

Quantum ($T=0$) superconductor-metal transitions in highly conducting films

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Quantum superconductor-metal (insulator) transitions take place at $T=0$ as functions of external parameters

Theoretical possibilities

- a. **Quantum S-wave superconductor-metal transition in an external magnetic field**
- b. **Quantum S-wave superconductor-metal transition as a function of disorder**
- c. **Quantum D-wave superconductor-metal transition as a function of disorder**
- d. **If the electron-electron interaction constant has random sign the system may exhibit quantum superconductor-metal transition**

Examples of experimental data

Experiments suggesting existence of quantum superconductor-insulator transition

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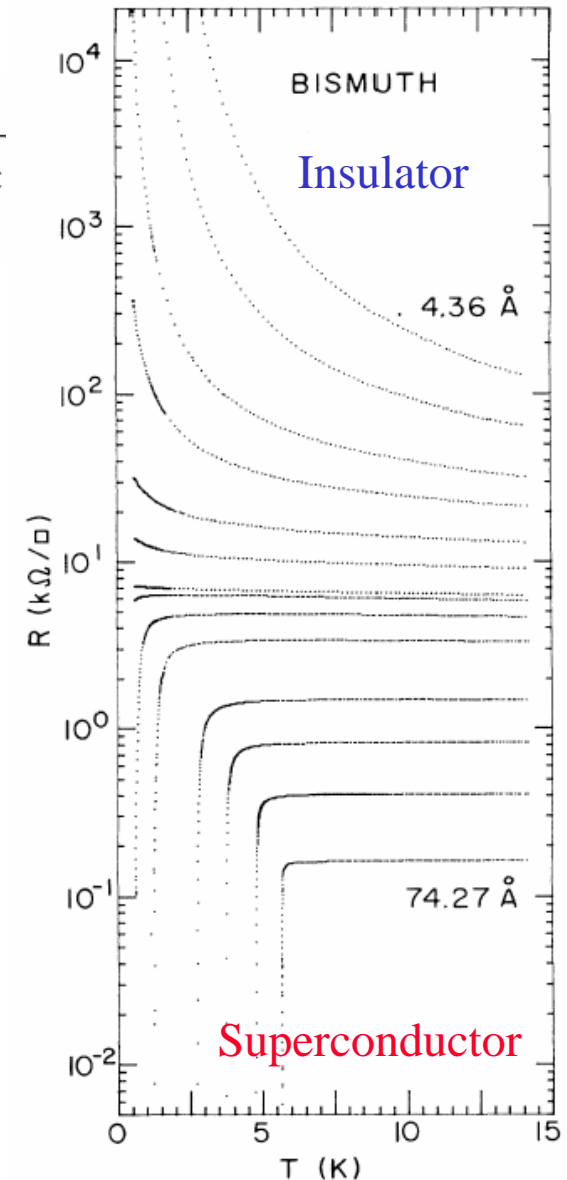
PHYSICAL REVIEW LETTERS

Onset of Superconductivity in the Two-Dimensional Limit

D. B. Haviland, Y. Liu, and A. M. Goldman

$$R_c = 4 h/e$$

Bi layer on amorphous Ge.
Disorder is varied by changing film thickness.



Experiments suggesting existence of quantum superconductor-metal transition

PHYSICAL REVIEW B

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Threshold for superconductivity in ultrathin amorphous gallium films

H. M. Jaeger, D. B. Haviland, and A. M. Goldman

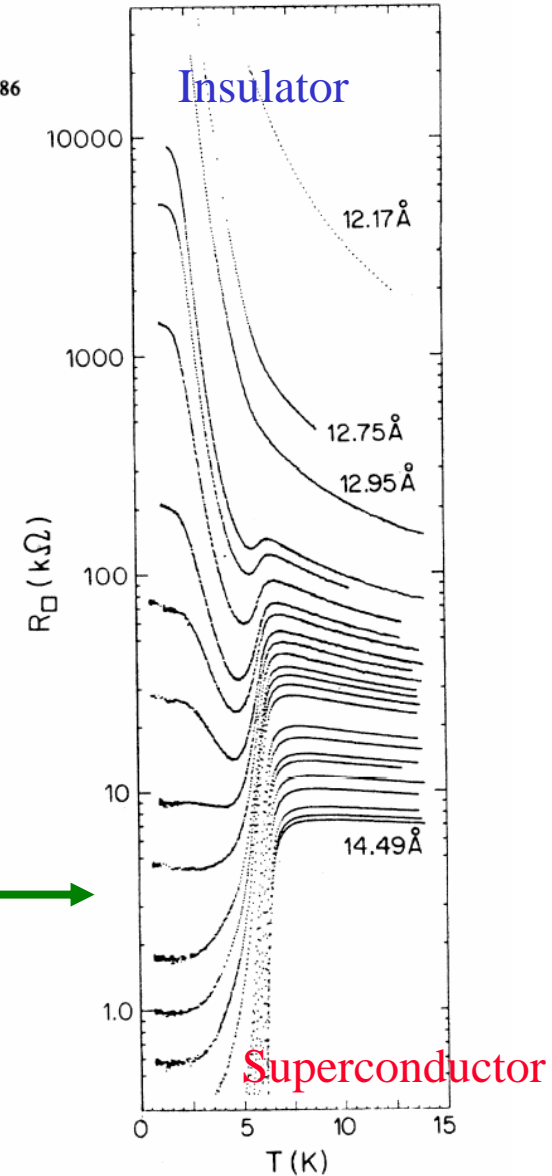
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

B. G. Orr

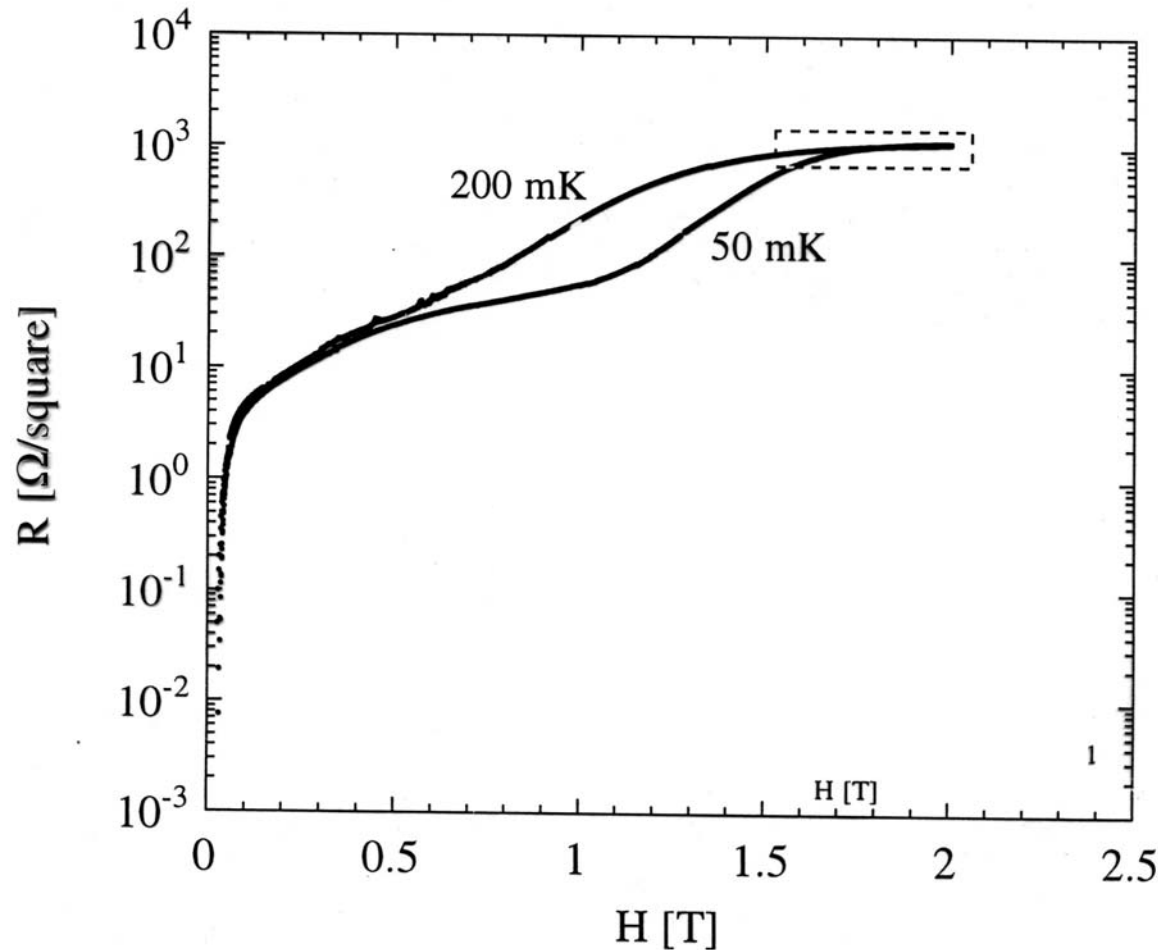
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

Ga layer grown on amorphous Ge

A metal?



