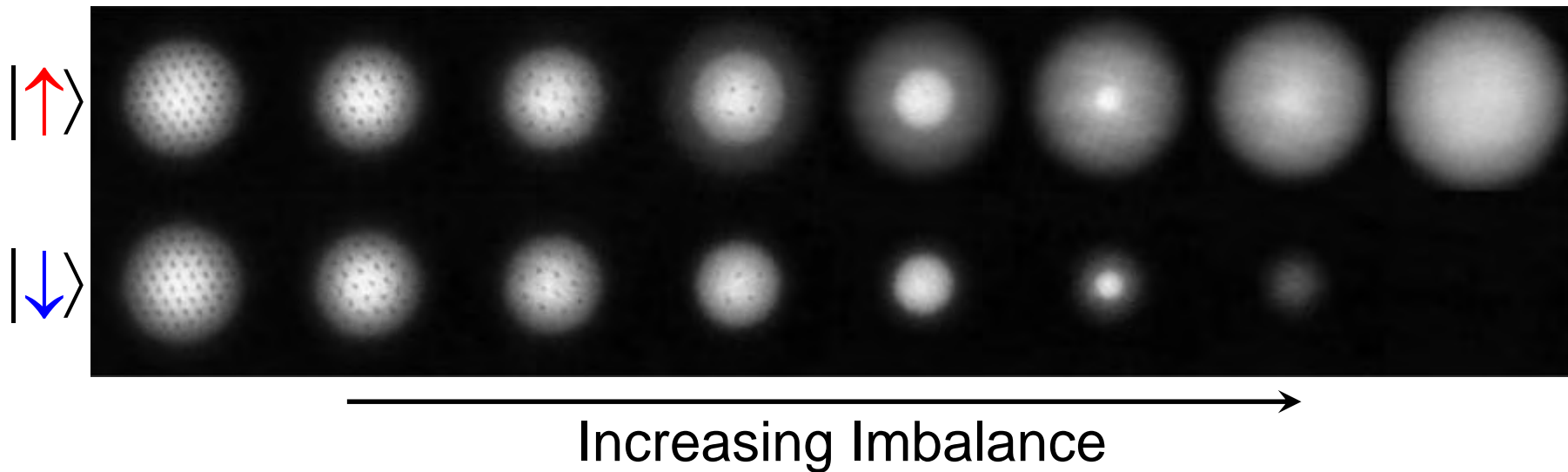


*A. I. Larkin Memorial Conference, Landau Institute, Chernogolovka, Russia
June 24, 2007*

The Ground State of Imbalanced Fermi Mixtures

Martin Zwierlein

MIT, Cambridge, USA and Johannes Gutenberg-Universität Mainz



*Massachusetts Institute of Technology
Center for Ultracold Atoms at MIT and Harvard*

\$\$\$: NSF, ONR, NASA



Pairing and Superfluidity of Fermions

Ubiquitous Phenomenon

Condensed Matter Physics

Superconductivity
Superfluidity in ^3He

Nuclear Physics

Pairing inside nuclei
p-p, n-n, n-p(?)

Astrophysics

Early Universe: Dense quark matter
Neutron stars, Quark (Color)
Superfluidity

Pairing and Superfluidity of Fermions

Atomic Physics: Provides a Model System

A mixture of **strongly interacting** fermionic atoms

Ultradilute

$$n = 10^{11} \dots 10^{15} \text{ cm}^{-3}$$

Ultracold

$$T_F = 0.1 \dots 10 \text{ } \mu\text{K}$$

- No impurities
- Highly controllable:

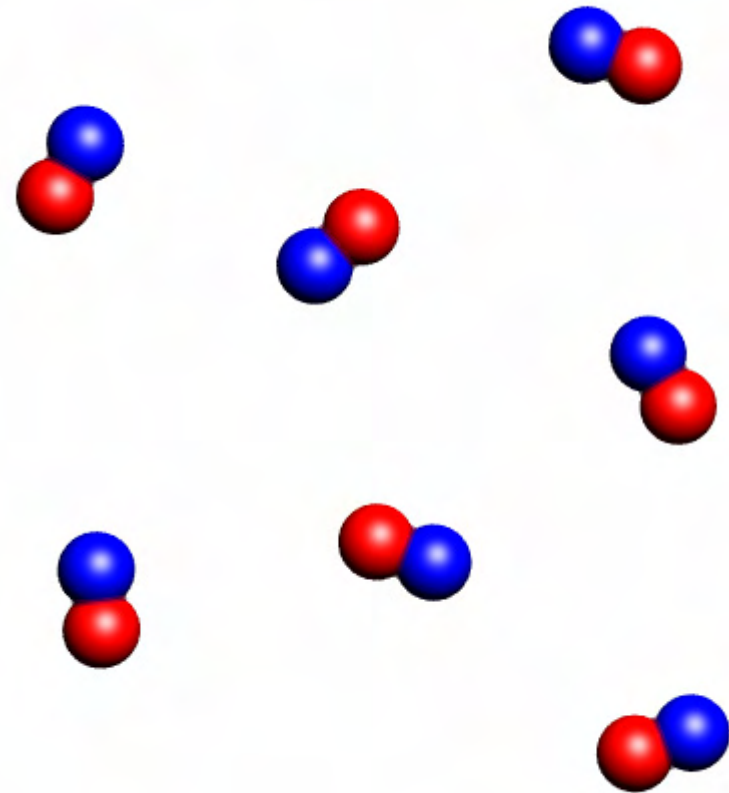
Density

Temperature

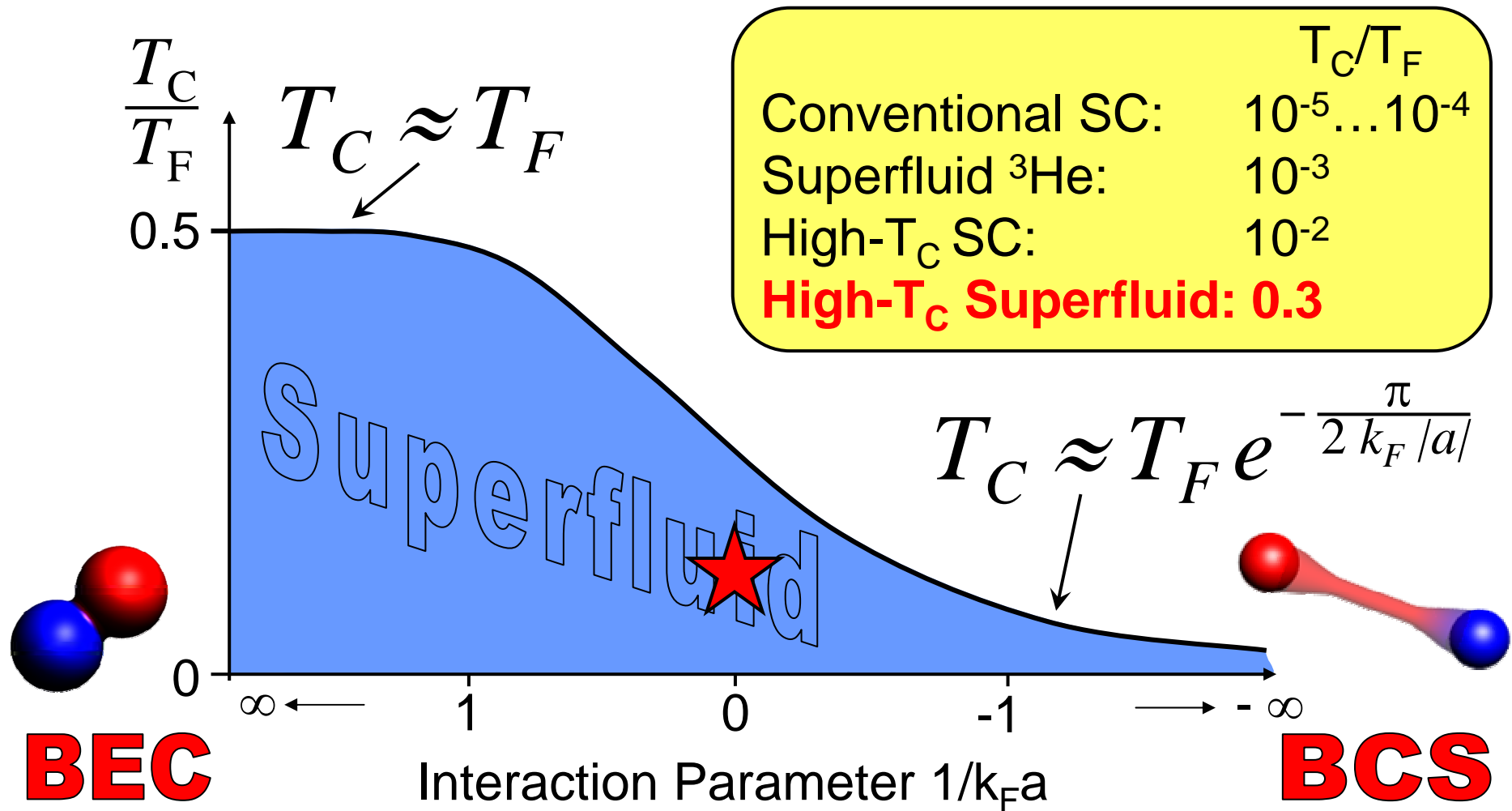
Dimensionality

Interaction strength

→ High T_C !



Critical Temperature for Fermionic Superfluidity



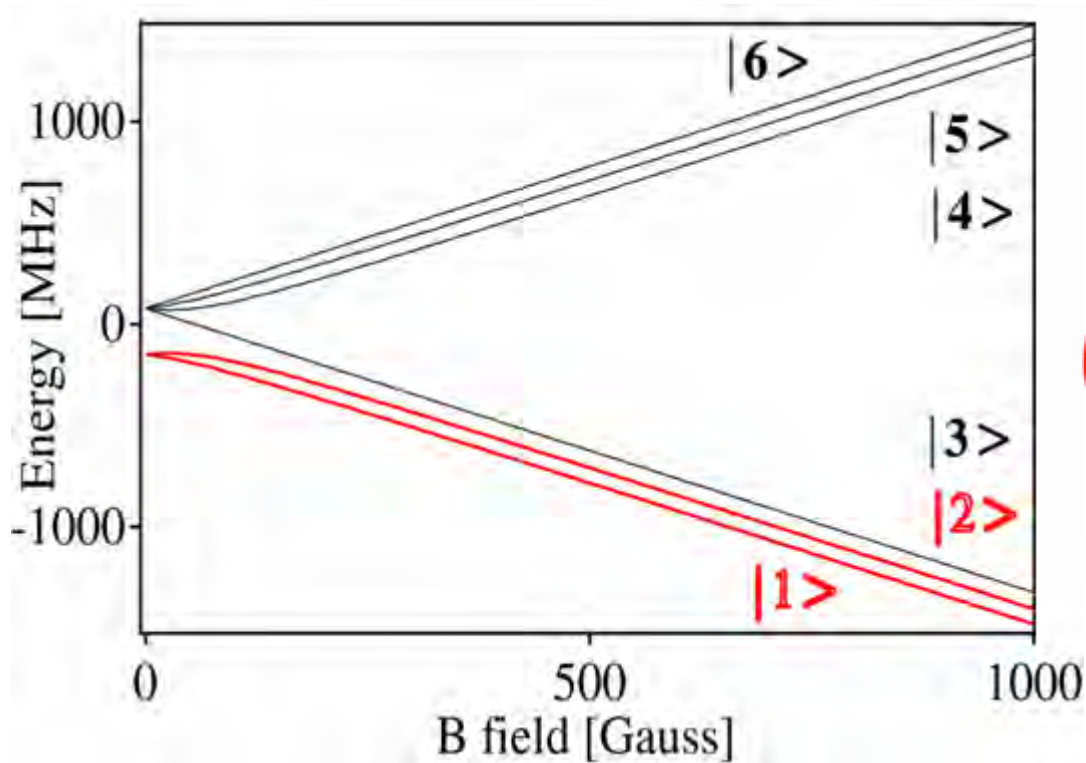
Scaled to the density of electrons in solids:

Superconductivity far above room temperature!

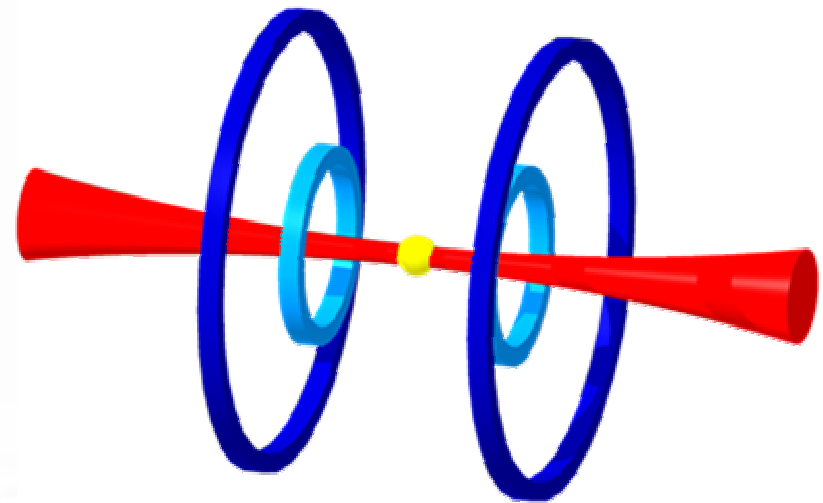
Preparation of an interacting Fermi system in Lithium-6

$$\left. \begin{array}{l} |\uparrow\rangle = |1\rangle \\ |\downarrow\rangle = |2\rangle \end{array} \right\}$$

Lowest two hyperfine states of the ${}^6\text{Li}$ atom



Optical trapping @ 1064 nm



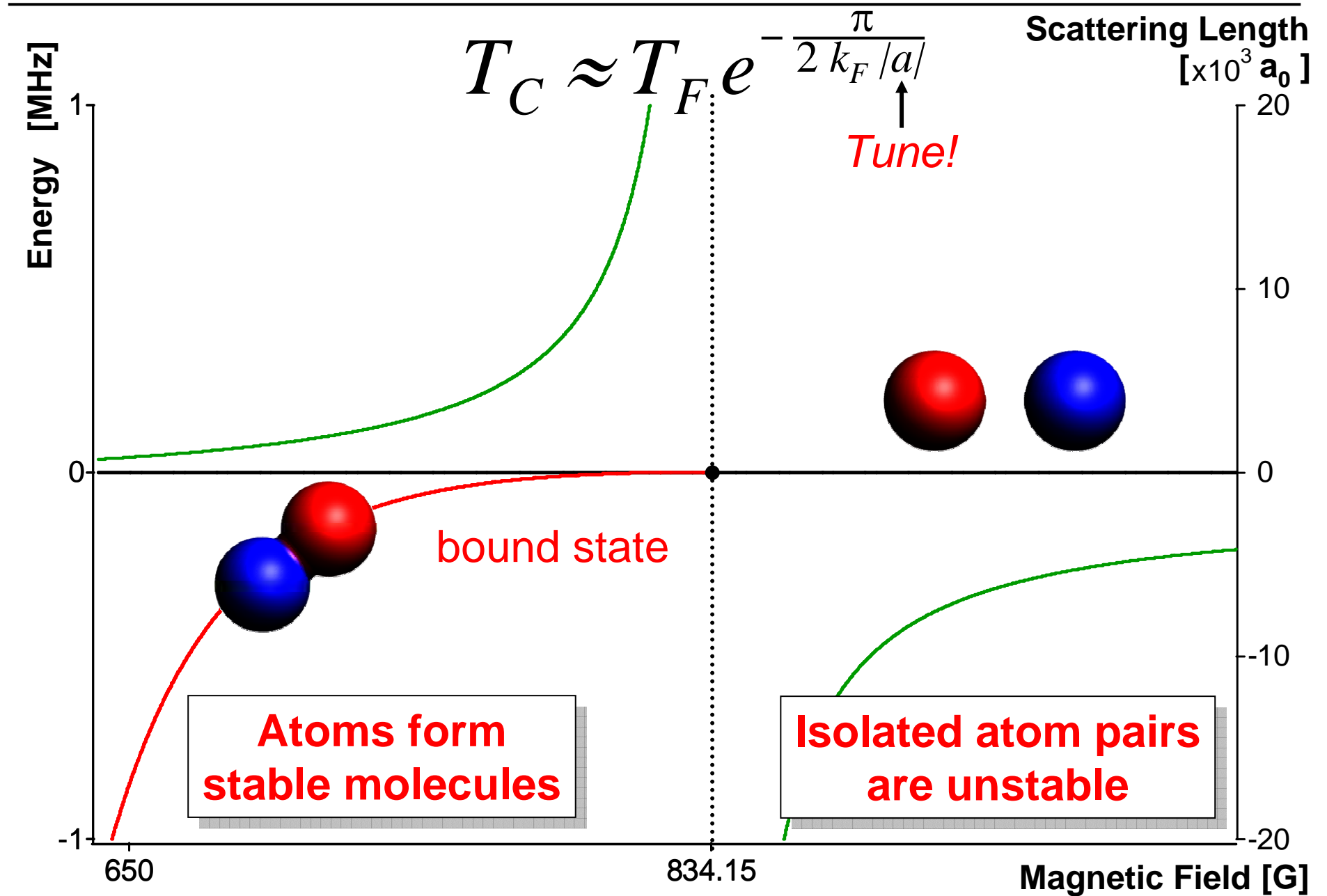
$$\begin{aligned} \nu_{\text{axial}} &= 10\text{-}20 \text{ Hz} \\ \nu_{\text{radial}} &= 50\text{-}200 \text{ Hz} \\ E_{\text{trap}} &= 0.5 - 5 \mu\text{K} \end{aligned}$$

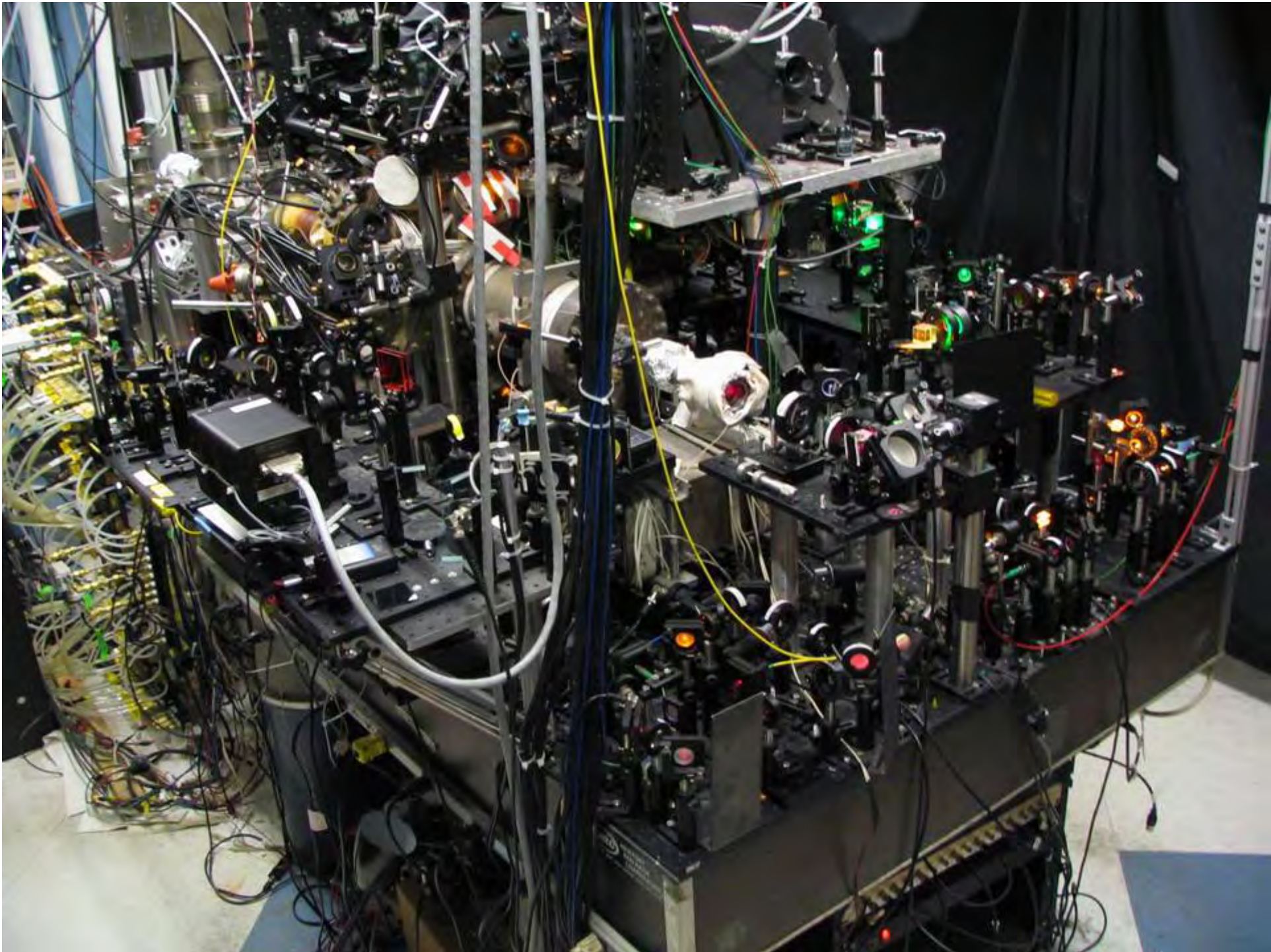
High T_C !

At high fields, states $|1\rangle$ and $|2\rangle$ have large and negative scattering length

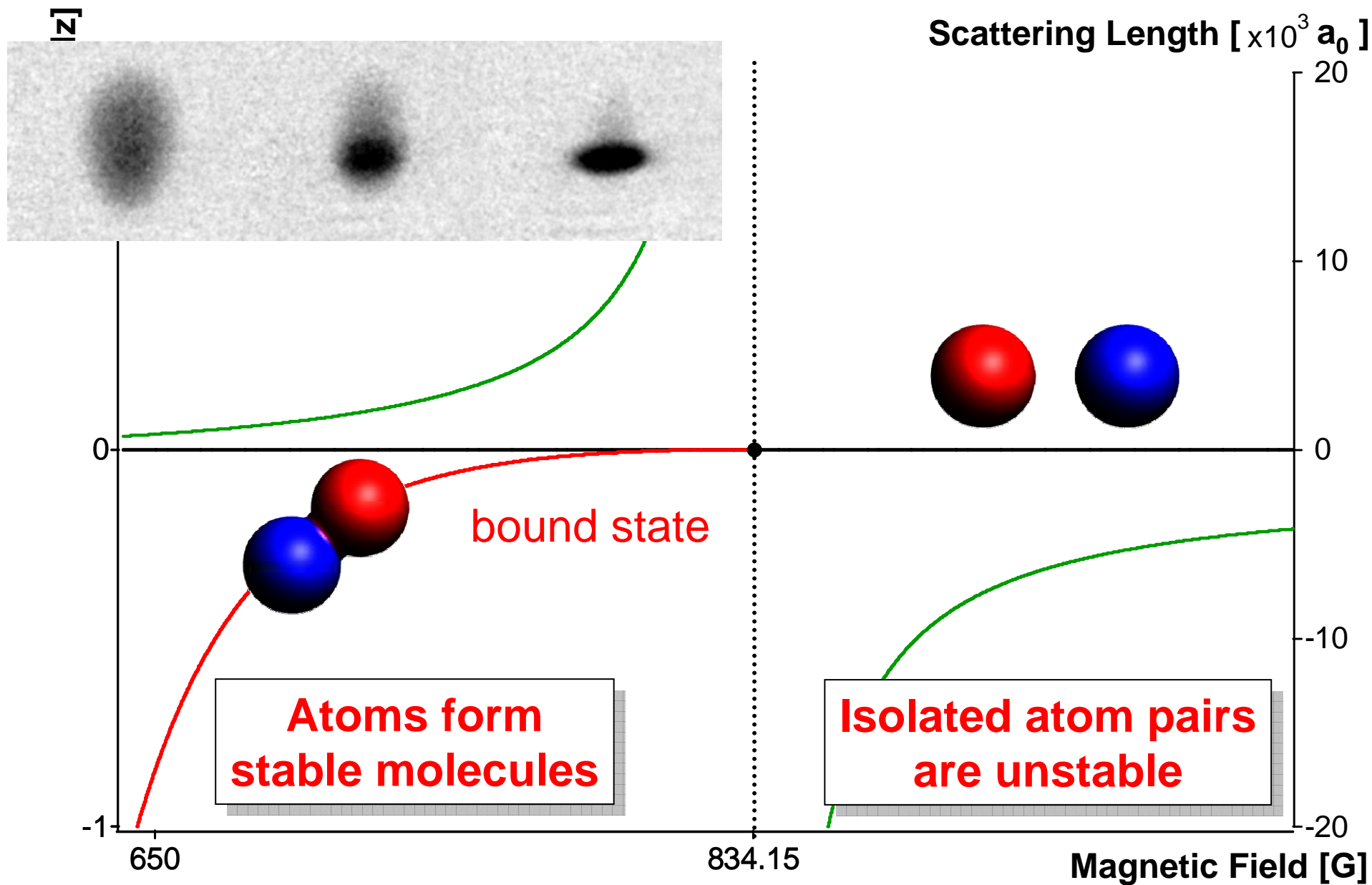
$$a_{12} = -2100 a_0$$

Feshbach Resonances

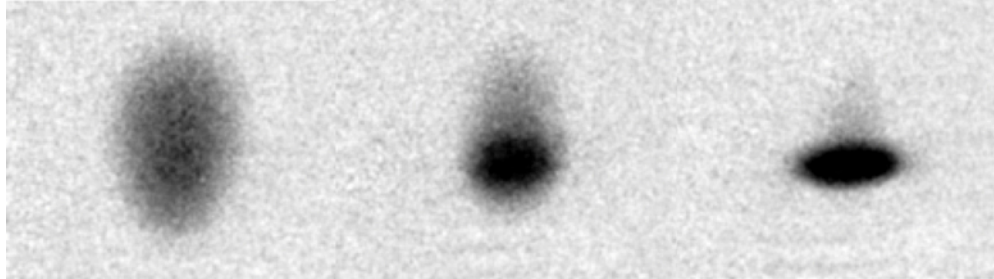




Feshbach Resonances



BEC of Fermion Pairs (Molecules)

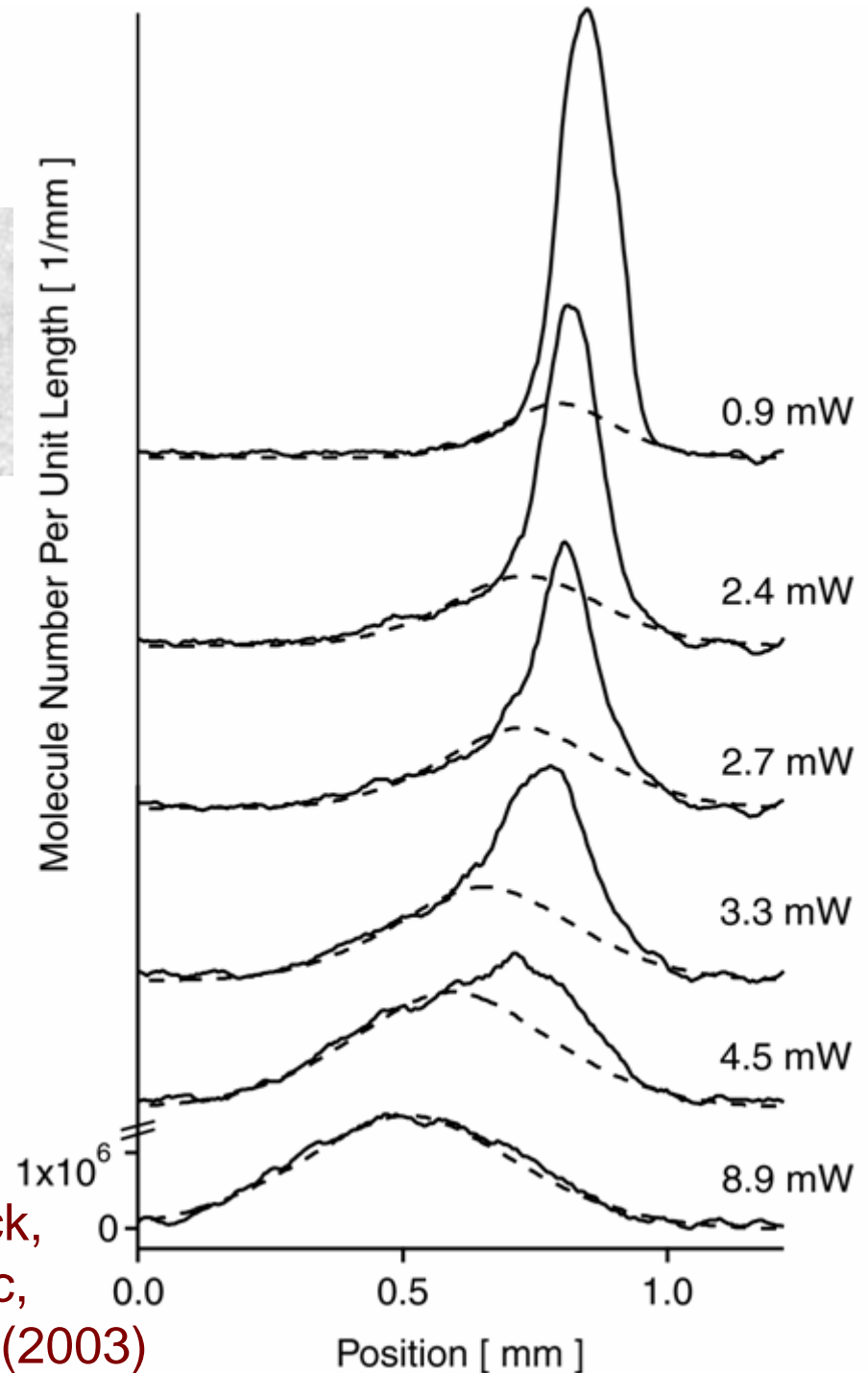


$T > T_C$ $T < T_C$ $T \approx T_C$

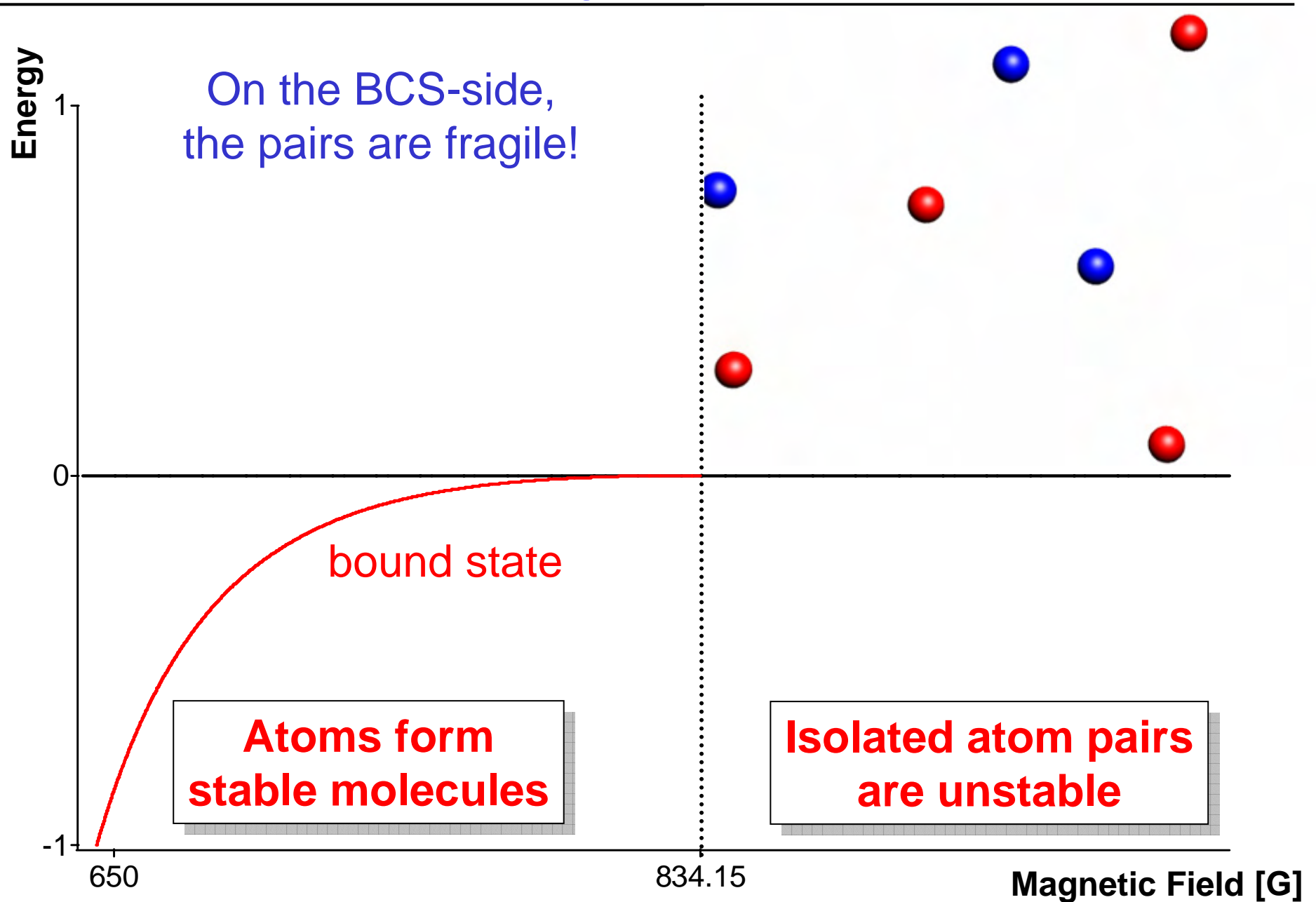
These days: Up to 10 million condensed molecules

