Larkin-Imry-Ma state of ^{3}He-A in aerogel

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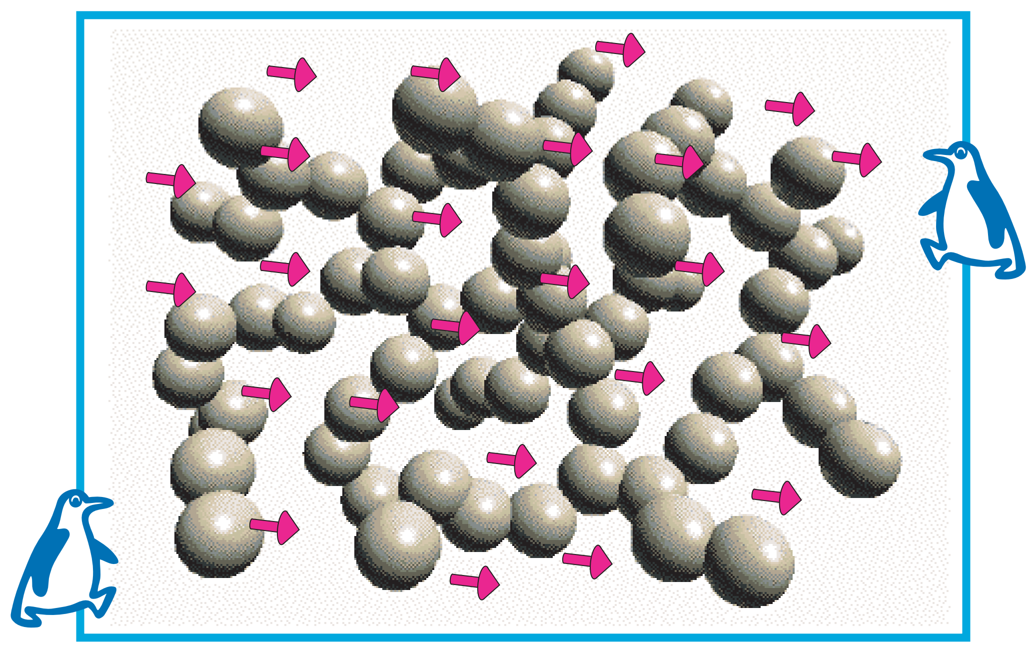
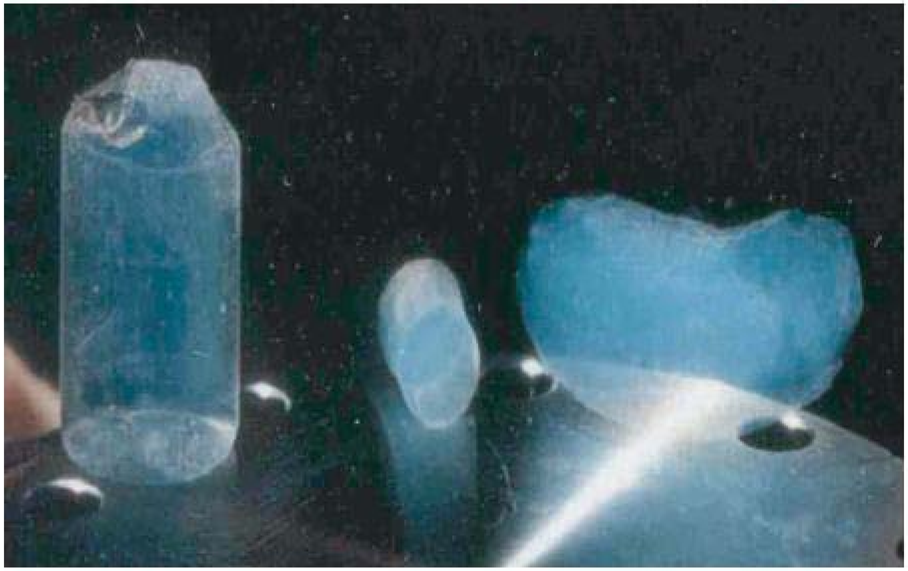


Aerogels: stiff foams composed of up to 99.8% air
Silica aerogel is the world's lowest-density solid: 1 mg/cm^3

2.38 g piece of aerogel supports a 2.5 kg brick.

Aerogels hold 15 different records for material properties, including best insulator

Impurity suppression of unconventional Superfluidity



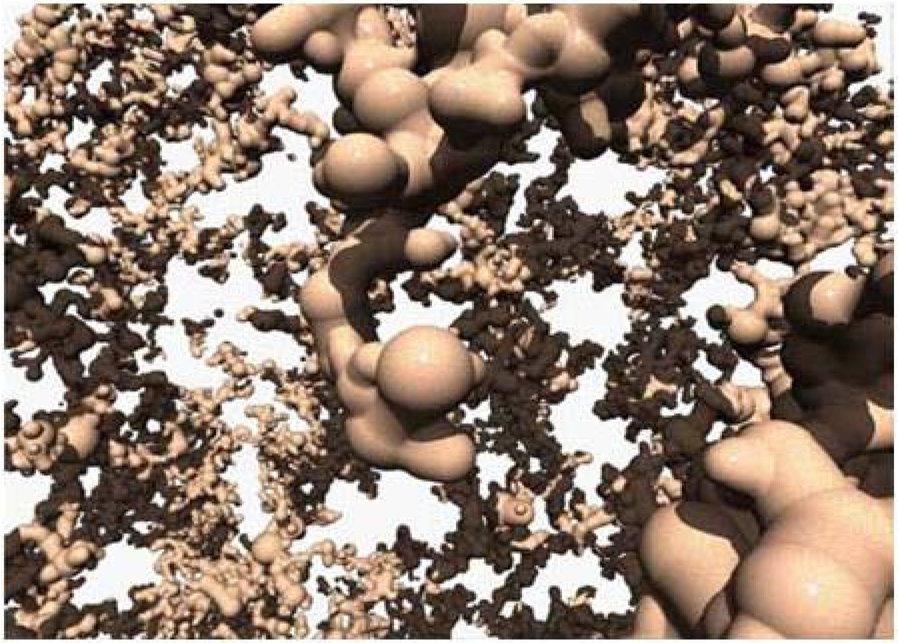
Unconventional Superfluids :

heavy fermions (UPt3),

ruthenates, ….

Cuprates

3He



Disorder can be introduced in
superfluid 3He by
high porosity silica aerogel

doubly anisotropic superfluid liquid ^{3}He-A

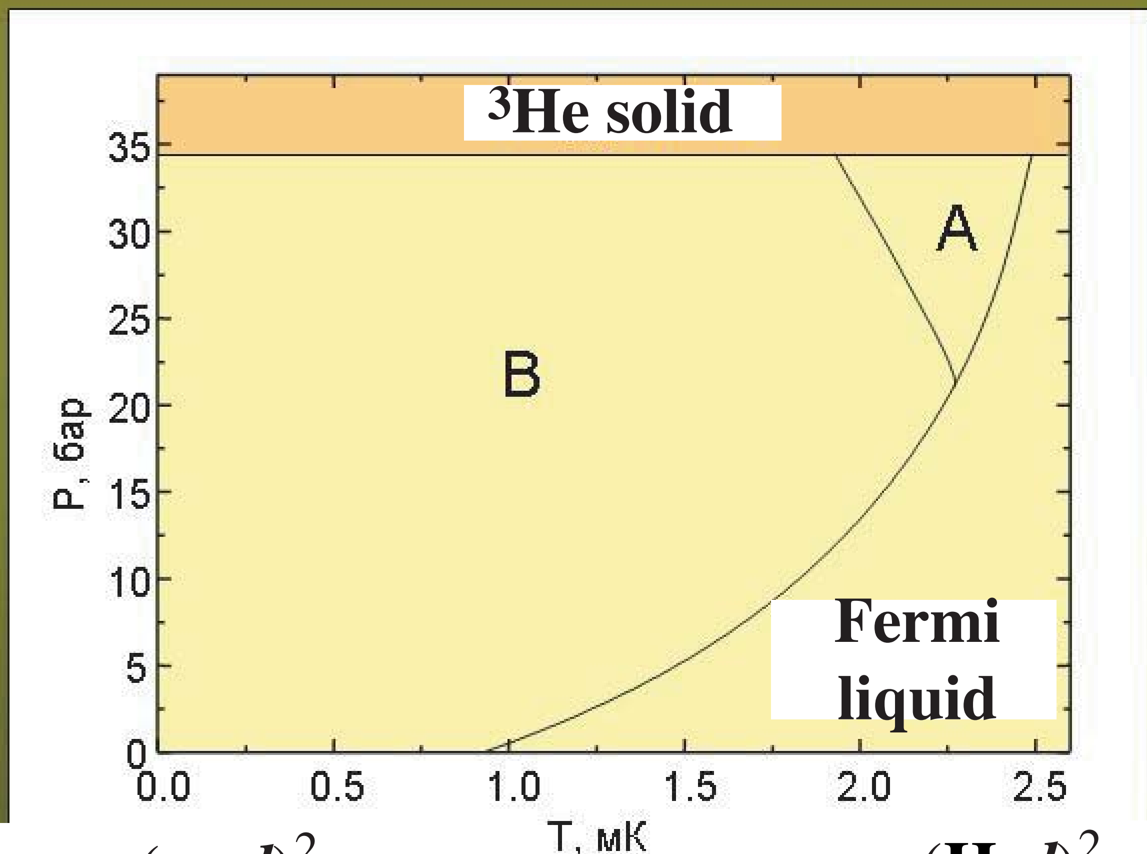
^{3}He-A $A_{\mu i} = \Delta e^{i\varphi} d_\mu (m_i + i n_i)$ $p_x + ip_y$ *superconductor*

^{3}He-B $A_{\mu i} = \Delta e^{i\varphi} \mathbf{R}(\hat{\mathbf{n}}, \theta)_{\mu i}$ Sr_2RuO_4

$\boldsymbol{l} = \boldsymbol{m} \times \boldsymbol{n}$
unit vector
in orbital space

chiral orbital
ferromagnet

anisotropic
superfluid
density



d - unit vector
in spin space

spin
nematic

anisotropic
magnetic
succeptibility

$1/2\, \rho_{s\|}(\mathbf{v}_s \cdot \boldsymbol{l})^2 + 1/2\, \rho_{s\perp}(\mathbf{v}_s \times \boldsymbol{l})^2$ $1/2\, \chi_{\|}(\mathbf{H} \cdot \boldsymbol{d})^2 + 1/2\, \chi_{\perp}(\mathbf{H} \times \boldsymbol{d})^2$

two main problems for ^{3}He in aerogel

nature of state:

Vortex liquid ?

Robust phase ?

Larkin-Imry-Ma ?

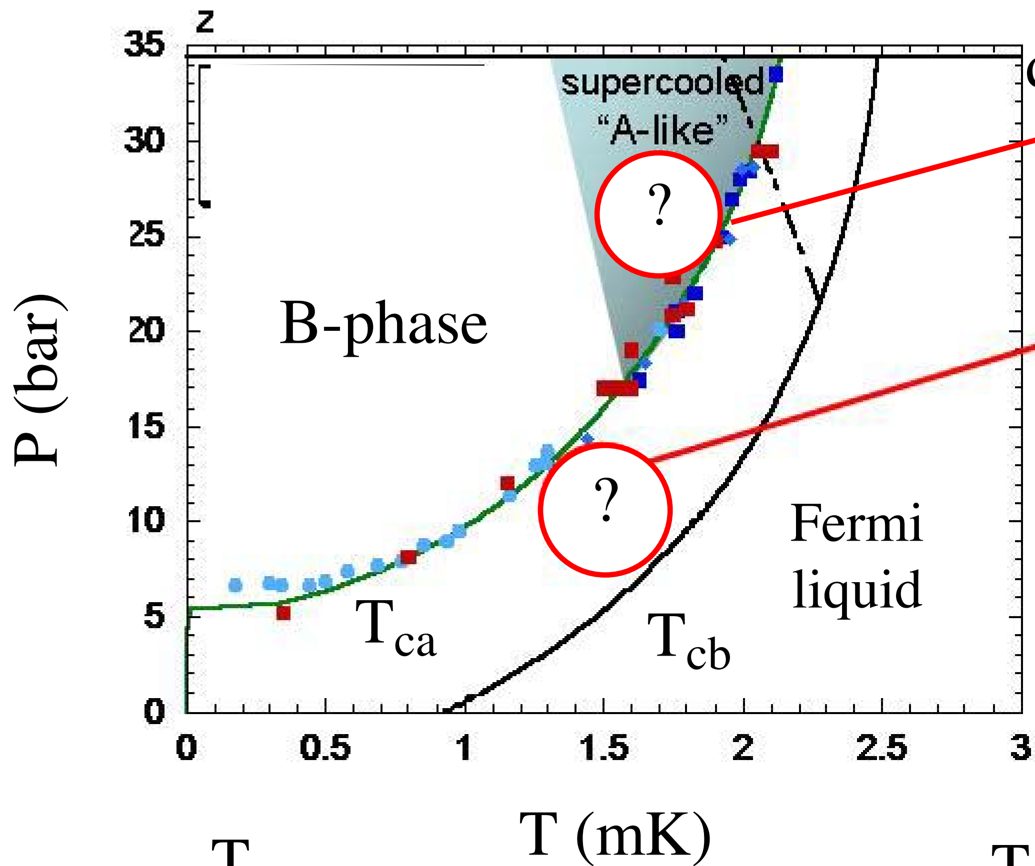
quasi long-range order?

life in between:

Griffiths phase ?

Pseudo-gap state?

Vortex liquid ?



T_{ca}
superfluid transition temperature in aerogel

T_{cb}
superfluid transition temperature in bulk 3He

Superfluid coherence length:
$\xi_o \approx$ 20 to 80 nm(34 to 0 bar)

Silica particle size diameter,
$\delta \approx$ 3 nm;

distance between strands

$$\xi_a \sim 20 \text{ nm}$$

is of order of correlation length

strands provide
local random anisotropy !