T=0 superconductor-metal transition in a perpendicular magnetic field



N. Masson,
A. Kapitulnik

There are conductors whose T=0 conductance is four order of magnitude larger than the Drude value.

Superconductor – glass transition in a magnetic field parallel to the film

W. Wu, P. Adams,

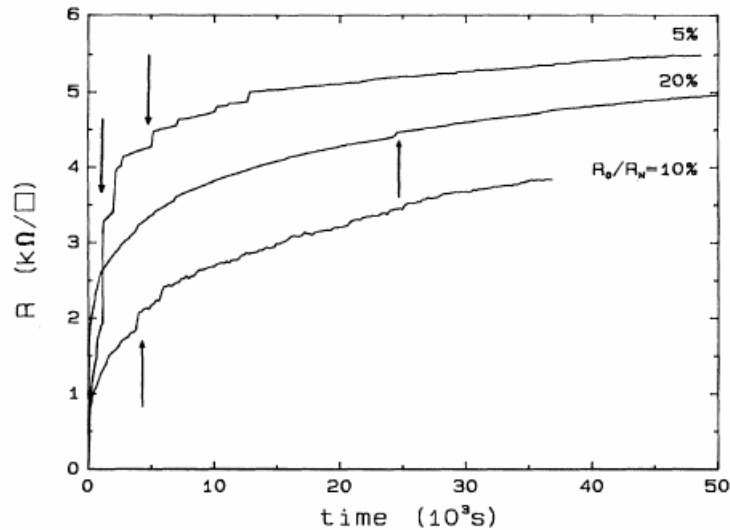
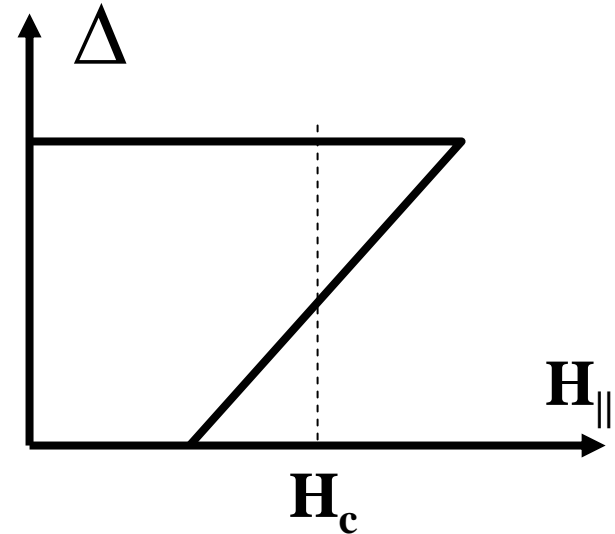


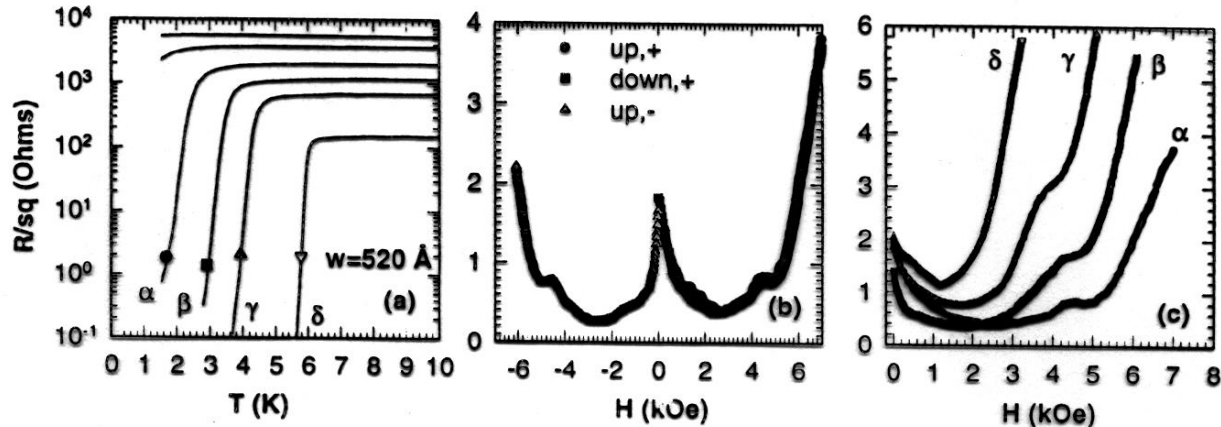
FIG. 2. R versus time after H_{\parallel} was held constant when R_0/R_N reached desired values during field-up sweeps. Arrows indicate some of the avalanches. Note that the $R_0/R_N = 5\%$ curve actually jumped above the $R_0/R_N = 20\%$ curve.



The mean field theory:
The phase transition is of first order.

There are long time (hours) relaxation processes reflected in the time dependence of the resistance.

The magneto-resistance of quasi-one-dimensional superconducting wires



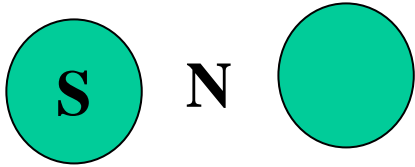
P. Xiong, A.V.Hertog, R.Dynes

The resistance of superconducting wires is much smaller than the Drude value

The magneto-resistance is negative and giant (more than a factor of 10)

A model:

Superconducting grains embedded into a normal metal



Effective action:

$$S = \sum_i \left\{ \alpha_i \int d\tau \left[\frac{(\gamma - \gamma_i)}{\gamma} |\Delta_i|^2 + \frac{|\Delta_i|^4}{\Delta_0^2} \right] + \beta_i \int d\tau d\tau' \frac{|\Delta_i(\tau) - \Delta_i(\tau')|^2}{(\tau - \tau')^2} \right\} + \\ + \sum_{ij} J_{ij} (\Delta_i \Delta_j^* + c.c)$$

γ_i , $\alpha_i \approx \nu V_i$ and $\beta_i \approx \frac{\alpha_i}{\Delta_0}$ are random variables

$V_i = L_i^2 d$ is the volume of i_{th} grain.

Does this action exhibit a superconductor-metal or superconductor-insulator transition ?

Correlation function of the order parameter of an individual grain

The case $(\gamma - \gamma_i) > 0$

Dynamics determined by the Cooper phenomenon

$$P_C(\omega) = \frac{1}{v\tau_0(-i|\omega| + (\gamma - \gamma_i))} \Rightarrow$$

$$P_C(t) = \langle \Delta(0)\Delta^*(t) \rangle_C \propto \frac{1}{t^2}$$

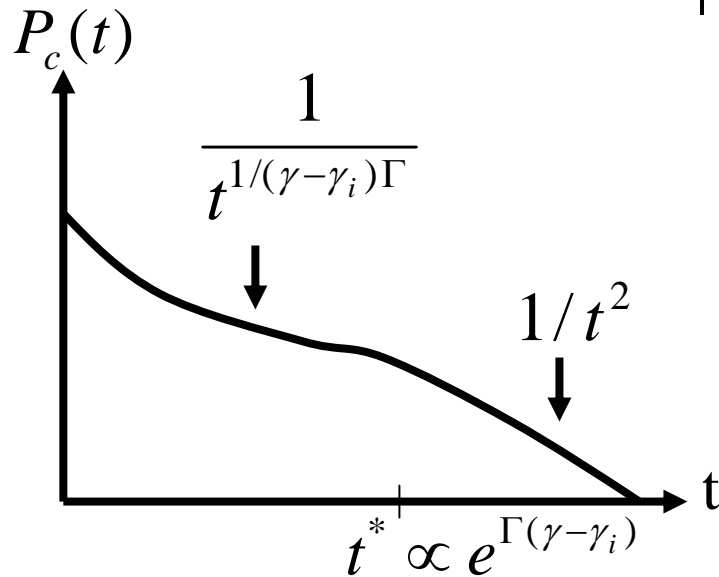
$$P_R(t) = \langle \Delta(0)\Delta^*(t) \rangle_R \propto \exp(-t\Delta_0(\gamma - \gamma_i))$$