Boulder Nov '03
Innsbruck Nov '03, Jan '04
MIT Nov '03
Paris March '04
Rice, Duke

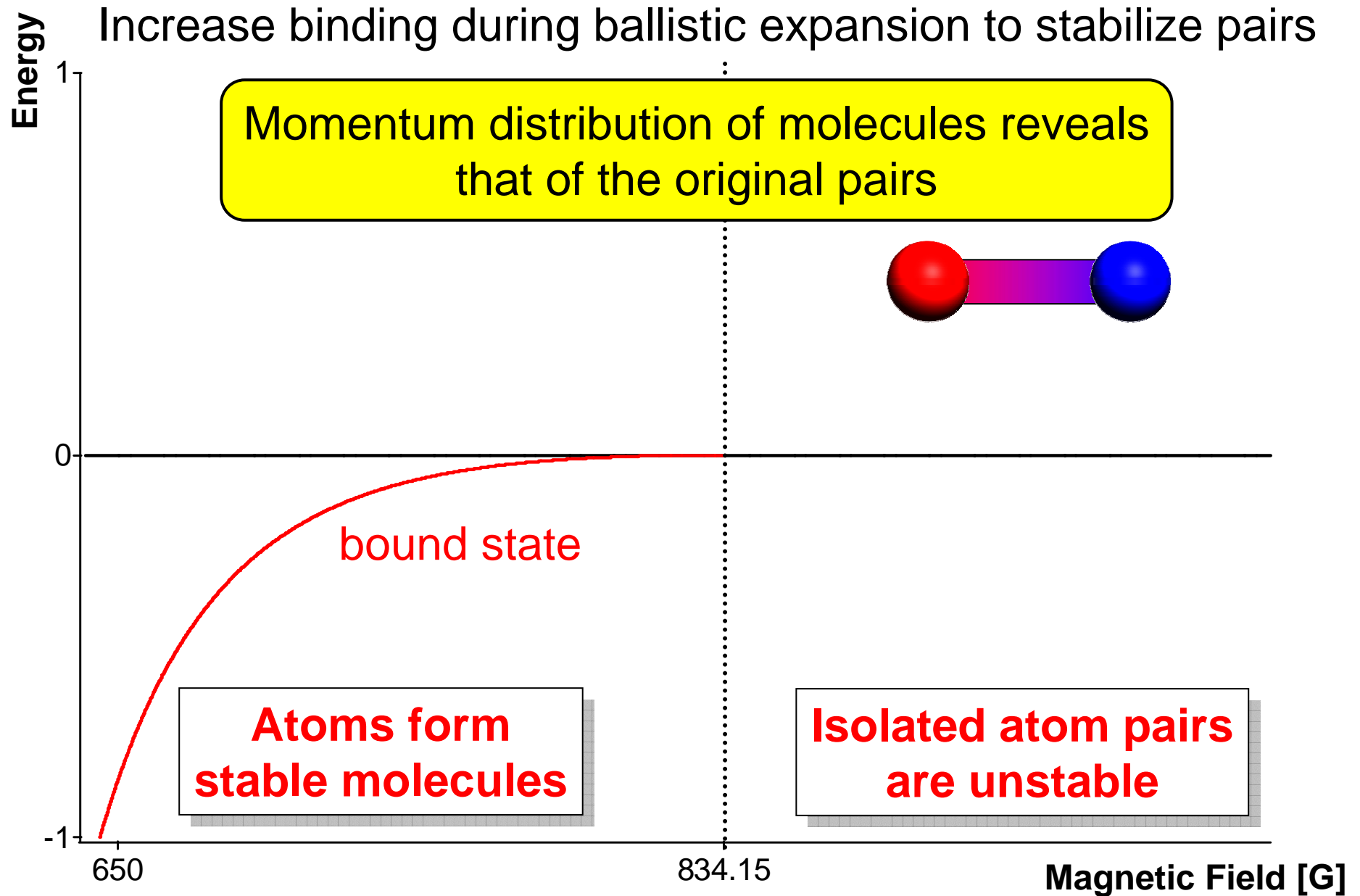
M.W. Zwierlein, C. A. Stan, C. H. Schunck,
S.M.F. Raupach, S. Gupta, Z. Hadzibabic,
W. Ketterle, Phys. Rev. Lett. 91, 250401 (2003)



Stabilizing Fermion Pairs



Stabilizing Fermion Pairs



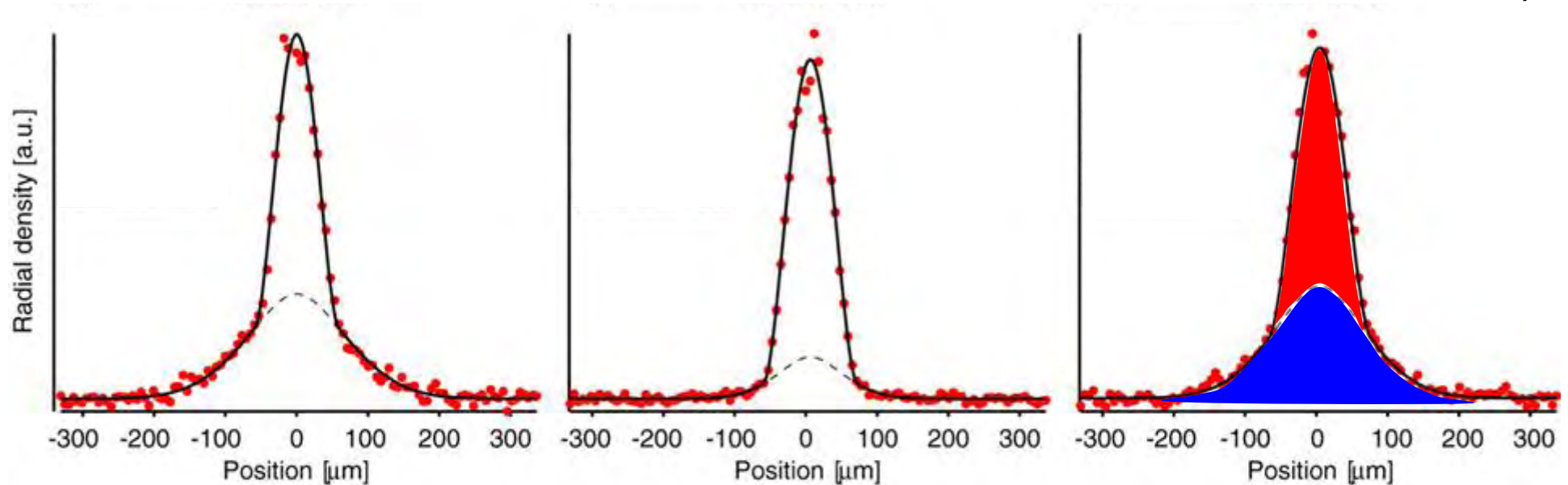
Observation of Pair Condensates

BEC-Side

Resonance

BCS-Side

(above dissociation
limit for molecules)

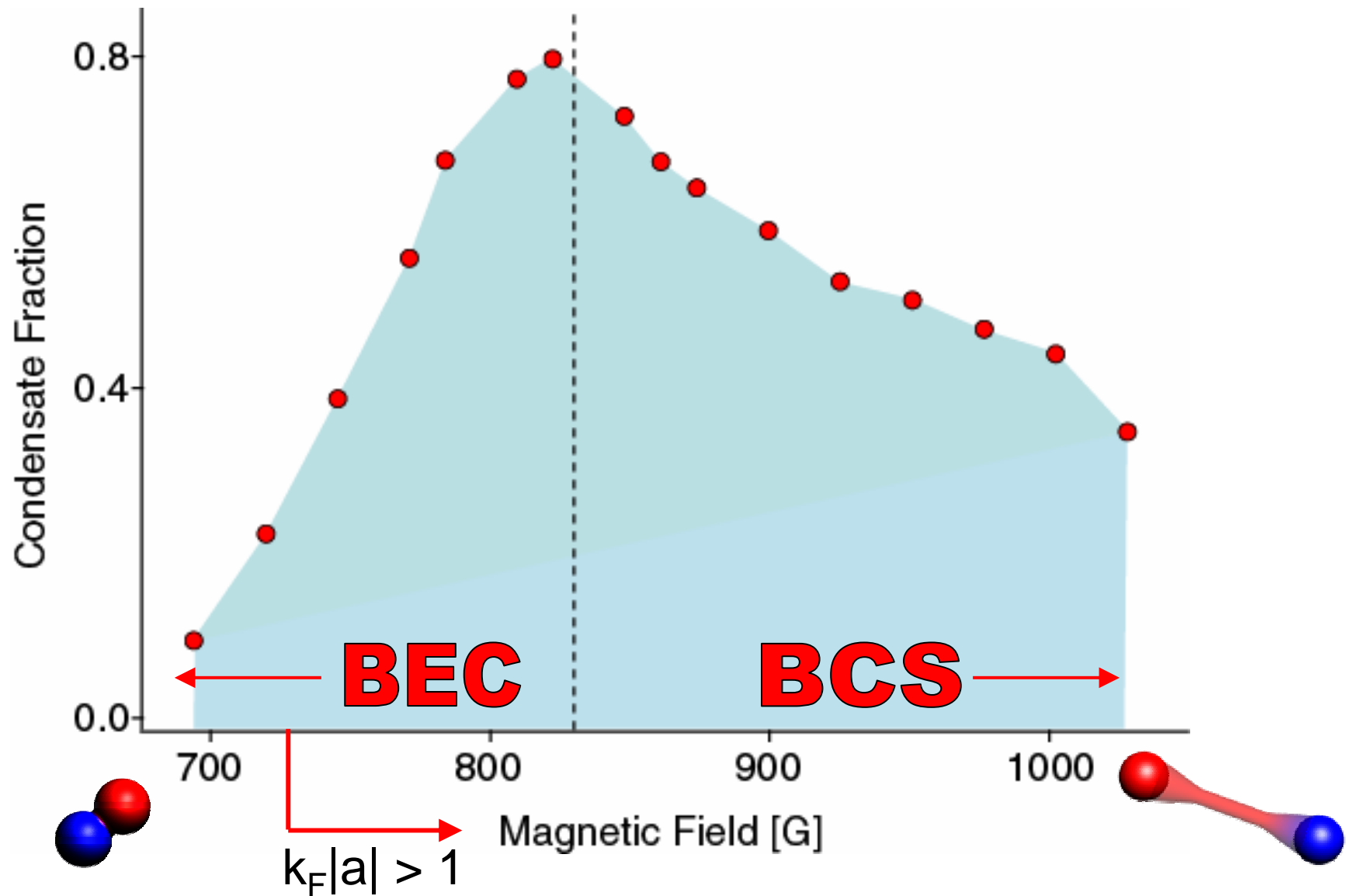


Thermal + condensed pairs

First observation: C.A. Regal et al., Phys. Rev. Lett. **92**, 040403 (2004)

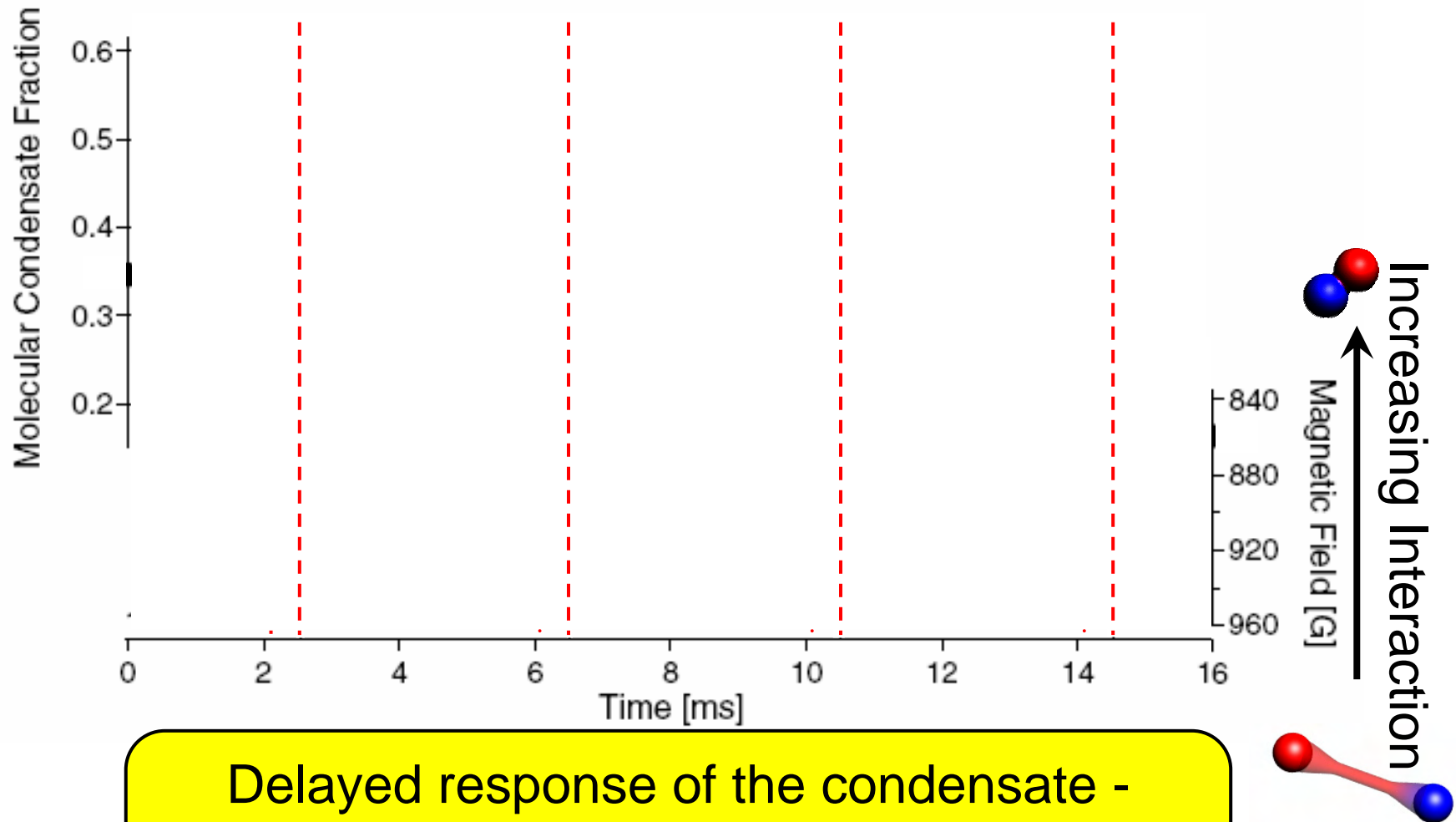
M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman,
W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

Condensate Fraction vs Magnetic Field



M.W. Zwierlein, C.A. Stan, C.H. Schunck, S.M.F. Raupach, A.J. Kerman, W. Ketterle, Phys. Rev. Lett. **92**, 120403 (2004).

Formation Time of the Condensate



Delayed response of the condensate -
Build-up of the order parameter takes time!
Quasi-particle redistribution time: $\tau \approx \hbar E_F / \Delta^2$

M.W. Zwierlein, C.H. Schunck, C.A. Stan, S.M.F. Raupach, W. Ketterle,
Phys. Rev. Lett. **94**, 180401 (2005).

**How can we show that these
gases are superfluid?**

Rotating Fluids



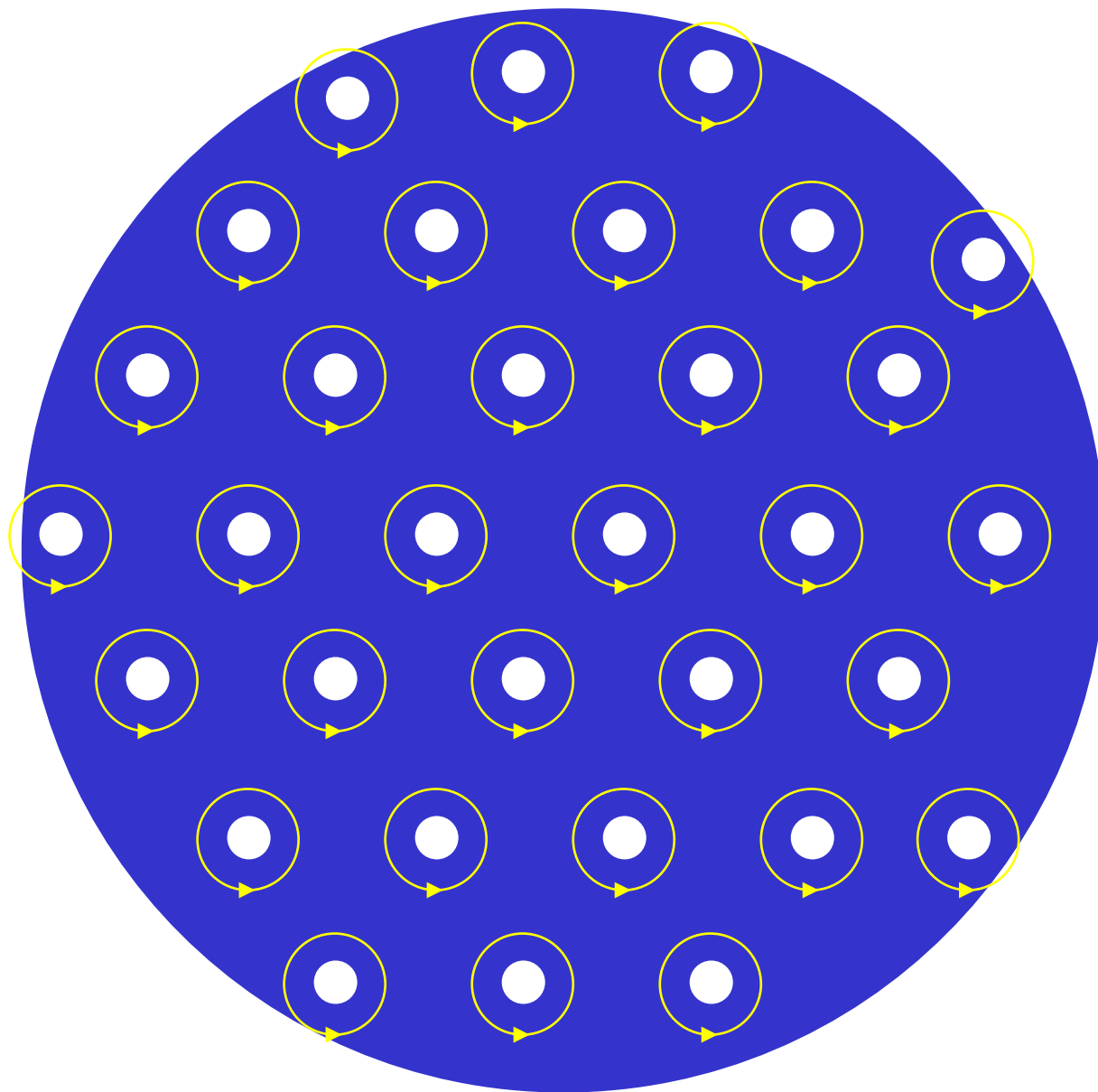
Normal



Quantum

Fluid

Abrikosov Lattice

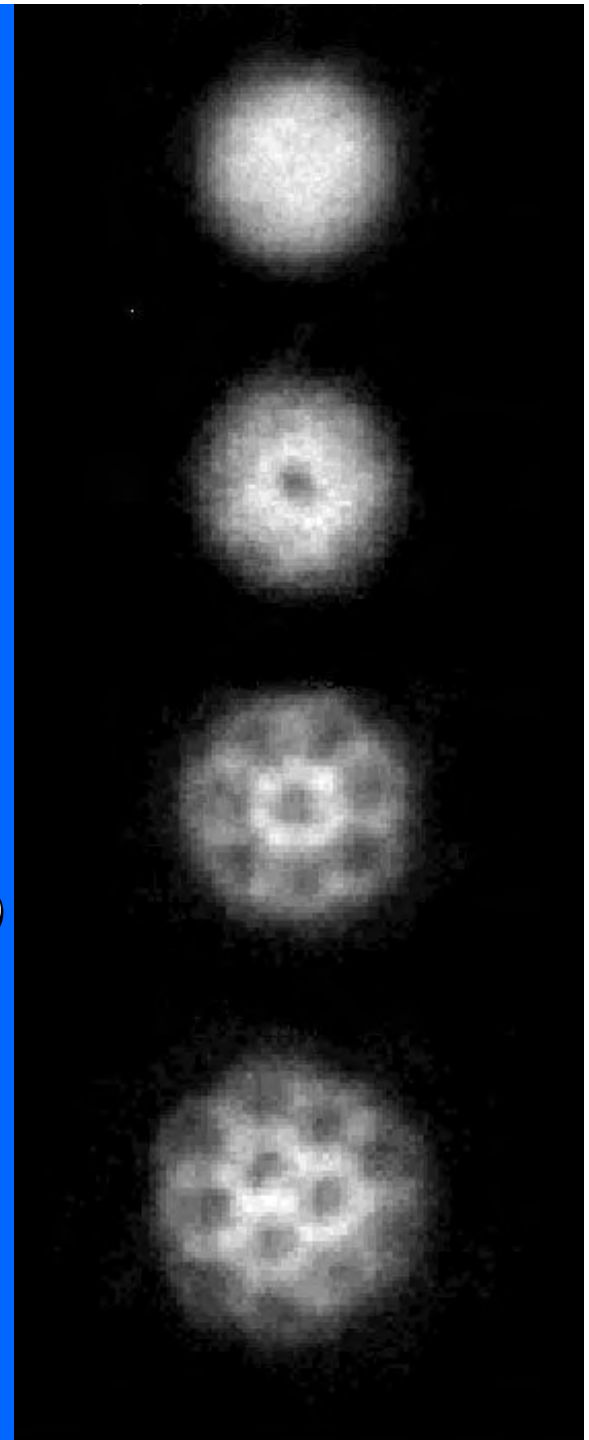
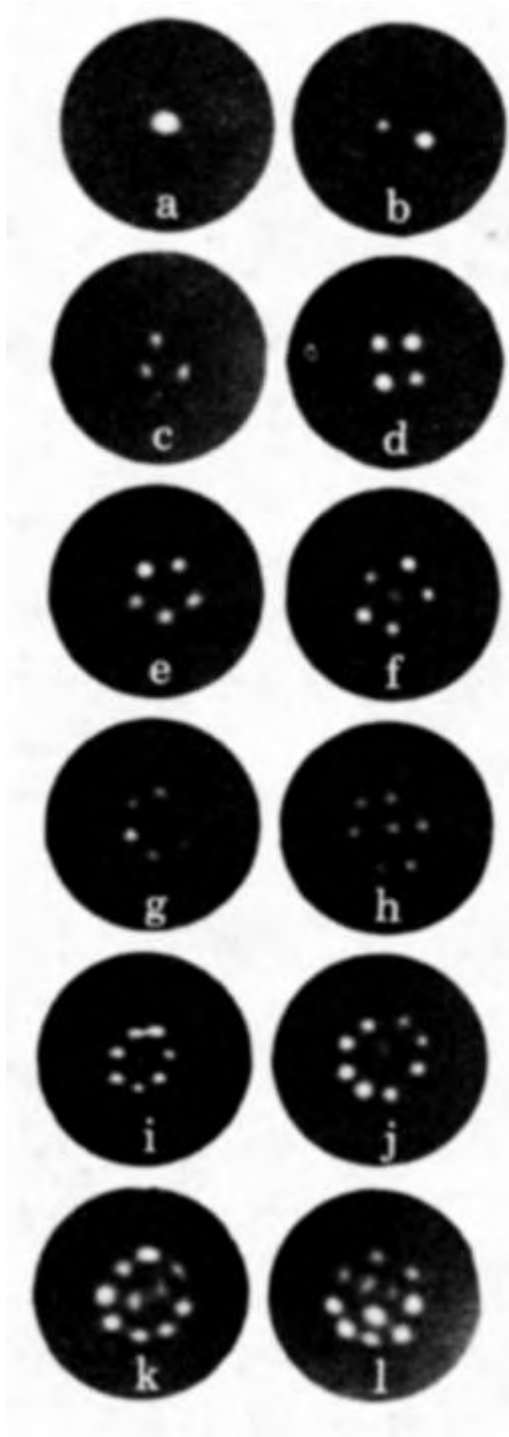


Vortex Arrays *in Bosonic Gases / Fluids*

Berkeley
(R.E. Packard, 1979)
Helium-4

ENS
(J. Dalibard, 2000)
Rubidium BEC

*Also: Phase engineering
of single vortices in BEC:
JILA (1999)*



THE DIRECT OBSERVATION OF INDIVIDUAL FLUX LINES IN TYPE II SUPERCONDUCTORS

U. ESSMANN and H. TRÄUBLE

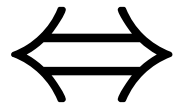
*Institut für Physik am Max-Planck-Institut für Metallforschung, Stuttgart and
Institut für theoretische und angewandte Physik der Technischen Hochschule Stuttgart*

Received 4 April 1967

Neutral superfluids under rotation

$$F = 2m (\mathbf{v} \times \boldsymbol{\Omega})$$

Coriolis force in rotating frame



Superconductors in magnetic field

$$F = q (\mathbf{v} \times \mathbf{B})$$

Lorentz Force

U. Essmann and H. Träuble,
Physics Letters A, **24**, 526 (1967)

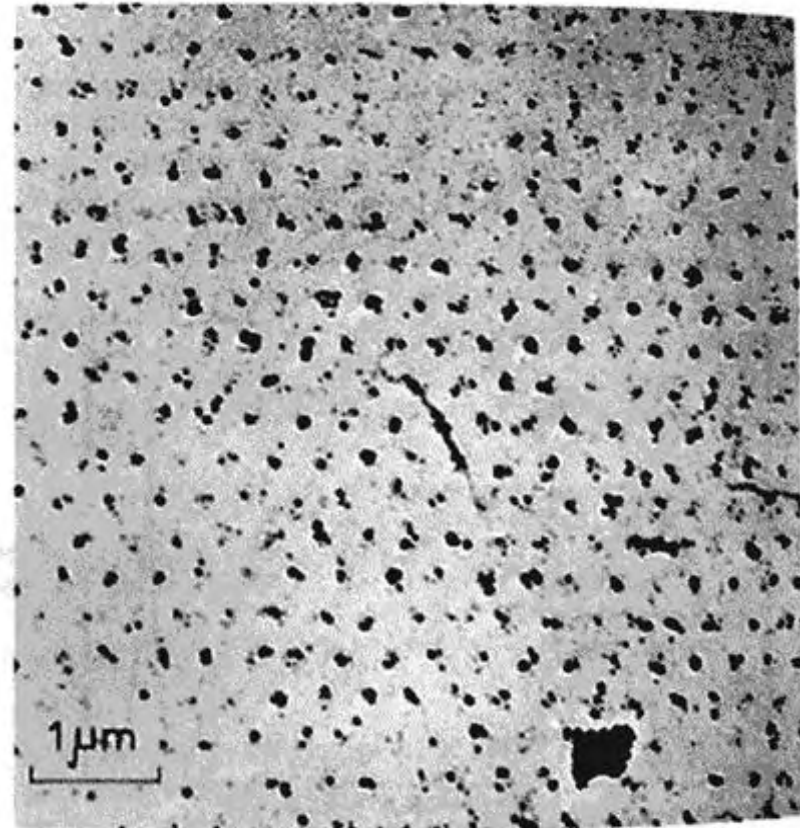
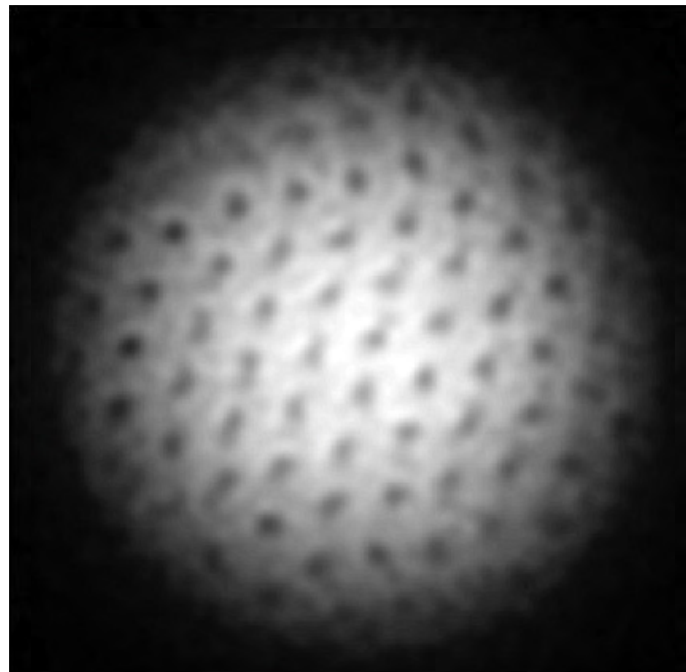
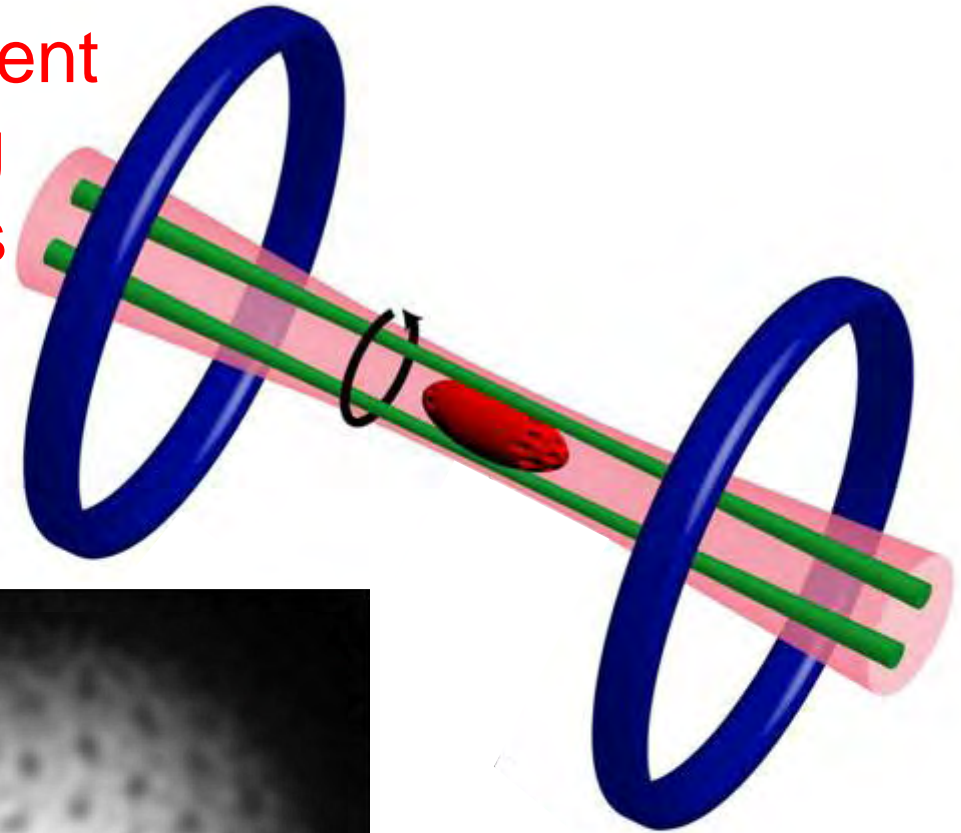


Fig. 1. "Perfect" triangular lattice of flux lines on the surface of a lead-4at% indium rod at 1.1°K. The black dots consist of small cobalt particles which have been stripped from the surface with a carbon replica.

