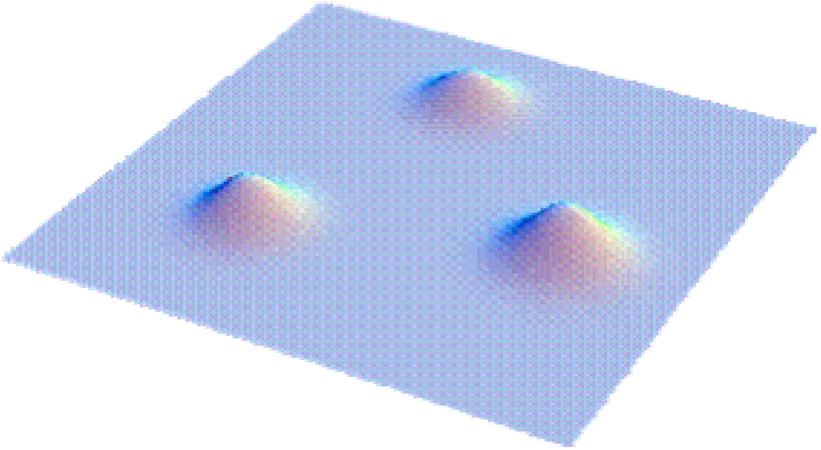
disorder dependent!

- **Mean field**

$$\langle\Delta(\mathbf{r})\rangle = \lambda \int \langle K(\mathbf{r} - \mathbf{r}')\rangle\, \langle\Delta(\mathbf{r}')\rangle\, d\mathbf{r}'$$

Uniform solutions: $\Delta(\mathbf{r}) = \text{const}$

- **Exact solution:** localized superconducting droplets at $T > T_c^{\text{MF}}$.



mesoscopic disorder at $T = 0$, $H \to H_{c2}(0)$:

Spivak, Zhou (1995)

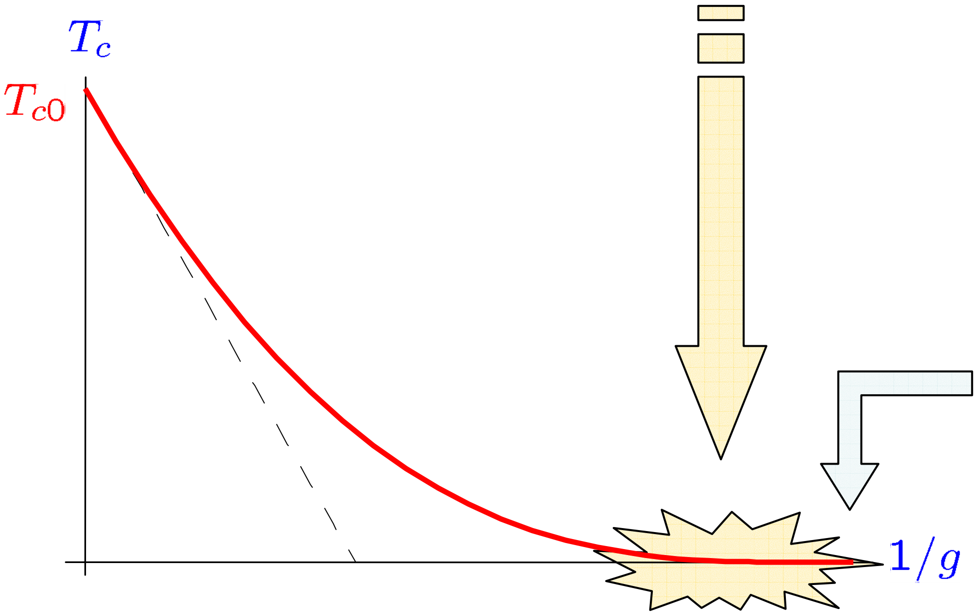
Galitski, Larkin (2001)