$$\int dt P_c(t) < \infty$$

Correlation function of the order parameter of an individual grain

The case $(\gamma_i - \gamma) > 1/\Gamma$

$$|\Delta_i|^2 = \Delta_0^2 (\gamma_i - \gamma)$$



Kosterlitz 1976

$$\int dt P_c(t) < \infty$$

Superconducting susceptibility of an individual grain

$$\chi_i \propto \exp[\Gamma_i(\gamma - \gamma_i)], \quad \Gamma_i \approx V_i v \Delta_0$$

r- dependence of the Josephson coupling

$$J_{ij} \propto \frac{1}{|r_i - r_j|^2} f(r_i, r_j, T)$$

Both in D-wave case and in the case when superconductivity is suppressed by the magnetic field the exchange energy has random sign (or phase)

Quantities r_i and γ_i are randomly distributed

The distribution function of γ_i

$$F(\gamma_i) \propto e^{-\frac{(\gamma_i - \langle \gamma_i \rangle)^2}{\Sigma^2}}$$

Criterion for superconductivity:

$$J_{ij} \chi_i \propto 1$$

There is a superconductor-metal quantum phase transition. It is of a quantum percolation type.

$$\gamma_c - \langle \gamma \rangle \approx \Sigma^2 \beta \Delta_0^2; \quad R \propto \exp(\Sigma^2 \beta^2 \Delta_0^4)$$

$$\Sigma^2 \beta^2 \Delta_0^4 \gg 1$$

The case of S-superconducting film in a perpendicular magnetic field

$$L_i = L_{H_{c2}}$$

$$\frac{H_{c2} - \langle H_{c2} \rangle}{\langle H_{c2} \rangle} \approx G \sigma_{H_{c2}}^2$$

$$R^* \approx \xi_0 \exp(\sigma_{H_{c2}}^2 G^2)$$

$\sigma_{H_{c2}}$ is the variance of the critical magnetic field on the grains

G is the conductance of the film

$$\sigma_{H_{c2}} = \sigma_{H_{c2}}^{(int)} + \sigma_{H_{c2}}^{(cl)}; \quad \sigma_{H_{c2}}^{(int)} \approx \frac{1}{G}, \quad \sigma_{H_{c2}}^{(cl)} \propto \frac{A}{L_{H_{c2}}}$$

A comparison with a theory of Larkin Feigelman and Skvortsov; and Larkin and Galitskii

$$\chi \propto e^{\sqrt{G}}, \quad D = 2 \text{ (Larkin Feigelman Skvortsov)}$$

Our results are valid if

$$\frac{1}{G} < \sigma_{H_{c2}} < \frac{1}{G^{1/4}}$$

The case of D-wave disordered superconductor

$$L_i \propto \sqrt{D / \Delta_{opt}}$$

$$\frac{\xi_0 - l_c}{l_c} \approx \frac{\Gamma^{1/2}}{G}$$

$$R^* \approx \xi_0 e^\Gamma \gg \xi_0$$

$$\Gamma \approx v \xi_0^2 d \Delta_0 < G$$

l_c is the critical elastic mean free path

A mean field approach

$$\Delta(\vec{r}) = \lambda(\vec{r}) \int d\vec{r}' K(\vec{r}, \vec{r}') \Delta(\vec{r}') + a |\Delta|^2 \Delta$$

$$K(\vec{r}, \vec{r}') \propto \frac{1}{|\vec{r} - \vec{r}'|^2} \quad (d = 2)$$

The parameters in the problem are:

grain radius R ,

grain concentration N ,

interaction constants $\lambda_N < 0$, $\lambda_S > 0$.

If $R < \xi$ there are no superconducting solutions in an individual grain.

In this case a system of grains has a quantum ($T=0$) superconductor-metal transition even in the framework of mean field theory!

At $T=0$ in between the superconducting and insulating phases there is a metallic phase.

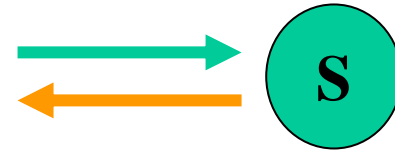
Properties of the exotic metal near the quantum metal-superconductor transition

- a. Near the transition at $T=0$ the conductivity of the “metal” is enhanced.
- b. The Hall coefficient is suppressed.
- c. The magnetic susceptibility is enhanced

**In which sense such a metal is a Fermi liquid?
For example, what is the size of quasi-particles ?
Is electron focusing at work in such metals ?**

Are such “exotic” metals localized in 2D?

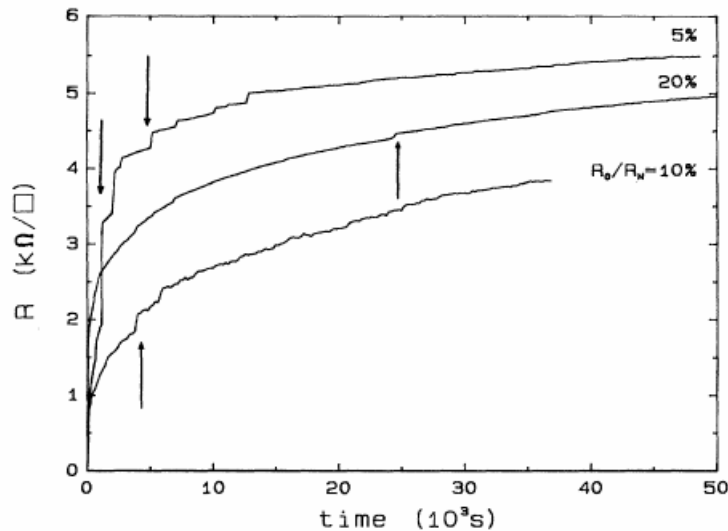
There is a new length associated with the Andreev scattering of electrons on the fluctuations of Δ



$$L_\varphi = \frac{1}{NR_c^2} \frac{\Delta_0^2}{\langle |\Delta|^2 \rangle} \approx \frac{1}{NR_c^2} \Delta_0^2 \nu \tau_0; \quad (|R - R_c| \ll R_c)$$

This length is non-perturbative in the electron interaction constant

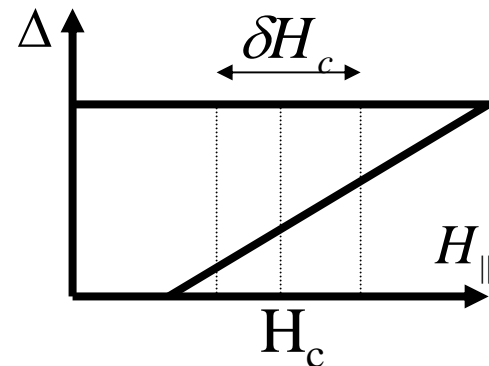
Is it a superconductor-glass transition in a parallel magnetic field?



W. Wo, P. Adams,
1995

FIG. 2. R versus time after H_{\parallel} was held constant when R_0/R_N reached desired values during field-up sweeps. Arrows indicate some of the avalanches. Note that the $R_0/R_N = 5\%$ curve actually jumped above the $R_0/R_N = 20\%$ curve.

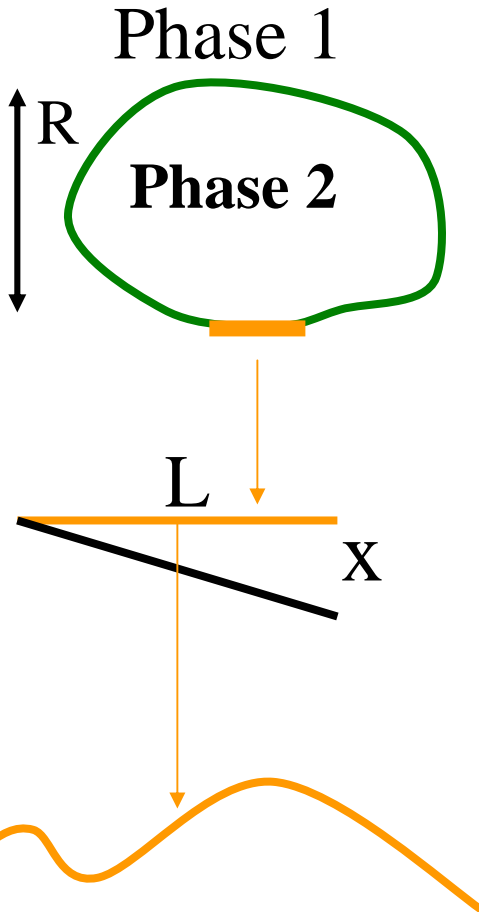
In the pure case the mean field $T=0$ transition as a function of H_{\parallel} is of first order.