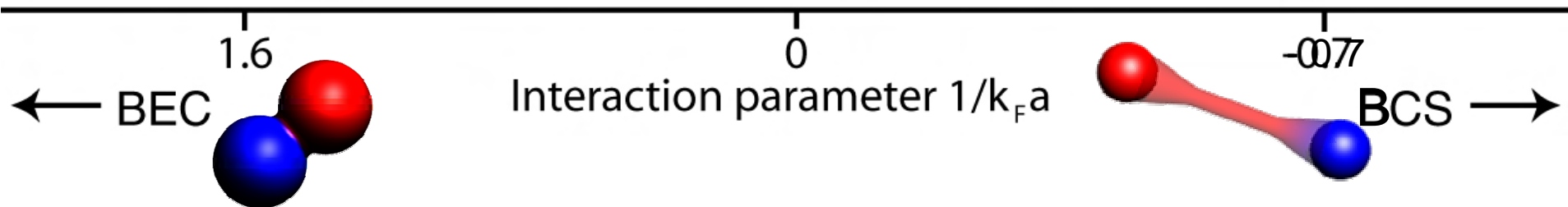
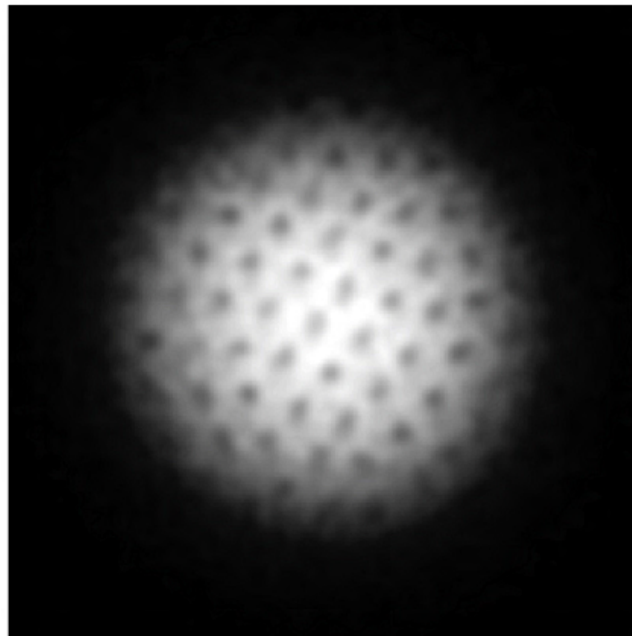
Spinning a strongly interacting Fermi gas

The rotating bucket experiment
with a strongly interacting
Fermi gas, a million times
thinner than air



Vortex lattices in the BEC-BCS crossover

Establishes *superfluidity* and *phase coherence*
in gases of **fermionic atom pairs**



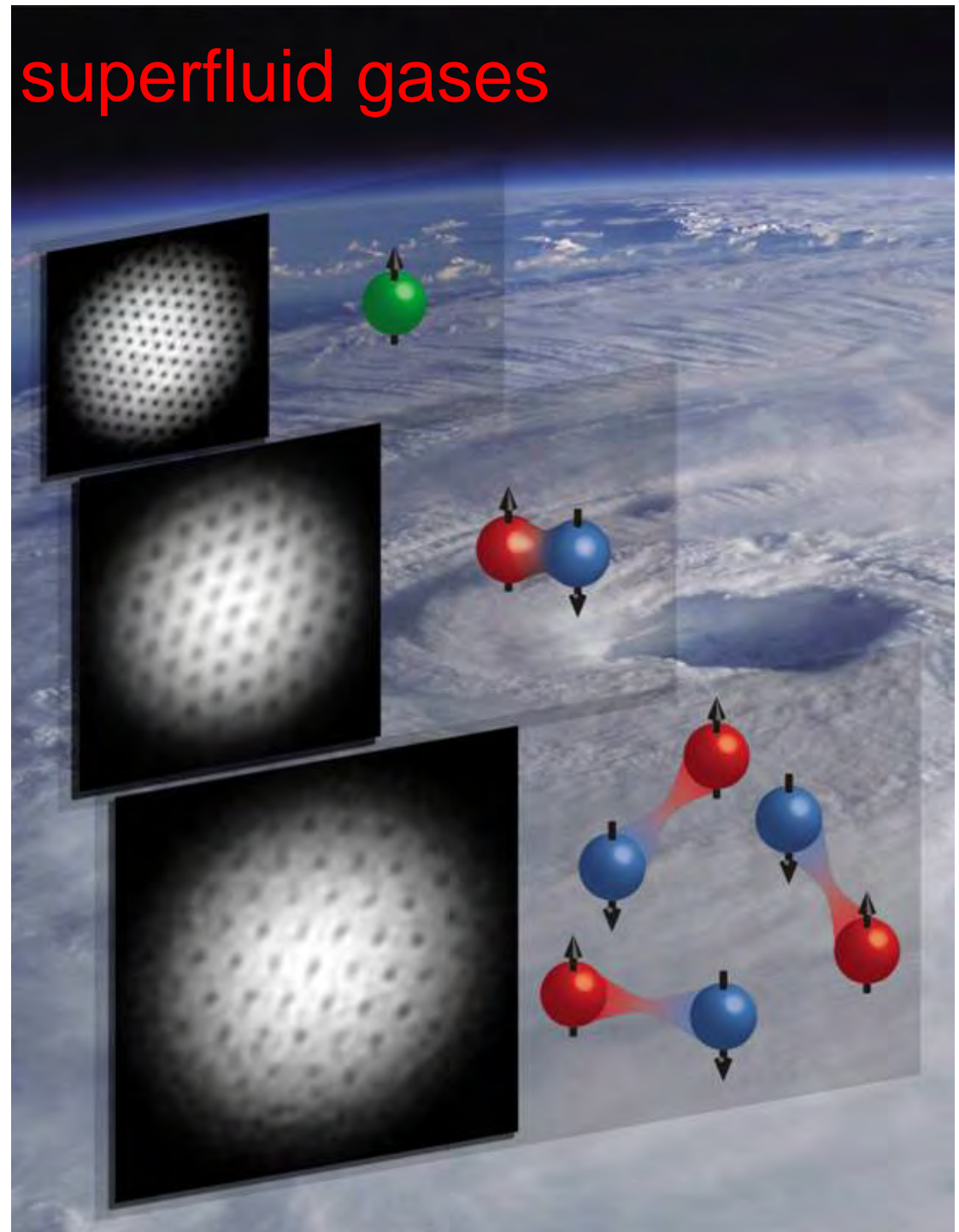
M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

Gallery of superfluid gases

Atomic Bose-Einstein condensate (sodium)

Molecular Bose-Einstein condensate (lithium ${}^6\text{Li}_2$)

Pairs of fermionic atoms (lithium-6)

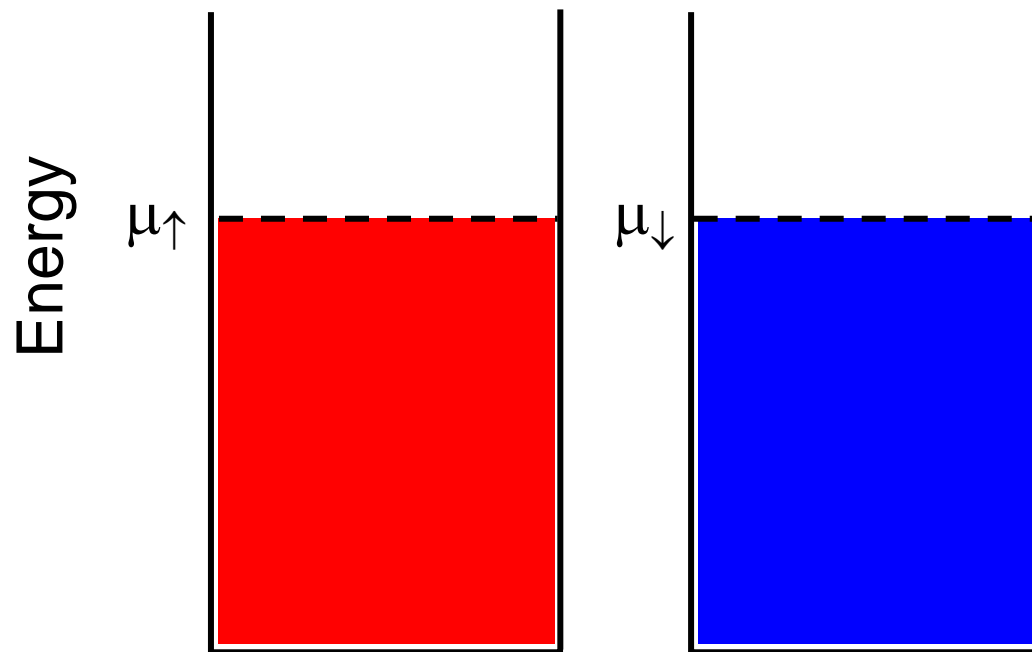


Fermionic Superfluidity with Imbalanced Spin Populations



What if there are more boys than girls...?

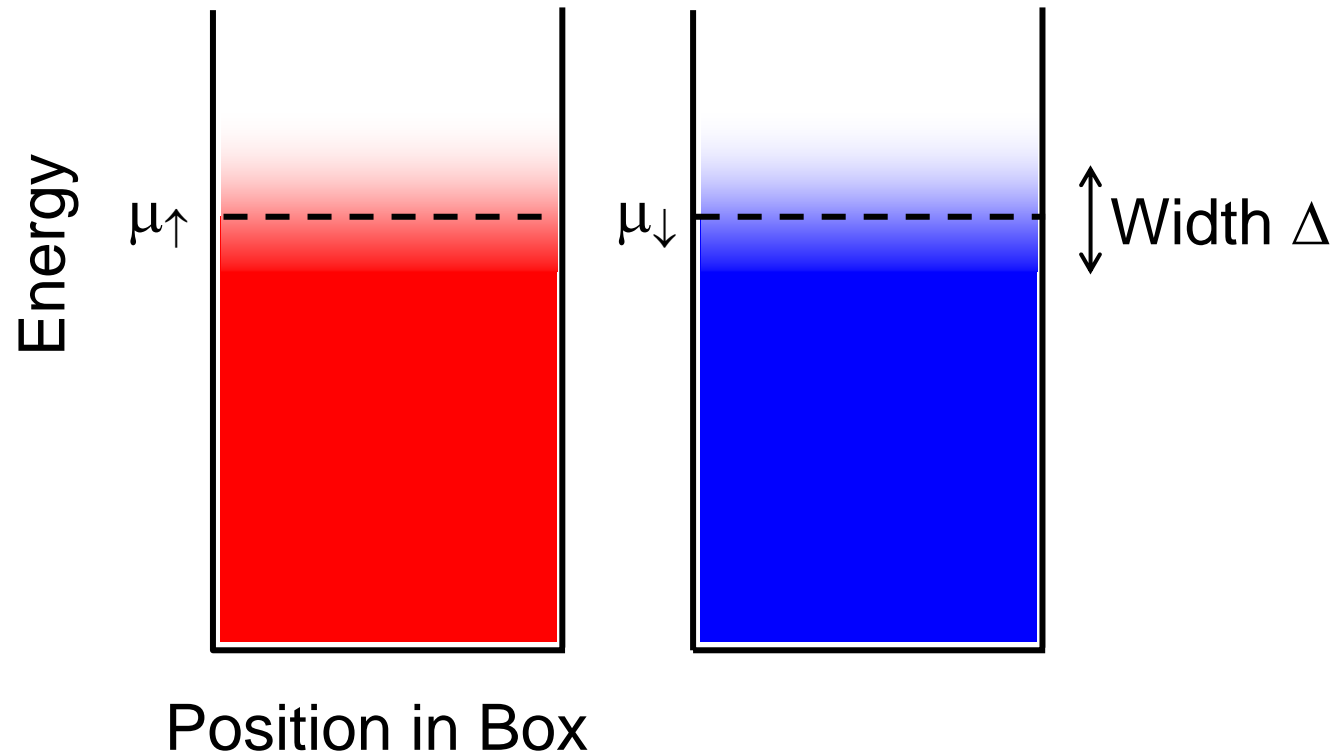
BCS Pairing of Fermions



BCS Pairing of Fermions

Pairing costs kinetic energy,
but there is gain in potential energy

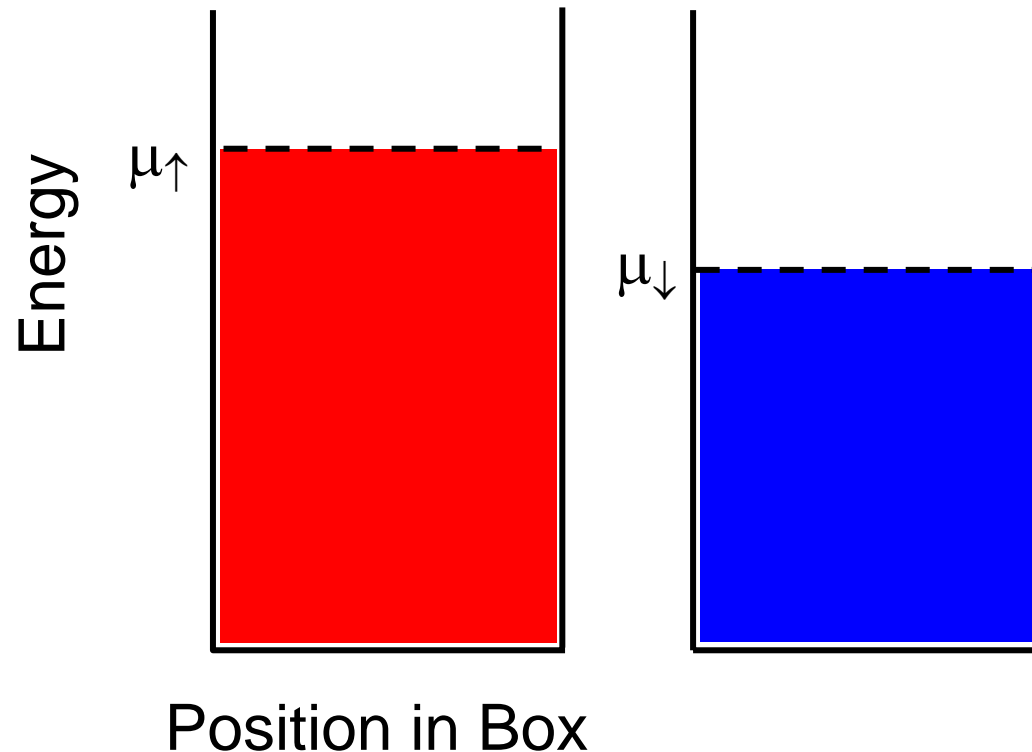
Pairing energy Δ



BCS Pairing of Fermions

Unequal Fermi surfaces, non-interacting case
(example: Apply magnetic field to a normal conductor)

“Magnetic Field” $\delta\mu = \mu_{\uparrow} - \mu_{\downarrow}$

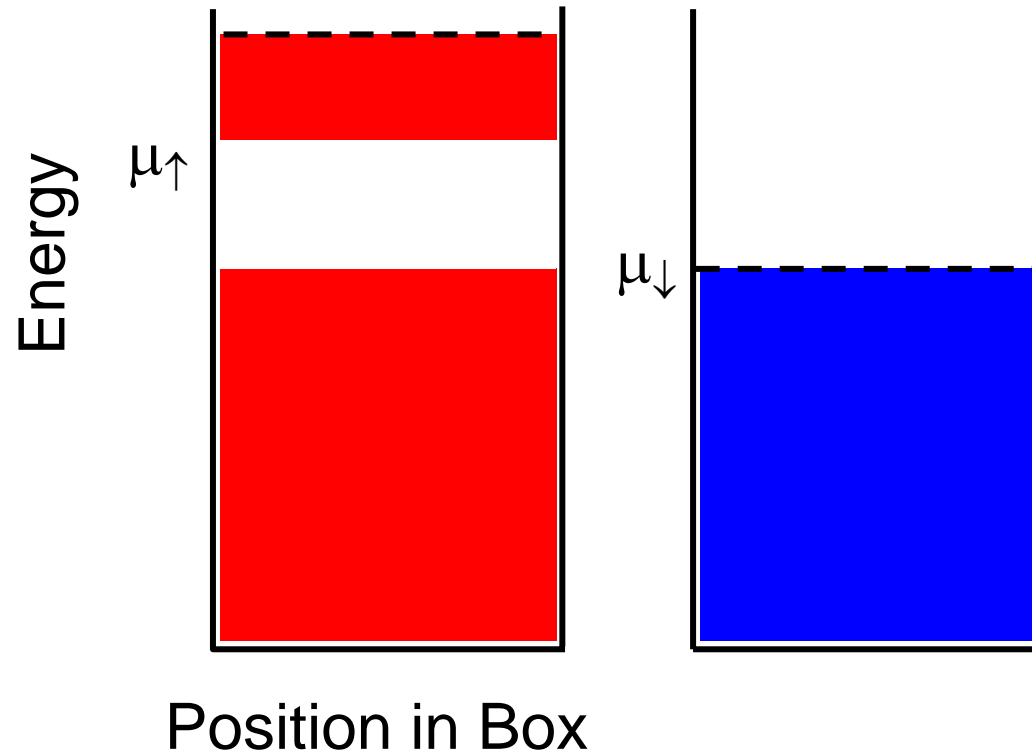


BCS Pairing of Fermions

Interacting case:

Sarma Phase (Sarma 1962), Breached Pairing Phase

Energy cost: $N \frac{\delta\mu}{E_F} \Delta$



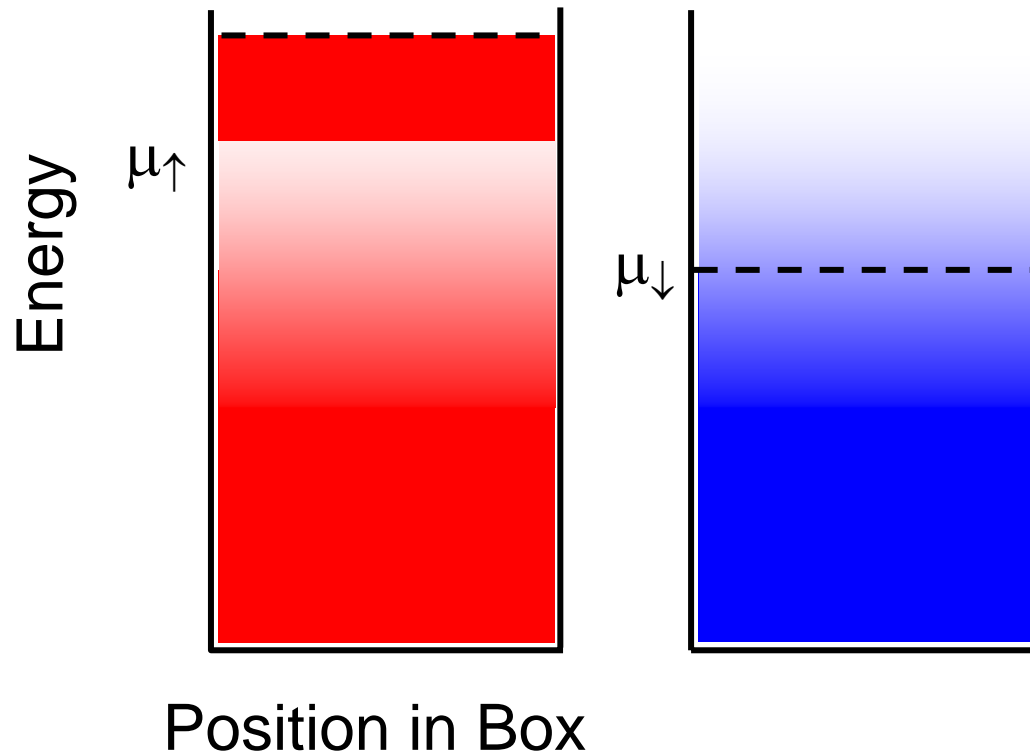
BCS Pairing of Fermions

Interacting case:

Sarma Phase (Sarma 1962), Breached Pairing Phase

Energy cost: $N \frac{\delta\mu}{E_F} \Delta$ Energy gain: $N \frac{\Delta}{E_F} \Delta$

Breakdown when: $\delta\mu > \Delta$



BCS Pairing of Fermions

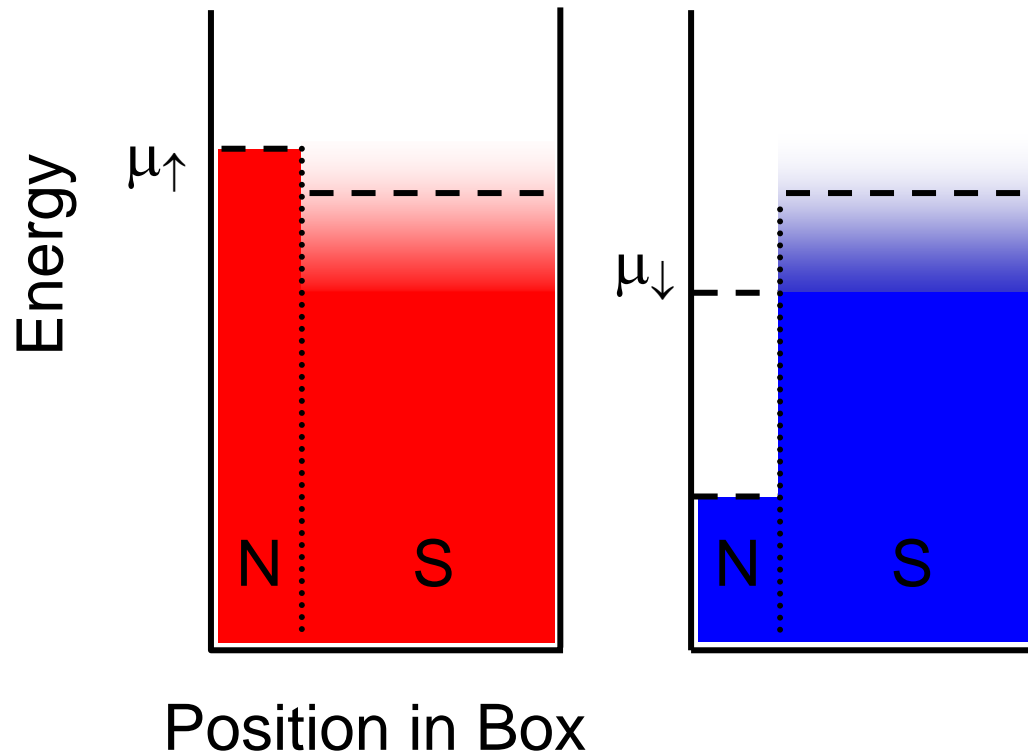
Energetically favorable solution:

Phase separation! (Bedaque, Caldas, Rupak 2003)

A fraction $x = \frac{\delta\mu}{\Delta}$ of the gas is a normal, polarized mixture

Breakdown of the BCS state
when $\delta\mu > \Delta$

*Chandrasekhar,
Clogston 1962*



Superfluidity with unequal Fermi energies

- Superfluidity in quarks involves unequal Fermi surfaces *due to different quark masses, charges and hence different densities*

- New superfluid states predicted:
e.g. Larkin-Ovchinnikov, Fulde-Ferrell–state
Cooper pairs with non-zero momentum
Generally: Inhomogeneous order parameter

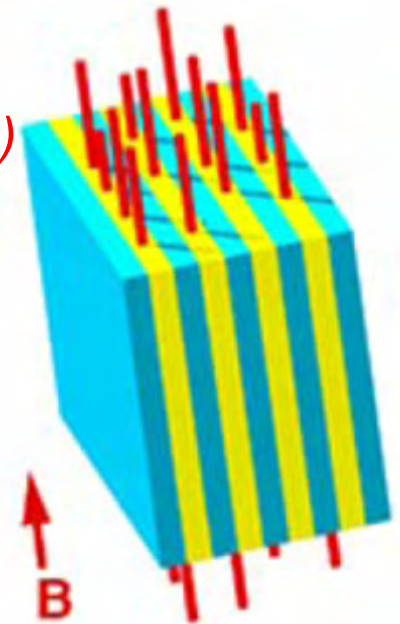
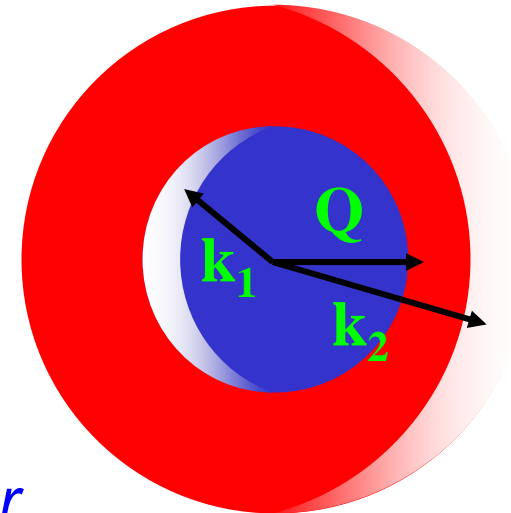
*A. I. Larkin, Yu. N. Ovchinnikov, “Inhomogeneous state of superconductors”, Zh. Eksp. Teor. Fiz. **47**, 1136 (1964)*

- Possible novel superconducting states in systems where vortices are suppressed:

ex.: Heavy fermion superconductor CeCoIn₅

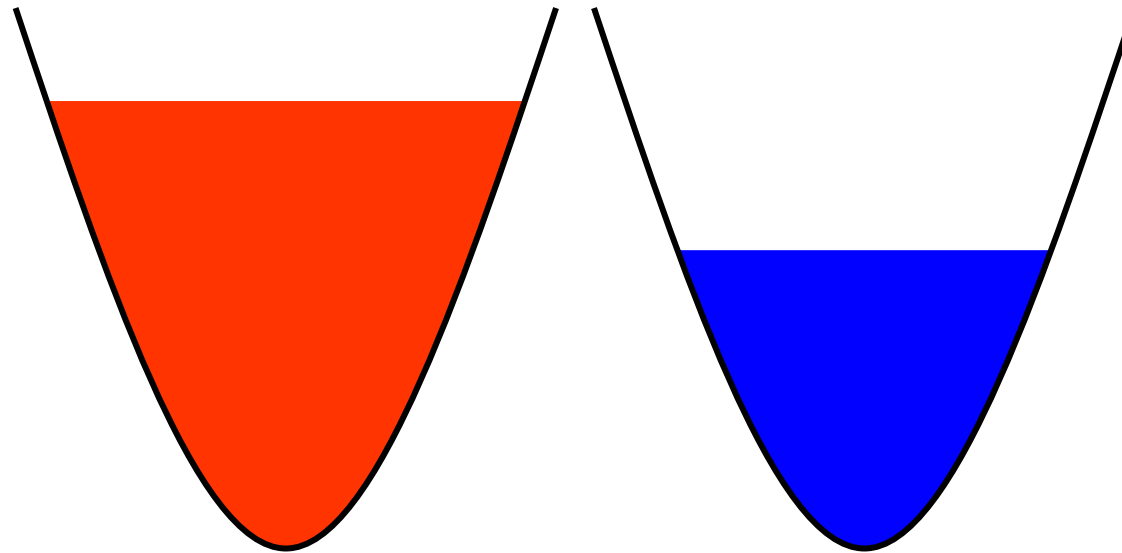
*Bianchi et al., PRL **91**, 187004 (2003)*

*Radovan et al., Nature **425**, 51 (2003)*



Trapped, Imbalanced Fermi Mixture

- Two spin states trapped in a 3D harmonic potential
- Population imbalance (or *Polarization*): $P = \frac{N_2 - N_1}{N_2 + N_1}$

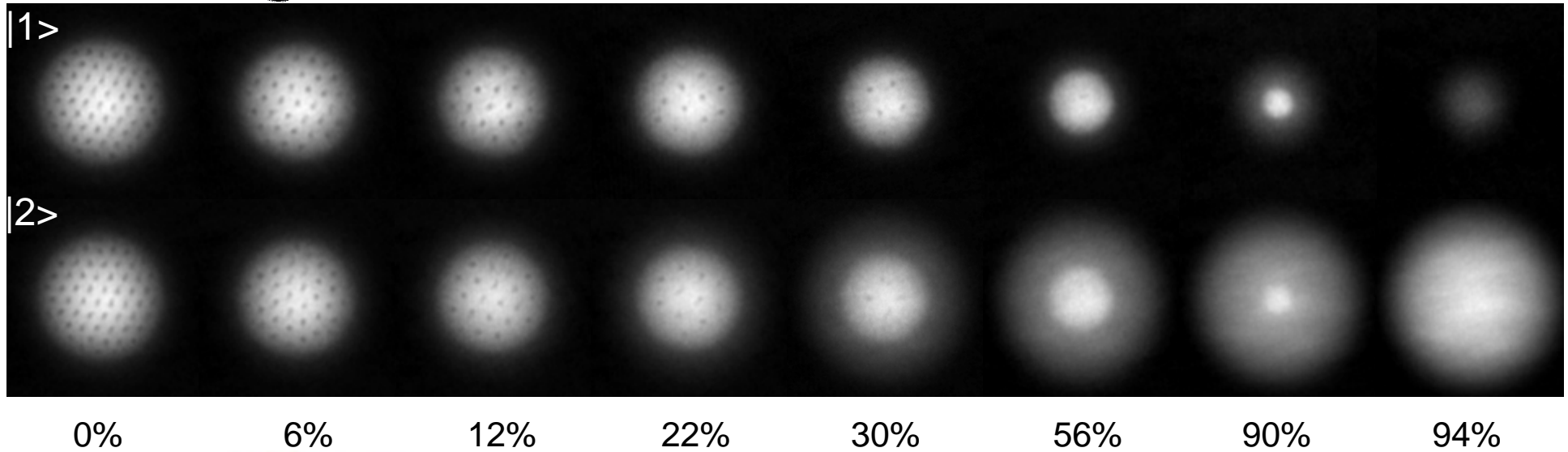


Can we have superfluidity?