Larkin-Imry-Ma length, at which long-range order of $\boldsymbol{l}$ is destroyed by local random anisotropy

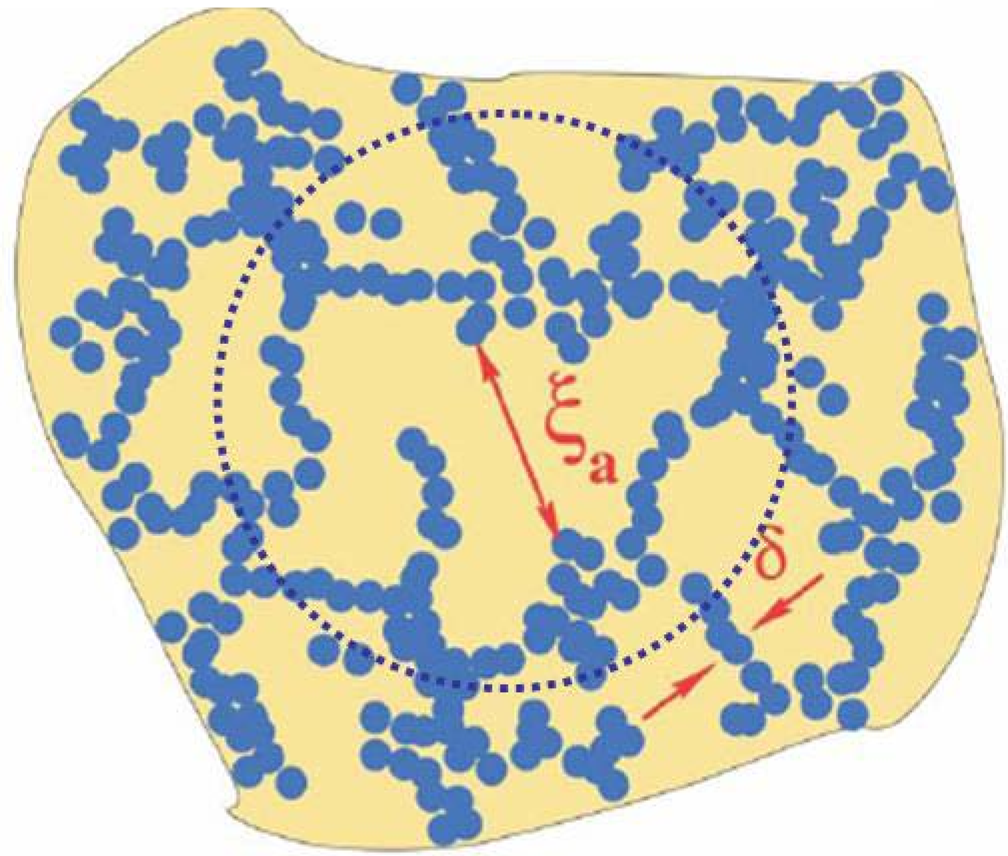
$$L \sim \xi_a \xi_0^2/\delta^2 \sim 1\ \mu\text{m}$$

(dipole) length of spin-orbit coupling

$$E_{so} = -\,g_D(\boldsymbol{l}\cdot\boldsymbol{d})^2$$

$$\xi_{so} \sim 10\ \mu\text{m}$$

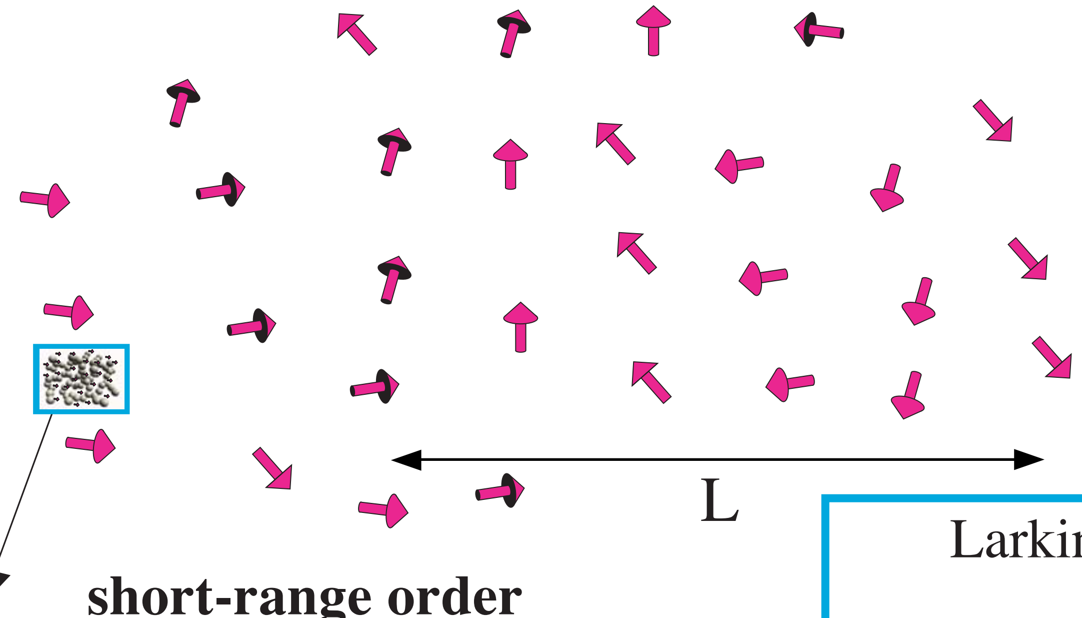
length scales in aerogel

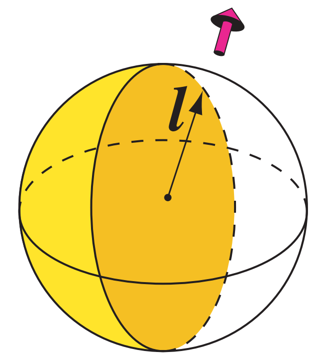


3 nm		20 nm		20-80 nm		1 μm		10 μm
δ	$\ll$	ξ_a	$\sim$	ξ_0	$\ll$	L	$<$	ξ_{so}

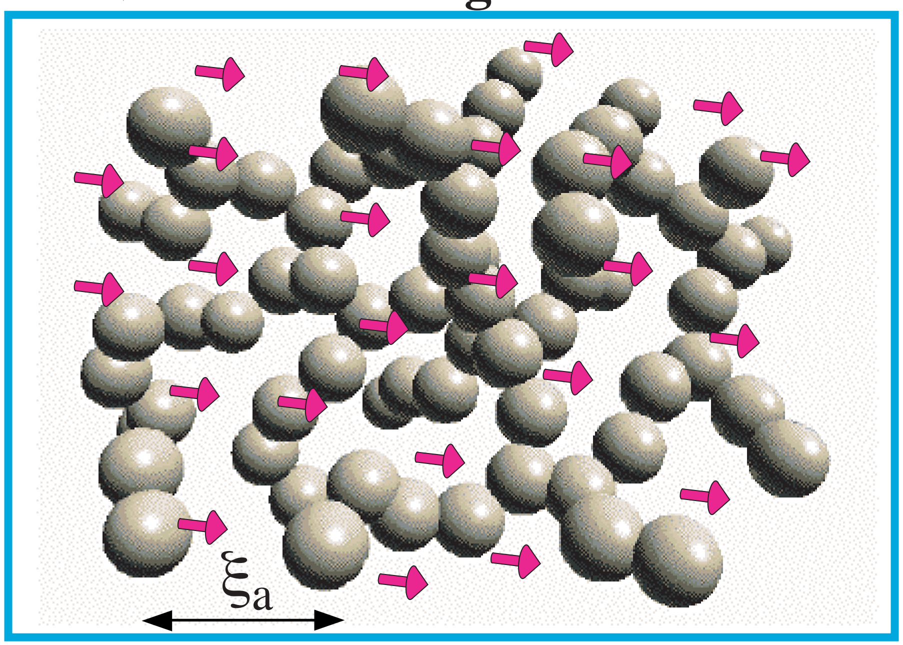
Larkin-Imry-Ma effect:

collective action of aerogel strings destroys orientational long-range order





short-range order



Larkin-Imry-Ma length

$$L \sim \xi_a (\xi_0^2/\delta^2) \sim 1\ \mu m$$

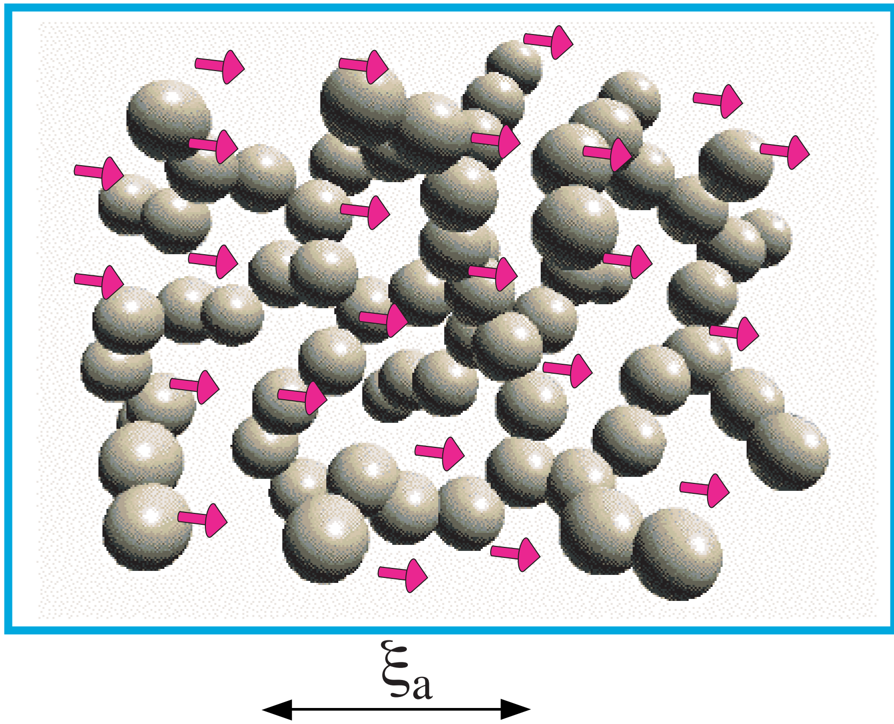
$$L \gg \xi_a \sim 20\ nm$$

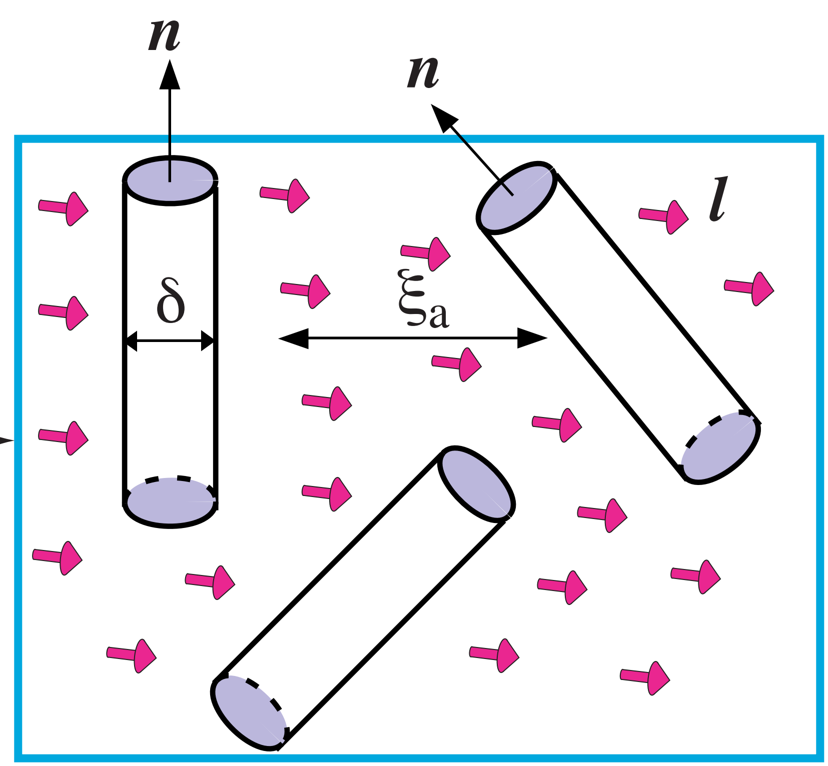
Larkin, JETP **31**, 784 (1970)
destruction of translational order
(melting of vortex lattice by impurities)

Imry & Ma, PRL **35**, 1399 (1975)
destruction of orientational order by random anisotropy

Larkin-Imry-Ma effect

model for strands: ***randomly oriented cylinders***





orientational energy acting on cylinder from $\boldsymbol{l}$

$$E(\boldsymbol{l},\boldsymbol{n}) = E_{a\ (anisotropy)}\left[(\boldsymbol{l}\cdot\boldsymbol{n})^2 - 1/3\right]$$

If diameter is smaller than coherence length $\delta \ll \xi_0$

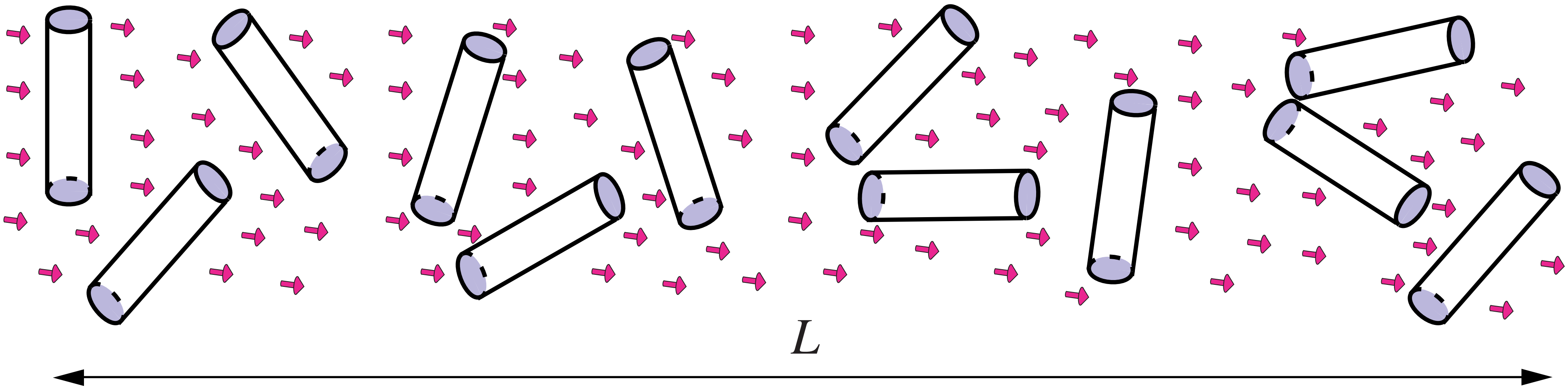
D. Rainer & M. Vuorio, J.Phys. **10** (1977) 3093

$$E_a \sim \sigma\, k_F^2\, (\Delta^2 / T_c) > 0$$

$\sigma = \xi_a \delta$ is scattering cross-section of a cylinder

Larkin-Imry-Ma effect:

collective effect of fluctuations of random anisotropy of strands



fluctuation of energy of random anistotropy

(finite size effect in the box $L \times L \times L$)

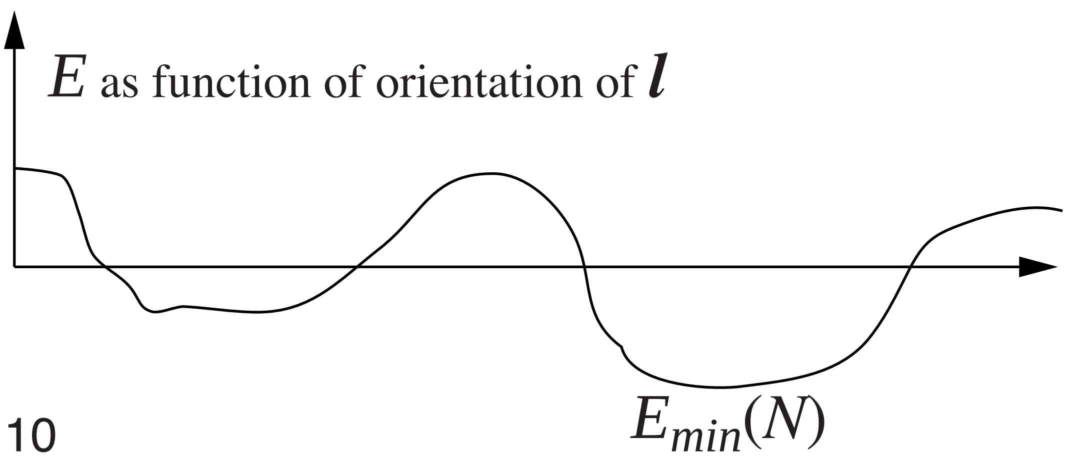
$<E(N)> = 0$; variance $= < E^2 > = N E_a^2$