In the absence of the spin-orbit scattering spatial fluctuations of the critical magnetic field is entirely due to interference effects

$$\frac{\langle (\delta H_c)^2 \rangle}{\langle H_c \rangle^2} \propto \frac{1}{G}$$

In 2D arbitrarily disorder destroys the first order phase transition. (Imry, Wortis).



A naive approach ($H=H_c$).

$$E_s = 2\pi R\sigma - A((\pi R)^2)^{1/2} = R(2\pi\sigma - A\pi^{1/2})$$

b. More sophisticated drawing:

$$\delta E_s = \frac{x^2}{L}\sigma - A(xL)^{1/2}, \quad x \approx L(A/\sigma)^{2/3} \Rightarrow \delta E_s = -LA^{4/3}\sigma^{1/3} = -L\delta\sigma$$

$\delta\sigma = A^{4/3}\sigma^{1/3}$ is independent of the scale L

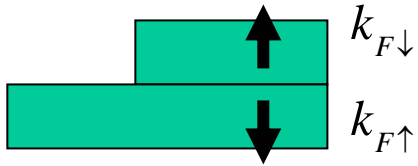
This actually means that

$$\delta\sigma \propto -\ln L \text{ and } \sigma \rightarrow 0 \text{ when } L \rightarrow \infty.$$

The surface energy vanishes and the first order phase transition is destroyed!

The critical current and the energy of SNS junctions with spin polarized electrons changes its sign as a function of its thickness L

Bulaevski, Buzdin



$$I = I_c \sin \varphi; \quad I_c \propto \exp\left(-\frac{L}{L_I}\right) \cos \frac{L}{L_I}$$

$$L_I = \left(\frac{Dm}{k_F (k_{F\uparrow} - k_{F\downarrow})} \right)^{1/2}$$

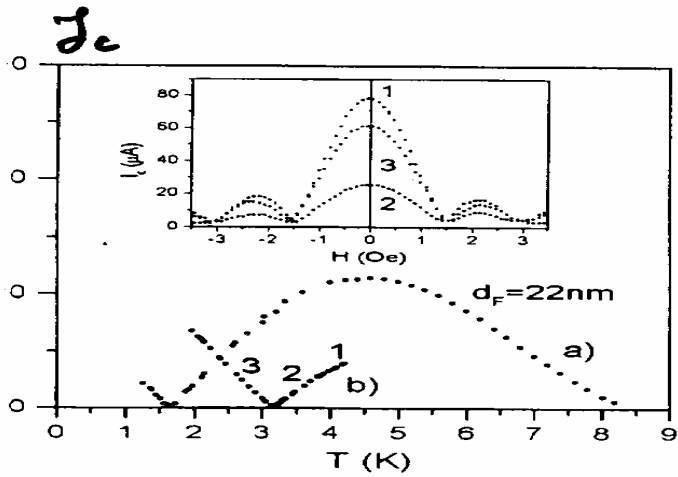


Figure 3: Critical current I_c as a function of temperature T for two junctions with $\text{Cu}_{0.48}\text{Ni}_{0.52}$ and $d_F = 22 \text{ nm}$ [17]. The inset shows the dependence on magnetic field H for the temperatures (1) $T = 4.19 \text{ K}$, (2) $T = 3.45 \text{ K}$, (3) $T = 2.7 \text{ K}$, the crossover to the π -state as indicated on the plot.

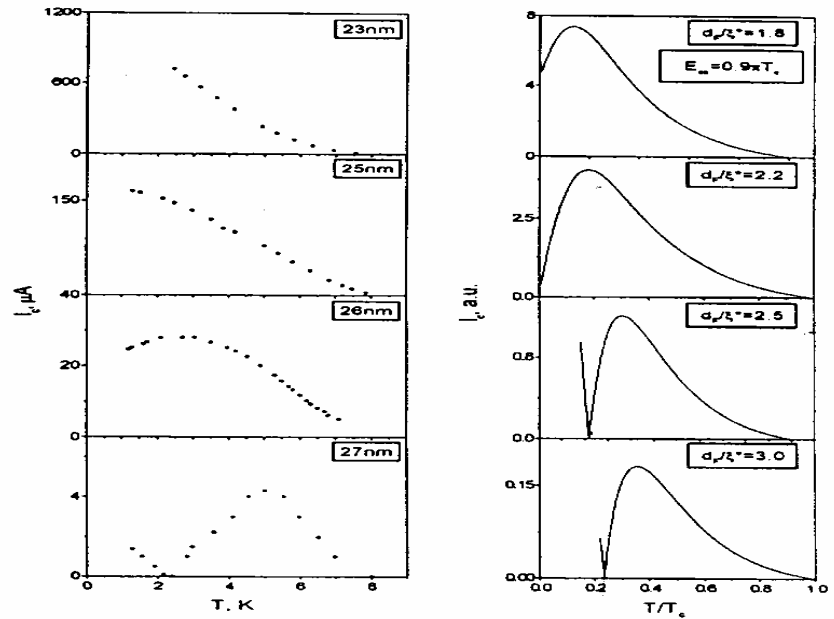
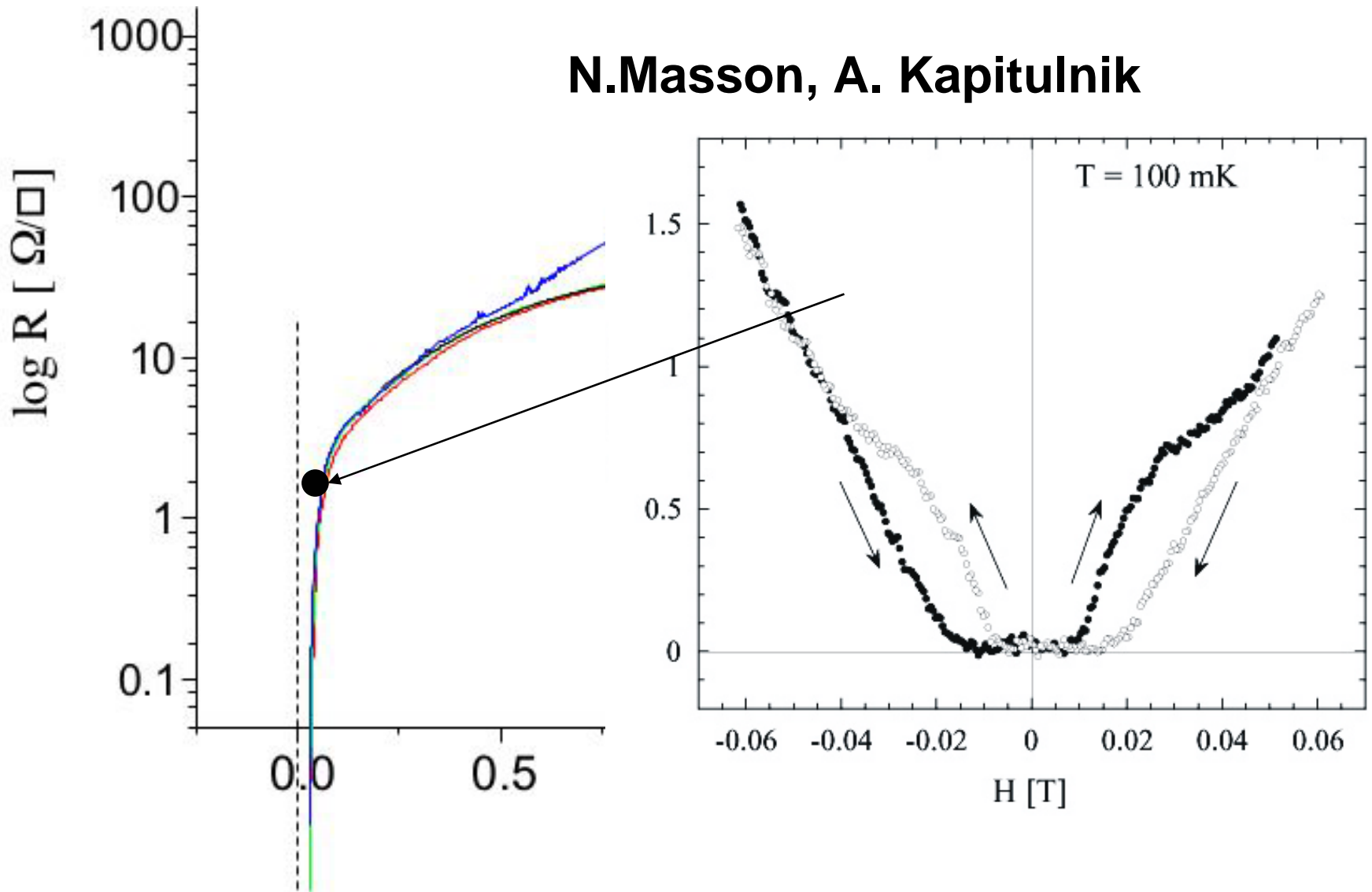