Fermionic Superfluidity with Imbalanced Spin Populations

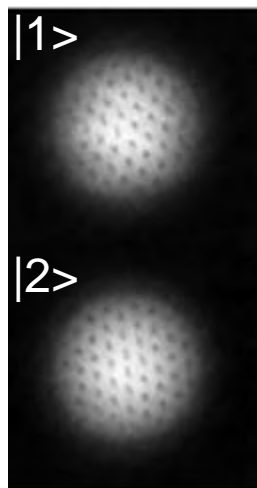
BEC-Side 

$$1/k_F a = 0.2$$



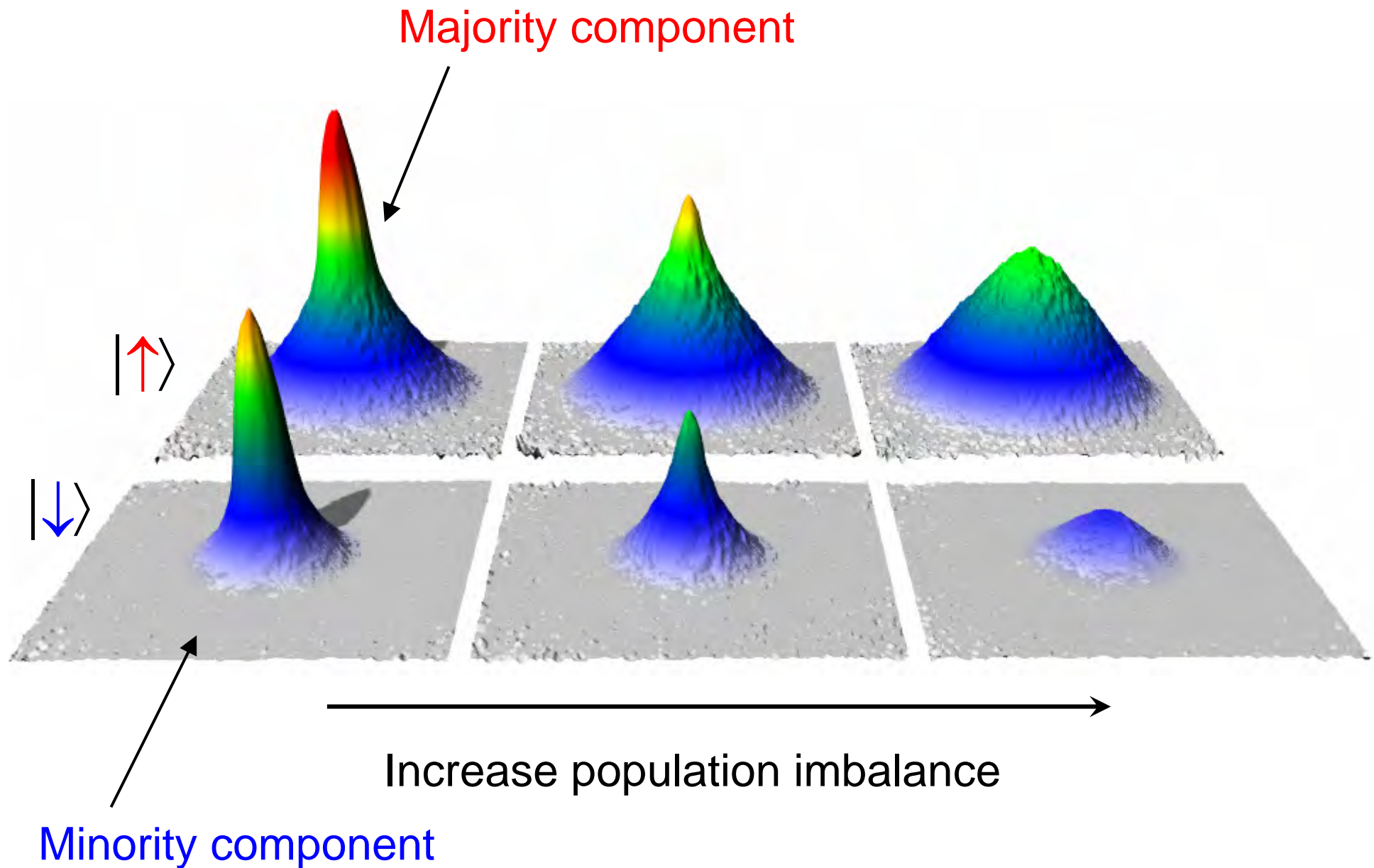
BCS-Side 

$$1/k_F a = -0.15$$

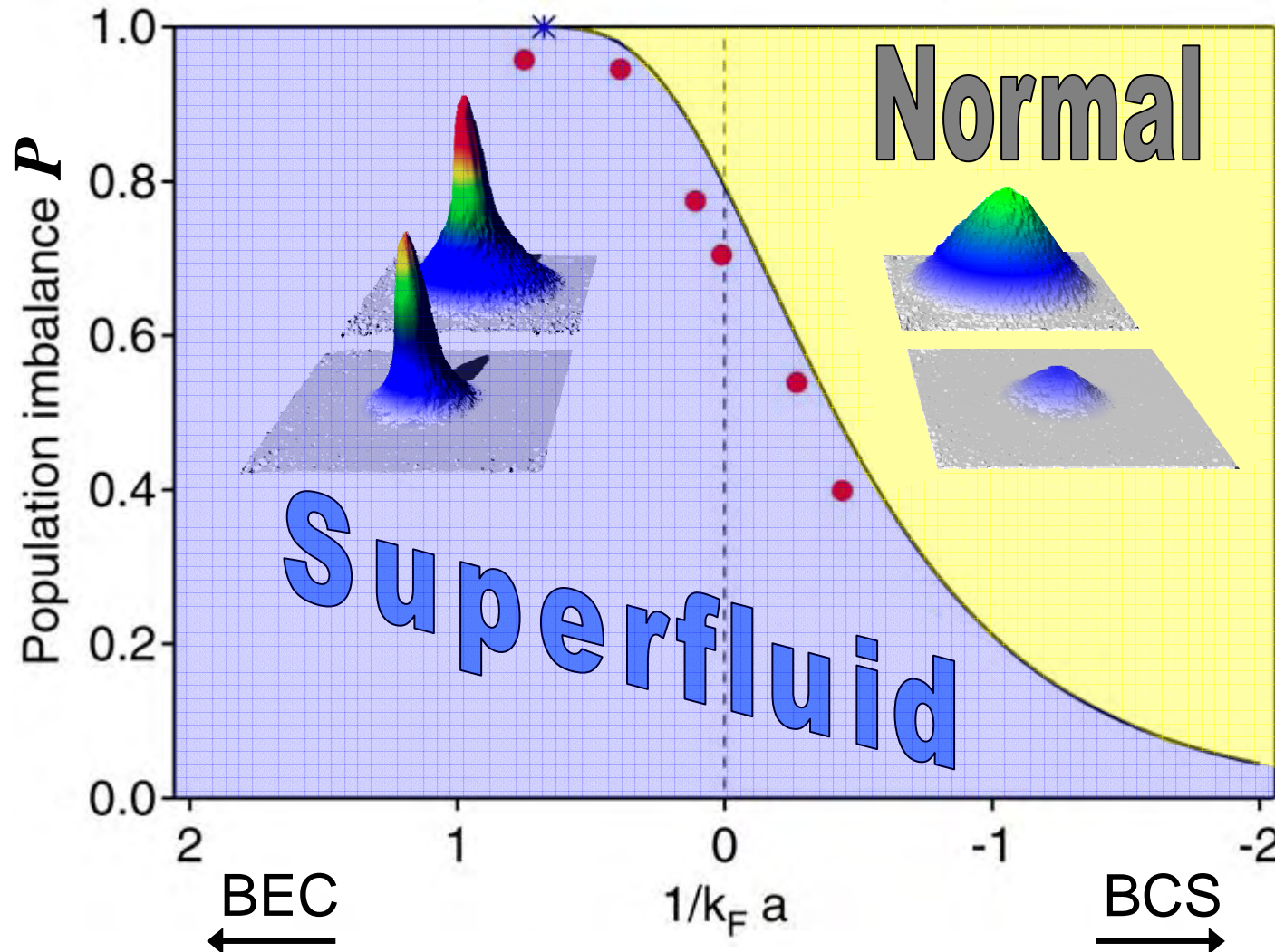


0%

The Clogston-Chandrasekhar limit



Phase Diagram for Unequal Mixtures



Breakdown: Critical polarization $P_c \propto \text{Gap } \Delta$

M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle,
Science 311, 492 (2006)

Critical Polarization

Breakdown: Critical polarization $P_c \propto \text{Gap } \Delta$

Critical Polarization on resonance (experiment): $P_c = 0.74$ (5)

PHYSICAL REVIEW A 74, 063628 (2006)

Universal phase diagram of a strongly interacting Fermi gas with unbalanced spin populations

F. Chevy

Laboratoire Kastler Brossel, École Normale supérieure, Paris, France

(Received 31 May 2006; published 29 December 2006)

PRL 97, 200403 (2006)

PHYSICAL REVIEW LETTERS

week ending
17 NOVEMBER 2006

Normal State of a Polarized Fermi Gas at Unitarity

C. Lobo, A. Recati, S. Giorgini, and S. Stringari

Dipartimento di Fisica, Università di Trento and CNR-INFN BEC Center, I-38050 Povo, Trento, Italy

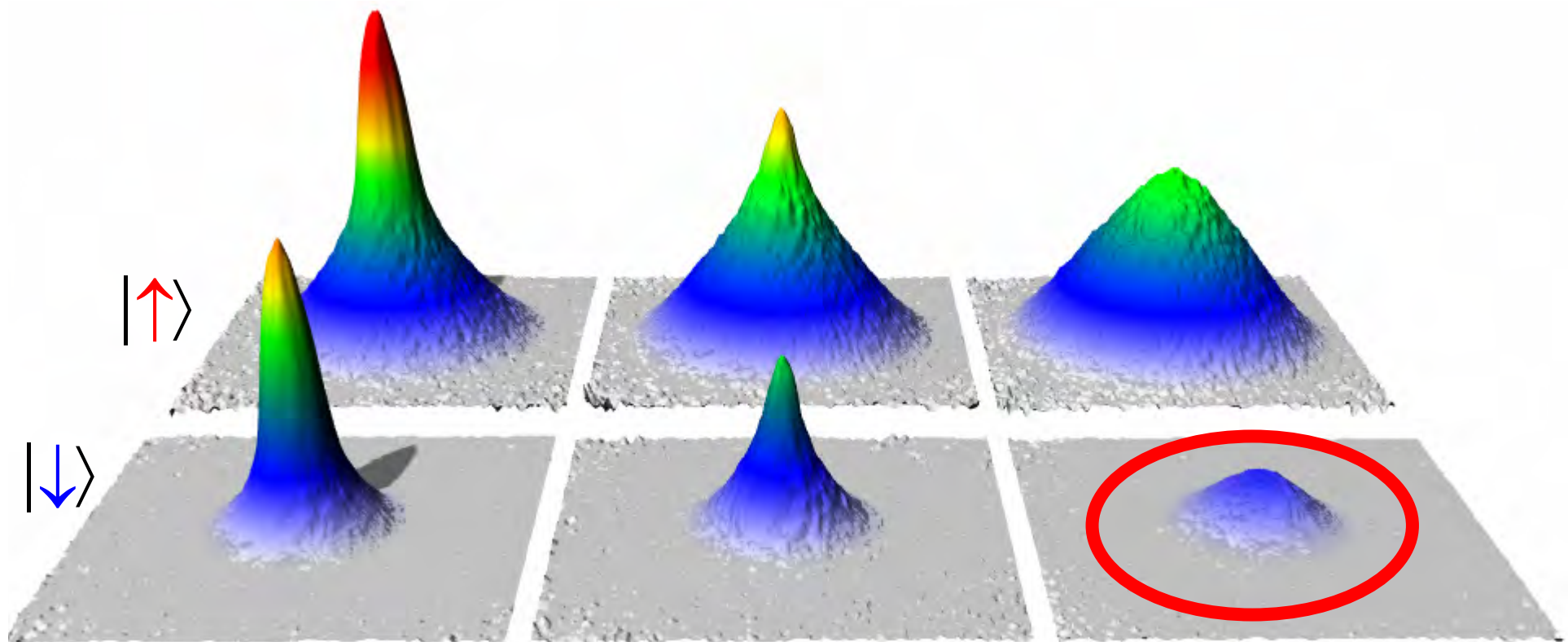
(Received 27 July 2006; published 13 November 2006)

We study the Fermi gas at unitarity and at $T = 0$ by assuming that, at high polarizations, it is a normal Fermi liquid composed of weakly interacting quasiparticles associated with the minority spin atoms. With a quantum Monte Carlo approach we calculate their effective mass and binding energy, as well as the full equation of state of the normal phase as a function of the concentration $x = n_{\downarrow}/n_{\uparrow}$ of minority atoms. We predict a first order phase transition from normal to superfluid at $x_c = 0.44$ corresponding, in the presence of harmonic trapping, to a critical polarization $P_c = (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow}) = 77\%$. We calculate the radii and the density profiles in the trap and predict that the frequency of the spin dipole mode will be increased by a factor of 1.23 due to interactions.

Critical polarization (theory): 77 %

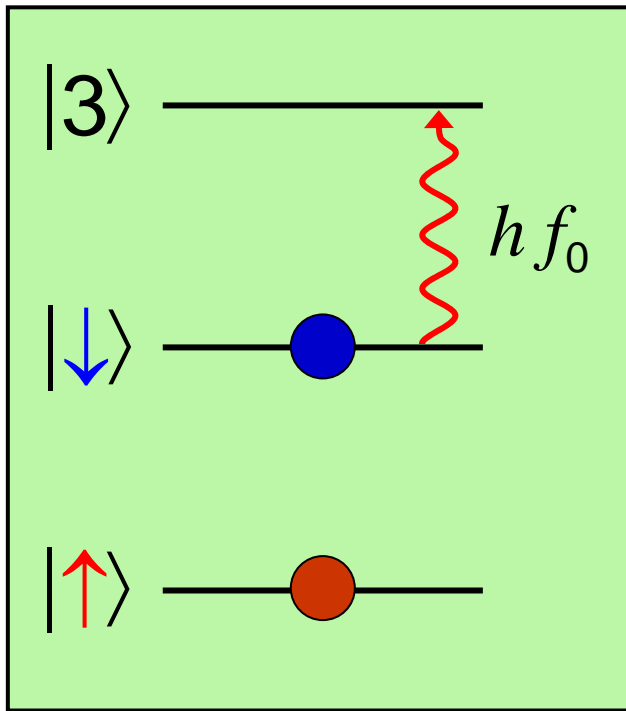
What happens at the Clogston-Chandrasekhar limit?

Also called the “Pauli pair breaking limit”

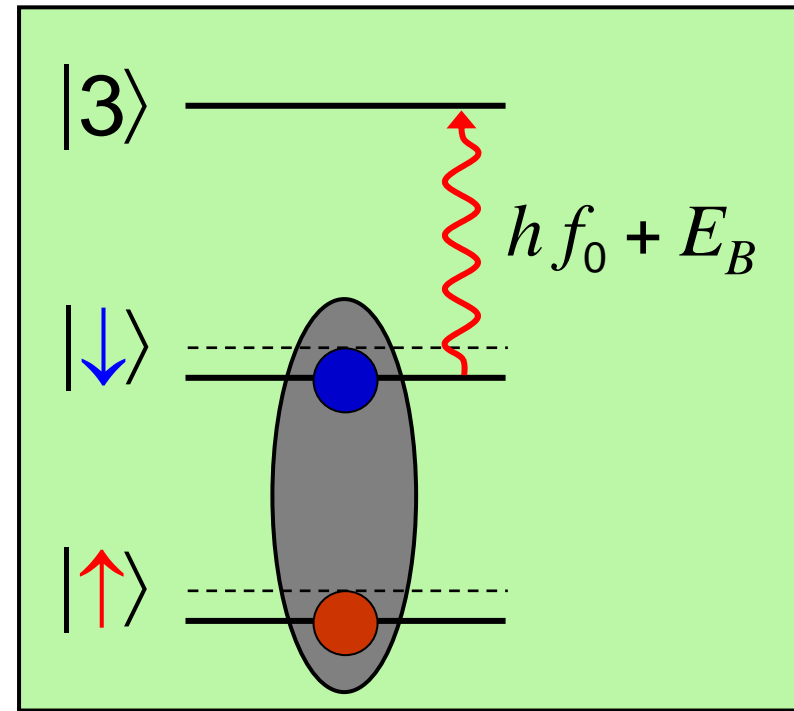


RF spectroscopy in a Fermi mixture

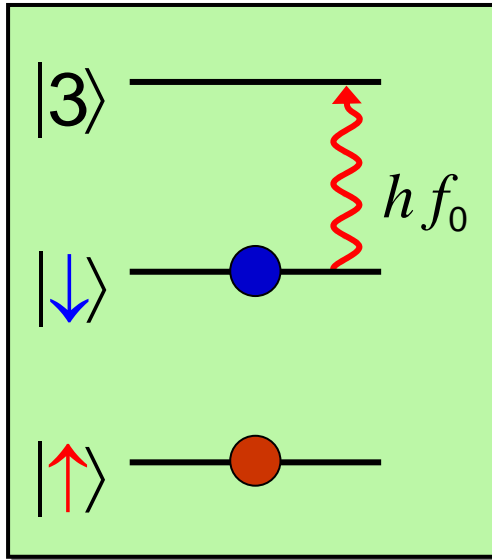
No interactions



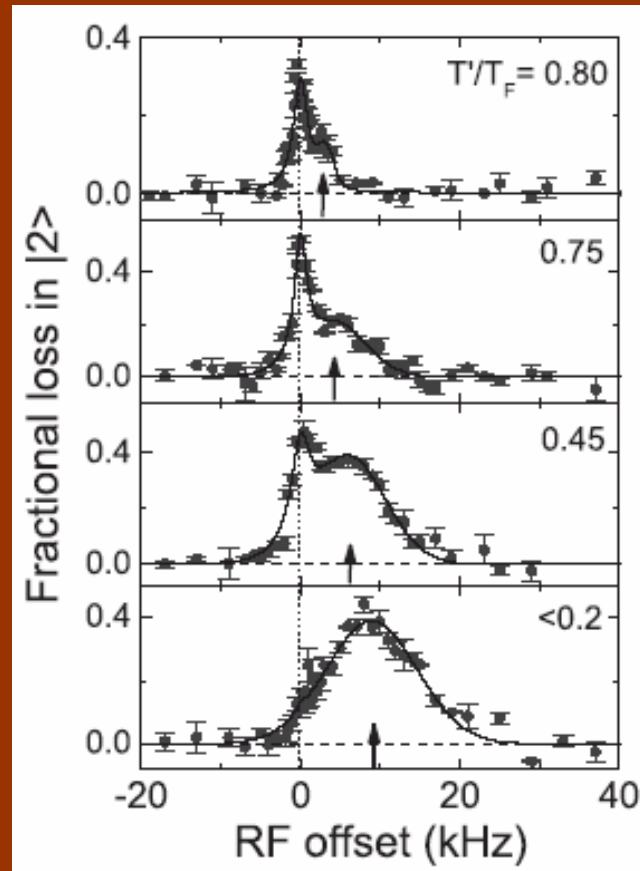
Pairing



Observation of a pairing gap in a *balanced* Fermi mixture



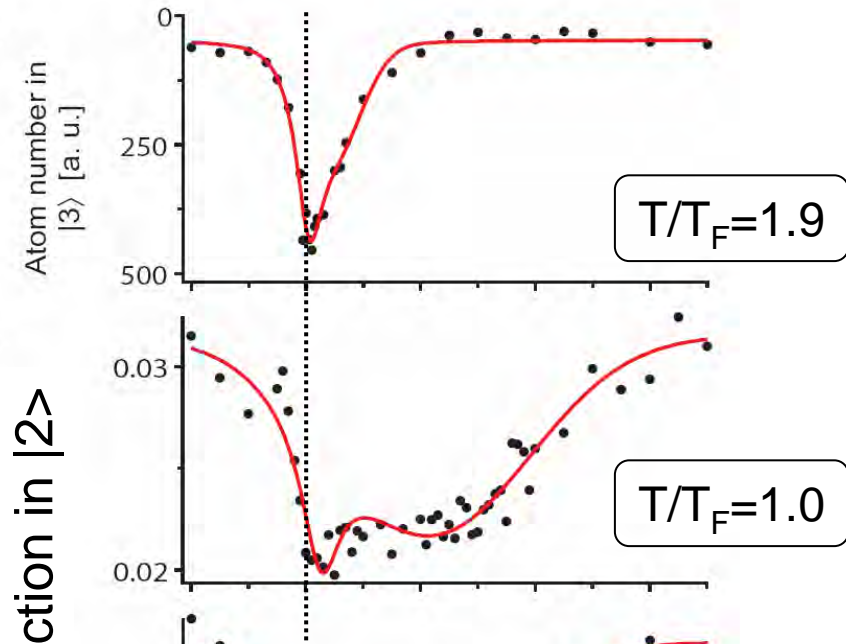
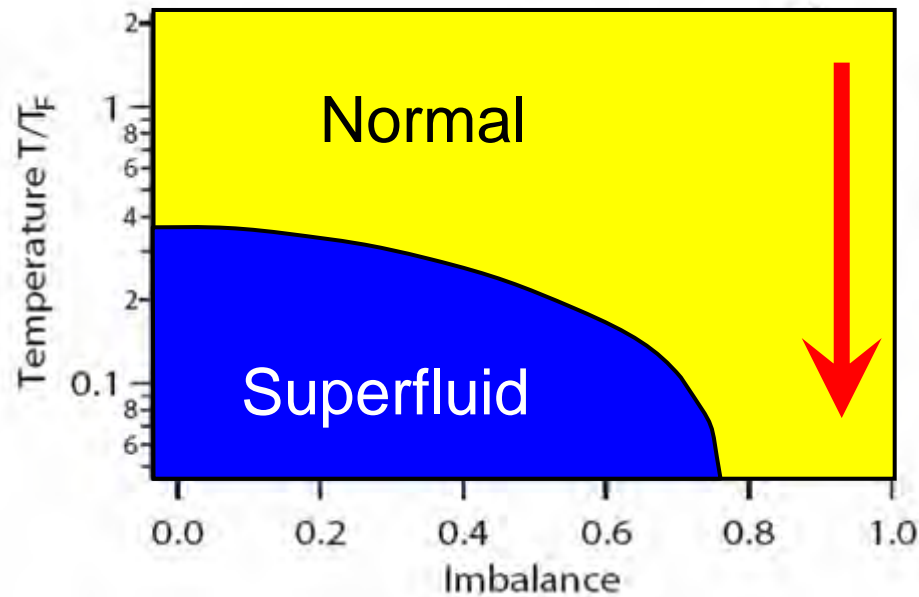
In balanced mixture:



C. Chin et al., Science **305**, 1128 (2004)

Double peak structure indicates the presence of pairing.

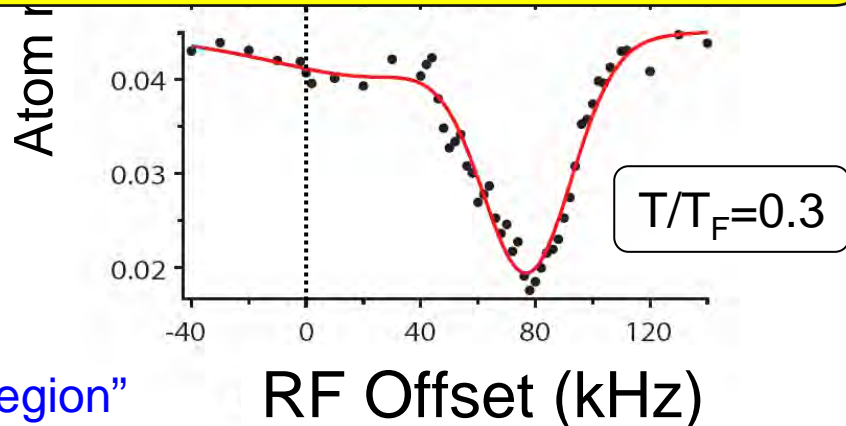
RF spectroscopy in an *imbalanced* Fermi mixture



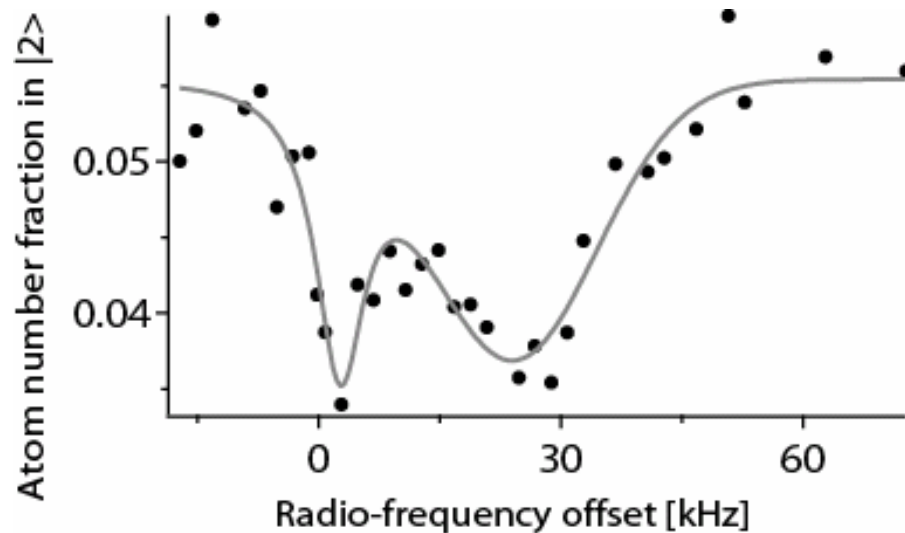
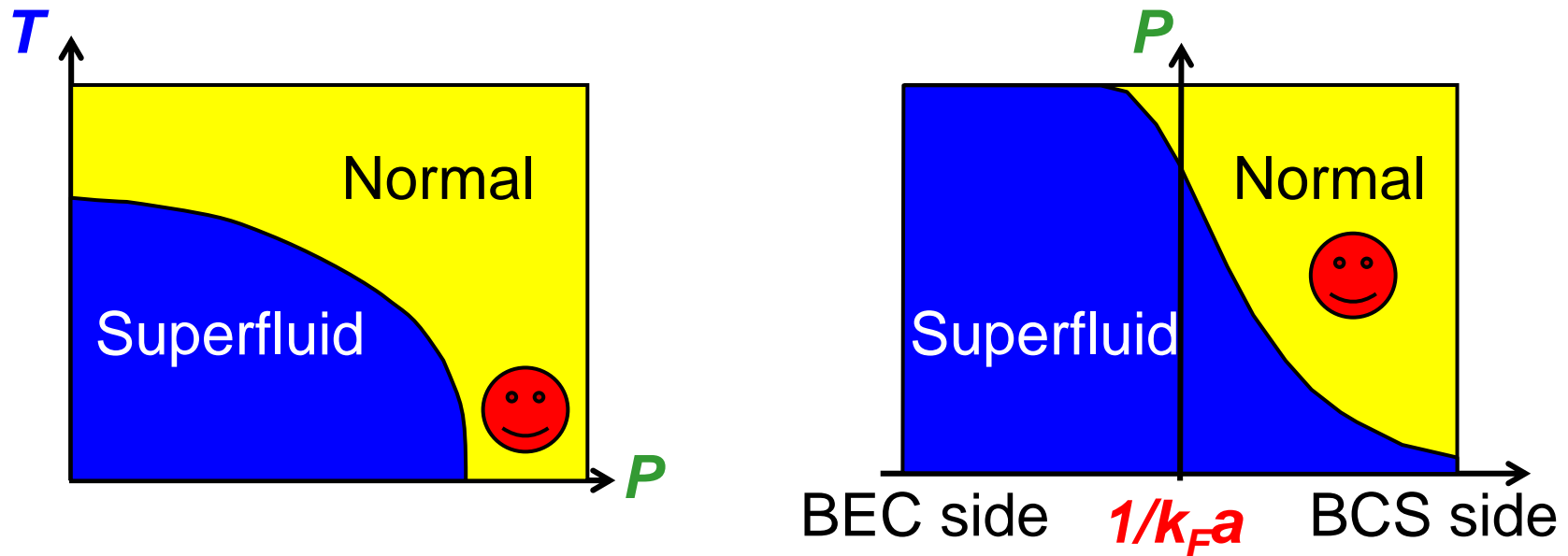
Fermion pairs *cannot* condense even at T=0
Pairing is *not* necessarily a precursor of **Superfluidity**.

C.H. Schunck, Y. Shin, A. Schirotzek,
 M.W. Zwierlein, and W. Ketterle,
 Science **316**, 867 (2007)

I. L. Aleiner, B. L. Altshuler,
 Phys. Rev. Lett. **79**, 4242 (1997):
 Fermion pairs exist “far above the transition region”



Pairing without Superfluidity



On the BCS side (936G)

Imbalance $P \sim 90\%$

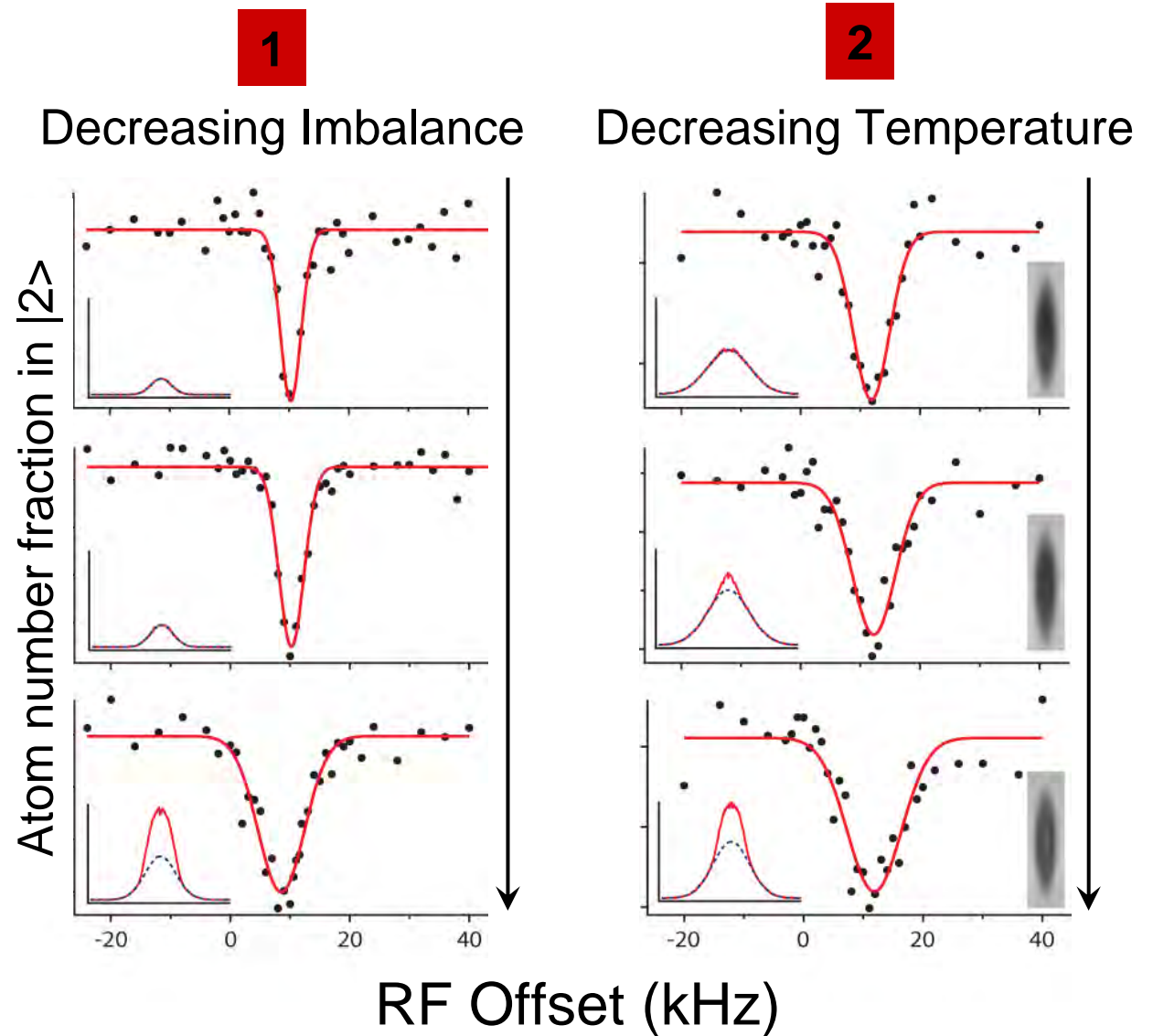
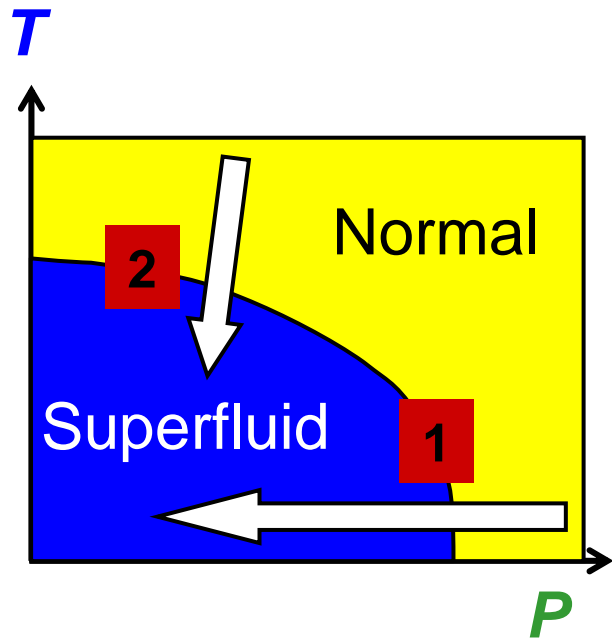
$T/T_F \sim 0.3$ $E_F/h \sim 280$ kHz

$1/k_F a = -0.18$

$P_{c,exp} \sim 60\%$

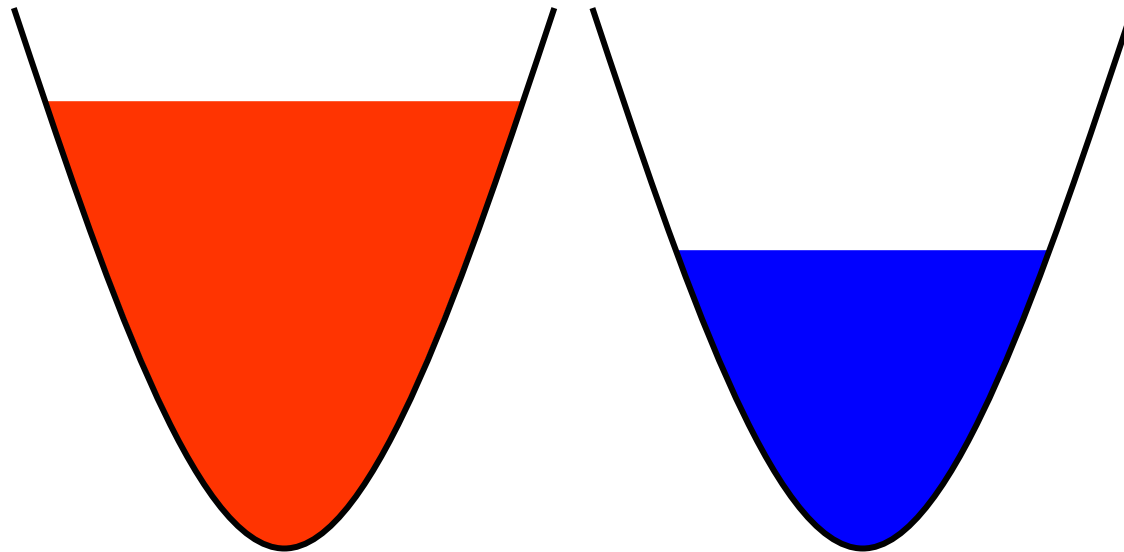
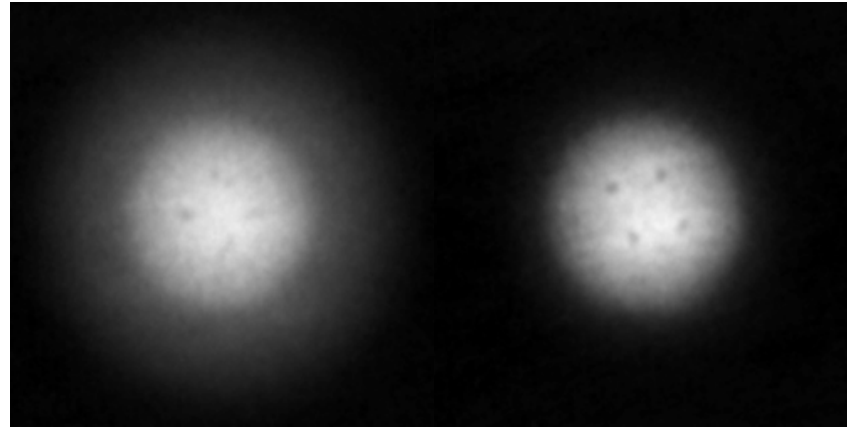
C.H. Schunck, Y. Shin, A. Schirotzek, M.W. Zwierlein, and W. Ketterle,
Science 316, 867 (2007)

Rf spectroscopy across the phase transition



Rf-spectroscopy does not reveal superfluidity

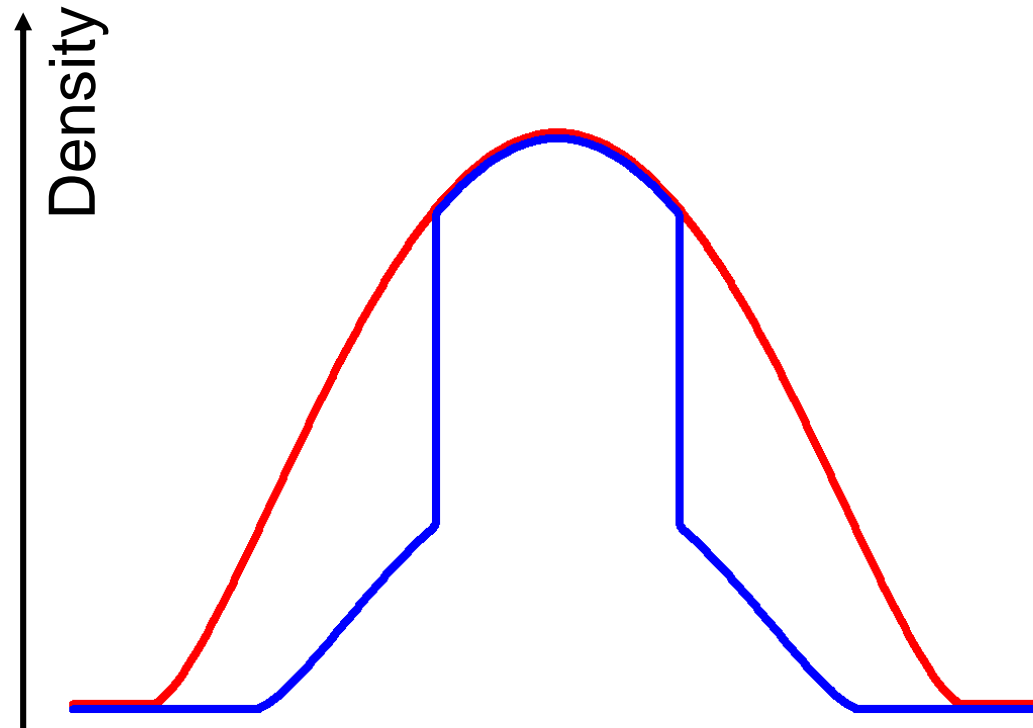
What is the Nature of the Superfluid State?