$N = L^3/\xi_a^3$

total number of cylinders of length ξ_a in volume L^3

$E_a \sim \xi_a \delta k_F^2 (\Delta^2 / T_c)$

orientational energy of cylinder of length ξ_a

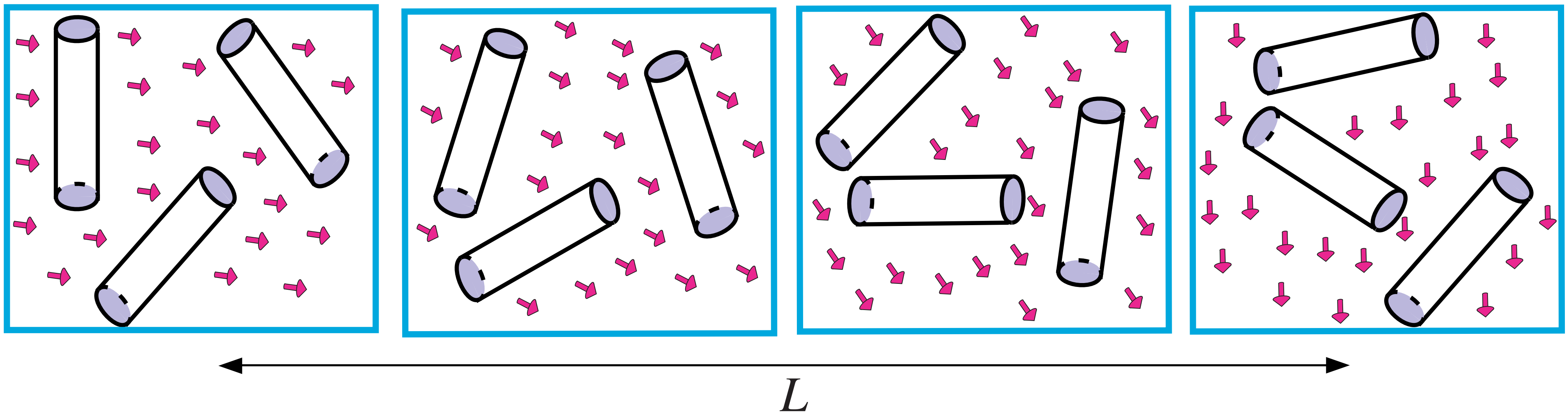


vector $\boldsymbol{l}$ prefers orientation mimimizing energy

$E_{min}(N) = - < E^2(N) >^{1/2} = - N^{1/2} E_a$

estimation of Larkin-Imry-Ma length

in the neighboring boxes $\boldsymbol{l}$ prefers different orientations, this is opposed by gradient energy



competition between orientational energy of fluctuation and gradient energy

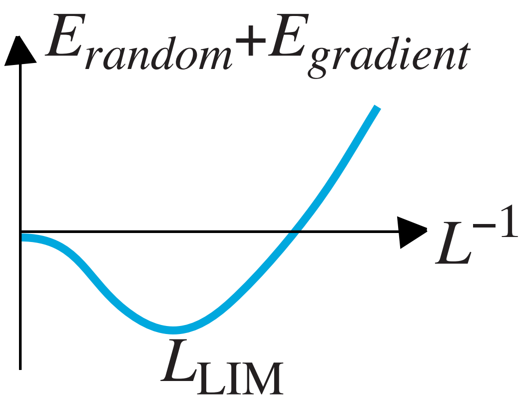
energy density of random anistotropy

$$E_{ran} \sim - < E^2(N) >^{1/2} L^{-3} \sim - E_a \, \xi_a^{-3/2} L^{-3/2}$$

+

density of opposing gradient energy

$$E_{gr} \sim K \, (\partial \boldsymbol{l})^2 \sim (k_F^3/m)(\Delta / T_c)^2 L^{-2}$$

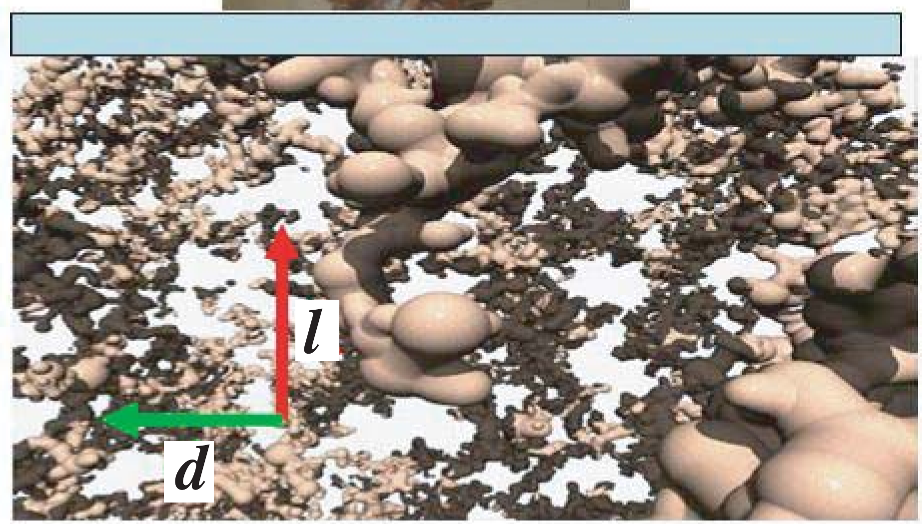


$E_{random}(L) + E_{gradient}(L)$
has minimum at Larkin-Imry-Ma length

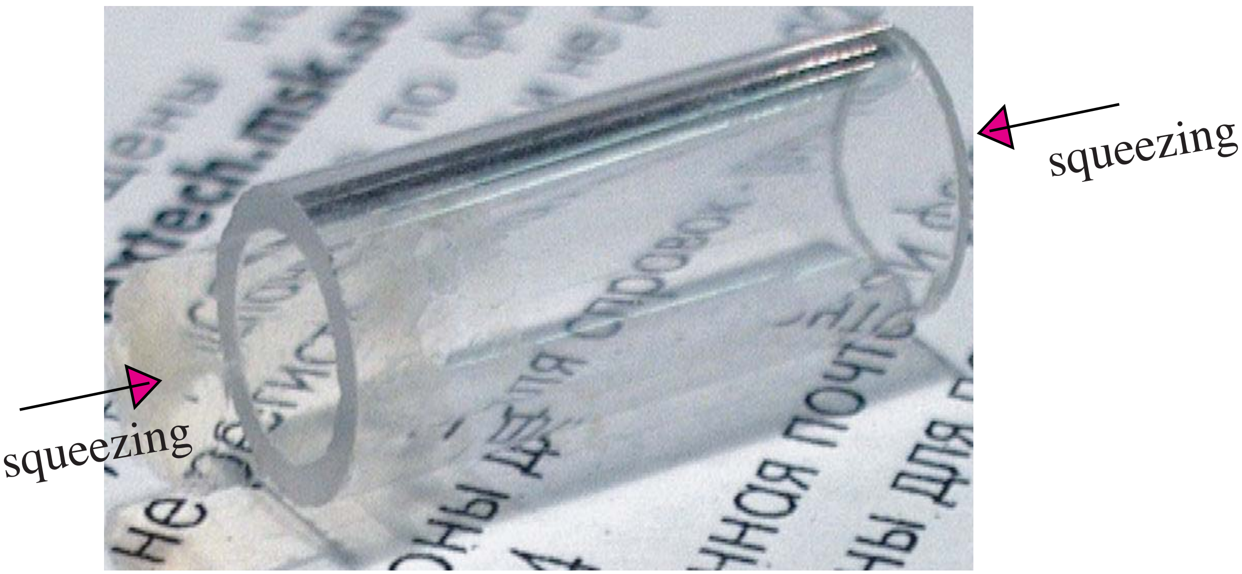
$$L_{LIM} \sim \xi_a (\xi_0^2/\delta^2) \sim 1 \ \mu m$$

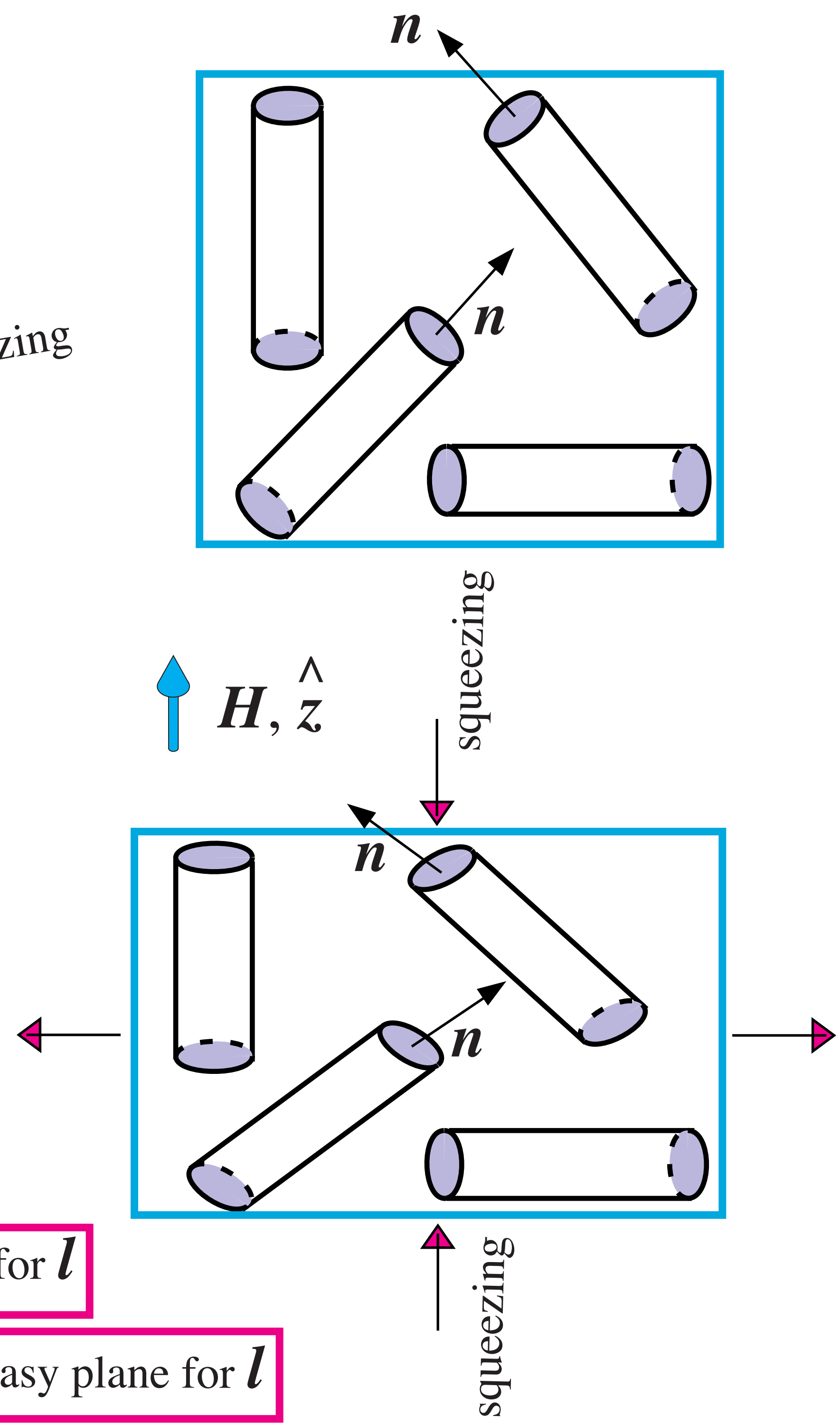
small deformation of aerogel destroys subtle
LIM effect & leads to global orientation of
^{3}He-A

Kunimatsu et al.
JETP Lett. **86** (2007), cond-mat/0612007
courtesy of Bunkov



regular anisotropy
in deformed aerogel





uniaxial deformation → regular uniaxial anisotropy

$$E_{reg\ an} = \langle E(\boldsymbol{l},\boldsymbol{n})\rangle \sim \frac{\Delta l}{l} E_a \xi_a^{-3} (\boldsymbol{l}\cdot\hat{z})^2$$

$$E(\boldsymbol{l},\boldsymbol{n}) = E_a\left[(\boldsymbol{l}\cdot\boldsymbol{n})^2 - 1/3\right]$$

squeezing: $\Delta l < 0$ → easy plane for $\boldsymbol{n}$ → easy axis for $\boldsymbol{l}$

stretching: $\Delta l > 0$ → easy axis for $\boldsymbol{n}$ → easy plane for $\boldsymbol{l}$

regular anisotropy may destroy Imry-Ma effect

energy density of uniaxial anisotropy

$$E_{regular} \sim \frac{\Delta l}{l} \; E_a \; \xi_a^{-3}$$

energy density of random anisotropy

$$E_{random} \sim E_a \, \xi_a^{-3/2} L^{-3/2}$$

Larkin-Imry-Ma length

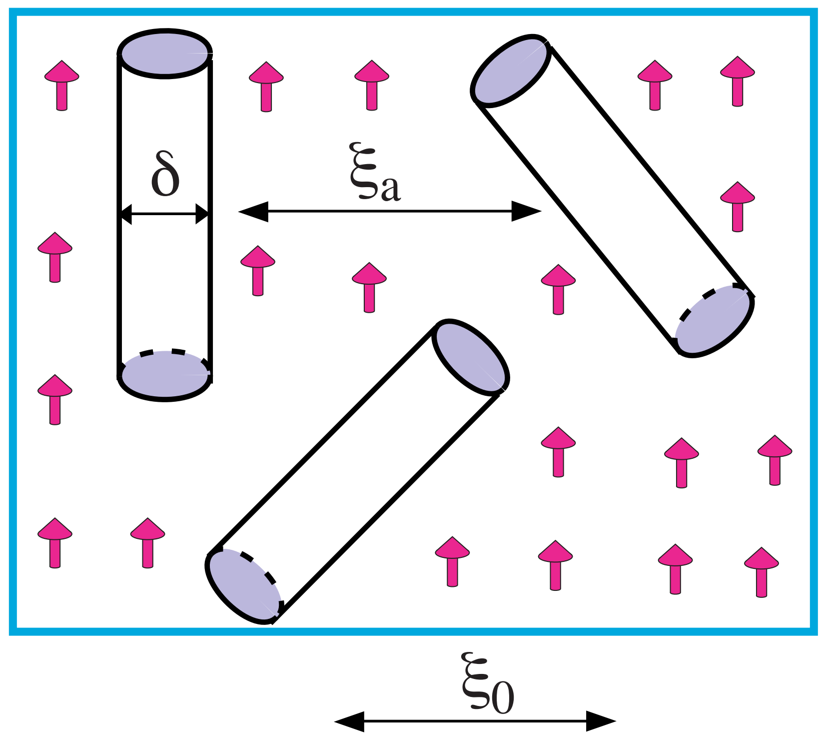
$$L \sim \xi_a (\xi_0^2/\delta^2)$$

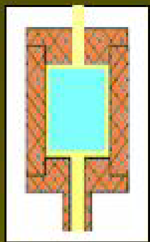
Larkin-Imry-Ma state is destroyed by small deformation:

$E_{regular} > E_{random}$ when

$$\frac{\Delta l}{l} > (\xi_a/L)^{3/2} \sim \delta^3/\xi_0^3 \sim 10^{-3}\text{-}10^{-2}$$

3 nm		20 nm		20-80 nm		1 μm		10 μm
δ	$\ll$	ξ_a	$\sim$	ξ_0	$\ll$	L	$<$	ξ_{so}

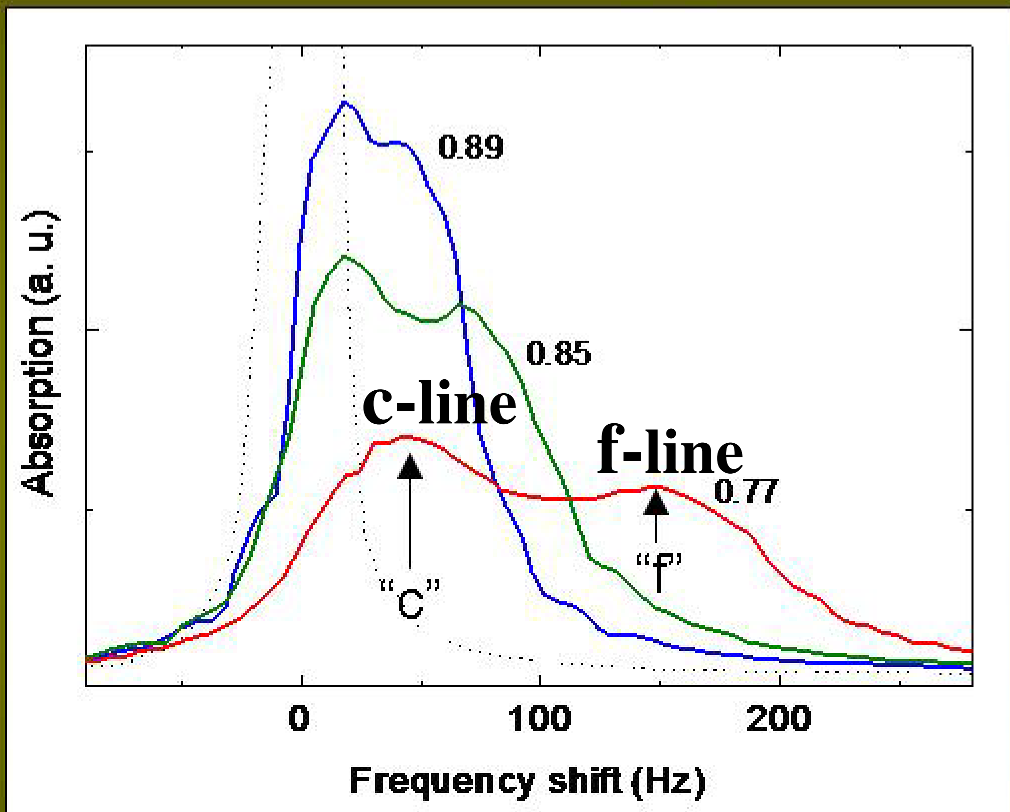




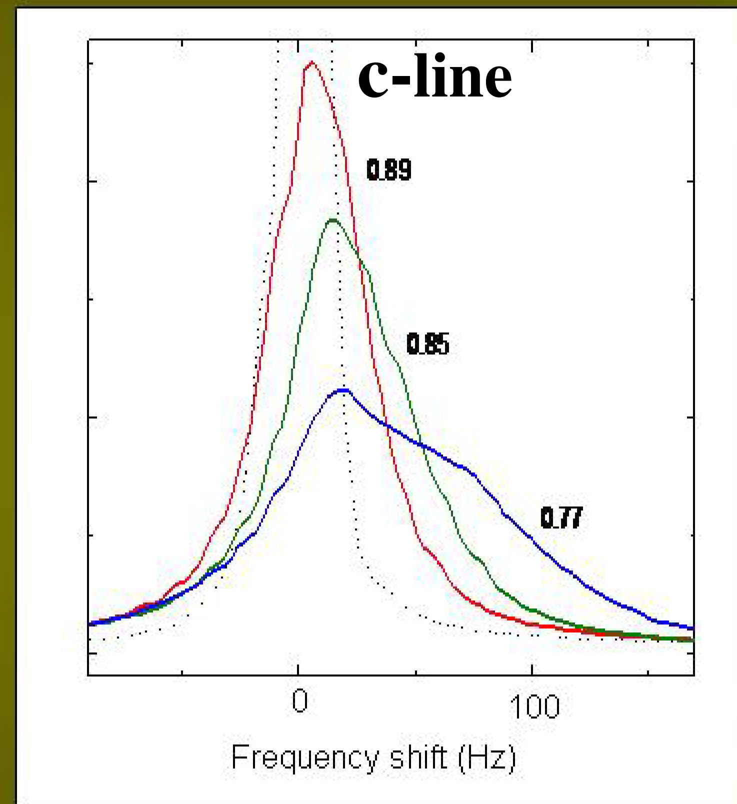
NMR probe of Larkin-Imry-Ma state

Dmitriev et al. JETP Lett. **84**, 461 (2006)

Охлаждение из норм. фазы



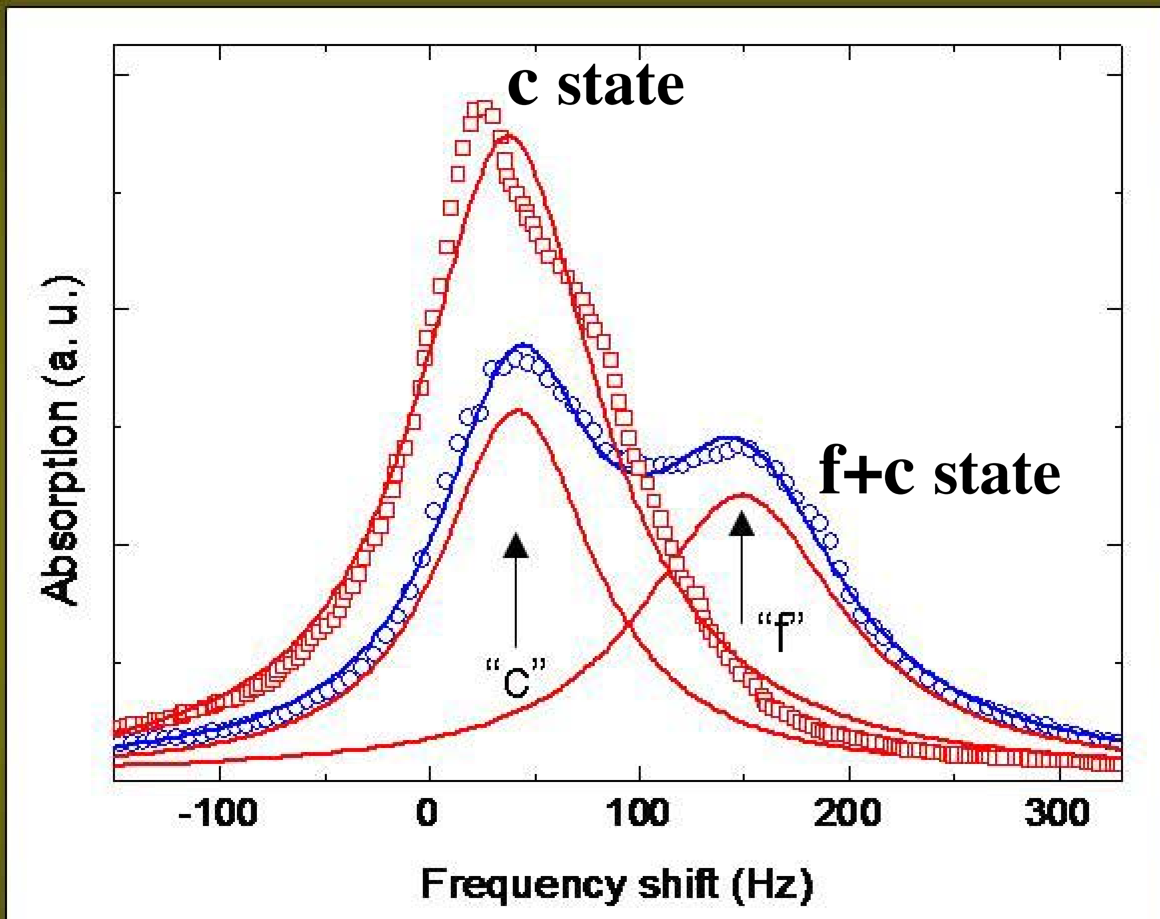
180 град. импульсы в районе T_{ca}



cooling through T_c
gives 2 lines in NMR spectrum
c (close) & **f** (far)

180^o pulse
removes **f**-line

NMR lines: in mixed f+c state and in pure c state

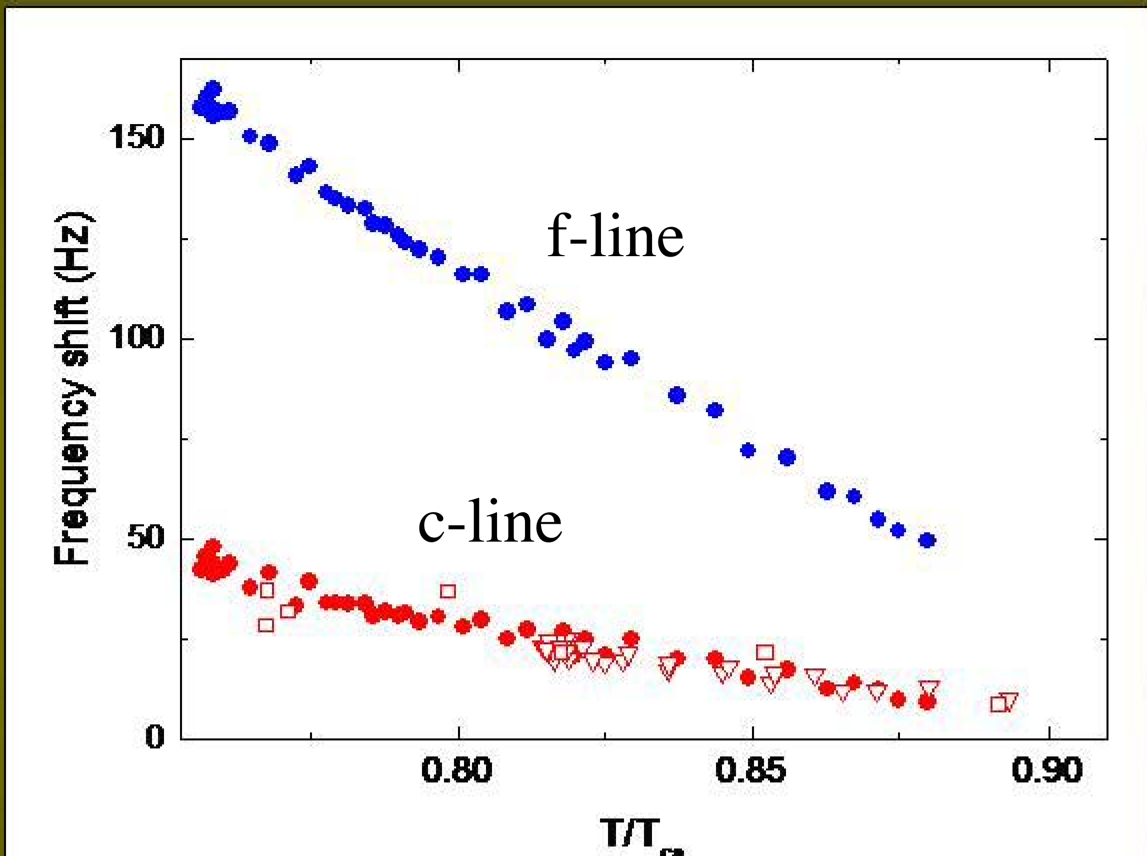


$T=0.77\ T_{ca}$,

P=29.3 бар

Dmitriev et al.

temperature dependence of NMR frequency shift

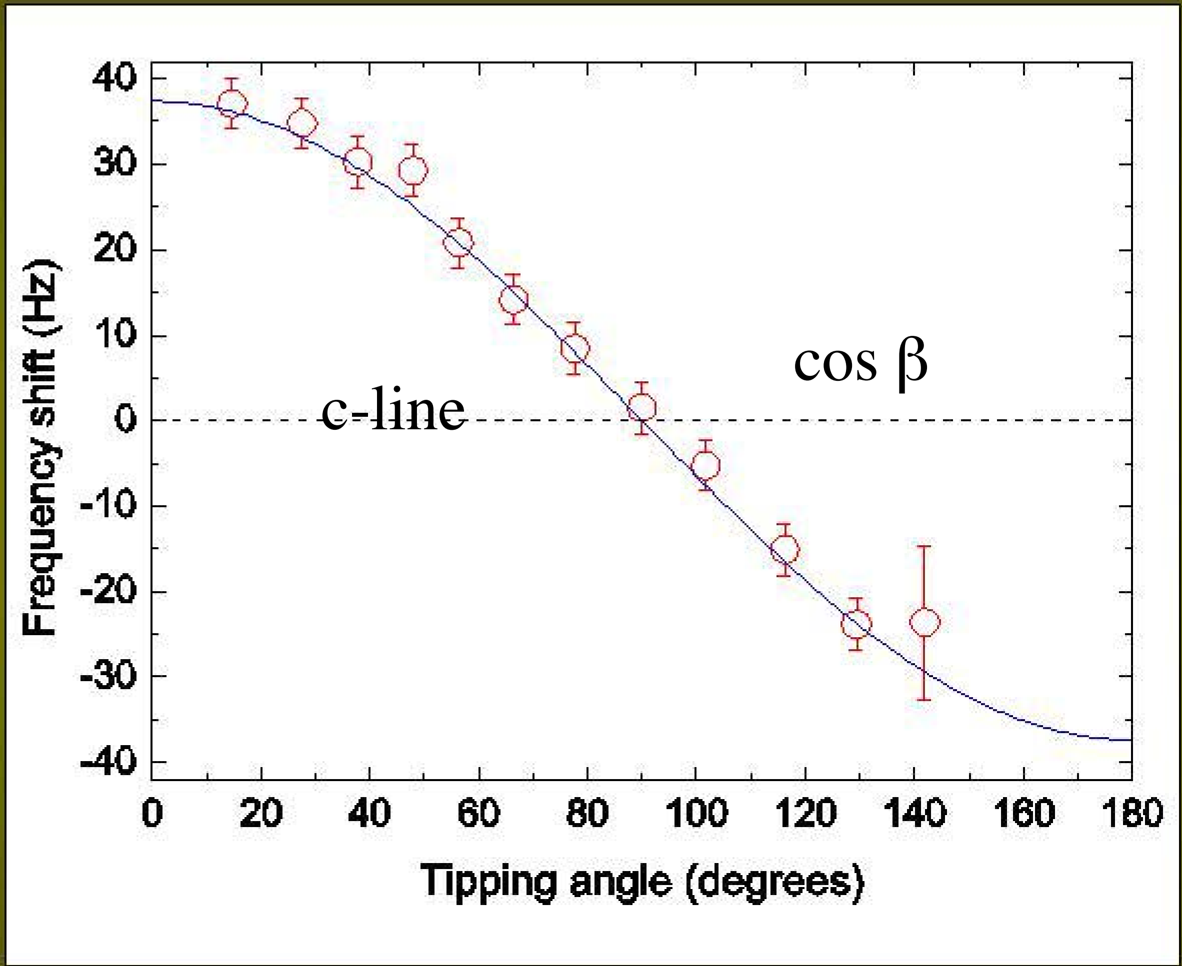


Dmitriev et al.