Figure 4: Left : critical current I_c as function of temperature for $\text{Cu}_{0.48}\text{Ni}_{0.52}$ junctions with different F-layer thicknesses between 23 nm and 27 nm as indicated. Right : model calculations of the temperature dependence of the critical current in an SFS junction for $E_{c\omega} = 0.9\pi T_c$ and various ratios of d_F/ξ^* , where $\xi^* = \sqrt{\hbar D / (2\pi k_B T_c)}$

Ryazanova et al.

Does the hysteresis in a perpendicular magnetic field indicate glassiness of the problem ?

N.Masson, A. Kapitulnik



Why is the glassy nature more pronounced in the case of parallel magnetic field?

Is this connected with the fact that in the pure case the transition is second order in perpendicular magnetic field and first order in parallel field?

Do we know what is the resistance of an SNS junction ?



The mini-gap in the N-region is of order $\delta(\phi) \sim D/L^2$

$$\frac{dS}{dt} \propto \frac{(\delta n)^2}{\tau_\varepsilon} \propto (d\phi/dt)^2 \tau_\varepsilon \propto GV^2 \Rightarrow G \propto \tau_\varepsilon$$

τ_ε is the energy relaxation time

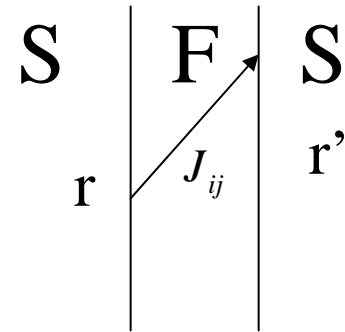
Conclusion:

In slightly disordered films there are quantum superconductor-metal transitions. Properties of the metallic phase are affected by the superconducting fluctuations

Superconductor-disordered ferromagnetic metal-superconductor junctions.

$$E = \int dr dr' J(r, r') \cos(\varphi_L(r) - \varphi_R(r'))$$

Mean field theory (A.Larkin, Yu.Ovchinnikov, L.Bulaevskii, Buzdin).



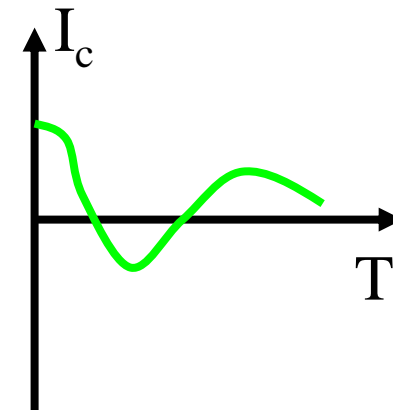
$$\langle J_c \rangle \propto \sin \dots \exp\left(-\frac{L}{L_I}\right); \quad L_I = (D/I)^{1/2};$$

Here I is the spin splitting energy in the ferromagnetic metal.

The quantity $J(r, r')$ exhibits Friedel oscillations. At $L \gg L_I$ only mesoscopic fluctuations survive.

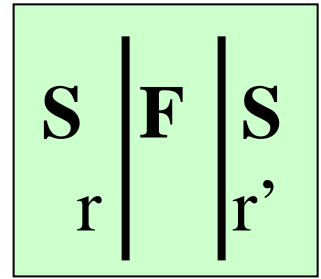
$$\langle J^2(r, r') \rangle \propto 1/|r - r'|^3; \quad (D = 2) \rightarrow \langle I_c^2 \rangle \propto 1/L$$

In the case of junctions of small area the ground state of the system corresponds to a nonzero sample specific phase difference $0 < \varphi_L - \varphi_R < \pi$. I_c is **not** exponentially small and exhibits oscillations as a function of temperature.



SFS junctions with large area

$$E = \int dr dr' J(r, r') \cos(\varphi_L(r) - \varphi_R(r')) + N_s \int dr V_s^2$$



The situation is similar to the case of FNF layers, which has been considered by Slonchevskii.

$$\varphi(r) = \langle \varphi \rangle + \delta\varphi$$

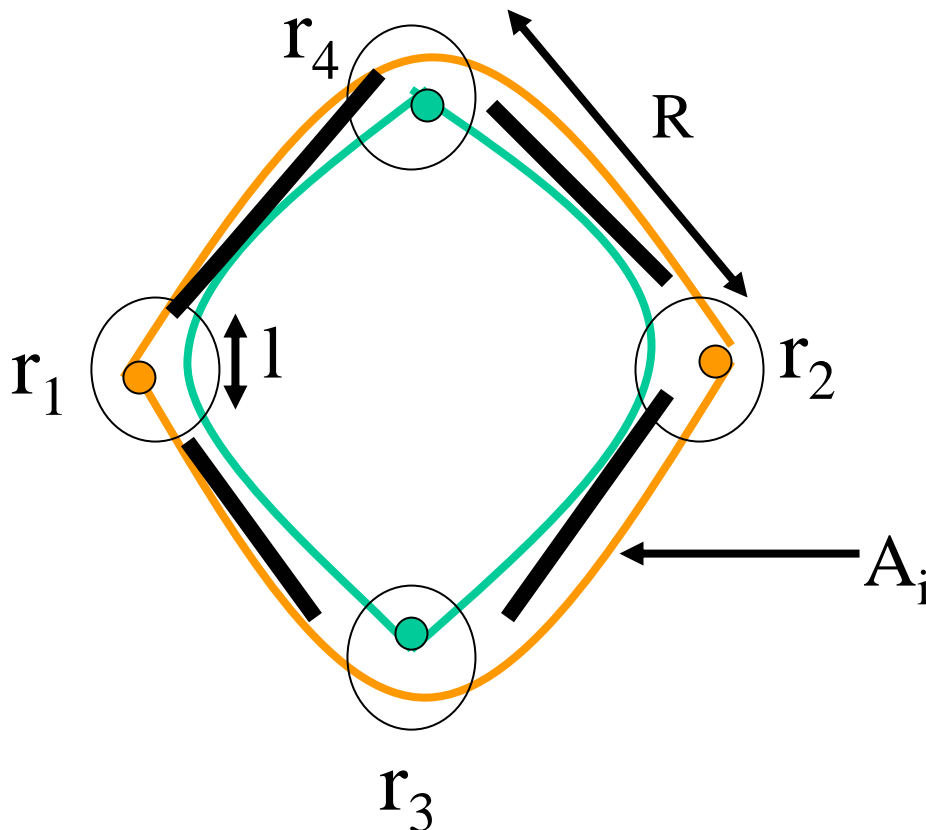
The ground state of the system corresponds to

$$\langle \varphi_L - \varphi_R \rangle = \pi / 2$$

A qualitative picture.

$$n(r, r') = \left| \sum_i A_i \right|^2; \quad \langle n(r_1, r_2) n(r_3, r_4) \rangle = \left\langle \sum_{ijkl} A_i A_j^* A_k A_l^* \right\rangle$$

The amplitudes of probabilities A_i have random signs. Therefore, after averaging of $\langle A A A A \rangle$ only blocks $\langle A_i^2 \rangle \sim 1/R$ survive.