Density distribution of *unequal* mixtures

Normal state: unequal densities

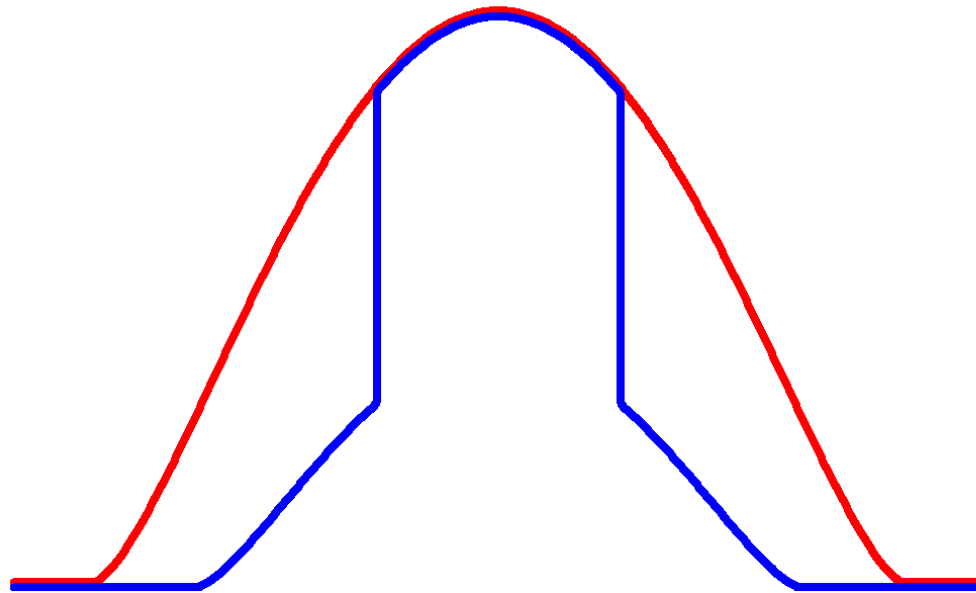
Superfluid: needs equal densities (Phase separation scenario)



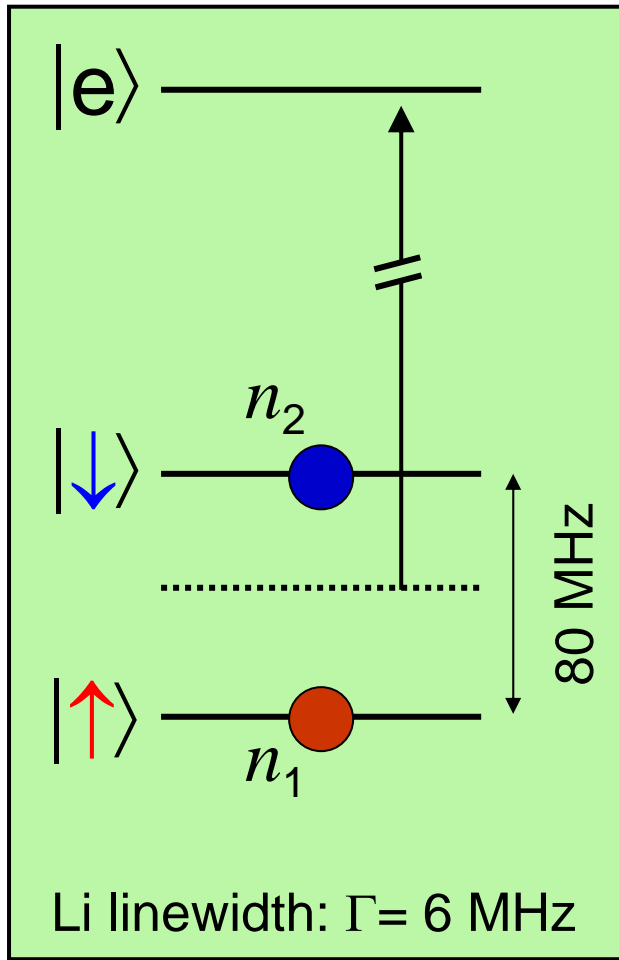
**At the Normal – Superfluid – Transition:
Density discontinuity or “kink” in the profiles!**

M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle,
Nature 442, 54 (2006)

How to image the density *difference*?



Phase Contrast Imaging



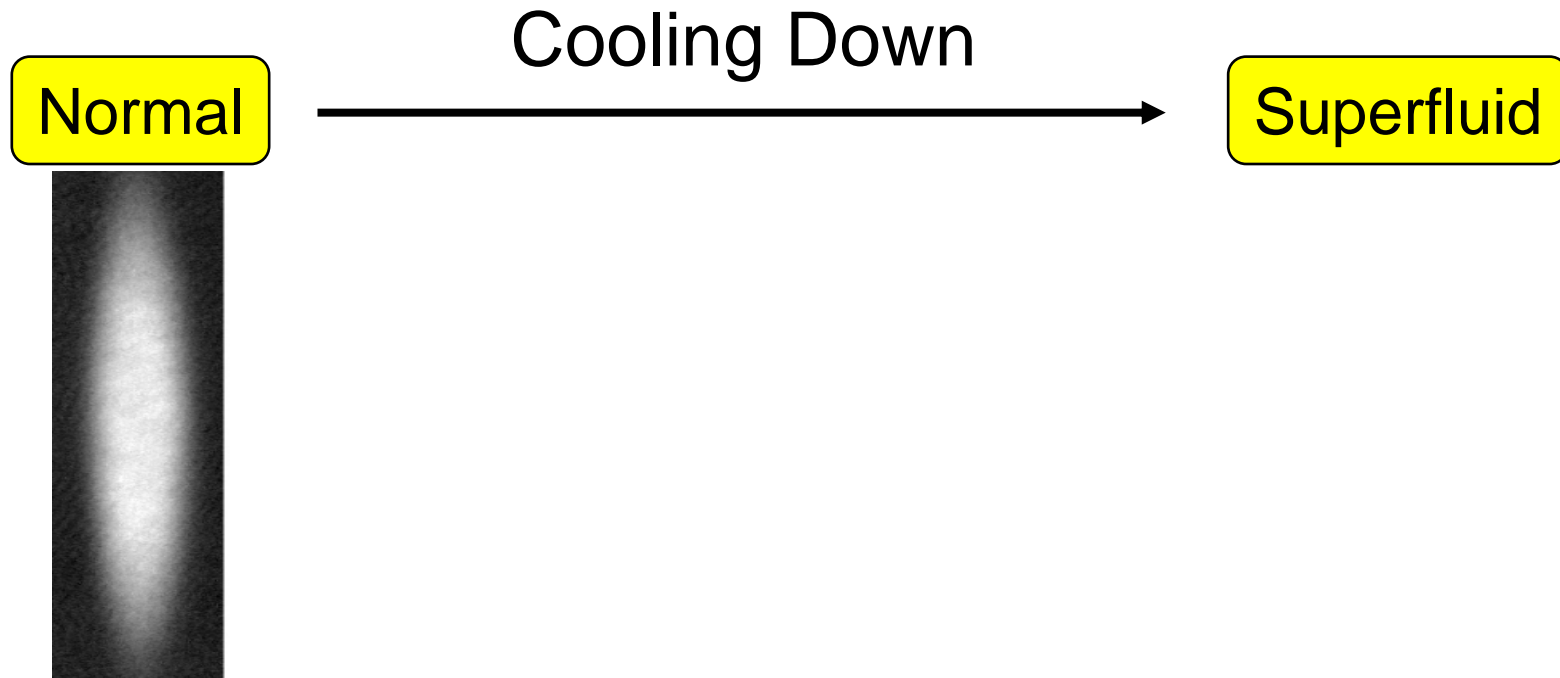
- Imaging beam **red**-detuned for $|\uparrow\rangle$
blue-detuned for $|\downarrow\rangle$

- Phase shift \propto Density difference

$$\Delta n = n_2 - n_1$$

In-trap images

Direct observation of the Phase transition



Shell structure: A hint for phase separation

Y. Shin, M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle,
PRL 97, 030401 (2006)

see also: M.W. Zwierlein, A. Schirotzek, C.H. Schunck, and W. Ketterle
Science **311**, 492 (2006)

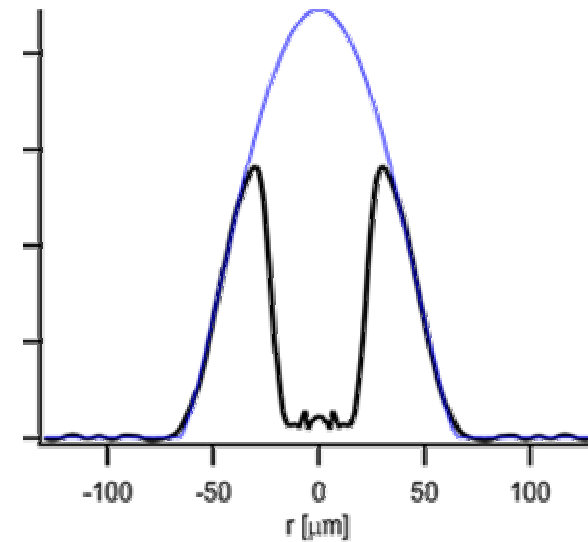
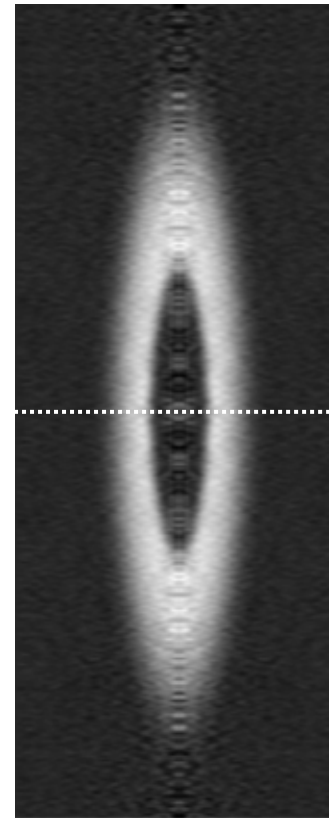
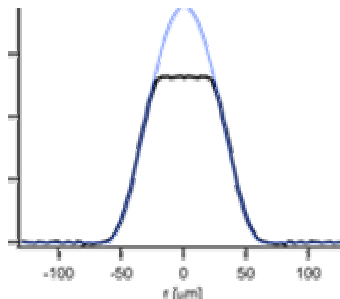
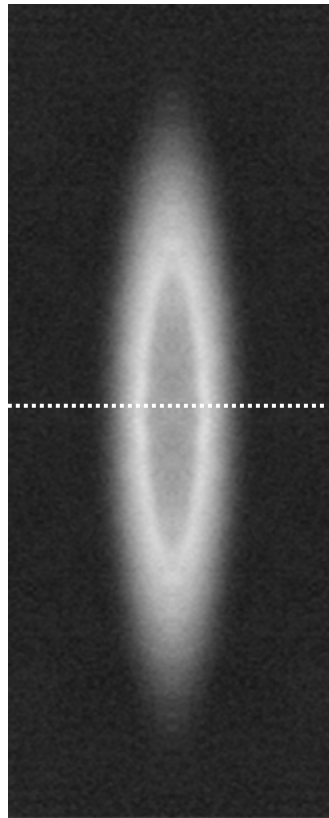
Nature **442**, 54 (2006)

G. B. Partridge *et al.*, Science **311**, 503 (2006)

Reconstruction of 3D density profile

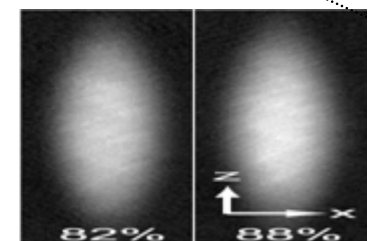
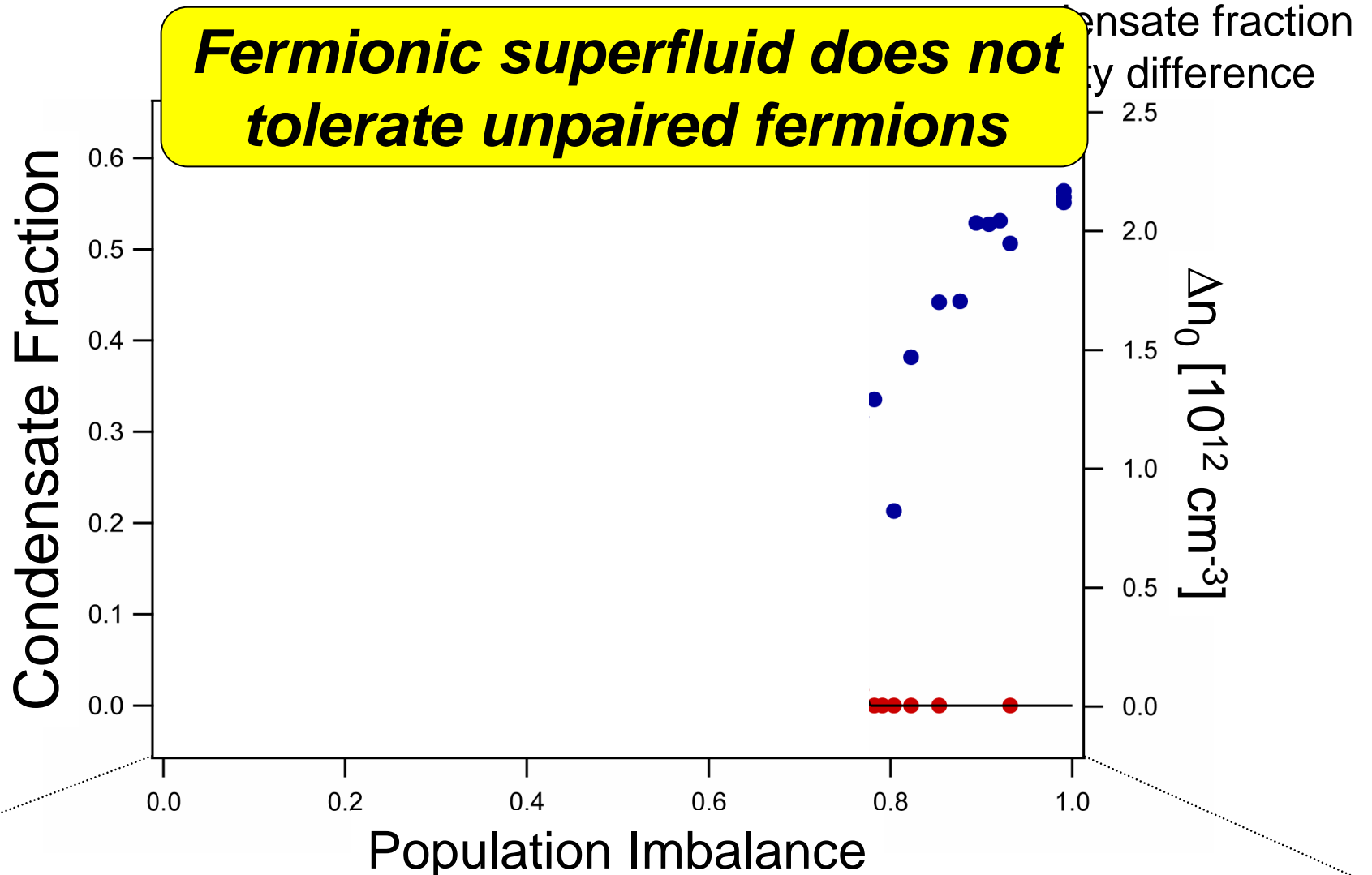
- Column density profile \rightarrow 3D density profile
- Only assumes cylindrical symmetry

$P = 0.6$

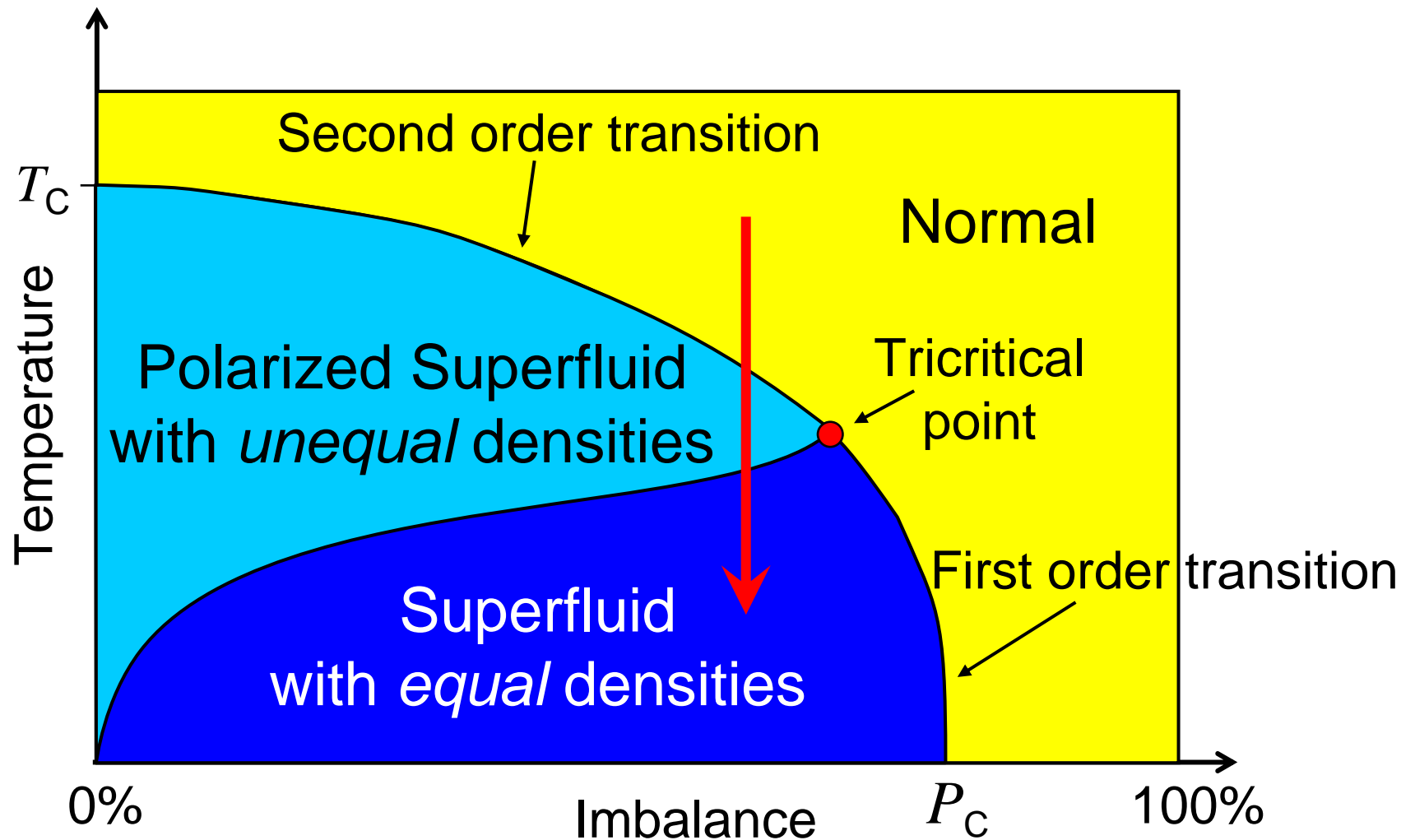


Observation of Phase Separation

Phase Separation in an Imbalanced Fermi Gas



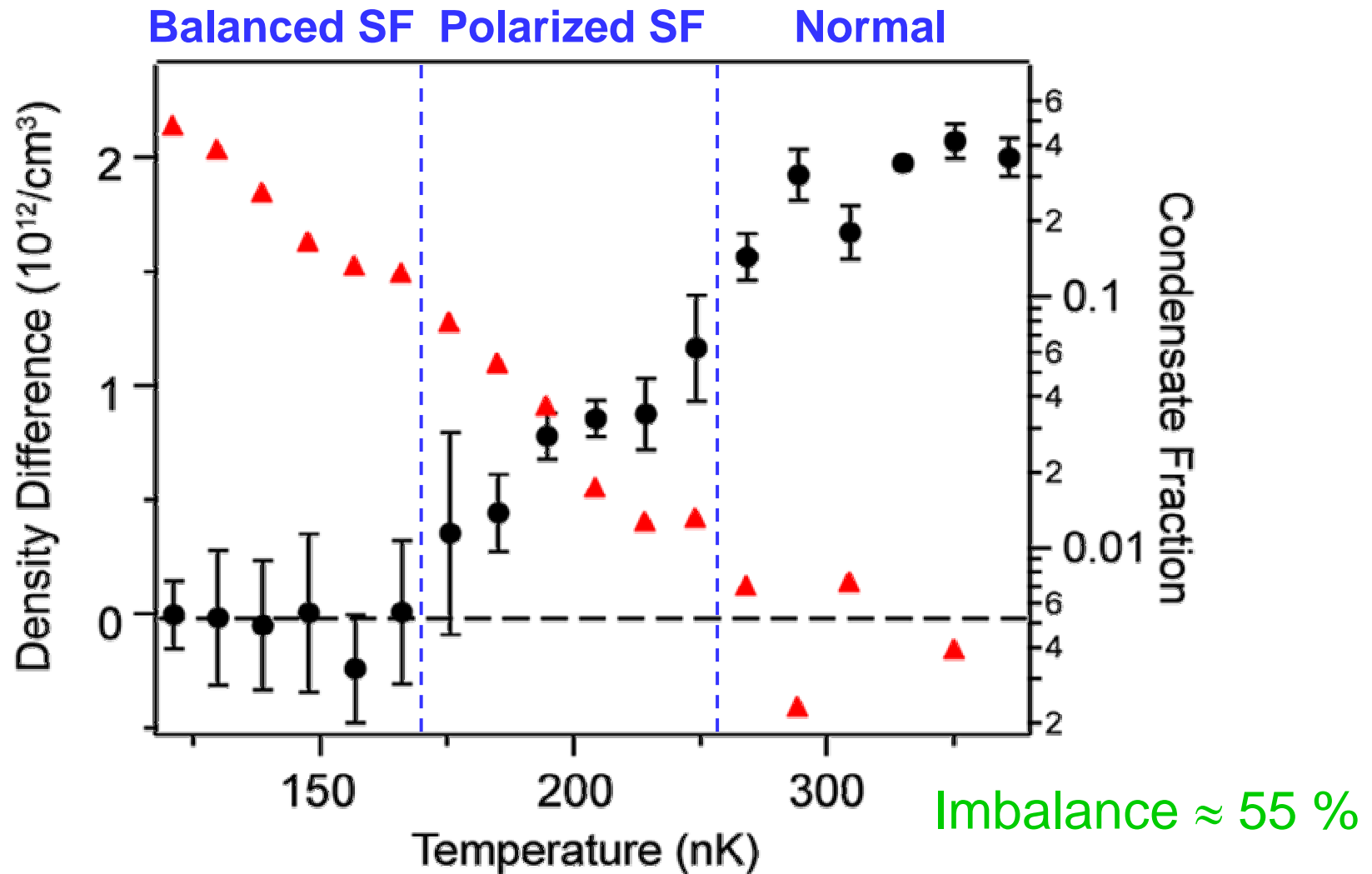
Finite Temperature Phase diagram



Proposed finite-T. phase diagrams:

Machida et al., PRL **97**, 120407 (2006), Liu and Hu, Eur.Phys.Lett. **75**, 364 (2006),
Gubbels et al., PRL **97**, 210402 (2006), Parish et al., Nature Physics **3**, 124 (2007),
He et al., PRA **75**, 021602 (2007), Duan et al., cond-mat 0706.1253 (2007),...

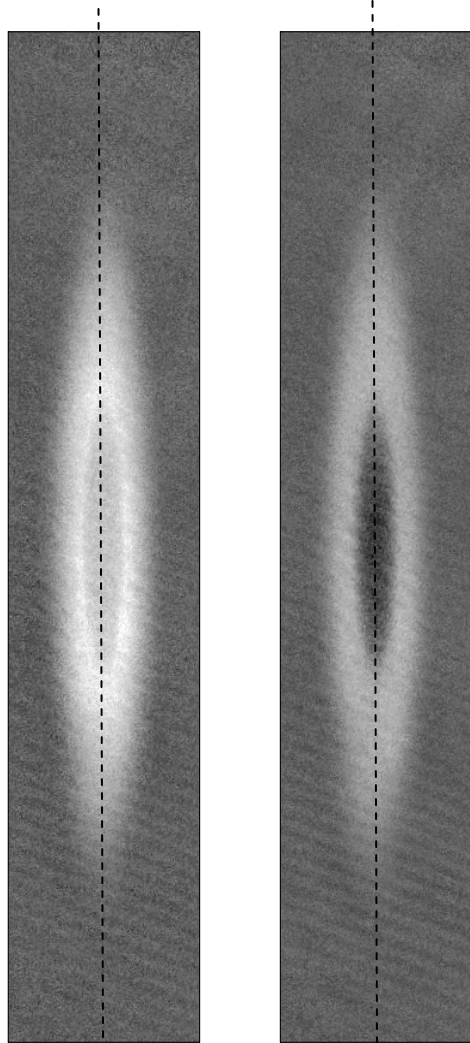
Finite Temperature Effects



- Temperature-induced polarized superfluid.
- Thermal population of quasiparticles ($k_B T > \Delta$)

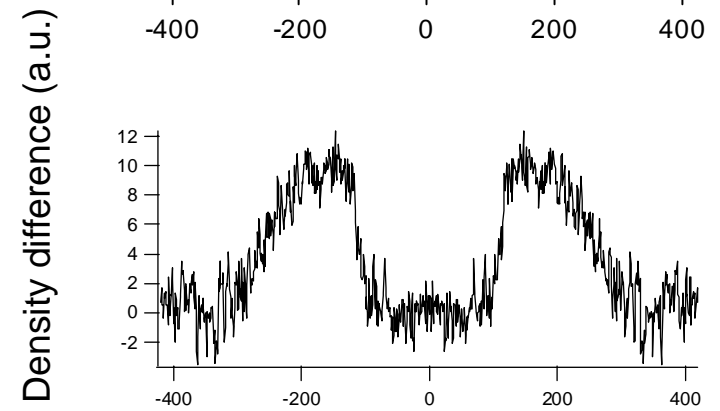
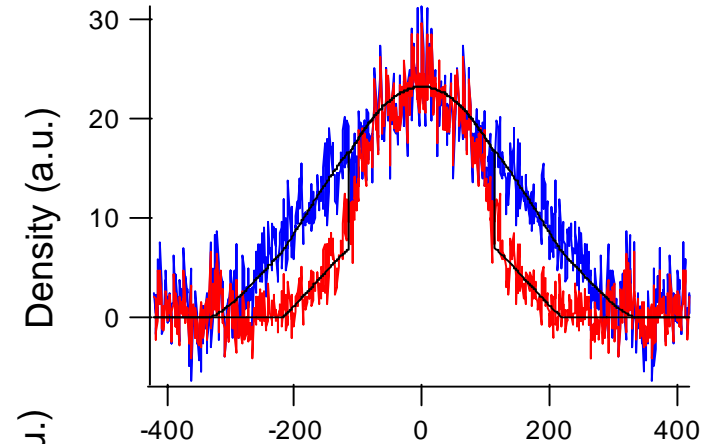
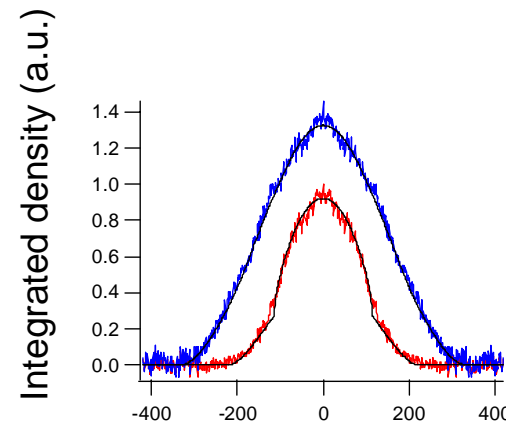
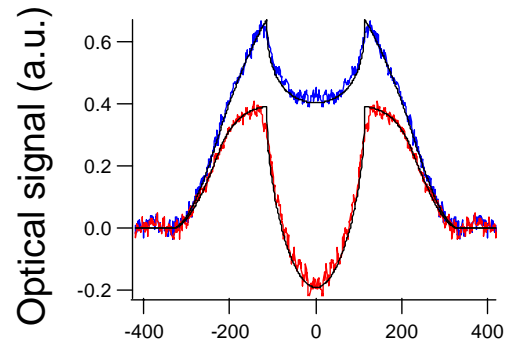
Phase separation: 1st order Phase transition?