f-line in mixed f+c state
c-line in mixed f+c state
c-line in pure c state

P=29.3 бар

dependence of frequency shift on tipping angle of precession

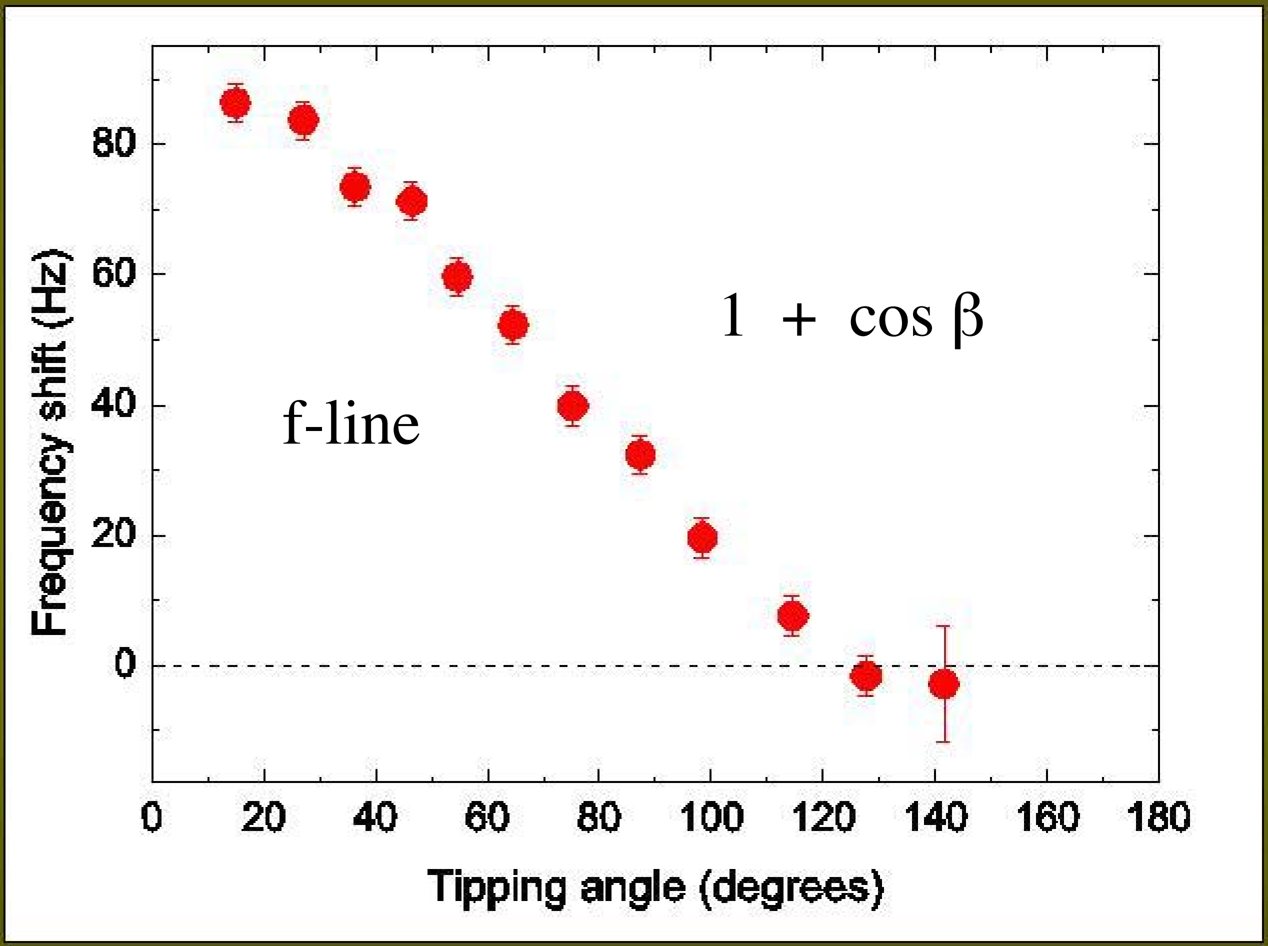


Сплошная линия:
$(\omega-\omega_L)/2\pi = A\cos\beta$
(A=37.4 Гц).

P=29.3 бар,
$T=0.76T_{ca}$.

Dmitriev et al.

dependence of frequency shift on tipping angle of precession

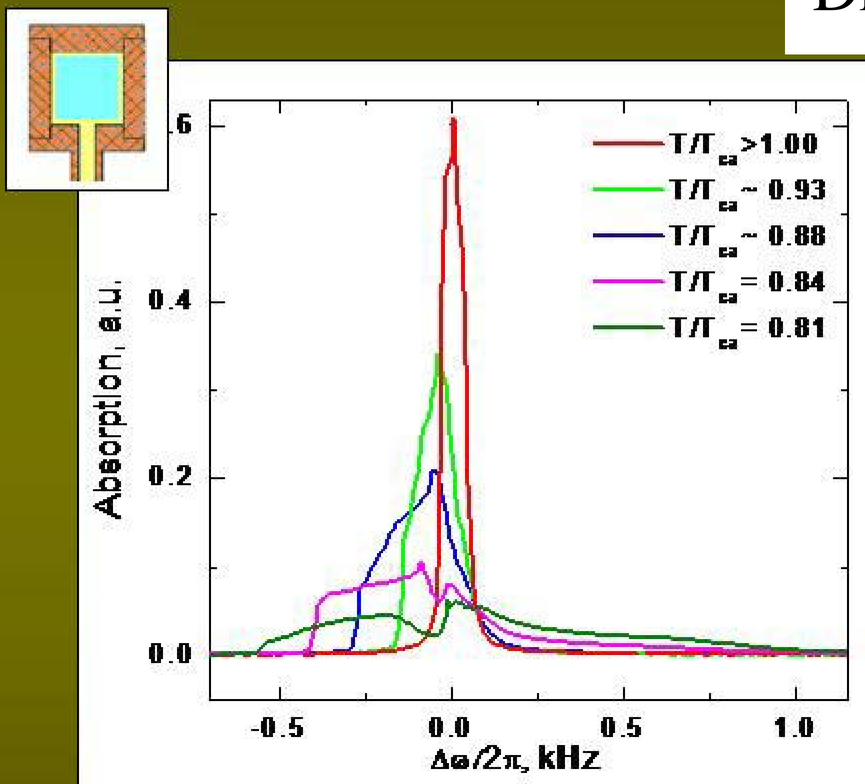


P=29.3 бар
$T=0.76T_{ca}$

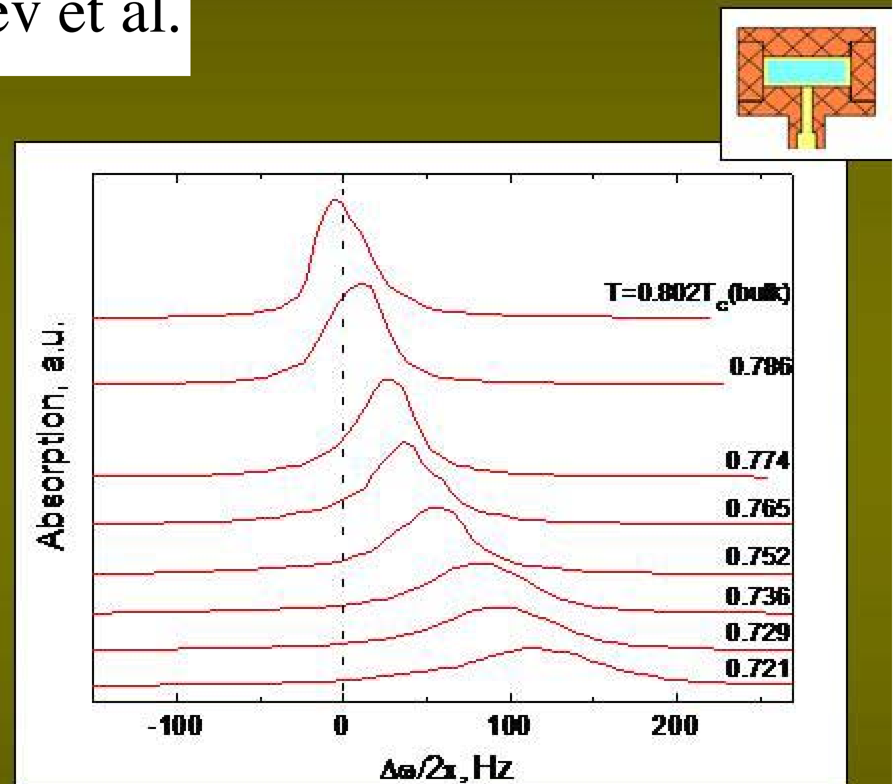
Dmitriev et al.

some samples exhibit large negative frequency shift

Dmitriev et al.



large negative frequency shift



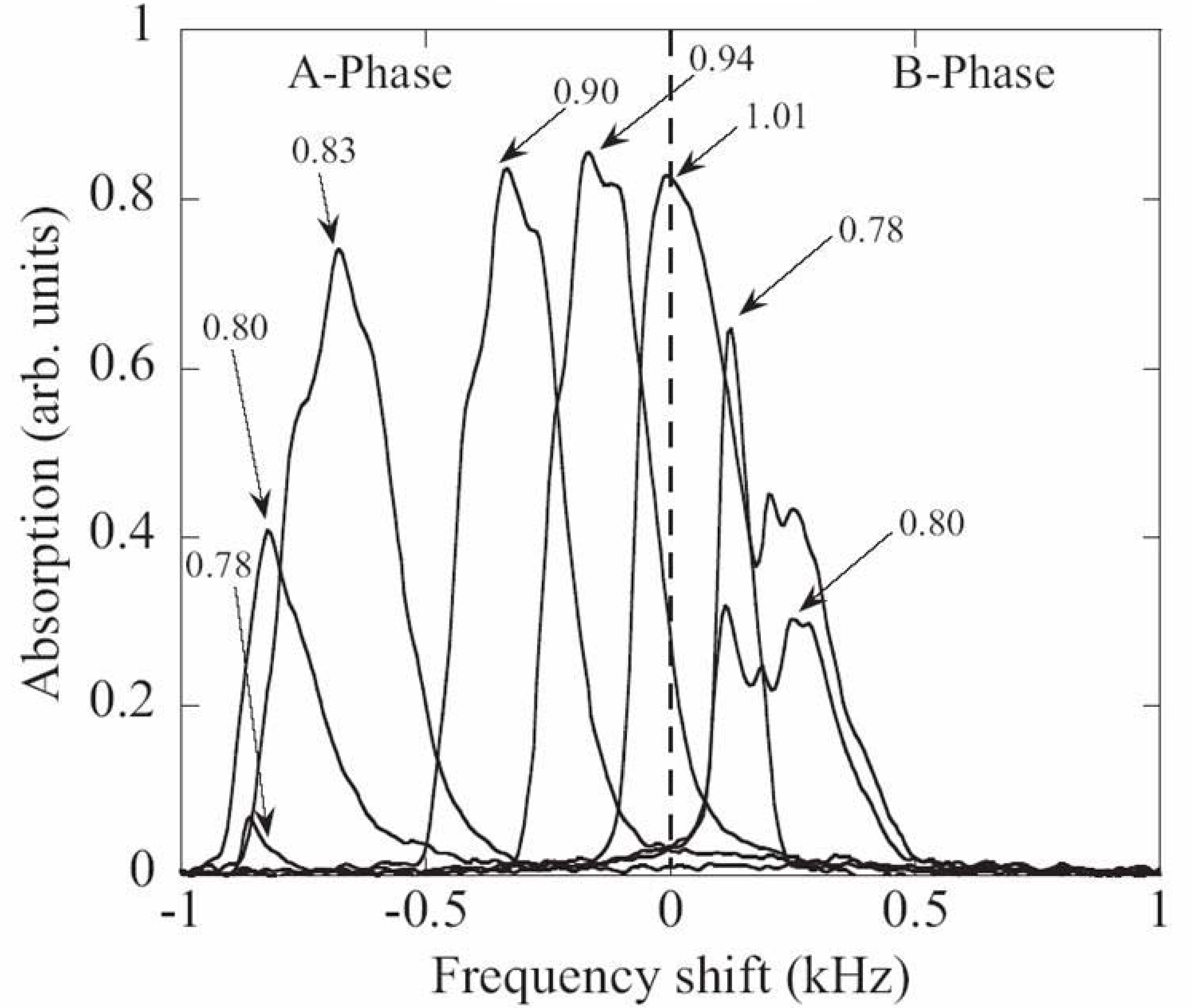
small positive frequency shift

H=284 Э, P=25.5 бар

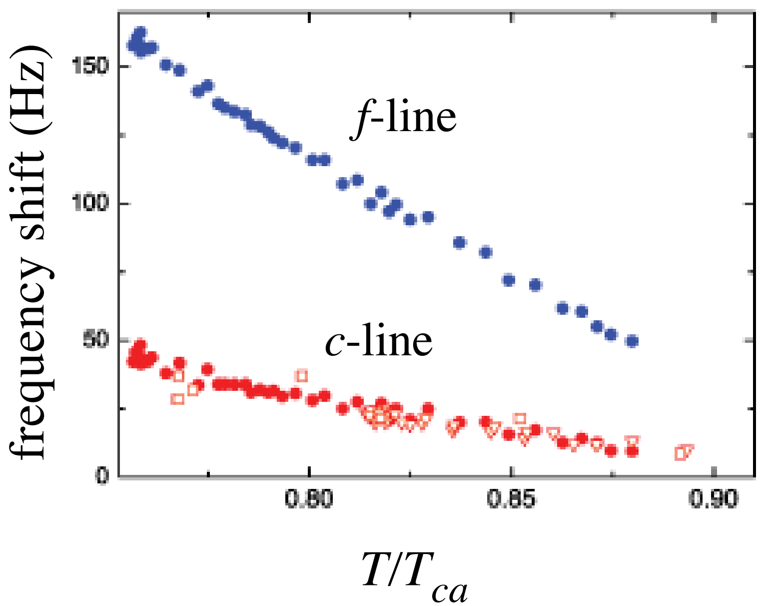
H=142 Э, P=26.0 бар

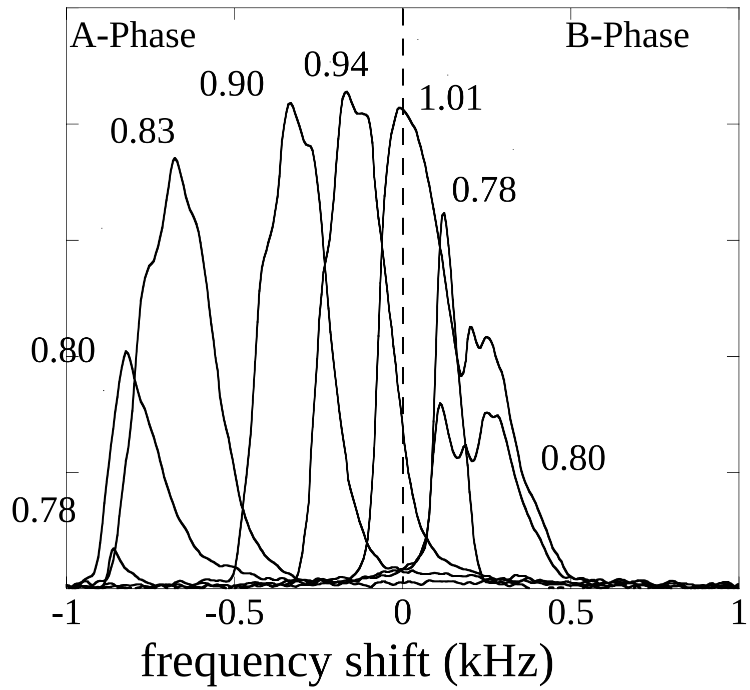
large negative frequency shift in deformed aerogel