$$\frac{\delta H_{c2}}{\overline{H}_{c2}} \sim \frac{1}{g}$$

Mesoscopic fluctuations near T_c are smaller than thermal fluctuations

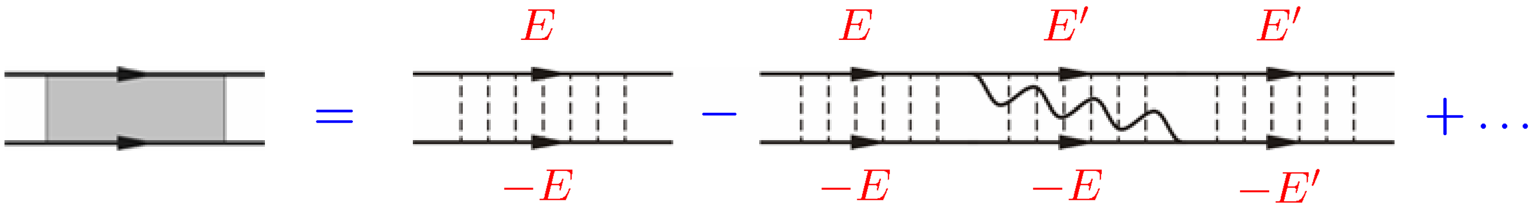
Can they become large?

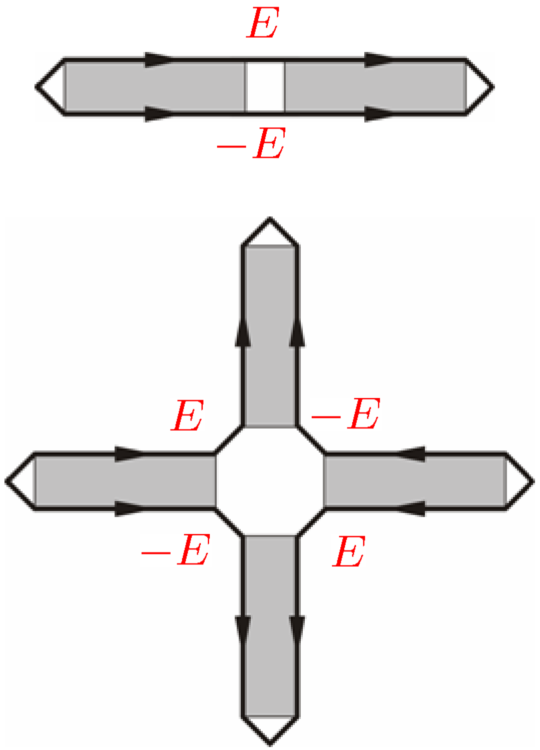
Yes, close to the Finkelstein's g_c



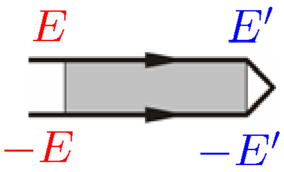
$$g_c = \frac{1}{2\pi} \ln^2 \frac{1}{T_{c0}\tau}$$

Ginzburg-Landau expansion

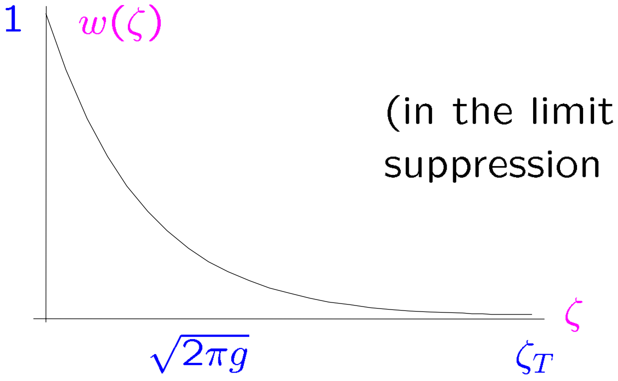




$$\frac{w(E)}{|E|} = T\sum_{E'}$$



A quasiparticle propagating at energy E feels the pairing potential $\Delta w(\zeta)$



(in the limit of large suppression of T_c)

Ginzburg-Landau expansion: result

$$F[\Delta] = \int \left[\alpha(T/T_c - 1)|\tilde{\Delta}|^2 + \gamma|\nabla\tilde{\Delta}|^2 + \frac{\beta}{2}|\tilde{\Delta}|^4\right] d\mathbf{r}$$