$\delta=55\%$



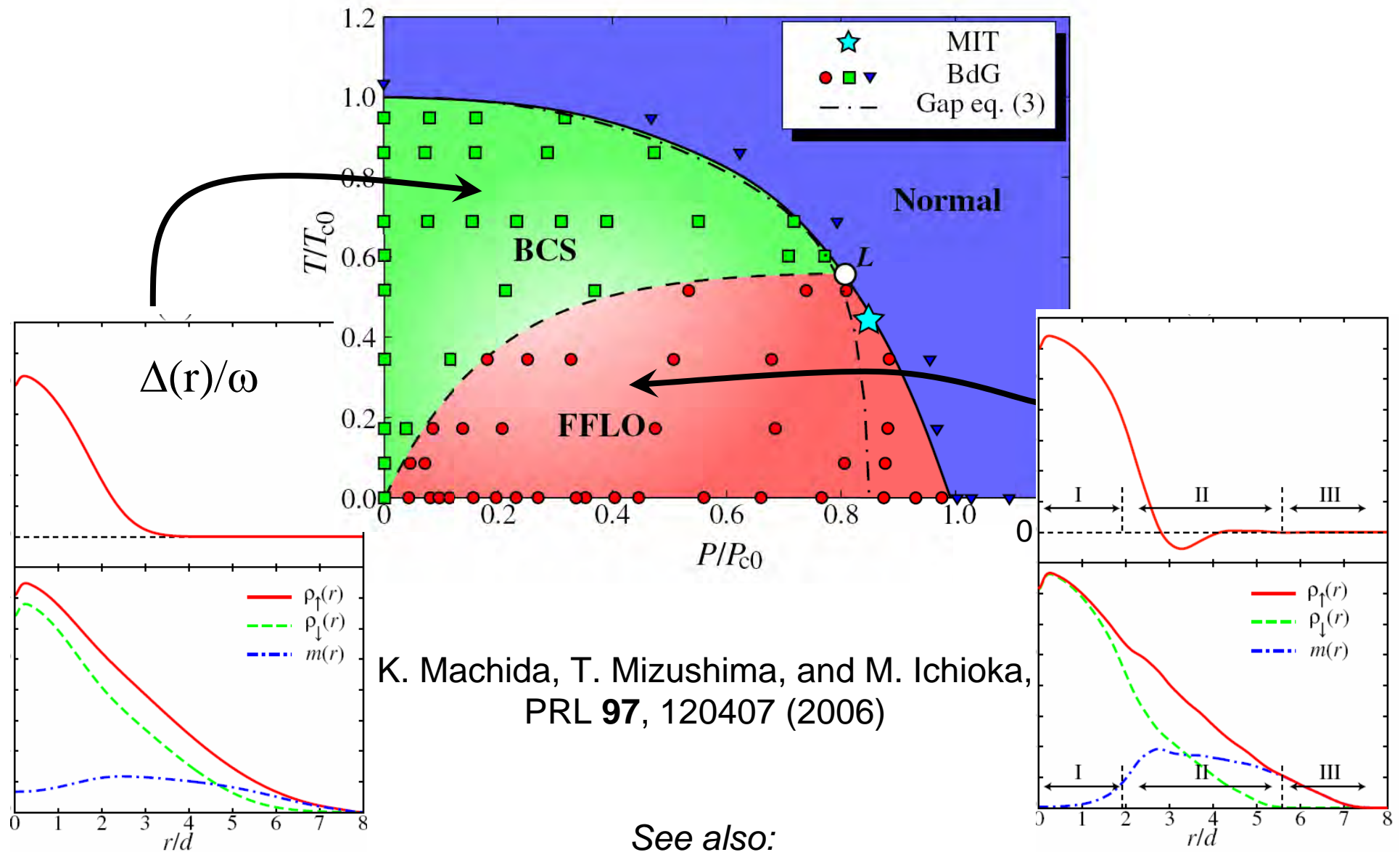
Signal = $n_1 - n_2$

$0.8n_1 - 1.4n_2$



Sharp transition between SF at equal densities and imbalanced, *partially* polarized state

Where is the LOFF-state?

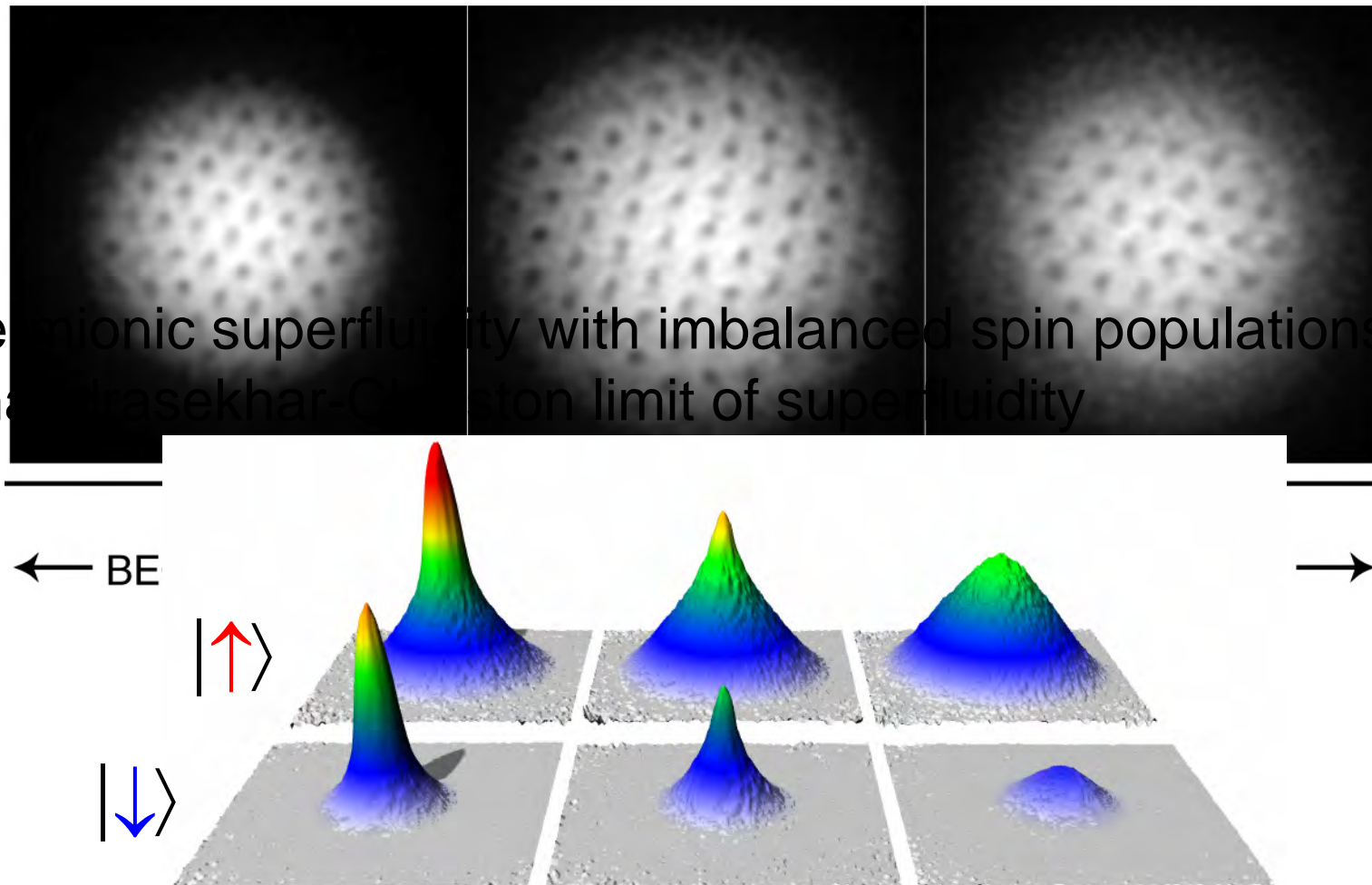


J. Kinnunen et al., PRL **96**, 110403 (2006), Liu and Hu, Eur.Phys.Lett. **75**, 364 (2006),
He et al., PRA **75**, 021602 (2007), Duan et al., cond-mat 0706.1253 (2007), Randeria et al.

Conclusions

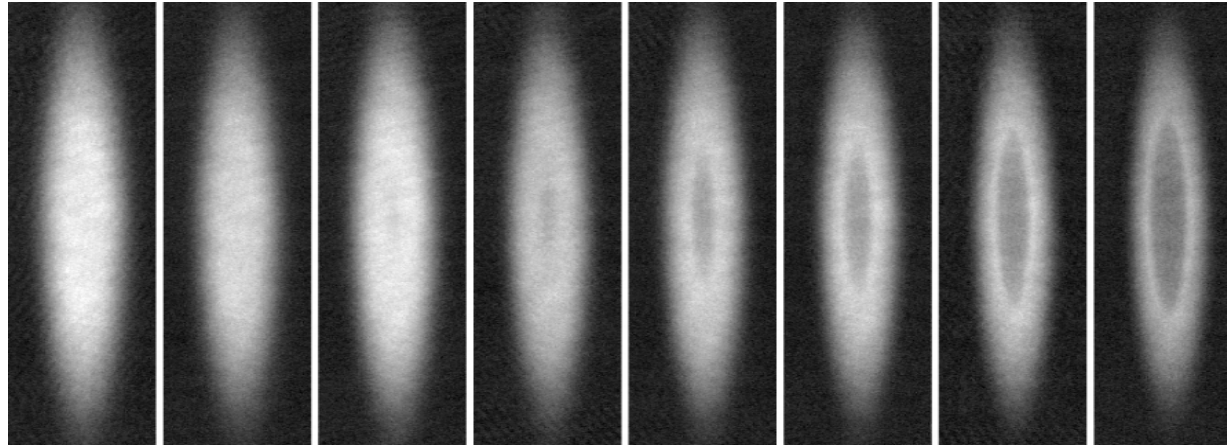
- Established **superfluidity** and **phase coherence** via observation of ordered vortex lattices

- Fermionic superfluidity with imbalanced spin populations
Chandrasekhar-Cramer limit of superfluidity

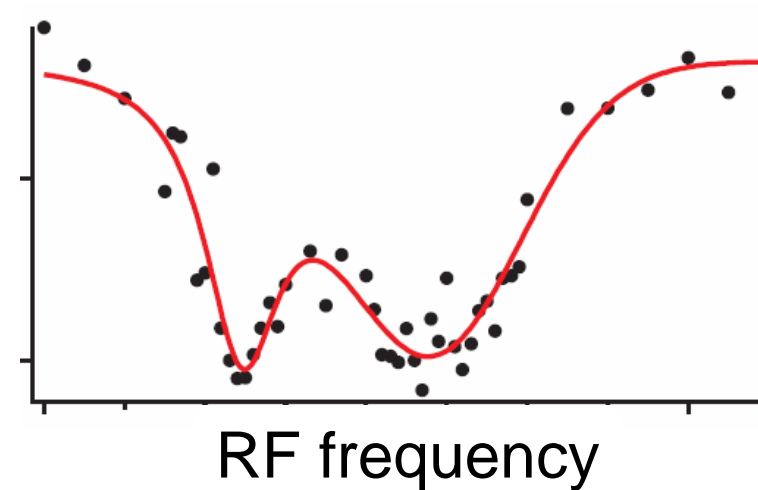
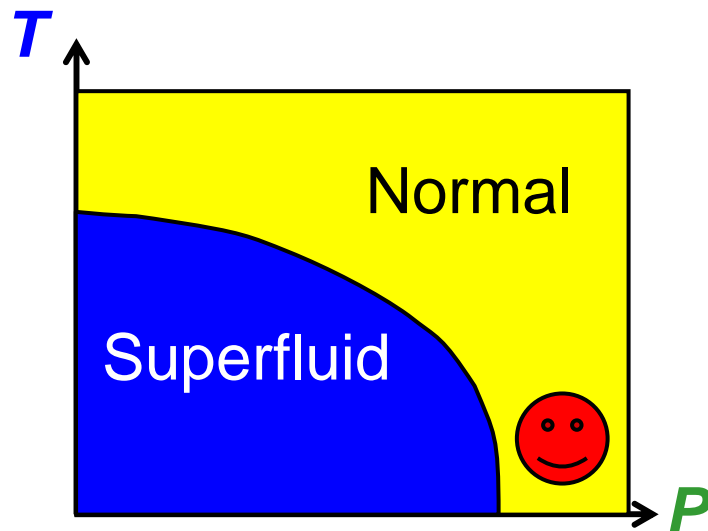


Conclusions

- **Direct** observation of the phase transition and phase separation



- **Pairing** without **Superfluidity**



Outlook

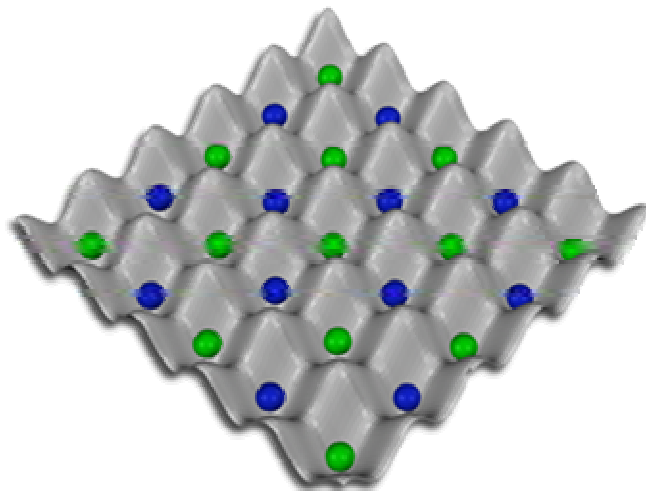
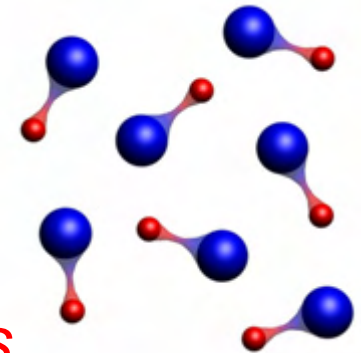
- Novel forms of superfluidity

FFLO-State

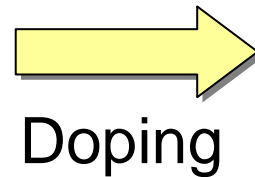
Superfluid pairing between different species

- Fermionic mixtures in optical lattices

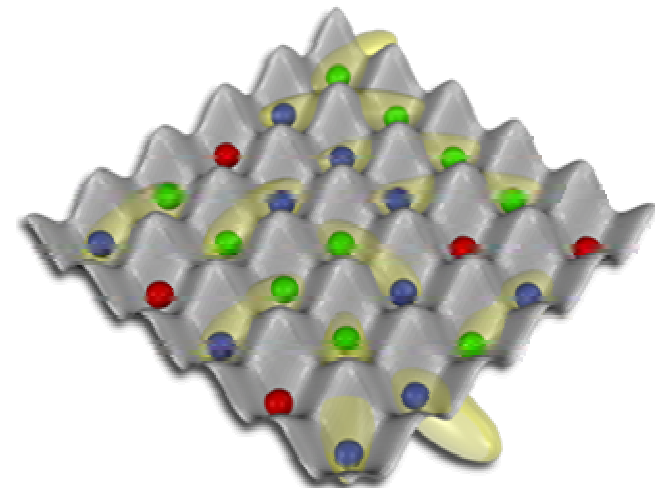
Model systems for condensed matter physics



***Antiferromagnetic
Insulator***



Doping



***Spin liquid
High- T_c Superconductor?***

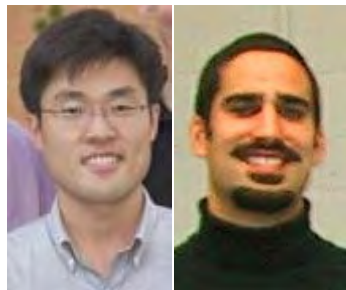
**Use the tools and precision of Atomic Physics
to study strongly correlated model systems**

The team



Subhadeep Gupta

Zoran Hadzibabic



Yong-Il Shin Jamil Abo-Shaeer



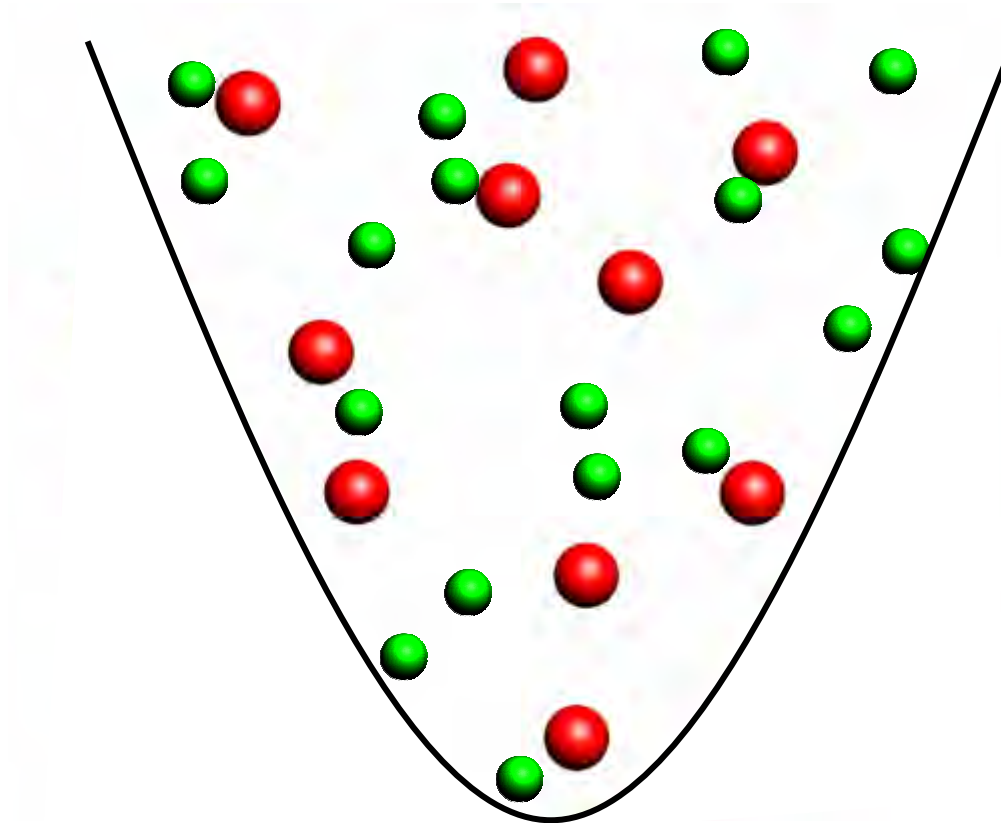
André Schirotzek

Christian Schunck

Wolfgang Ketterle

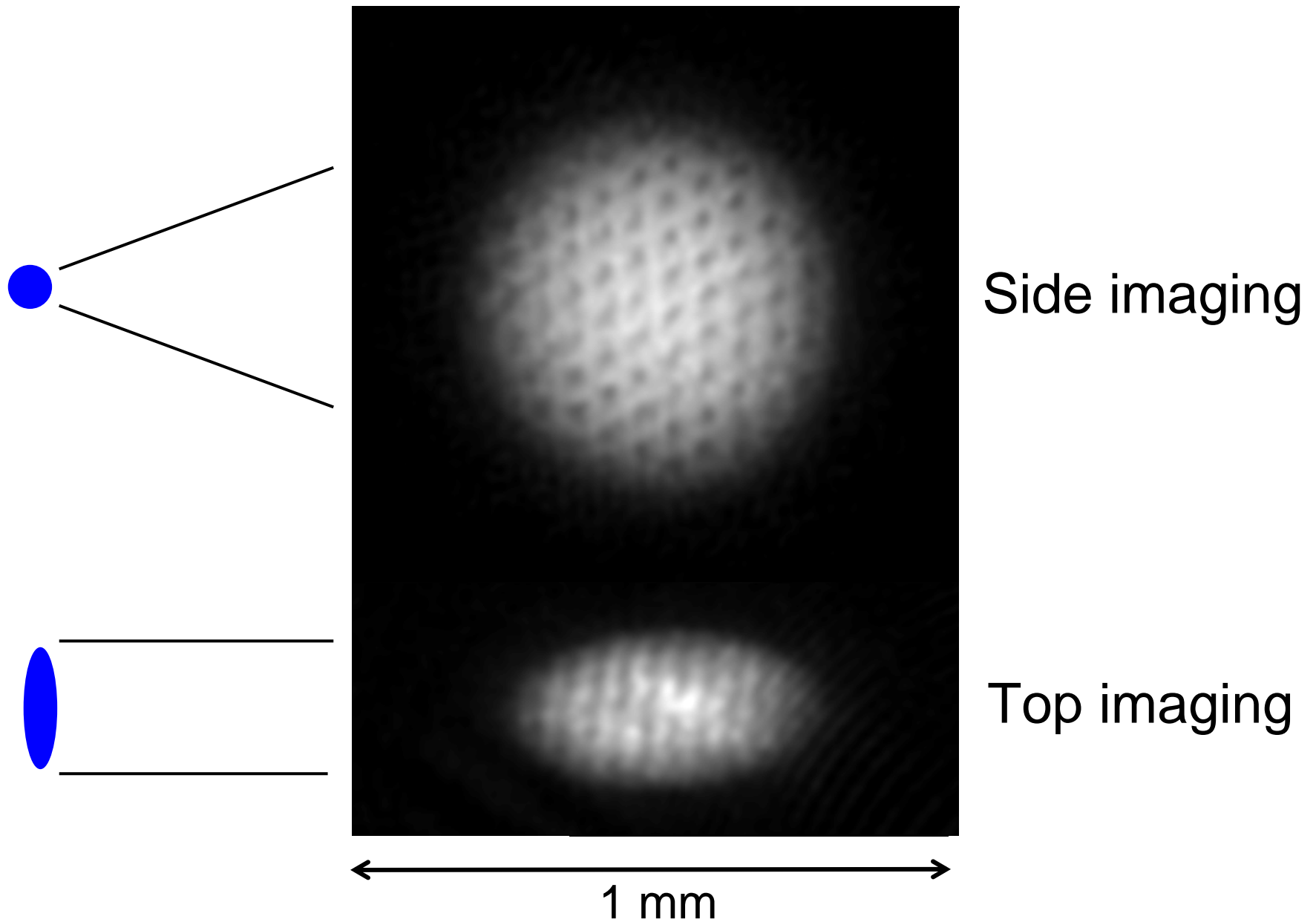
Source of ultracold fermions

- **Our choice:** Spin-polarized fermionic ${}^6\text{Li}$
- **The problem:** Evaporative cooling does not work!
At low temperatures: All but s-wave collisions frozen out
s-wave collisions *forbidden* due to Pauli principle
→ No thermalization
- **The solution:** Use a coolant! (*Our case: Sodium atoms*)



In trap

After expansion



Side imaging

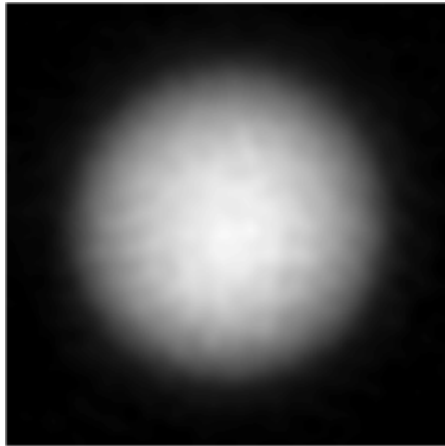
Top imaging

1 mm

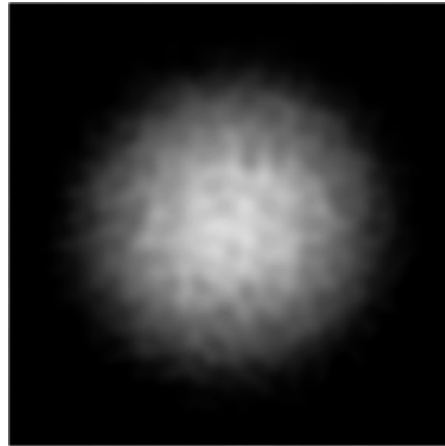
Formation of Vortex Lattice

Equilibration close to resonance (812 G)

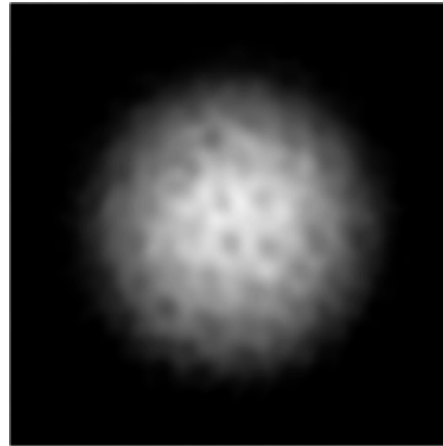
Initial state



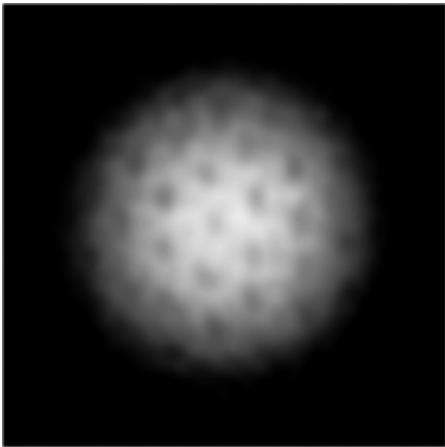
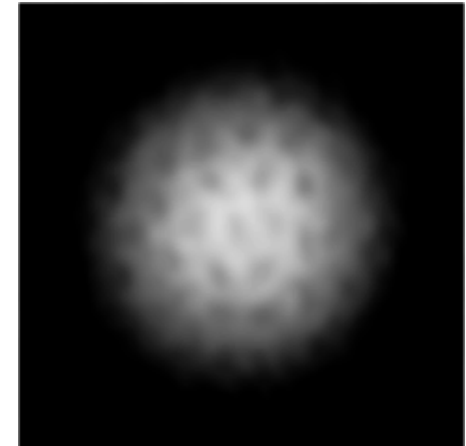
Stir + wait 40 ms



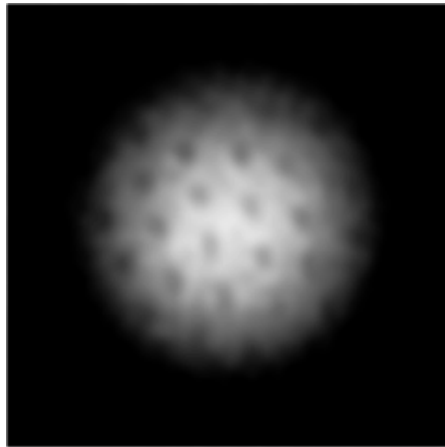
240 ms



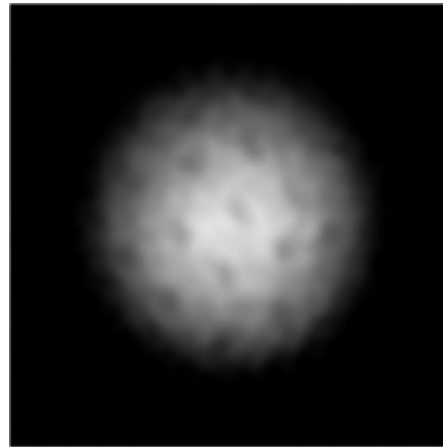
390 ms



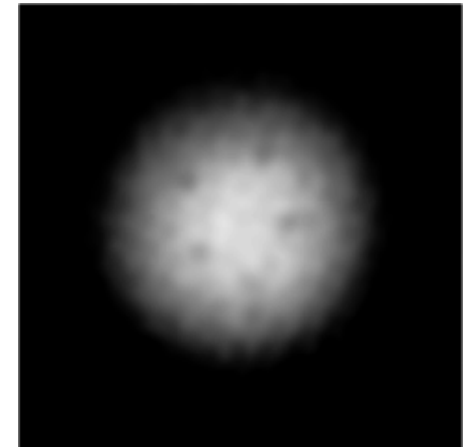
790 ms



1140 ms



1240 ms

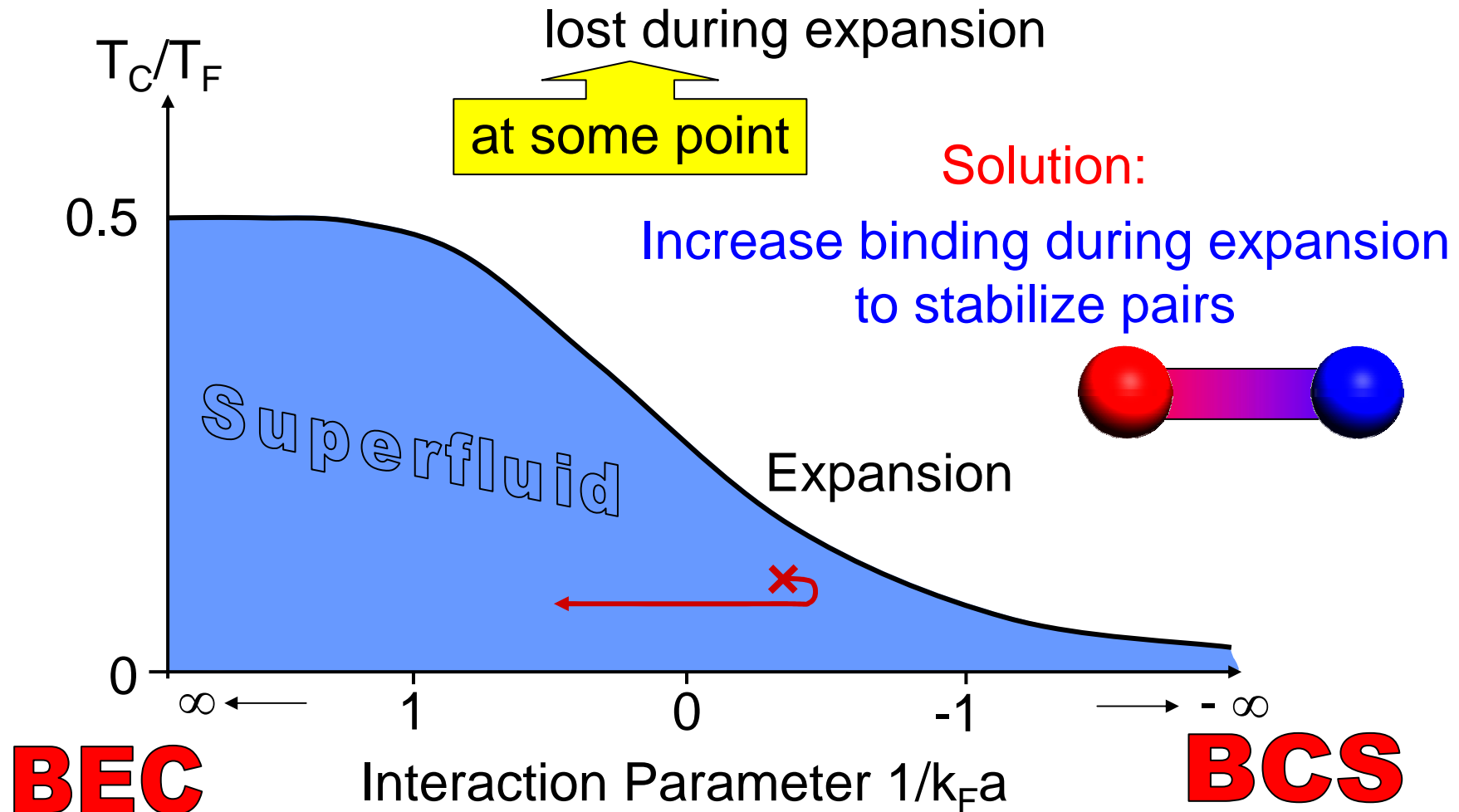


2940 ms

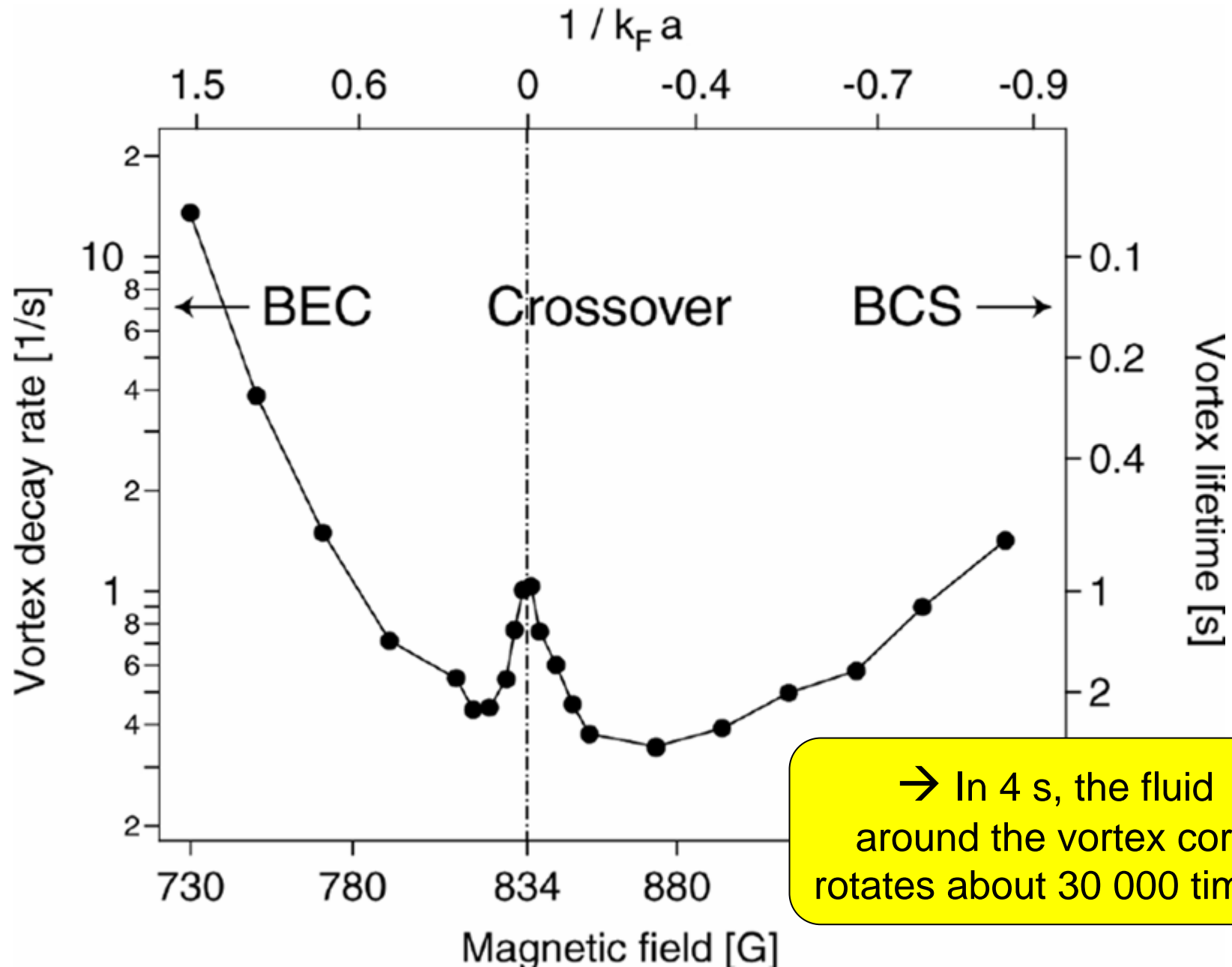
Superfluid expansion of fermion pairs

On the BCS-side, the superfluid state is fragile!