Bunkov, Kunimatsu, et al.



non-deformed vs deformed aerogel





$$\frac{\Delta\omega_{c\text{-}line}}{|\Delta\omega_{neg}|} \sim 0.03$$

negative frequency shift
in squeezed aerogel reaches

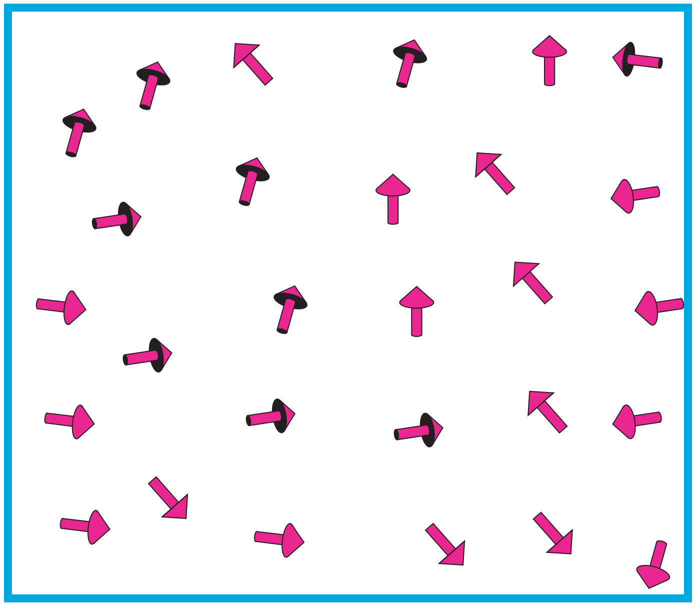
– 1 kHz

$$\frac{\Delta\omega_{c\text{-}line}}{|\Delta\omega_{neg}|} \sim 0.03$$

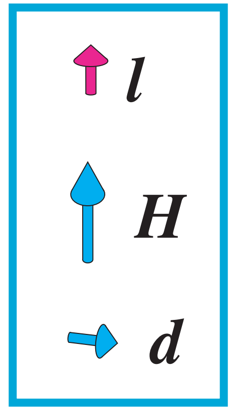
small frequency shift from Larkin-Imry-Ma effect

$$\Delta\omega = \Delta\omega_{max}[(\boldsymbol{l}\cdot\boldsymbol{d})^2 - (\boldsymbol{l}\cdot\boldsymbol{H})^2/H^2]$$

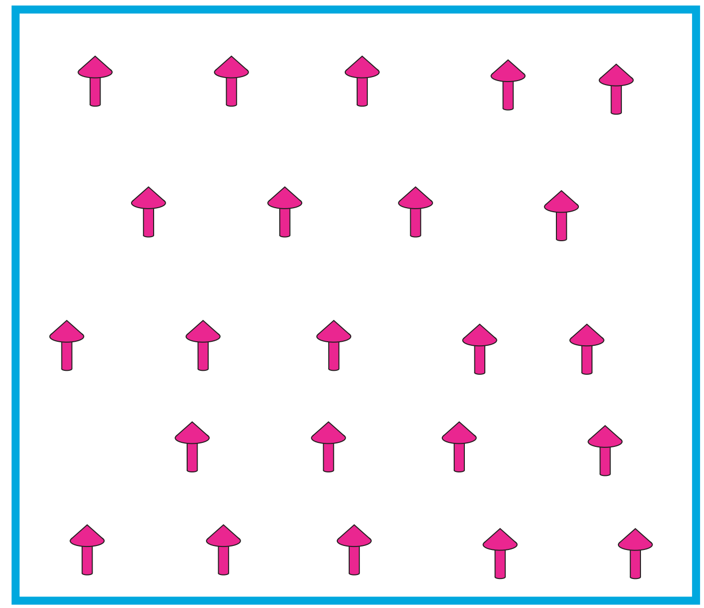
dependence of frequency shift $\Delta\omega$
on orientation of $\boldsymbol{l}$
with respect to magnetic field $\boldsymbol{H}$
and vector $\boldsymbol{d}$



for random $\boldsymbol{l}$
and
fixed $\boldsymbol{d} \perp \boldsymbol{H}$
$\Delta\omega = 0$



if $\boldsymbol{l}$ along $\boldsymbol{H}$
$\Delta\omega = -\Delta\omega_{max}$



state with c-line:
small positive shift in disordered
Larkin-Imry-Ma state

state with negative shift:
Larkin-Imry-Ma state
is destroyed
by deformation of aerogel

experimental determination of Larkin-Imry-Ma length

non-zero NMR shift in random $\boldsymbol{l}$ texture is due to spin-orbit interaction:
another regular anisotropy

$$E_{\mathrm{so}} = -\, g_D(\boldsymbol{l} \cdot \boldsymbol{d})^2$$

spin-orbit coupling slightly aligns $\boldsymbol{l}$ & $\boldsymbol{d}$ by amount

$$E_{\mathrm{so}} / E_{\mathrm{random}} \sim (L / \xi_{\mathrm{so}})^2$$

leading to small frequency shift

$$\frac{\Delta\omega^{\mathrm{theor}}_{c\text{-}line}}{|\Delta\omega_{neg}|} \sim (L / \xi_{\mathrm{so}})^2$$

$$\frac{\Delta\omega^{\mathrm{exp}}_{c\text{-}line}}{|\Delta\omega_{neg}|} \sim 0.03$$

→

$$(L / \xi_{\mathrm{so}})^2 \sim 0.03$$

3 nm		20 nm		20-80 nm		1-3 μm		10 μm
δ	$\ll$	ξ_a	$\sim$	ξ_0	$\ll$	L	$<$	ξ_{so}

dependence of frequency shift on tipping angle β in disordered states

$$\frac{\Delta\omega(\beta)}{|\Delta\omega_{neg}|} = b\cos\beta + a(1+\cos\beta)$$

$b = (1/2)(1 - 3\langle l_z^2 \rangle)$

$a = b = 0$ in full randomness,

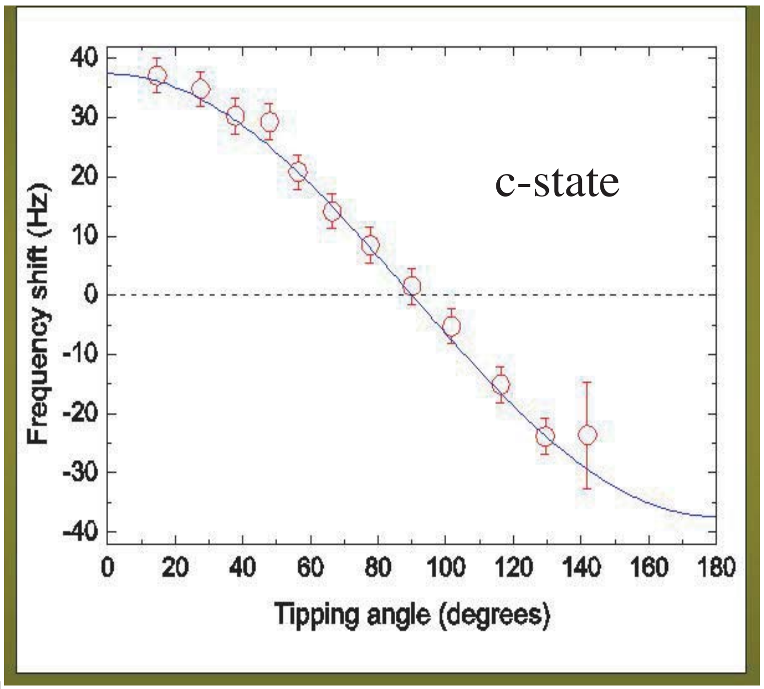
for finite L/ξ_{so} :

$a, b \sim (L/\xi_{so})^2 \ll 1$

$a = (1/6)(1 - 2\langle \sin^2\Phi \rangle)$

Φ - angle between $\boldsymbol{l}_\perp$ and $\boldsymbol{d}_\perp$ in transverse plane

interpretation of c-state: $b\cos\beta$



$\boldsymbol{l}_\perp$ random

$\boldsymbol{d} \perp \boldsymbol{H}$ regular

(a =0),

uniaxial anisotropy due to $\boldsymbol{H}$

(b >0)

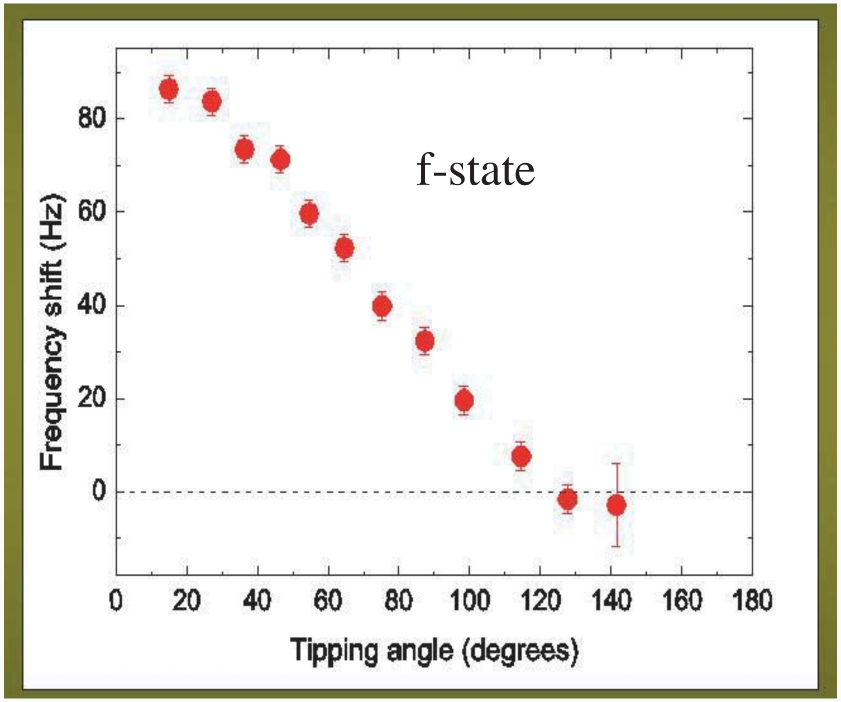
interpretation of f-state: $a(1+\cos\beta)$

$\boldsymbol{l}$ random

(b =0),

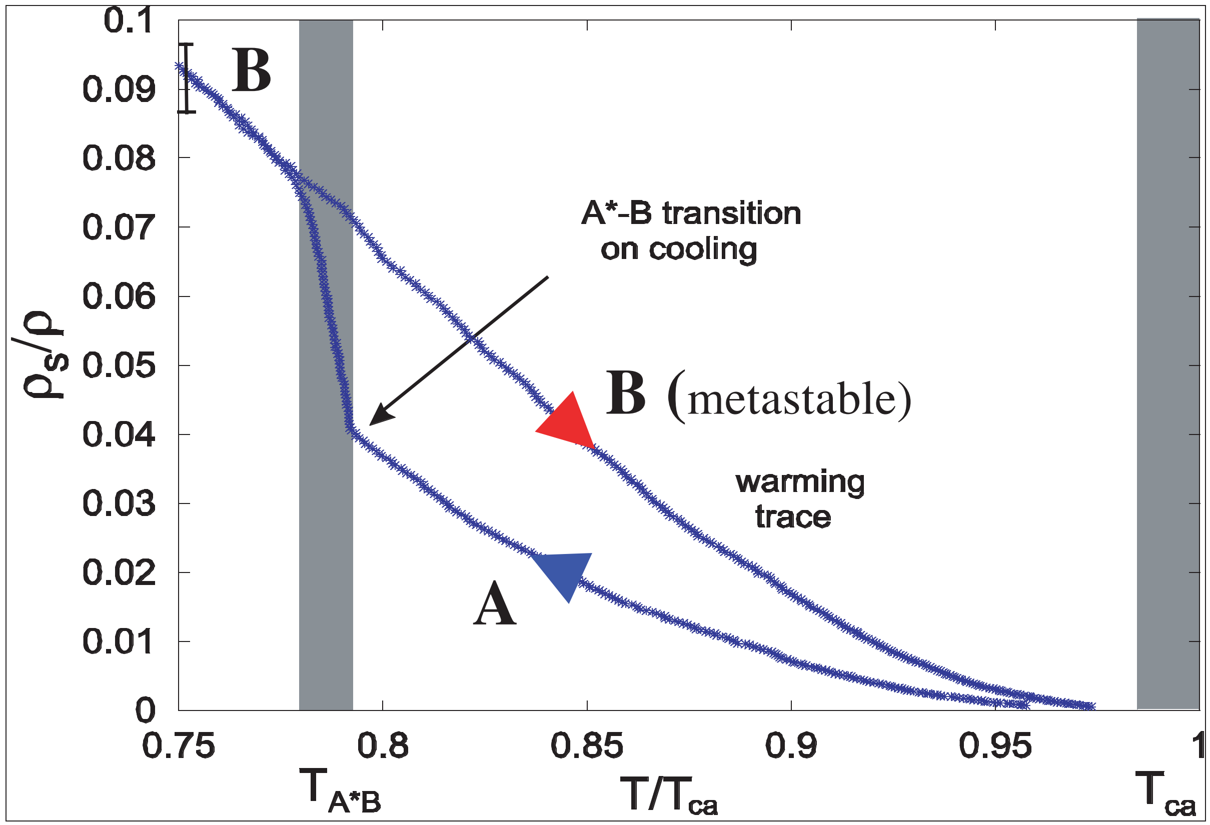
$\boldsymbol{d}_\perp$ and $\boldsymbol{l}_\perp$ are not independent