α, β and γ are the usual GL parameters for dirty superconductors, and

$$\tilde{\Delta} = \Delta w(\zeta_{T_c}) = \frac{\Delta}{\cosh\left(\frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T\tau}\right)}$$

Gi is the same as in the absense of the Coulomb repulsion:

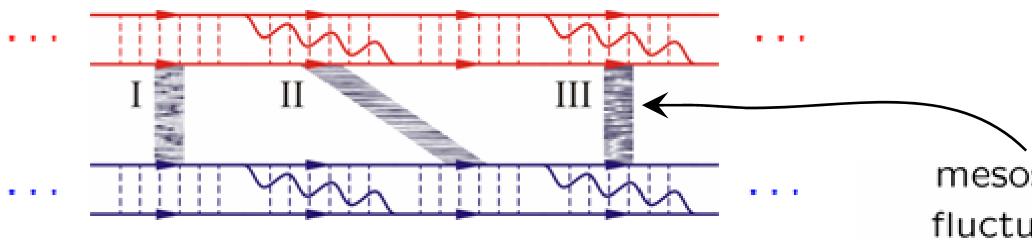
$$\text{Gi} = \frac{\pi}{8\,g}$$

Mesoscopic fluctuations of the kernel *K*(r,r′)

$$F^{(2)} = \int d\mathbf{r}\, d\mathbf{r}'\, \Delta^*(\mathbf{r}) \left(\frac{\nu\delta(\mathbf{r}-\mathbf{r}')}{|\lambda_0|} - K(\mathbf{r},\mathbf{r}') \right) \Delta(\mathbf{r}')$$

$$\langle K(\mathbf{r}-\mathbf{r}')\rangle + \delta K(\mathbf{r},\mathbf{r}')$$

$$\langle\langle \delta K(\mathbf{r}_1,\mathbf{r}_2)\delta K(\mathbf{r}_3,\mathbf{r}_4)\rangle\rangle =$$



mesoscopic
fluctuations
of return probability

Length scales:

- $\langle\langle \delta K(\mathbf{r}_1,\mathbf{r}_2)\delta K(\mathbf{r}_3,\mathbf{r}_4)\rangle\rangle$: decays at $\mathbf{r}_i - \mathbf{r}_j \sim L_T = \sqrt{D/T_c}$
- superconductive coherence length $\xi(T) = L_T\sqrt{\frac{T_c}{T-T_c}} \gg L_T$

From the viewpoint of the superconductive system, fluctuations of $K(\mathbf{r},\mathbf{r}')$ are short-ranged and characterized by a single number:

$$C = \int \langle\langle \delta K(\mathbf{r}_1,\mathbf{r}_2)\delta K(\mathbf{r}_3,\mathbf{r}_4)\rangle\rangle\, d\mathbf{r}_2\, d\mathbf{r}_3\, d\mathbf{r}_4$$

Superconductor with fluctuating T_c

Superconductor with fluctuating T_c (Ioffe, Larkin (1981)):

$$F = \int \left\{ \left[\alpha(T/T_c - 1) + \delta\alpha(\mathbf{r})\right] |\tilde{\Delta}|^2 + \gamma|\nabla\tilde{\Delta}|^2 + \frac{\beta}{2}|\tilde{\Delta}|^4 \right\} d\mathbf{r}$$