- ⇒ Pairs dissociate as cloud expands
- ⇒ Information on the COM wavefunction is lost during expansion



Vortex lifetime in the Crossover

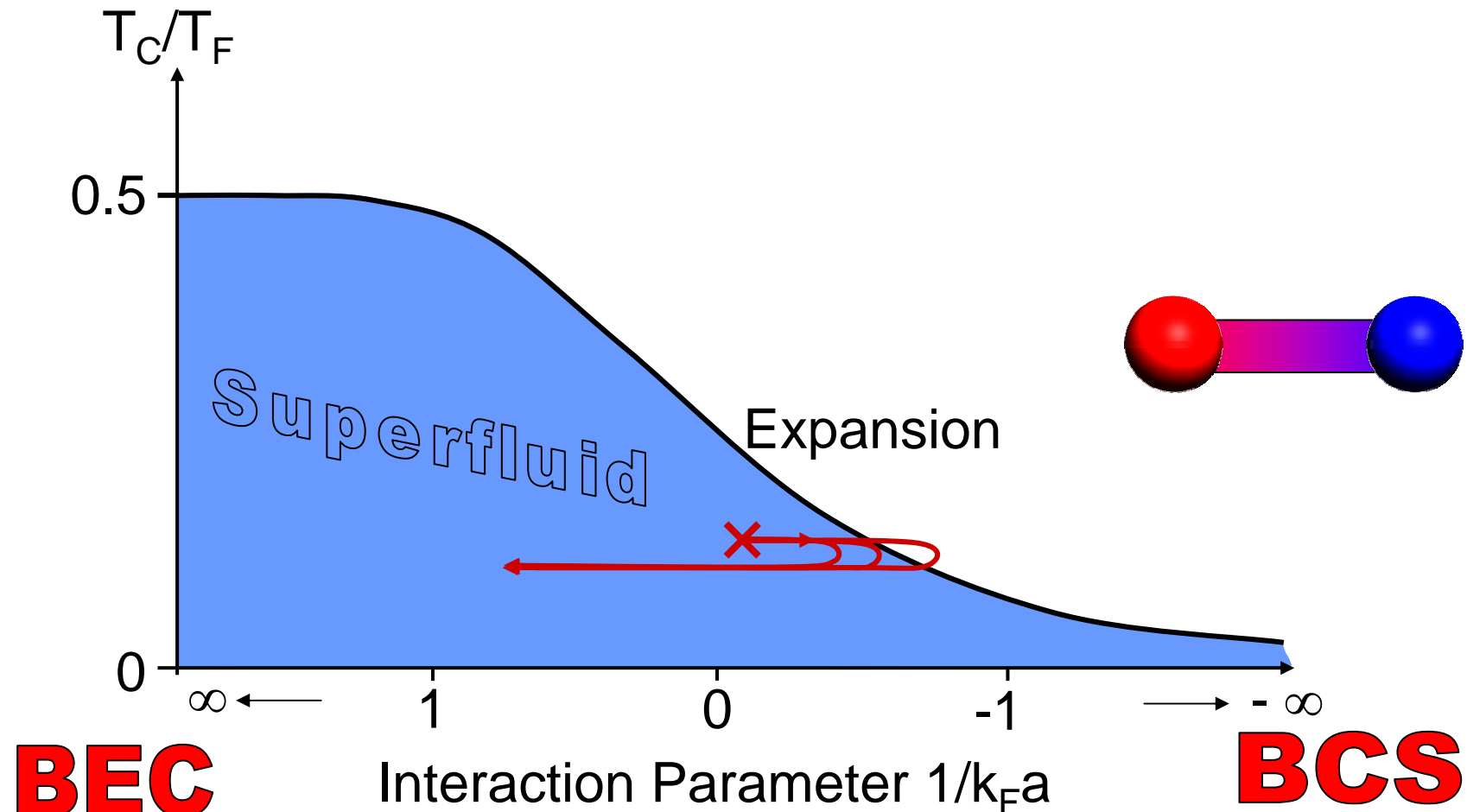


Superfluid expansion of fermion pairs

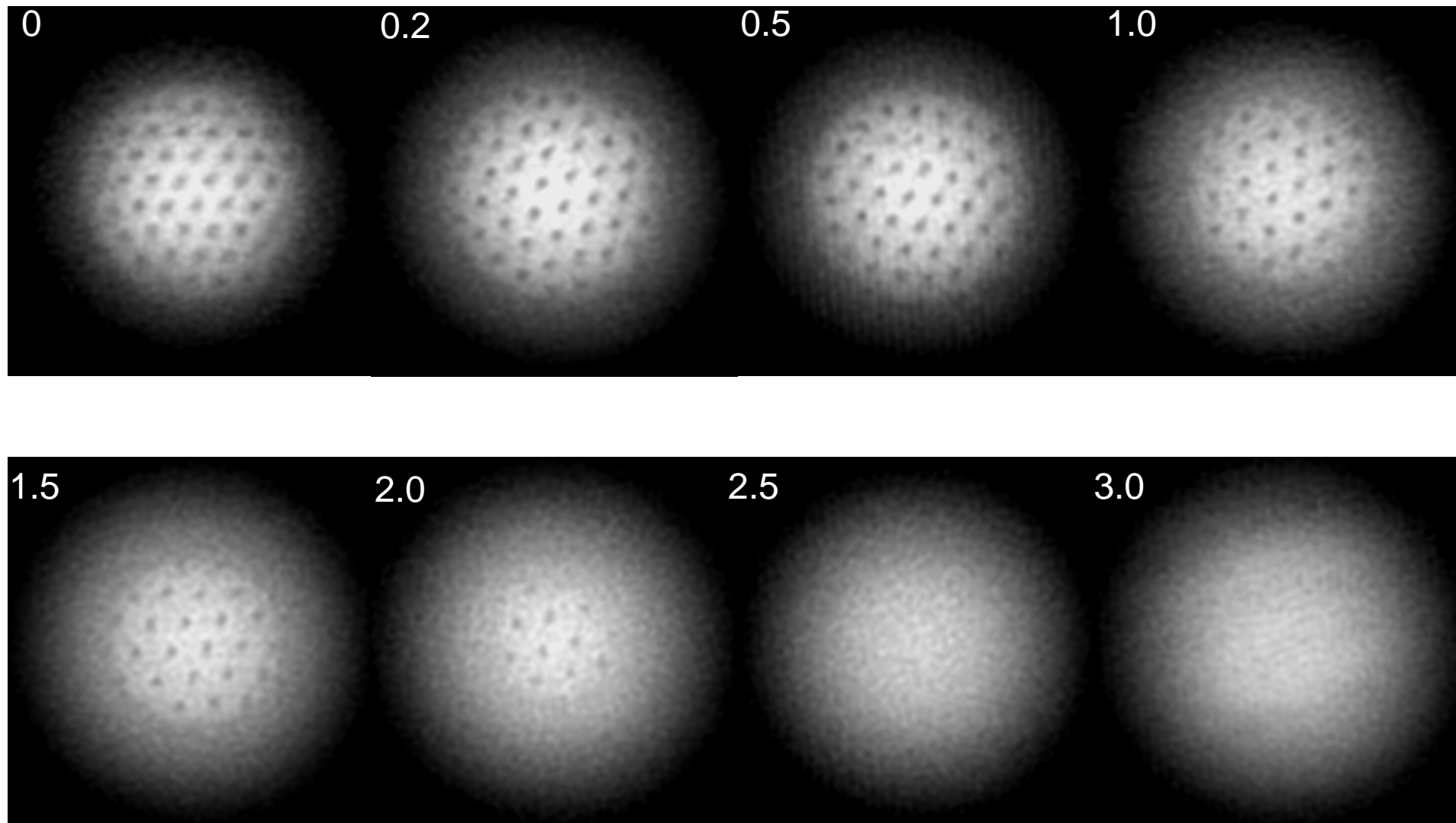
On the BCS-side, the superfluid state is fragile!

We can cross the phase boundary during expansion!

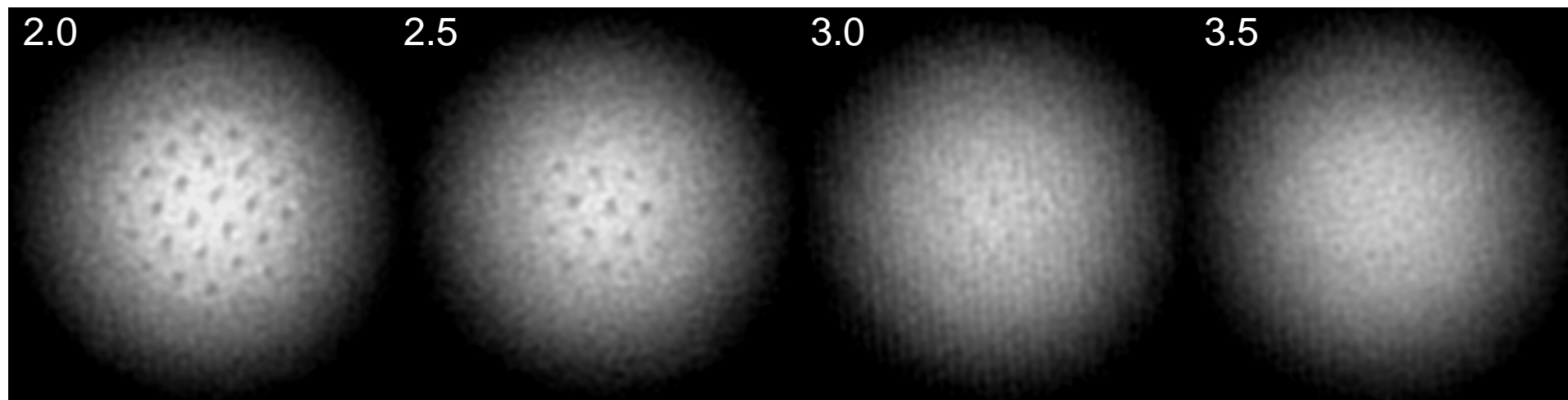
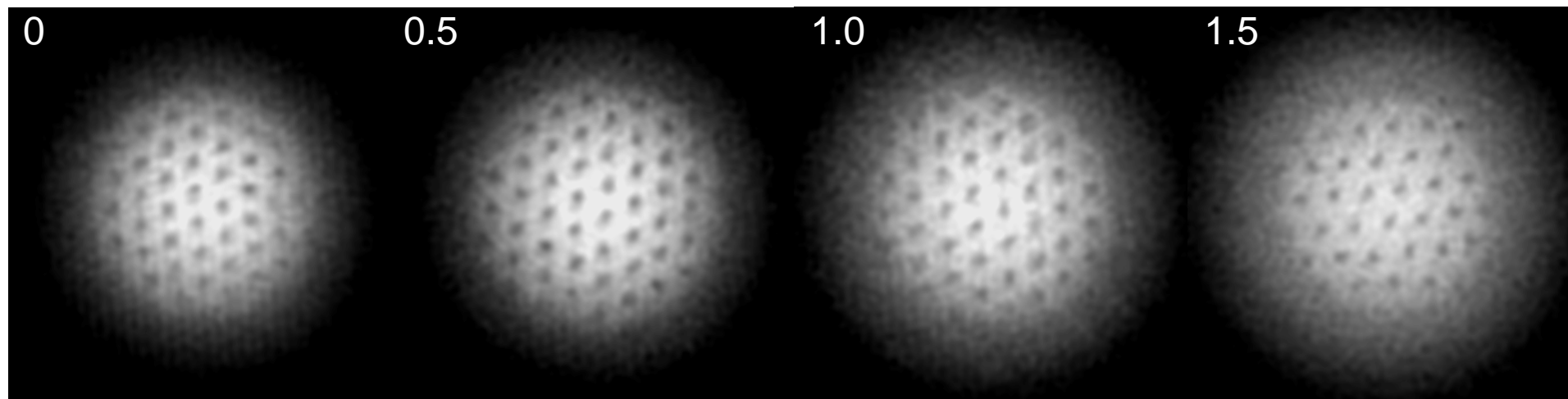
To observe this: Delay the “rescue” of fermion pairs



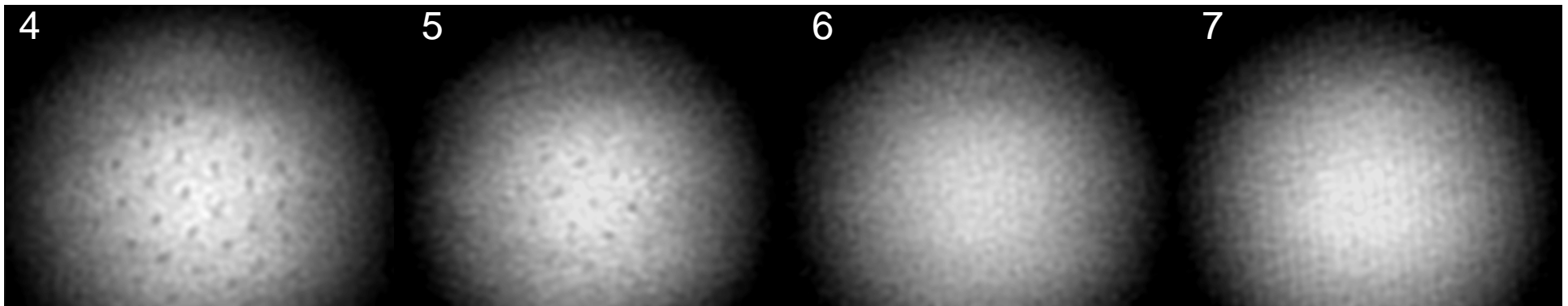
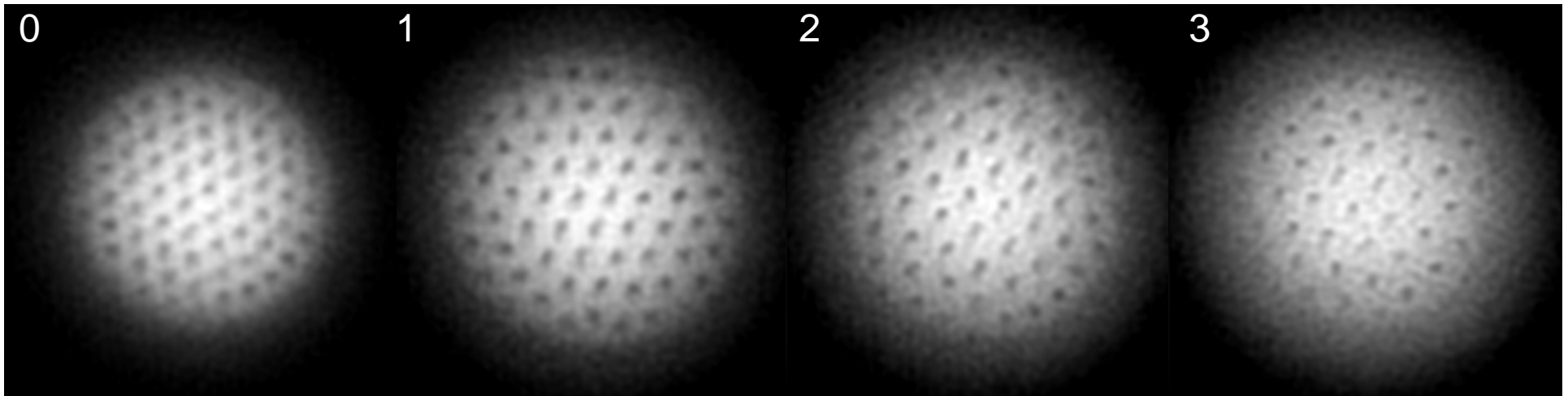
Pair breaking in TOF [ms] 930 G (small a)



Pair breaking in TOF [ms] 912 G (medium *a*)



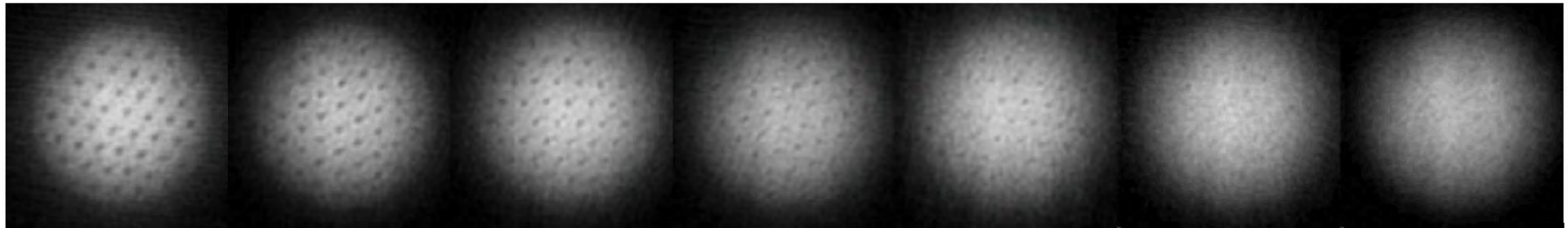
Pair breaking in TOF [ms] 875 G (large a)



Fermionic superfluid survives expansion!
Eventual breakdown at $k_F |a| \approx 1$

Superfluid expansion of fermion pairs on resonance

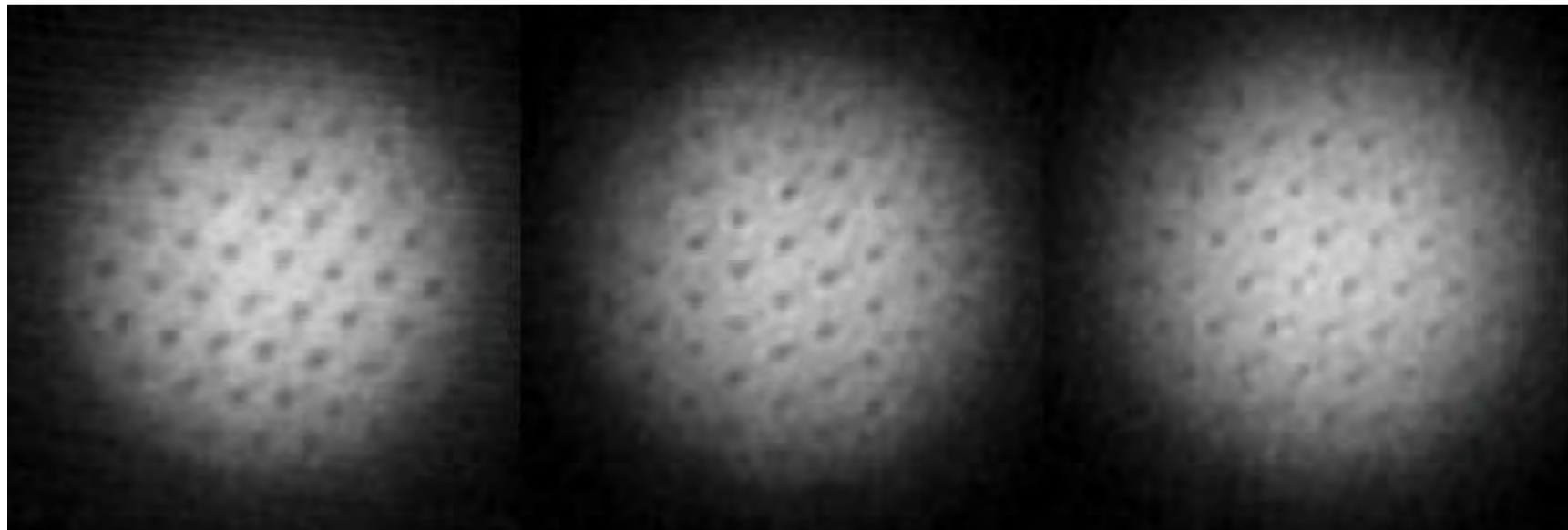
When $a \rightarrow \pm\infty$, T/T_C is *constant* during expansion



2 ms

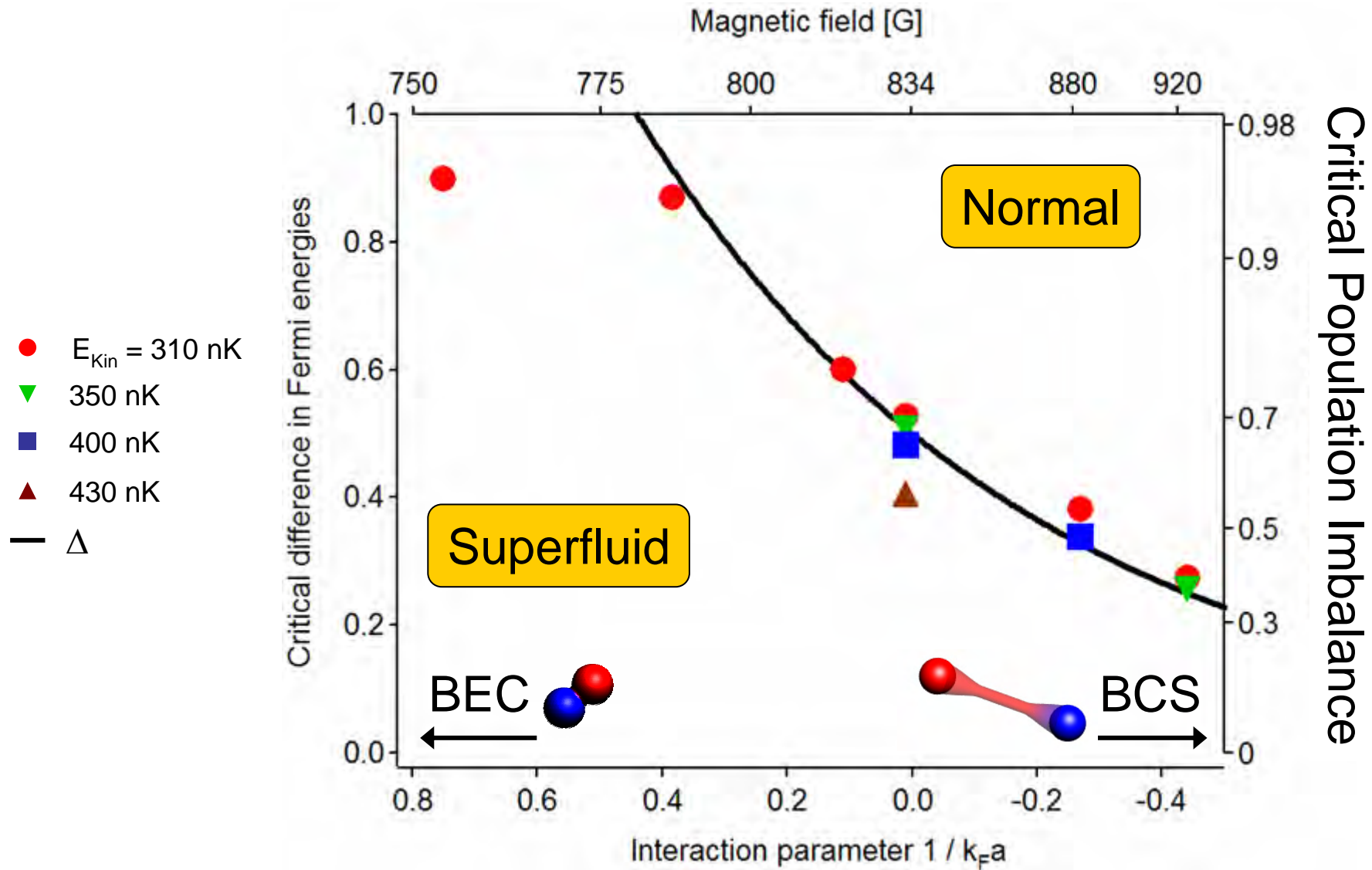
Ramp delay

6 ms



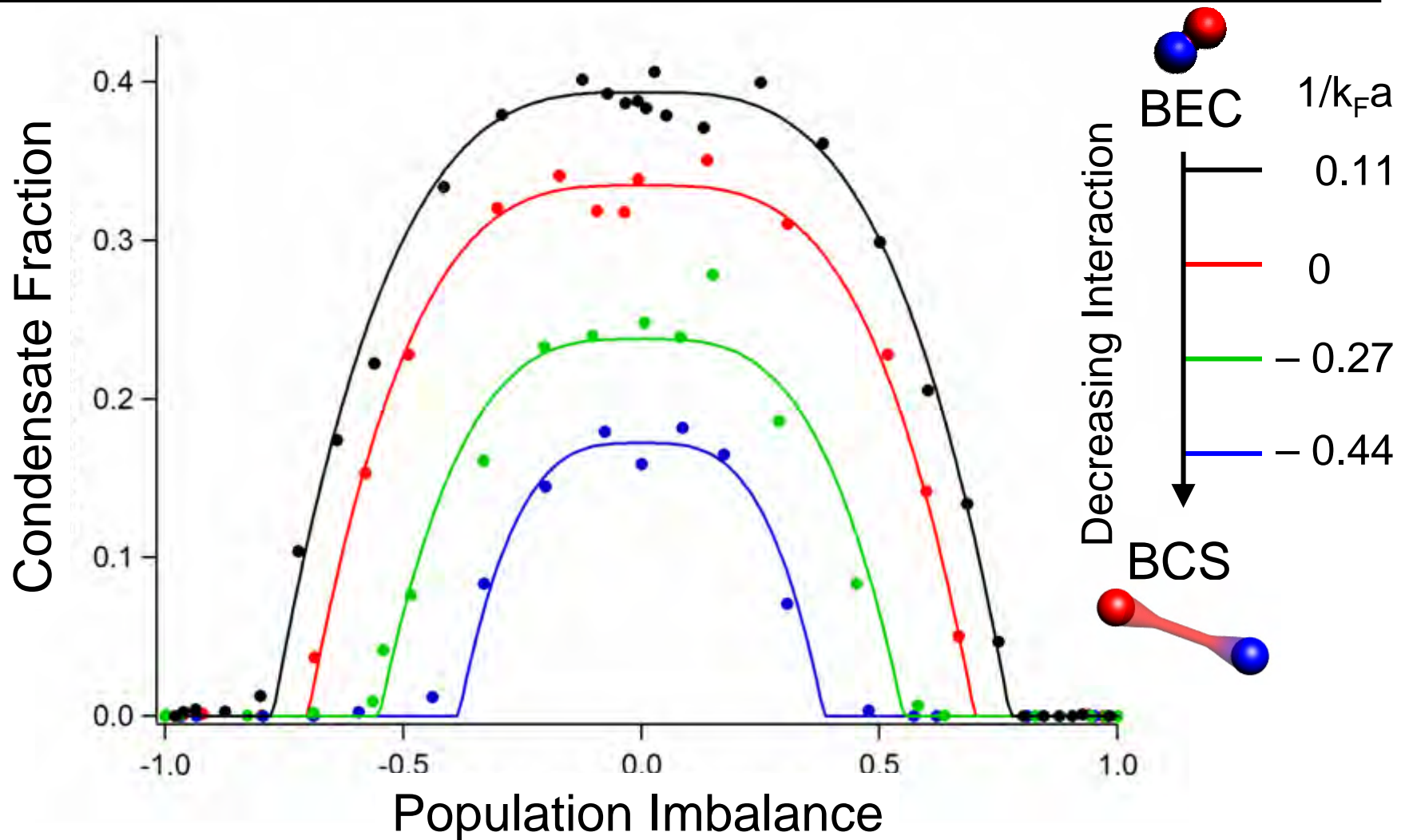
Vortices survive, but gradually fill in \rightarrow Vortex cores support more and more bound states for normal particles

Phase Diagram for Unequal Mixtures



Breakdown: Critical mismatch in Fermi energies $\Delta E_F \approx \text{Gap } \Delta$

The Window of Superfluidity



Superfluidity is robust in the strongly interacting regime!