Negative magneto-resistance in quasi-1D superconducting wires.

A model of tunneling junctions:



$$\rho \propto \int dI_c \exp\left(-\frac{E_c}{T}\right) F(I_c); \quad E_c = \frac{e}{h} I_c$$

At small T the resistance of the wire is determined by the rare segments with small values of I_c .

$$I_c \propto \left| A + B \exp\left(i \frac{\Phi}{\Phi_0}\right) \right|^2, \quad \Phi = HS, \quad \Phi_0 \text{ is the flux quanta.}$$

The probability amplitudes of tunneling paths A and B are random quantities uniformly distributed, say over an interval (-1,1).

Friedel oscillations in superconductors (mean field level):

$$\Delta(r) = \lambda \int dr' K(r, r') \Delta(r') + n.l. \text{ terms}$$

$$K(r, r') = \int d\varepsilon G_{\varepsilon}(r, r') G_{-\varepsilon}(r, r')$$

The kernel $K(r, r')$ is expressed in terms of the normal metal Green functions $G_{\varepsilon}(r)$ and therefore it exhibits the Friedel oscillations.

$$\langle G \rangle \propto \dots \exp\left(-\frac{|r-r'|}{l}\right) / |r-r'|, \quad \langle K \rangle \propto \exp\left(-\frac{|r-r'|}{L_H}\right) / |r-r'|^3,$$

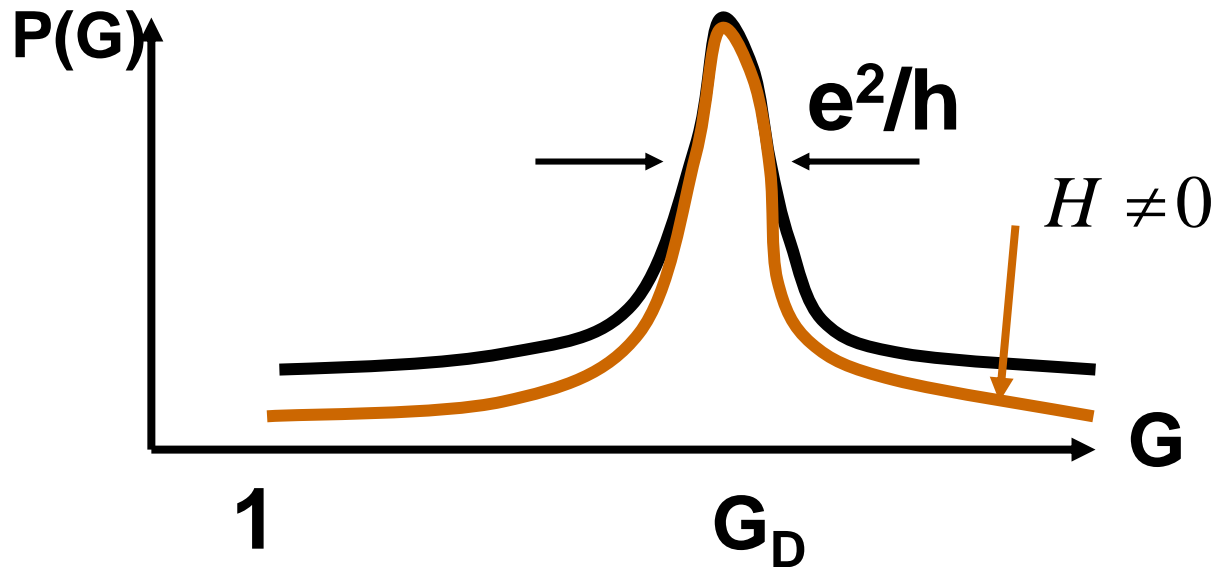
$$\langle K^2 \rangle \propto 1 / |r-r'|^6$$

An explanation of experiments :

The probability for I_c to be small is suppressed by the magnetic field. Consequently, the system exhibits giant magneto-resistance.

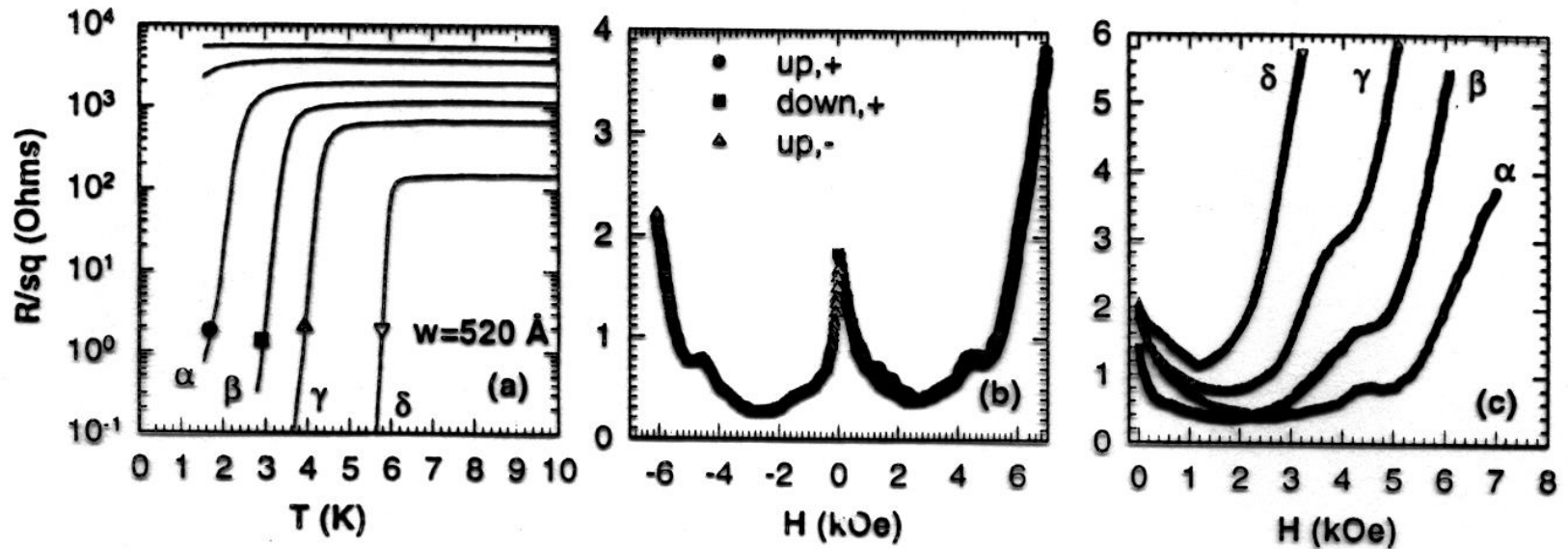
A problem: in the experiment $G \gg 1$.

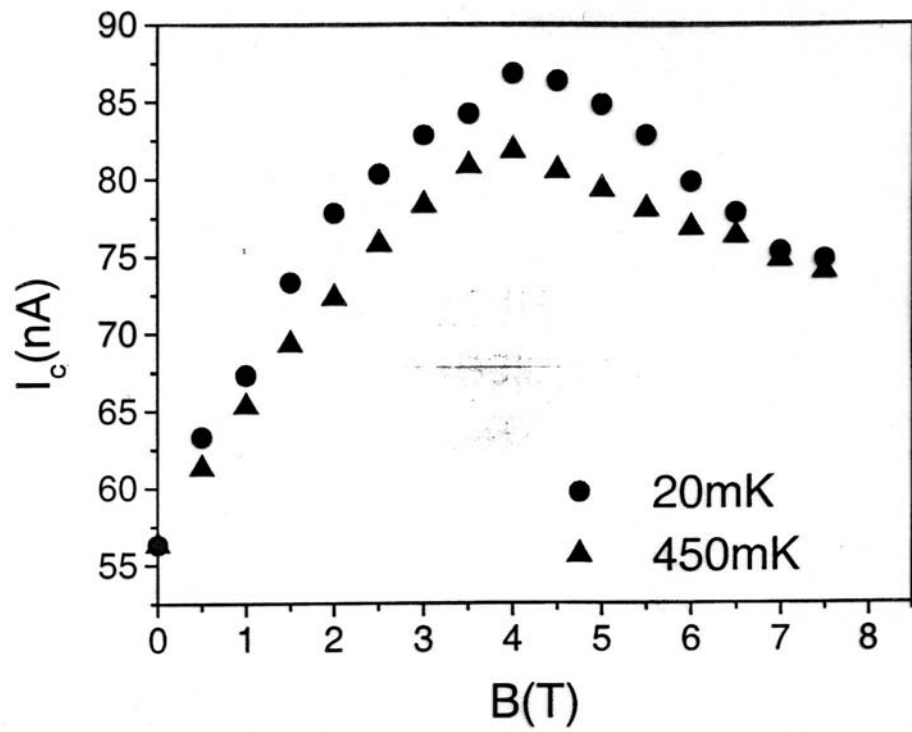
A question:



Negative magneto-resistance in 1D wires

P. Xiong, A.V.Hertog, R.Dynes





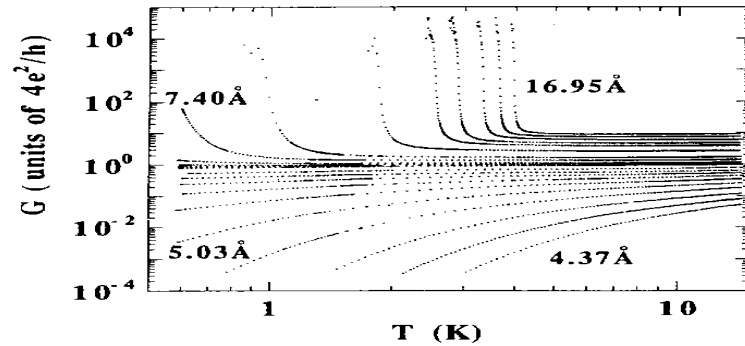


FIG. 2. Evolution for Bi films of the electrical conductance G in units of $4e^2/h$ as a function of temperature T . The thicknesses of a few selected films are indicated. Note that conductance and conductivity are identical in two dimensions. Only some of the data of the sequence of films is shown to avoid too high a density of data points.

Y. Liu, D. B. Haviland,
A. M. Goldman

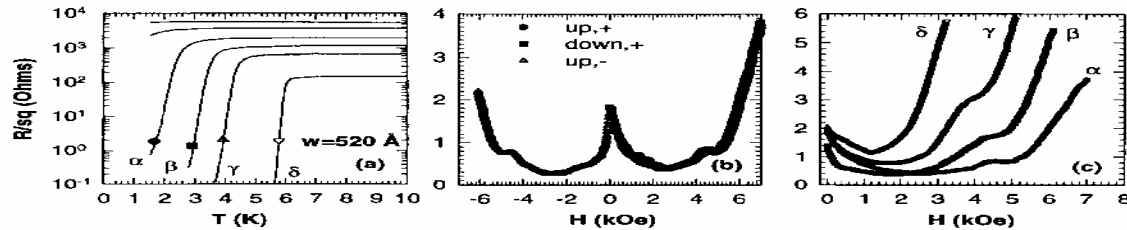


FIG. 2. (a) Resistive transitions for a 520 Å wide wire. The symbols represent the points at which we measured the magnetoresistance. (b) Three field sweeps at point α . (c) Magnetoconductance at points $\alpha - \delta$. The temperatures at the four points are 1.63, 2.90, 3.93, and 5.78 K, respectively.

dence of the negative MR on wire width is apparent; the negative MR increases with decreasing wire width.

The evolution of the negative MR, with wire cross-sectional area and temperature as described above follows the evolution of the zero-field excess broadening beyond that predicted by the LAMH theory. There appears to be a close correlation between the two effects. This

correlation is further evidence that the excess broadening is not due to edge roughness of the wires, but instead is an intrinsic dimensional effect. It also suggests that the negative MR is a manifestation of the suppression of the superconducting fluctuations by the magnetic field. To summarize, we have observed an enhancement of superconductivity and a suppression of superconducting fluctuations by a perpendicular magnetic field in thin homogeneous Pb wires, and the effect is a result of the dimensional crossover from 2D to 1D.

Negative MR was observed close to T_c in superconducting Sn stripes [11] and Al loops [12]. The negative MR was attributed to the nonequilibrium charge imbalance process [13] at normal-superconducting (N-S) boundaries, produced either intrinsically by spatially localized phase-slip centers due to high current density [11] or artificially by ion damage [12]. The magnetic field reduces the charge imbalance relaxation time and hence

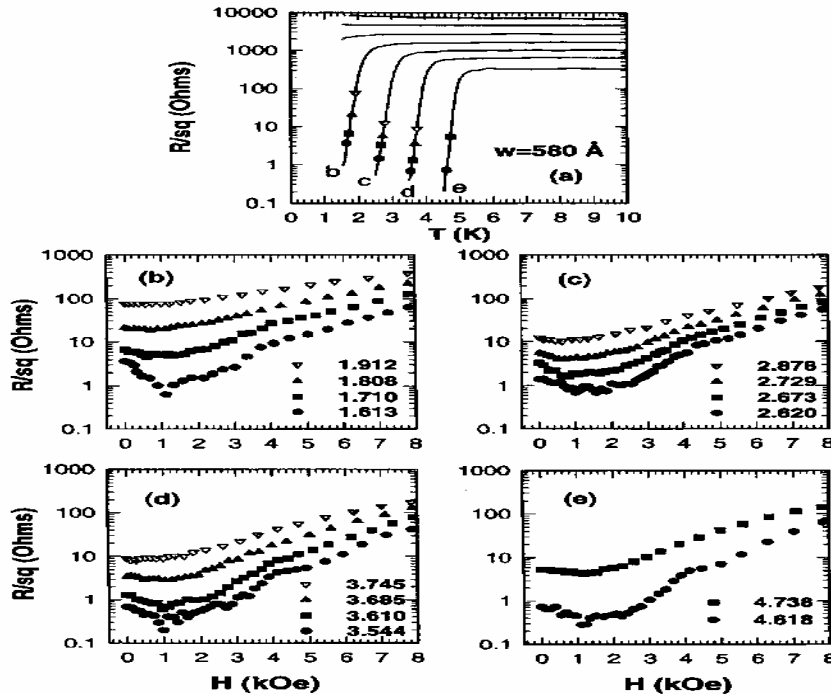


FIG. 3. (a) Resistive transitions for a 580 Å wide wire. The symbols represent the points at which we measure the magnetoresistance. (b)–(e) Magnetoconductance for the wire at different thicknesses. The numbers indicate the temperature in Kelvin.

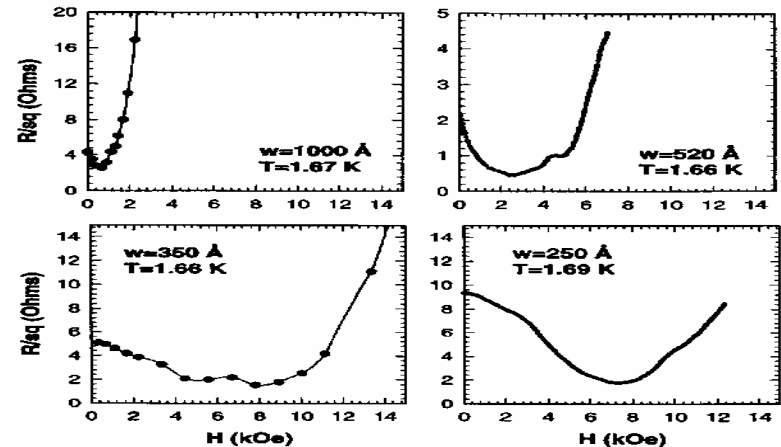


FIG. 4. Magnetoconductance for four wires of different widths: (a) 1000 Å; (b) 520 Å; (c) 350 Å; (d) 250 Å. The measurements were taken at approximately the same position along transitions of approximately the same T_c for each wire.