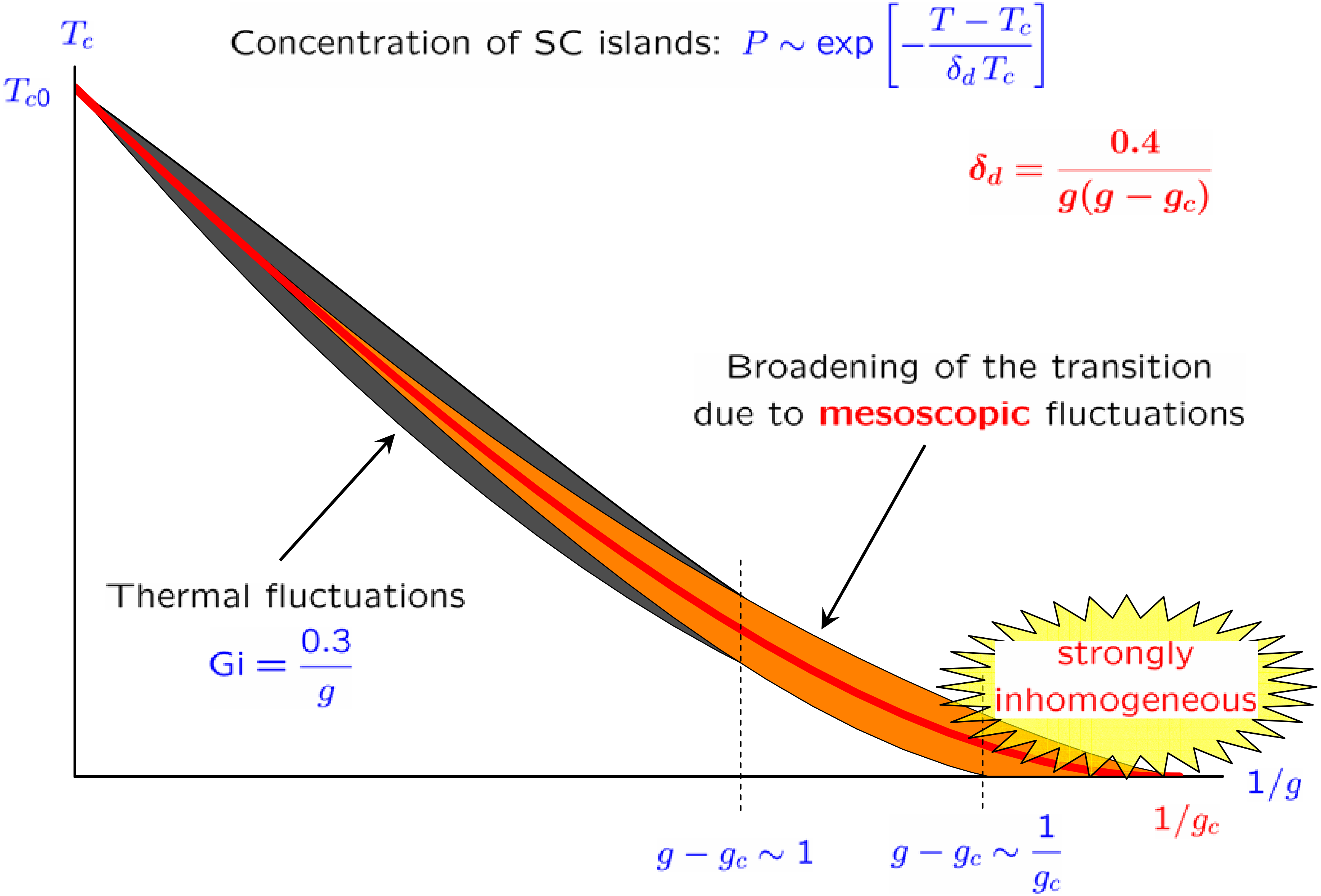
$$\langle \delta\alpha(\mathbf{r})\delta\alpha(\mathbf{r}') \rangle = \frac{7\zeta(3)}{8\pi^4 DT} \cosh^2\left(\frac{1}{\sqrt{2\pi g}} \ln\frac{1}{T\tau}\right) \delta(\mathbf{r}-\mathbf{r}')$$

GL equation looks like a Schrödinger equation with random potential
Zittartz, Langer; Halperin, Lax (1966)

⇓

Instantons

Mesoscopic vs. thermal fluctuations



Concentration of SC islands: $P \sim \exp\left[-\frac{T-T_c}{\delta_d T_c}\right]$

$$\delta_d = \frac{0.4}{g(g-g_c)}$$

Broadening of the transition due to **mesoscopic** fluctuations

Thermal fluctuations

$$\mathrm{Gi} = \frac{0.3}{g}$$

strongly inhomogeneous

Conclusions

- Quantum superconductor-metal transition at $R_\Box \approx \frac{h}{e^2} \frac{2}{\ln^2(b/d)}$ in a proximity-coupled array of islands

- Dephasing due to Andreev reflection off the superconducting islands dominates over AAK dephasing in a wide temperature range $T < G_A T_c$

- Giant mesoscopic fluctuations near T_c suppressed by Coulomb effects