(a >0)



Superfluid density in Larkin-Imry-Ma state

4-th sound measurements



E. Nazaretski, N. Mulders & J. M. Parpia
Pis'ma ZhETF **79**, 470 (2004)

$\frac{\rho_{sA}}{\rho_{sB}}$ *in aerogel* < $\frac{\rho_{sA}}{\rho_{sB}}$ *in bulk*

suppression of ρ_{sA} *in LIM state ?*

Larkin-Imry-Ma state as vortex state

l-texture produces continuous vorticity:

$$\nabla\times\mathbf{v}_s = (h/8\pi m)\, e_{ijk}\, l_i \nabla l_j \times \nabla l_k$$

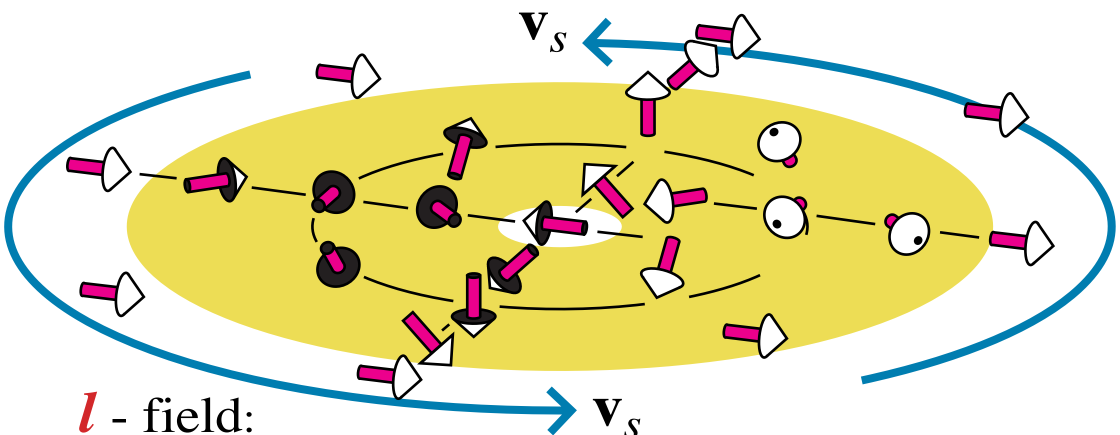
Continuous vortex - skyrmion in bulk ^{3}He-A

$$\int \mathbf{v}_s \cdot d\mathbf{r} = (h/m)(e_{ijk}/8\pi) \int d\mathbf{S} \cdot (l_i \nabla l_j \times \nabla l_k) = h/m\, N_2$$

N_2 - topological charge of skyrmion

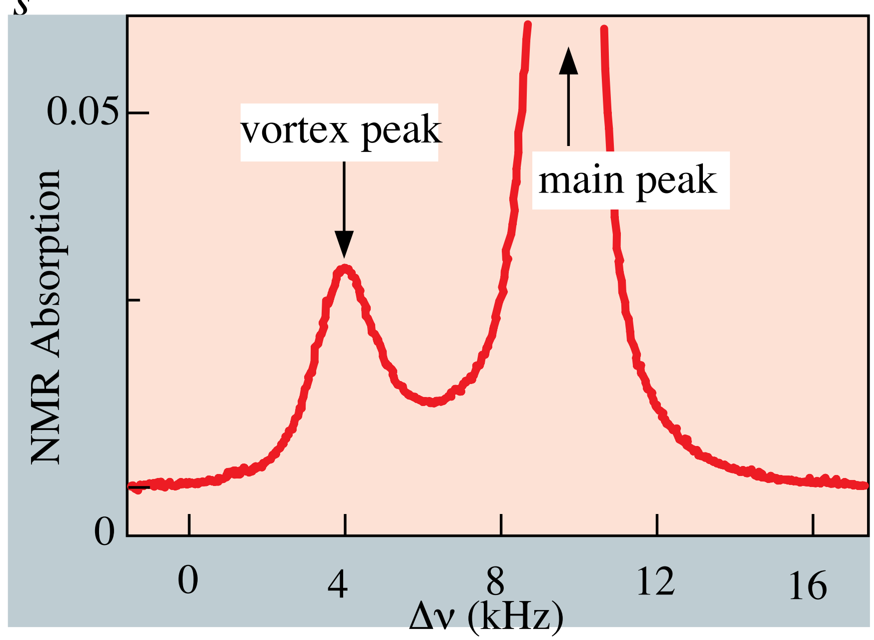
Circulation of $\mathbf{v}_s$
$N = 2N_2 = 2$

Doubly quantized continuous vortex



Position of vortex peak determines the type of vortex

Intensity of vortex peak determines number of vortices

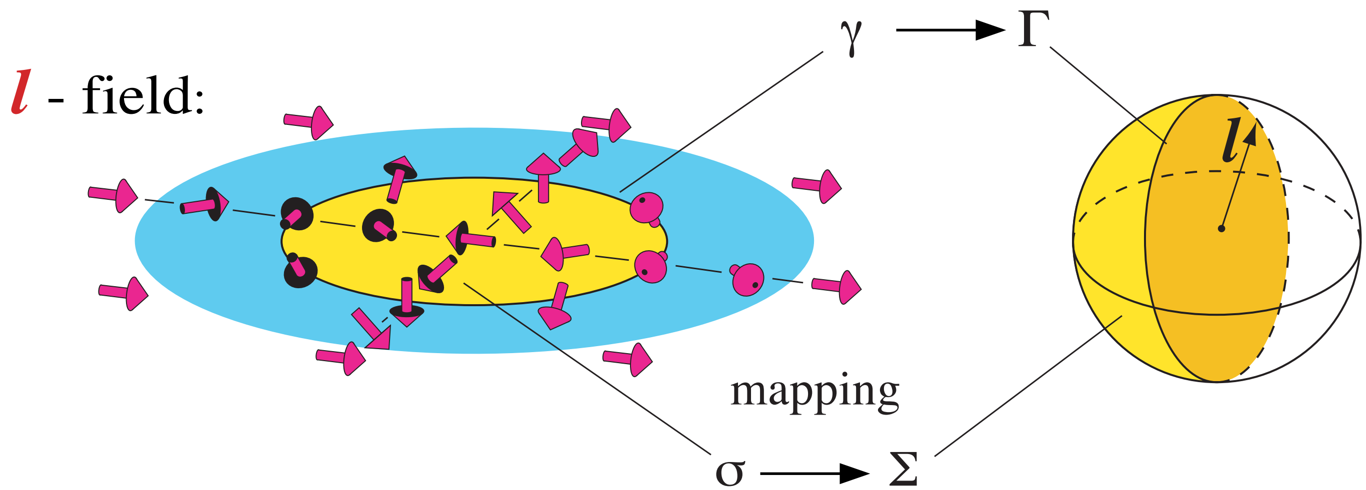


experimental observation by NMR in bulk ^{3}He-A
(Helsinki)

Topology of continuous vortex - skyrmion

classes of mapping $S^2 \longrightarrow S^2$

disk with one point at infinity $\longrightarrow$ unit sphere of **l**-vector



topological invariant for skyrmion

$$N_2 = \frac{1}{8\pi} e^{ijk} \int dS_i\, \boldsymbol{l}\cdot(\partial_j \boldsymbol{l} \times \partial_k \boldsymbol{l}) = 1$$

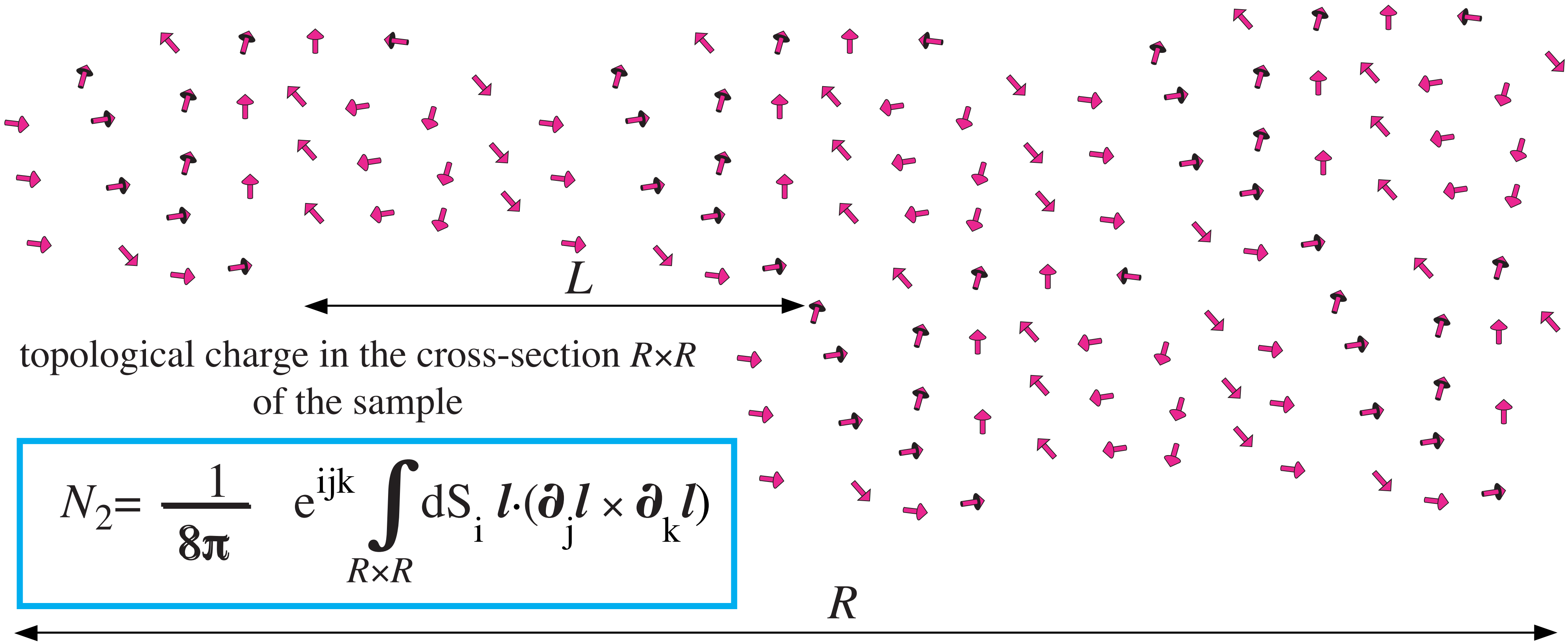
homotopy group

$\pi_2 = Z$

3He-A	***circulation***	**2-component BEC**
$N = 2N_2$		$N = N_2$

Larkin-Imry-Ma state as vortex state:

random distribution of continuous vortices with core size of order LIM length L



topological charge in the cross-section $R\times R$ of the sample

$$N_2 = \frac{1}{8\pi} e^{ijk} \int_{R\times R} dS_i \, \mathbf{l}\cdot(\partial_j \mathbf{l} \times \partial_k \mathbf{l})$$

number of vortex lines crossing the cross-section

$$N = 2N_2$$

$$\langle N \rangle = 0$$

$$\langle N^2 \rangle = R^2 / L^2$$

area law for $\langle N^2 \rangle$:
LIM state of superfluid ^{3}He-A looks similar to
the state ***above Berezinskii-Kosterlitz-Thouless transition***

is Larkin-Imry-Ma state superfluid ?

area law for $< N^2 >$:
LIM state of superfluid ^{3}He-A looks similar to vortex plasma state
above Berezinskii-Kosterlitz-Thouless superfluid transition

$\rho_{sA} = 0$ **if vortices can freely move**

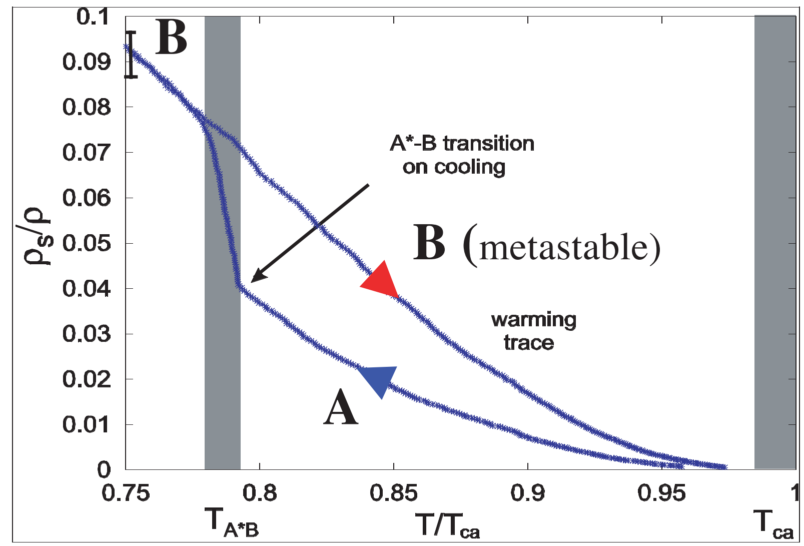
$\rho_{sA} > 0$ **if some pinning of vortices occurs**

for strong pinning:

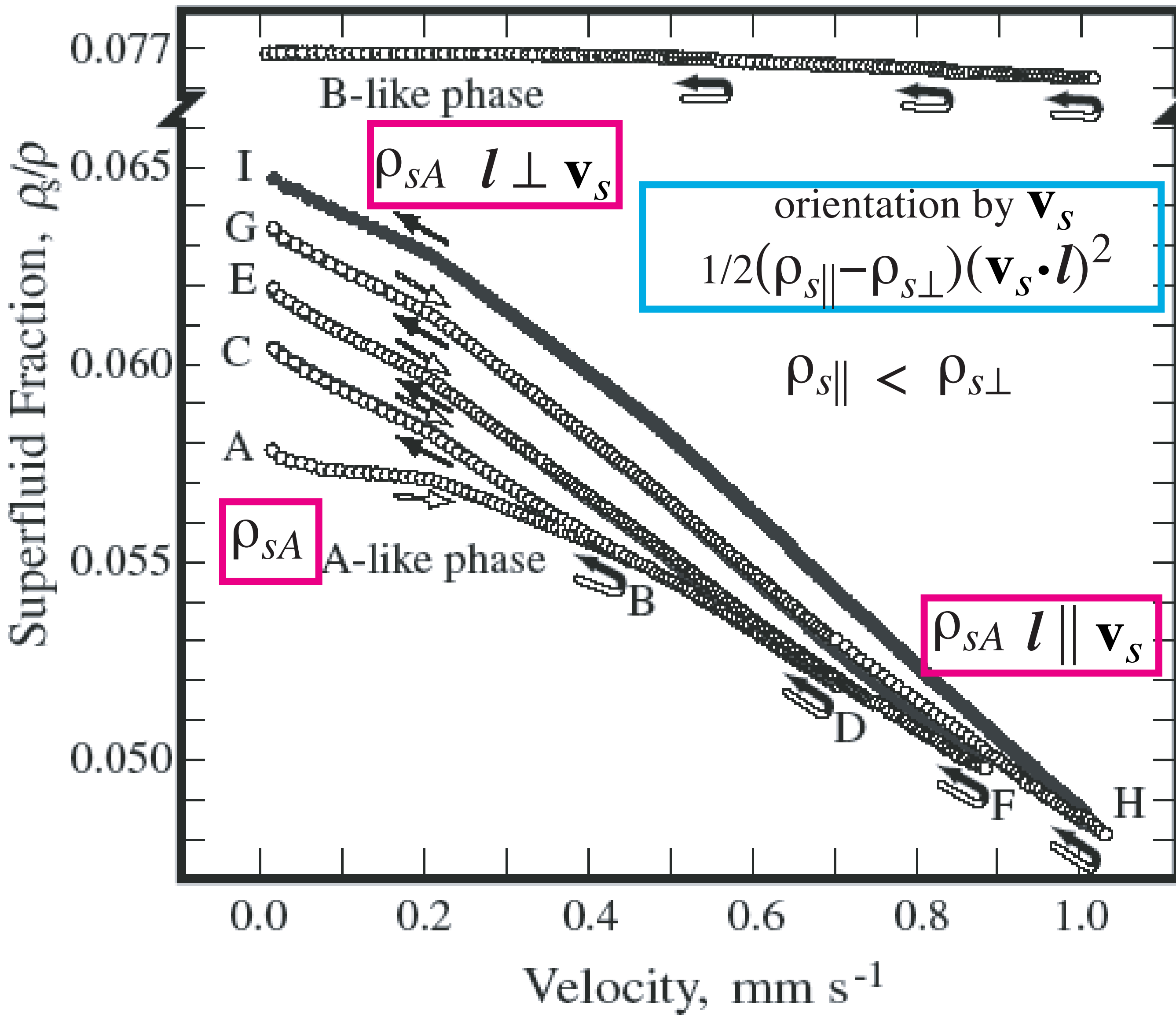
$$\frac{\rho_{sA}}{\rho_{sB}} \textit{ in aerogel} = \frac{\rho_{sA}}{\rho_{sB}} \textit{ in bulk}$$

in general:

$$\frac{\rho_{sA}}{\rho_{sB}} \textit{ in aerogel} < \frac{\rho_{sA}}{\rho_{sB}} \textit{ in bulk}$$



problems: calculate ρ_s in Larkin-Imry-Ma state
measure ρ_s in c-state
measure ρ_s in f-state
measure ρ_s in uniform states with $\boldsymbol{l} \perp \mathbf{v}_s$ & $\boldsymbol{l} \parallel \mathbf{v}_s$



estimation of LIM length L from critical velocity orienting $\boldsymbol{l}$:

$$v_c \sim 1\,\text{mm/s} \sim \hbar/mL$$
$$L \sim 10\ \mu\text{m}$$

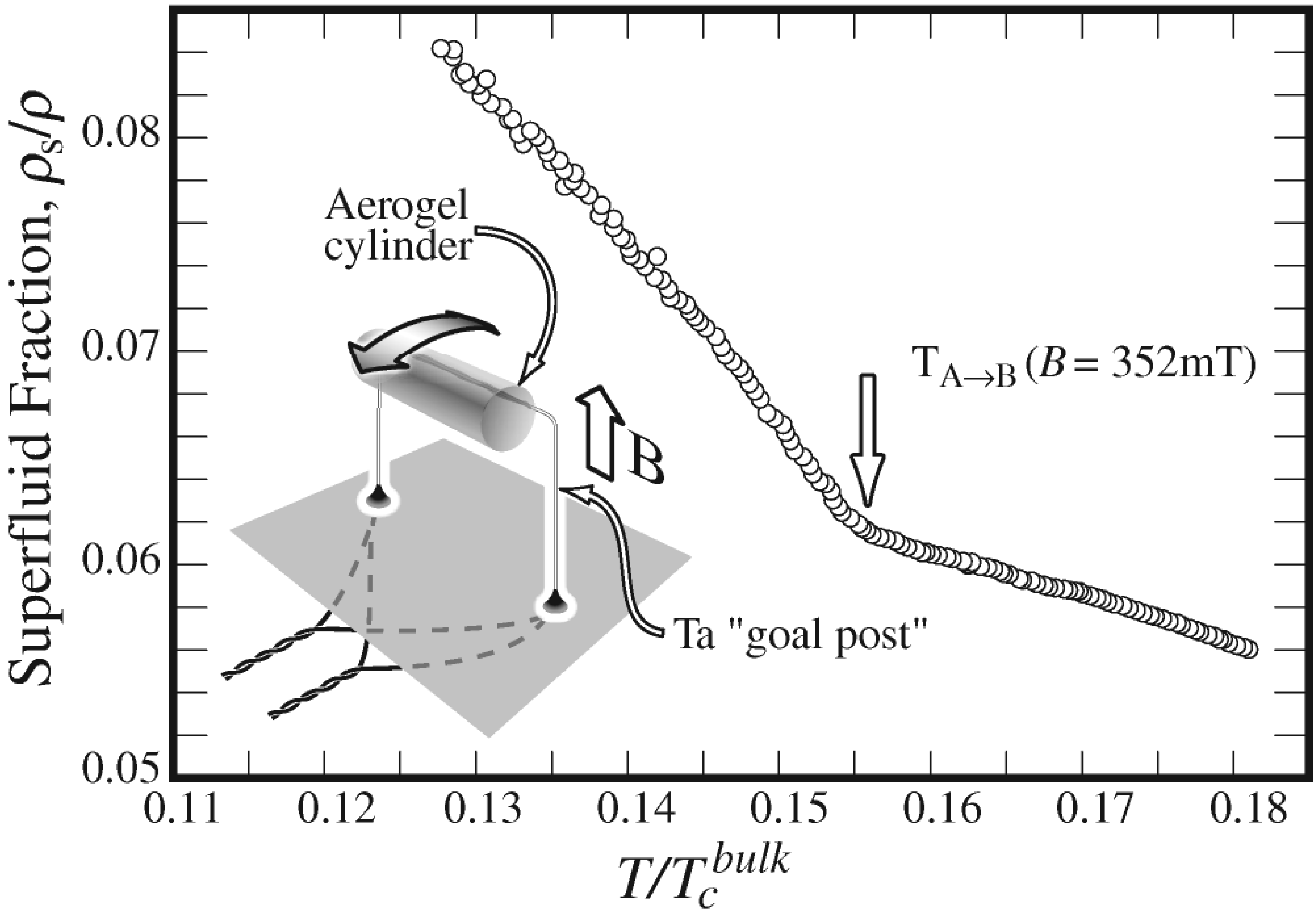
for random $\boldsymbol{l}$ texture:

$$\langle\rho_s\rangle = (1/3)\rho_{s\|} + (2/3)\rho_{s\perp}$$

$$\rho_{s\|} < \langle\rho_s\rangle < \rho_{s\perp}$$

agrees with pinned $\boldsymbol{l}$ texture

FIG. 2. The inferred superfluid fraction as a function of velocity at a temperature of $\sim 0.18T_c^{\text{bulk}}$.



Superfluid Fraction, ρ_s/ρ
0.05
0.06
0.07
0.08
0.11
0.12
0.13
0.14
0.15
0.16
0.17
0.18
T/T_c^{bulk}
Aerogel cylinder
B
Ta "goal post"
$T_{A\rightarrow B}$ (B = 352mT)

conclusion:

- ^{3}He-A in aerogel loses long-range orientational order
- long-range orientational order is restored by deformation of aerogel
- c-state is most probably Larkin-Imry-Ma state

problems:

experimental

- measure critical deformation
- identify f-state
- identify half-quantum vortices
  (they are stable if $\boldsymbol{l} \parallel \boldsymbol{H}$)
- measure superfluid density ρ_s
  (in Larkin-Imry-Ma state, in c-state, in f-state, in uniform states)

theoretical

- quasi long-range order:
  Emig, Bogner & Nattermann, PRL **83**, 400 (1999)
  "Nonuniversal Quasi-Long-Range Order in the Glassy Phase of Impure Superconductors"
- long-range correlation of silicon strands:
  Fedorenko & Kühnel, Phys. Rev. B **75**, 174206 (2007)
- role of topological defects
  (hedgehogs, singular vortices, solitons, skyrmions)
- NMR spectrum in all these states

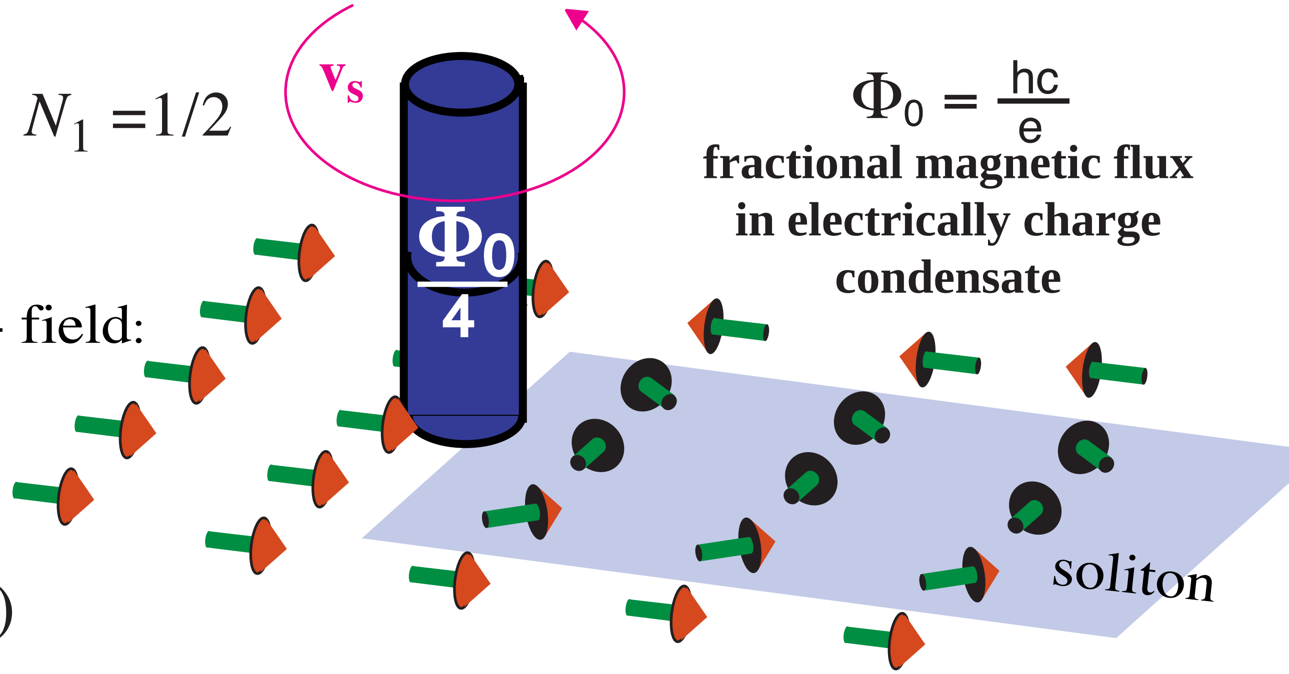
Half-quantum vortex

(Alice string)

$N_1 = 1/2$

$\Phi_0 = \frac{hc}{e}$

fractional magnetic flux in electrically charge condensate



vector-spinor order parameter (Higgs field)

d - field:

$$\Delta^{i}_{\alpha\beta} \sim e^{i\Phi}\, \sigma^{k}_{\alpha\beta} d^{k} (e^{i}_{1} + i e^{i}_{2})$$

d - unit vector in spin space $SO(3)$ $\quad$ $\mathbf{e}_1, \mathbf{e}_2, \mathbf{l} = \mathbf{e}_1 \times \mathbf{e}_2$ - triad in orbital space $SO(3)$

$SO(3) \Rightarrow U(1)$ $\quad$ $SO(3) \times U(1) \Rightarrow U(1)$

$G = SO(3) \times SO(3) \times U(1) \Rightarrow H = U(1) \times U(1) \times Z_2$

Z_2 symmetry: ***after circling*** $\mathbf{d} \Rightarrow -\mathbf{d}$ ***but*** $\Delta(\mathbf{r})$ ***must be single valued***

↓

this requires $\Phi \Rightarrow \Phi + \pi$ ***after circling***

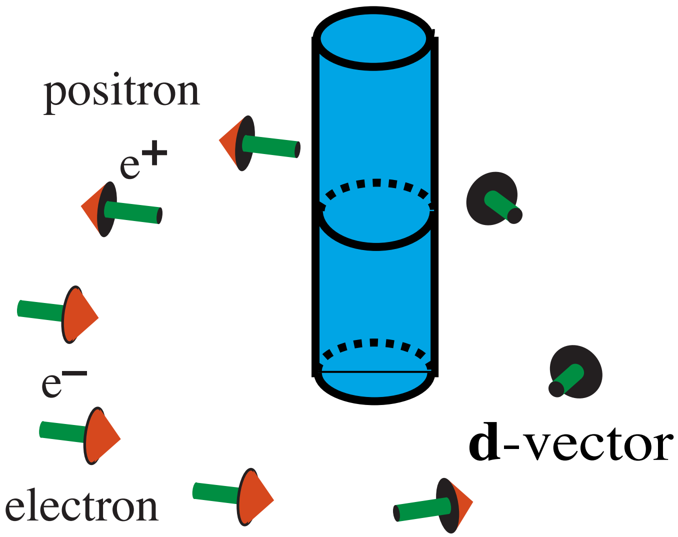
↓

$\Phi = \frac{\phi}{2}$ $\quad$ **half-quantum vortex**

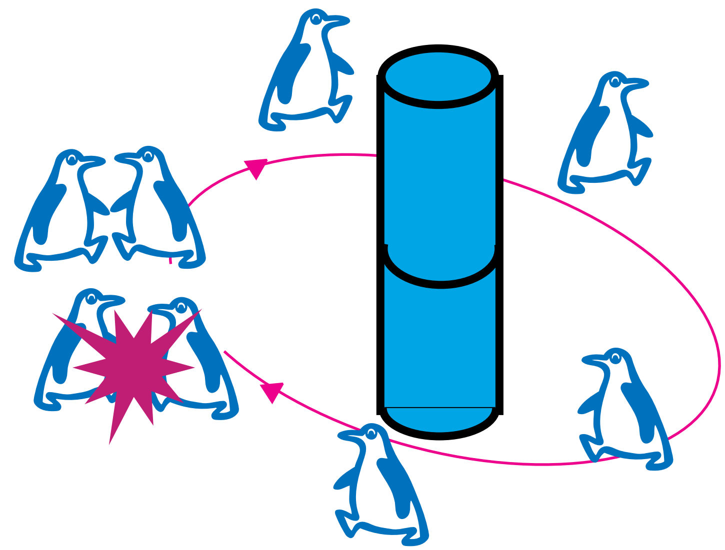
Alice string in spinor BEC

Alice string in GUT (A. Schwarz)

$$\Psi(\mathbf{r}) = \mathbf{d}(\mathbf{r})e^{i\phi}$$



particle continuously transforms
to antiparticle
after circling around the Alice string



person traveling around string
can annihilate
with person who was at home