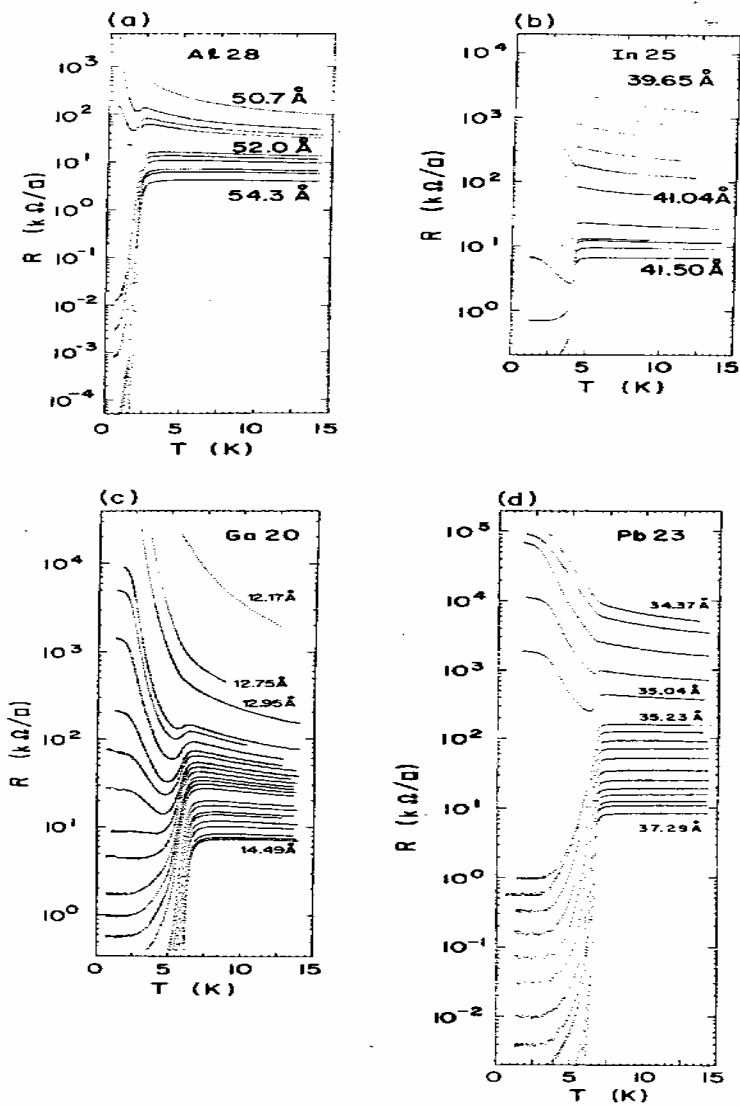


FIG. 2. The evolution of $R(T)$ curves for (a) Al, (b) In, (c) Ga, and (d) Pb, obtained *in situ* after successive increments of film thickness. Note that in all cases the entire evolution from insulating to globally superconducting spans an interval of nominal thickness of less than one monolayer.

D. B. Haviland,
M. Goldman

The case of large grains:

The effective action for a single tunnel junction

- There is a nonzero mean field solution for the order parameter on a single grain: amplitude fluctuations can be neglected
- The effective action for a single tunnel junction contains a dissipative term (Ambegaokar, Eckern and Schön, 1982)

$$S_D[\mathcal{G}(\tau)] = \frac{2}{\pi} G \int_0^\beta d\tau \int_0^\beta d\tau' \frac{\sin^2 \frac{1}{4} (\mathcal{G}(\tau) - \mathcal{G}(\tau'))}{(\tau - \tau')^2}$$

which reproduces the action of Caldeira and Leggett (1981) as a special case.

- The total action is

$$S = S^{(i)} + S_D + \int_0^\beta E_J \cos\theta(\tau) d\tau$$

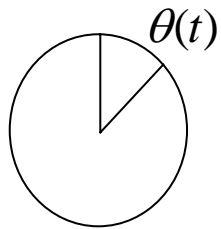
where $S^{(i)}$ is the charging energy.

The system of large superconducting grains embedded in a normal metal (Feigel'man and Larkin, 1998)

- The integral of the correlation function $C(t) \equiv \langle e^{i(\theta(t) - \theta(0))} \rangle_0$ converges due to the fact that at largest times $C(t) \propto 1/t^2$

- Notice:

If the time τ during which the phase undergoes a variation of 2π is finite, the asymptotic behavior of the corresponding retarded correlation function is

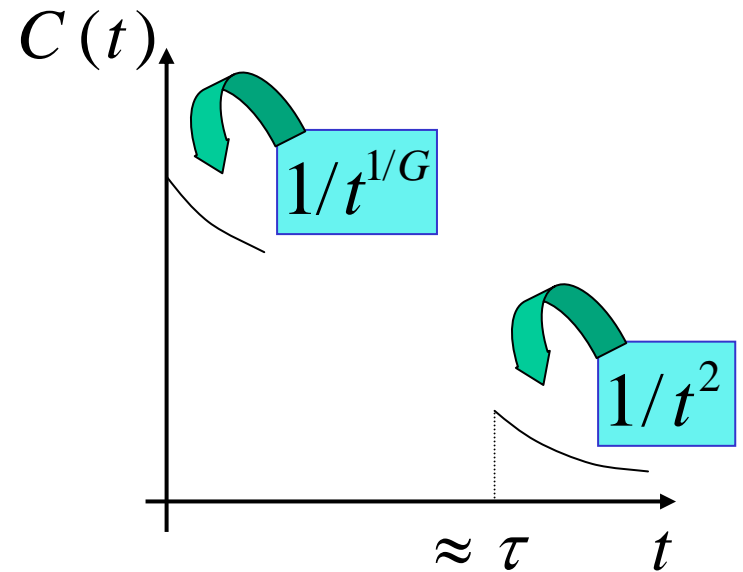


$$\langle e^{-(\theta(t) - \theta(0))^2 / 2} \rangle \propto e^{-t/\tau}$$

$$C(t) \propto 1/t^2$$

The time during which the phase undergoes a variation of 2π , in the model of Feigelman and Larkin

- τ can be found estimating the limits of validity of linearized (in $1/G$, $G \gg 1$) RG equations for the (inverse-square) X-Y model (Kosterlitz, 1976).



$$\int_0^\infty dt \langle e^{i(\theta(t) - \theta(0))} \rangle_0 \approx \tau \approx \exp(G)$$

The large grain results of Feigel'man and Larkin

- The perturbation theory in the interaction between grains $J_{ij} \propto (R/r_{ij})^{1+2\sqrt{\lambda_N}} \ln(R/r_{ij})$, $r_{ij} = |r_i - r_j|$ (taking account of Coulomb repulsion in the metal, $\lambda_N = \frac{4\pi\epsilon_0 N}{R}$) breaks down for

$$J \approx \left(\int_0^\infty dt \langle e^{i(\theta(t) - \theta(0))} \rangle_0 \right)^{-1} \sim \exp[-G]$$

$$J \propto N / \ln(N^{1/2} / R)$$

(where $\lambda_N = \frac{4\pi\epsilon_0 N}{R}$)

- As a result
 - There is a quantum superconductor – metal transition;
 - Critical concentration is exponentially small in dimensionless film conductance G

An alternative approach to the same problem in the same limit $R < \xi$



$$S = \sum_i S_i + \sum_{(i \neq j)} \int d\omega J_{ij} \Delta_{\omega}^{(i)} \Delta_{-\omega}^{(j)*}$$

S_i is the action for an individual grain “I”.

$$J_{ij} \propto \frac{R^2}{r_{ij}^2} \frac{1}{(1 + 2\nu |\lambda_N| \ln \frac{r_{ij}}{R})^2}$$

Perturbation on the metallic side of the transition $R < \xi$

The metallic state is stable with respect to superconductivity if

$$\sum_i J_{ij} \times \int dt P(t) < 1,$$

where $P(t) = \langle \Delta(0) \Delta^*(t) \rangle$

$$J_{ij} \propto \frac{R^2}{r_{ij}^2} \frac{1}{(1 + 2 \nu |\lambda_N| \ln \frac{r_{ij}}{R})^2}; \quad \sum_i J_{ij} < \infty$$

Quantum fluctuations of the order parameter in an individual grain ($T=0, R < R_c$)

The action describing the Cooper instability of a grain

$$S = \nu \tau_0 \int (-i|\omega| + \frac{1}{\tau}) |\Delta(\omega)|^2 d\omega;$$

$$\frac{1}{\tau} = \frac{R_c - R}{\tau_0 R}, \quad \tau_0 = \Delta_0^{-1} = \frac{\xi}{v_F}$$

At $R < R_c$ the metallic state is stable

$$\sum_i J_{ij} \times \int dt P(t) < \infty$$

At T=0 in between the superconducting and insulating phases there is a metallic phase.

However, in the framework of this model the interval of parameters where it exists is exponentially narrow.