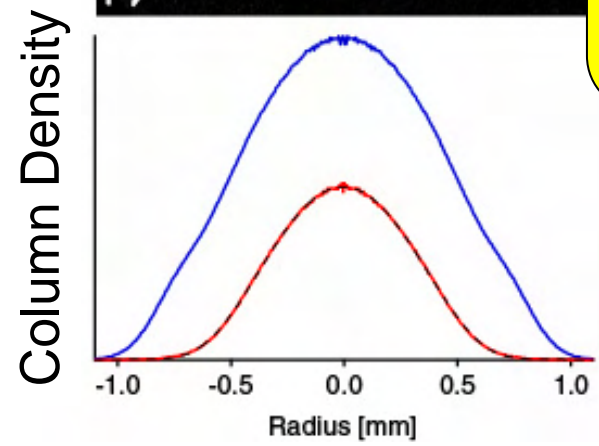
M.W. Zwierlein, A. Schirotzek, C.H. Schunck, W. Ketterle,
Science 311, 492 (2006)

HOT

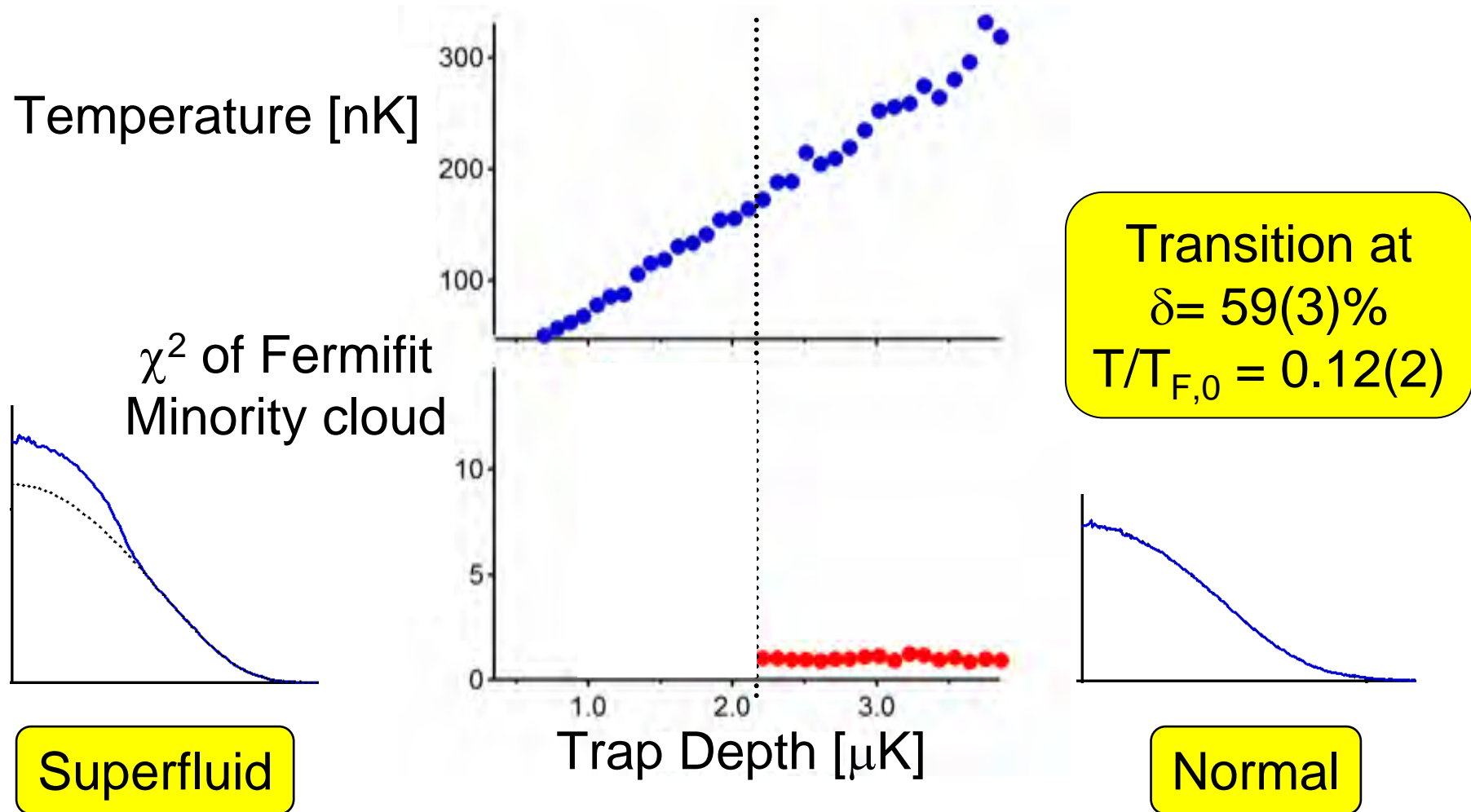
|1>

First direct obser

|2>



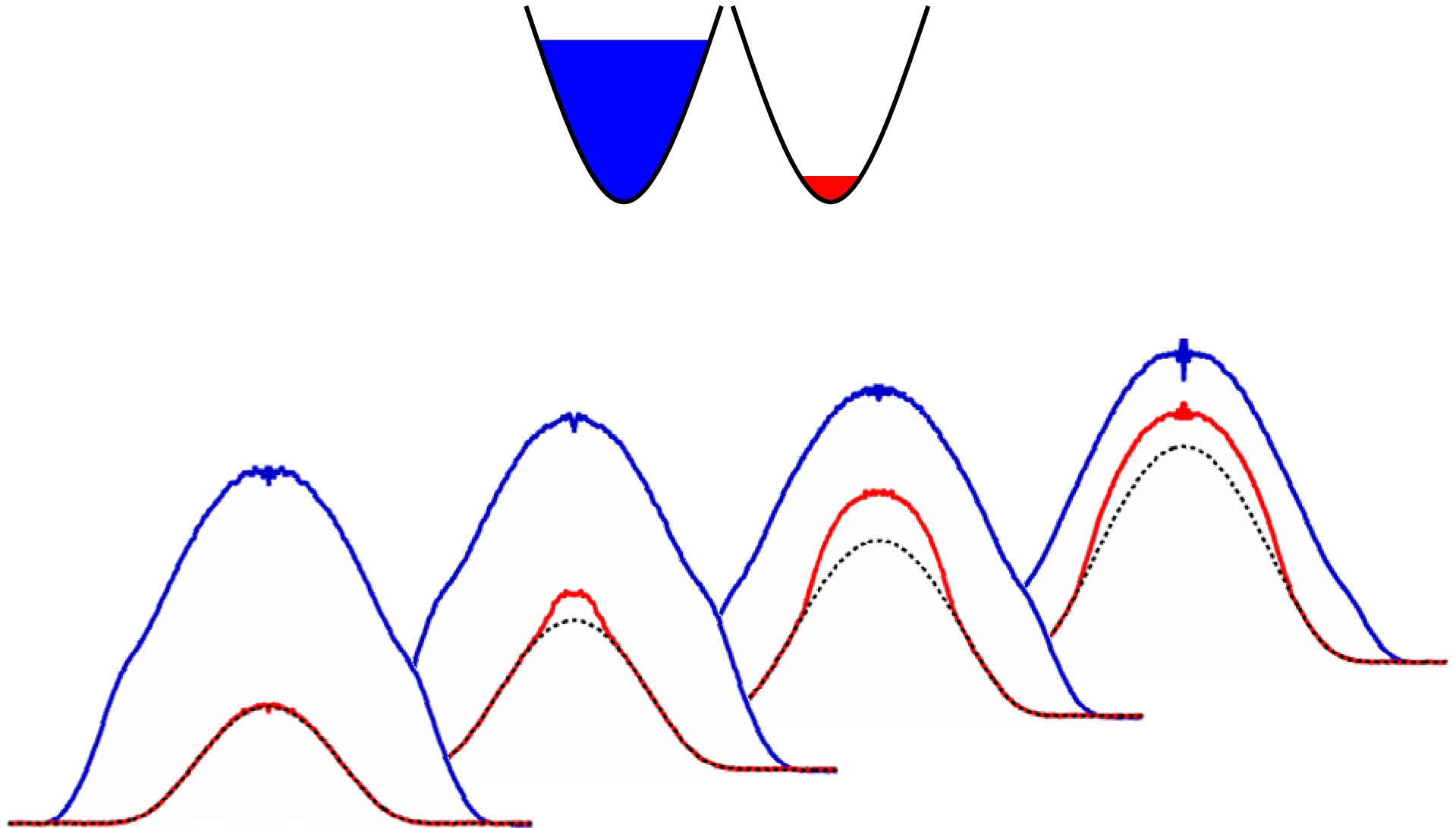
Direct Observation of the Phase Transition



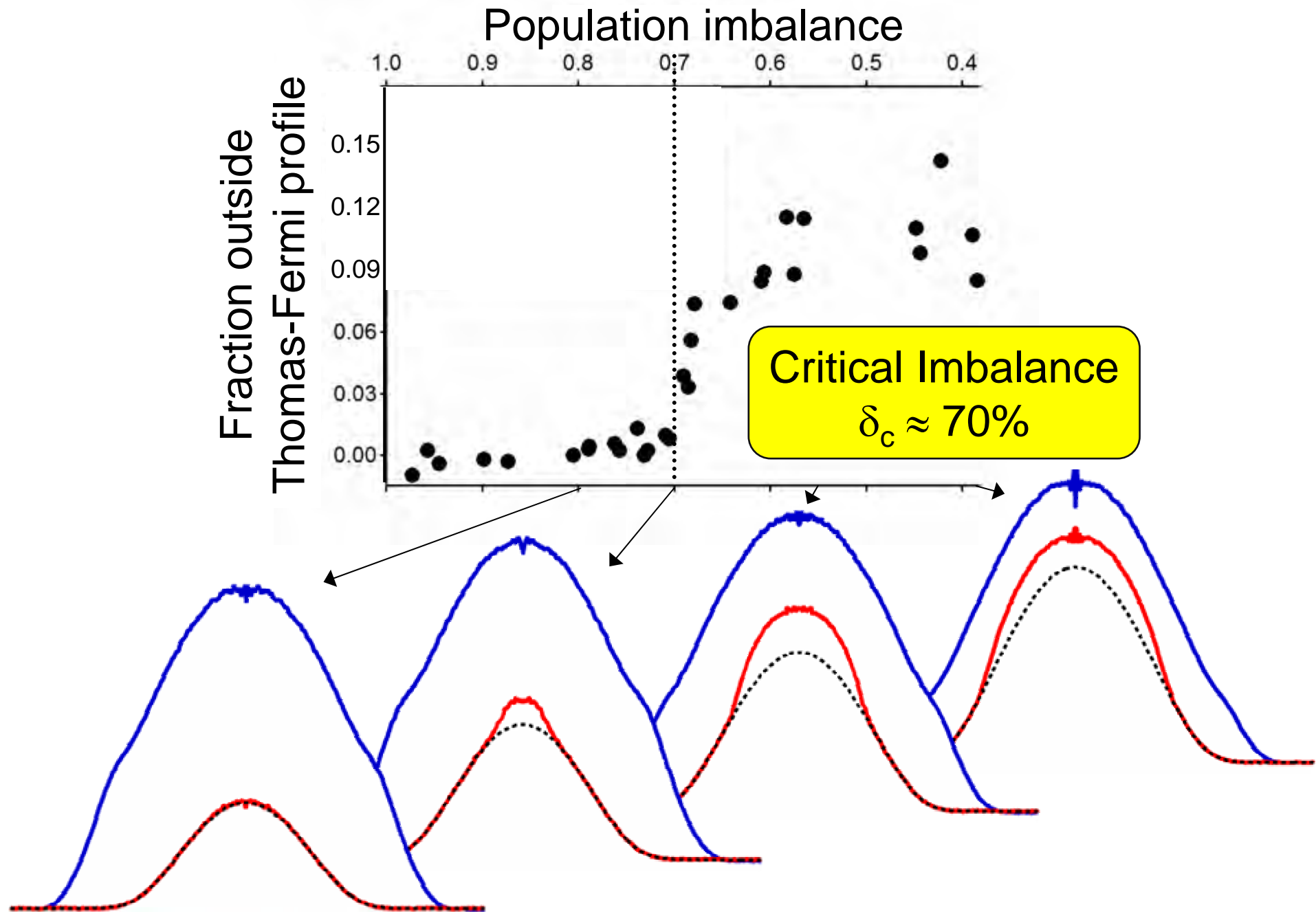
M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle,
Nature 442, 54 (2006)

A condensate emerges from the Fermi sea

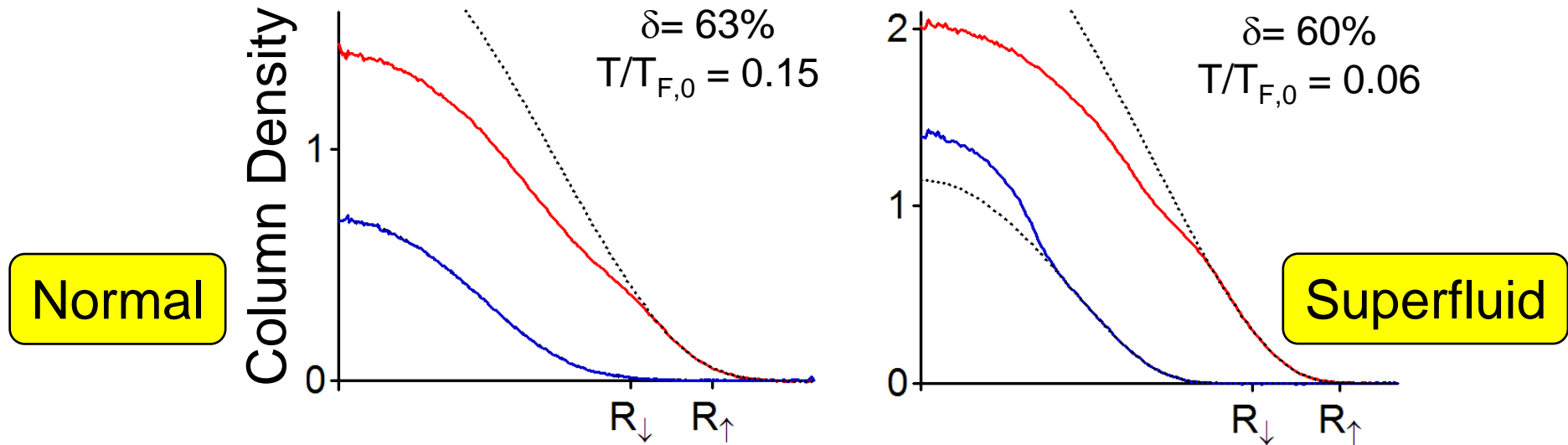
Increase atom number in smaller cloud



A condensate emerges from the Fermi sea

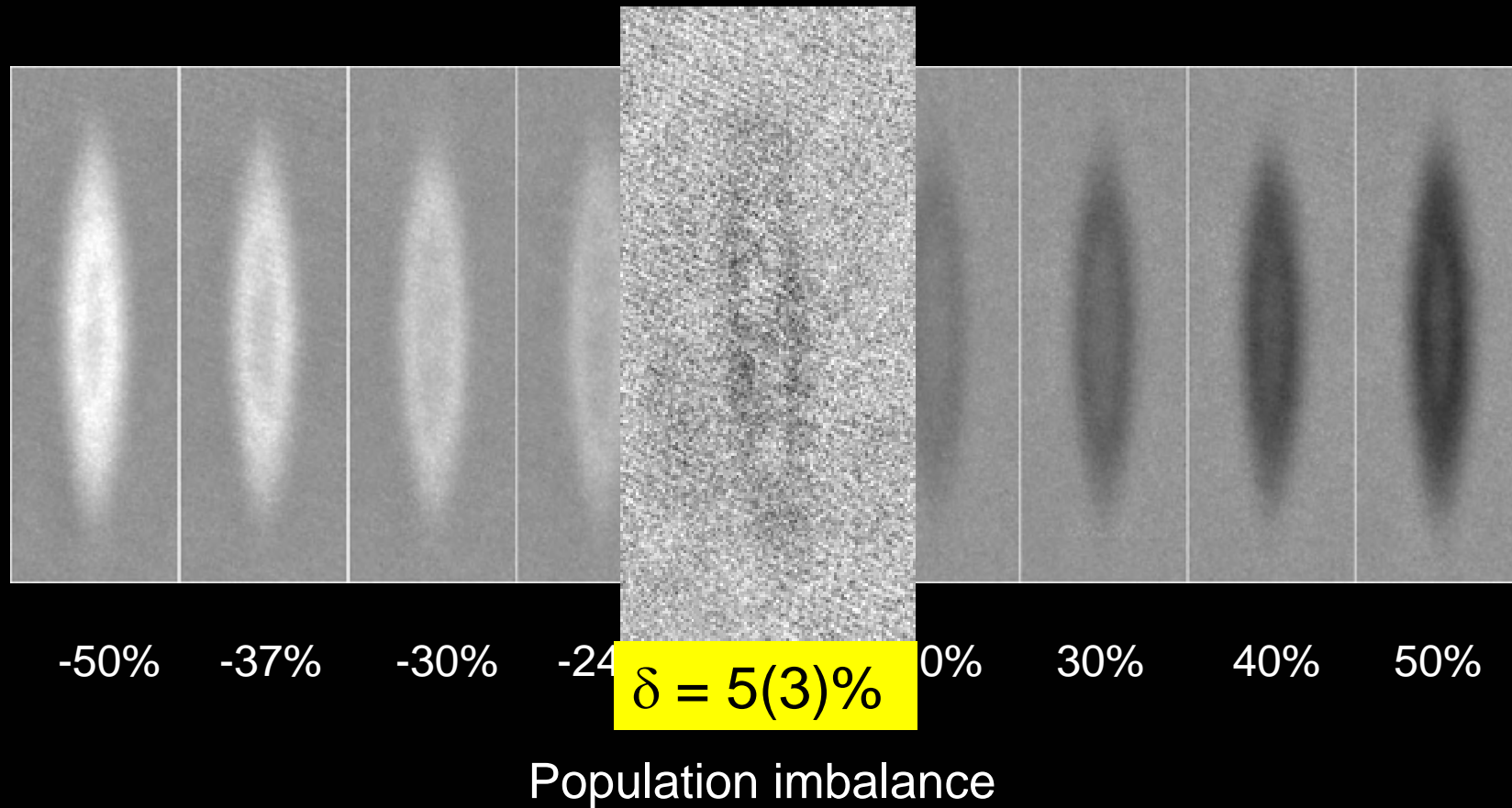


Density Profiles on Resonance: Unequal Mixtures



Depletion of excess fermions at the position of the condensate
→ Superfluid expells normal state; Phase separation?

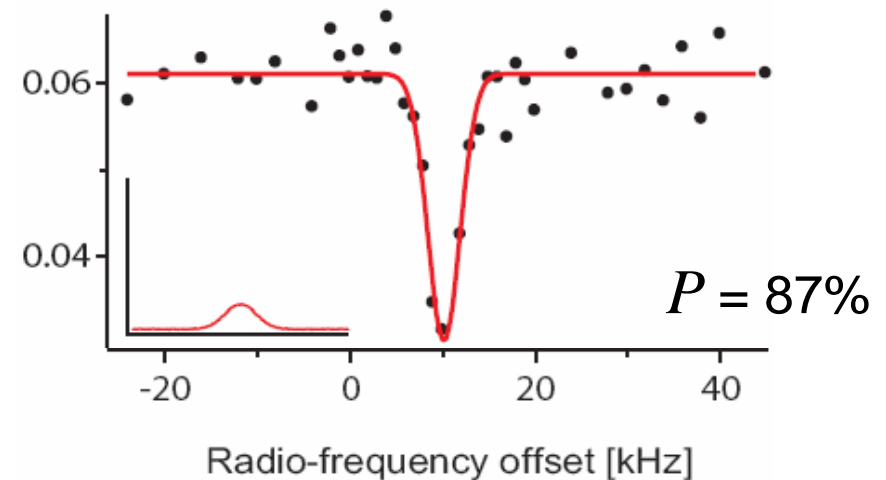
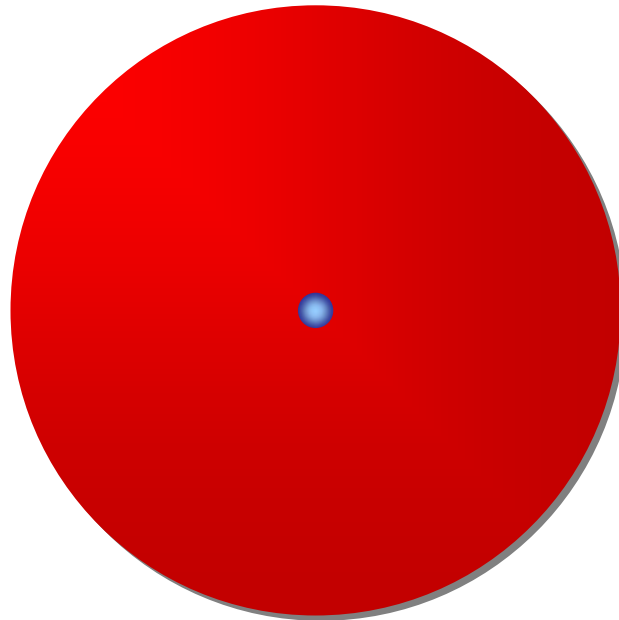
Shell Structure in an Imbalanced Fermi Gas



Y. Shin, M.W. Zwierlein, C.H. Schunck, A. Schirotzek, W. Ketterle,
PRL 97, 030401 (2006)

Limit of high imbalance

A single $|\downarrow\rangle$ atom immersed in a $|\uparrow\rangle$ cloud with unitarity limited interactions



$$\delta\nu_{\text{exp}} = -0.38$$

Binding energy must be universal

$$\mu_{\downarrow} = \gamma E_{F\uparrow}$$

$$\gamma = -0.6$$

F. Chevy PRA **74**, 063628 (2006), Variational Cooper pair Ansatz

C. Lobo, A. Recati, S. Giorgini, S. Stringari, PRL **97**, 200403 (2006), Monte-Carlo

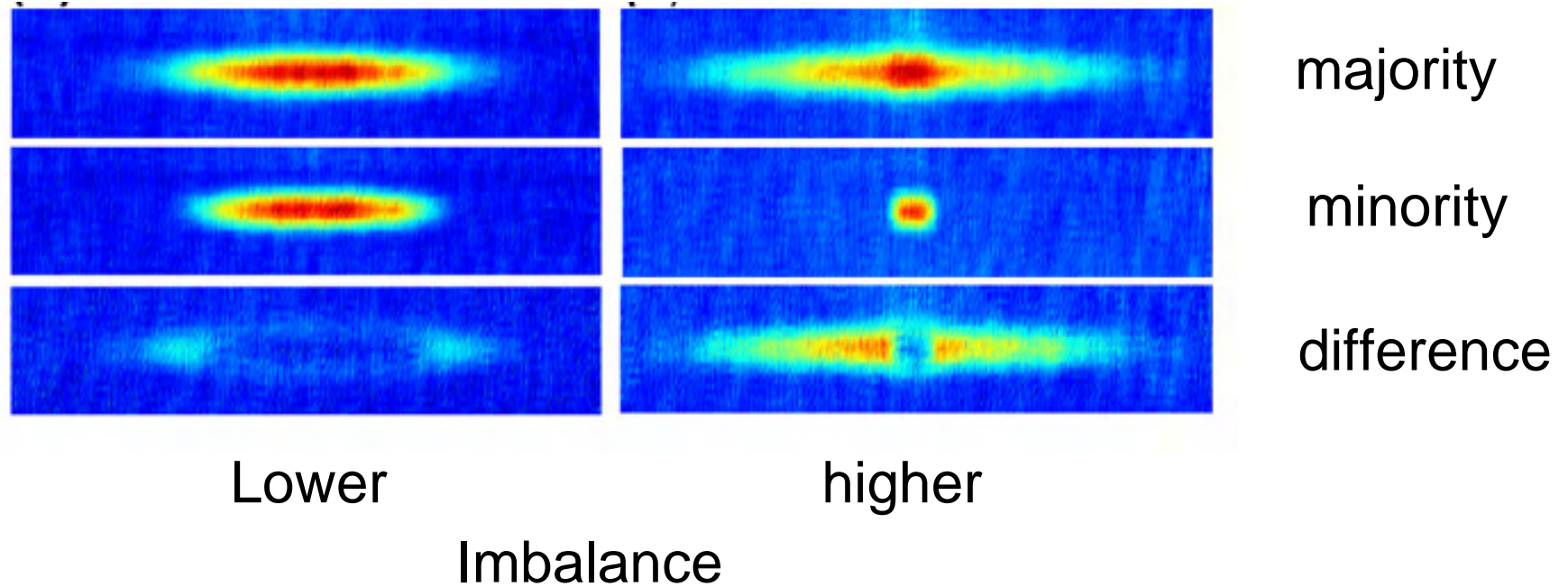
Deformation of a Trapped Fermi Gas with Unequal Spin Populations

G. B. Partridge, Wenhui Li, Y. A. Liao, and R. G. Hulet

Department of Physics and Astronomy and Rice Quantum Institute, Rice University, Houston, Texas 77251, USA

M. Haque and H. T. C. Stoof

Institute for Theoretical Physics, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands



Different system:

Very elongated trap (aspect ratio of 45 vs. 6)

Very small samples (200,000 vs. 10 million atoms)

Very strong finite size and surface energy effect

Strong deformation of the cloud

All conclusions are based on spatial deformations of the cloud

no observations of pair condensates

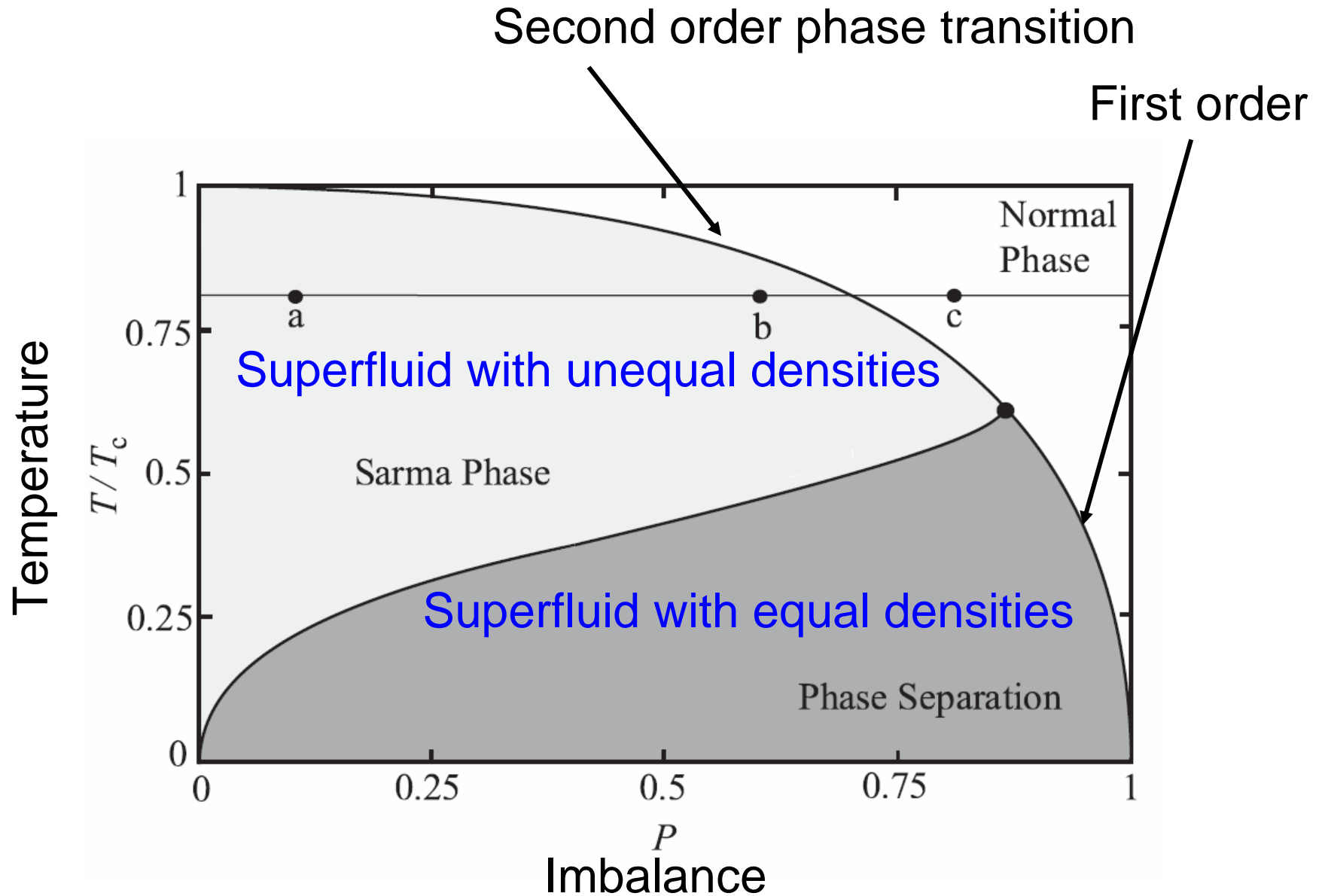
no observations of superfluidity (vortices)

no measurements in ballistic expansion

no observation of any phase transition vs. temperature

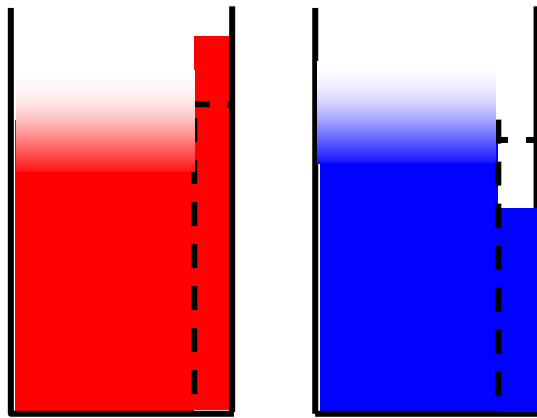
(data taken at only two different temperatures)

Phase separation

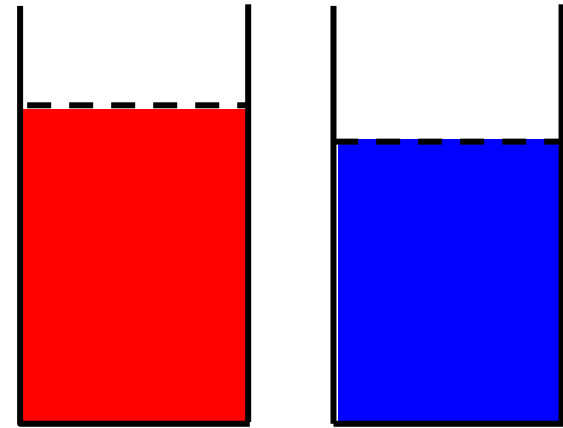


K.B. Gubbels, M.W.J. Romans, and H.T.C. Stoof, PRL 97, 210402 (2006)

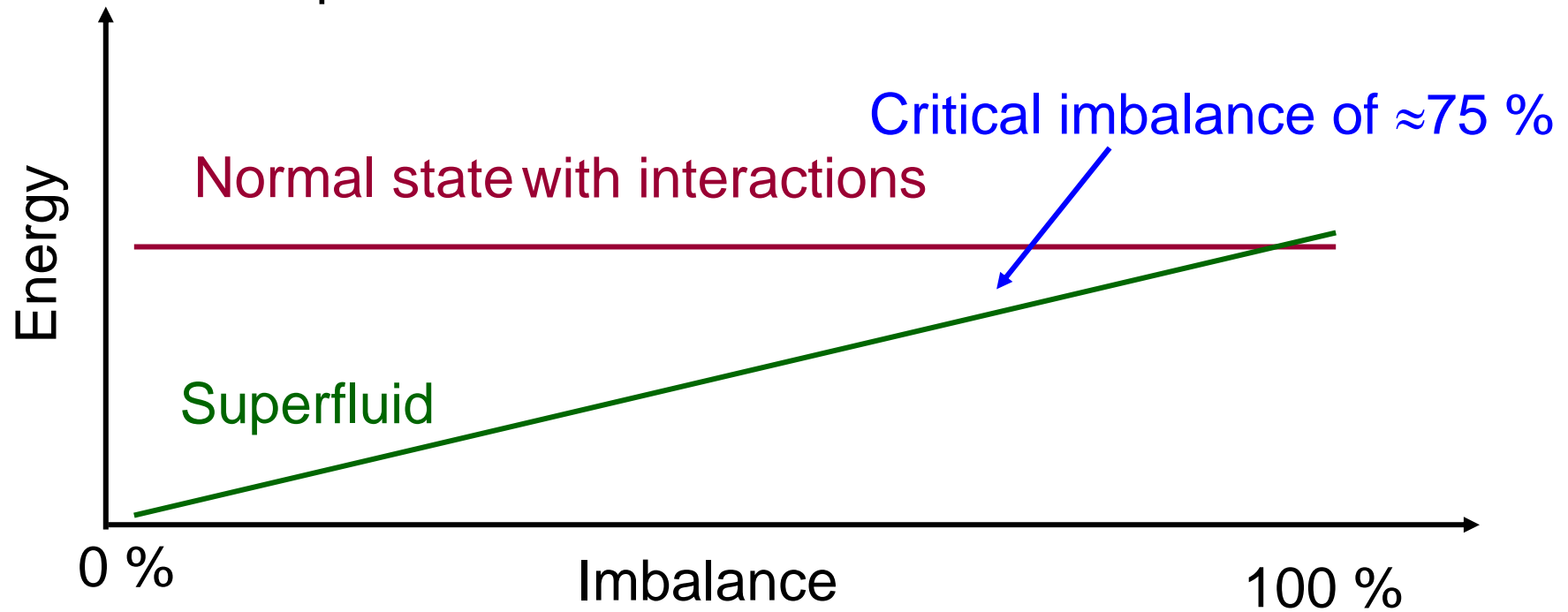
T=0



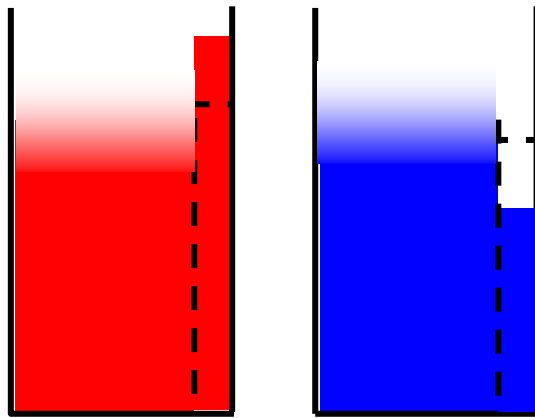
Superfluid state



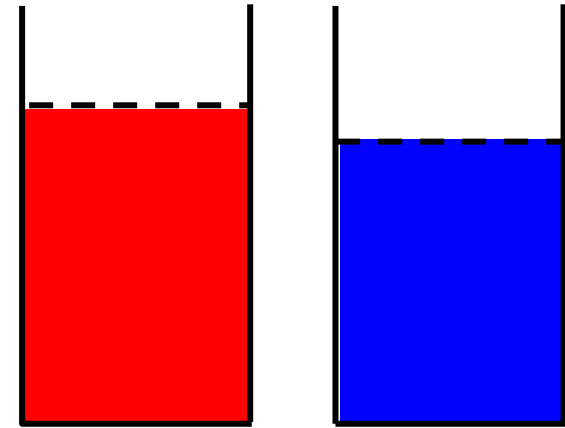
Normal state



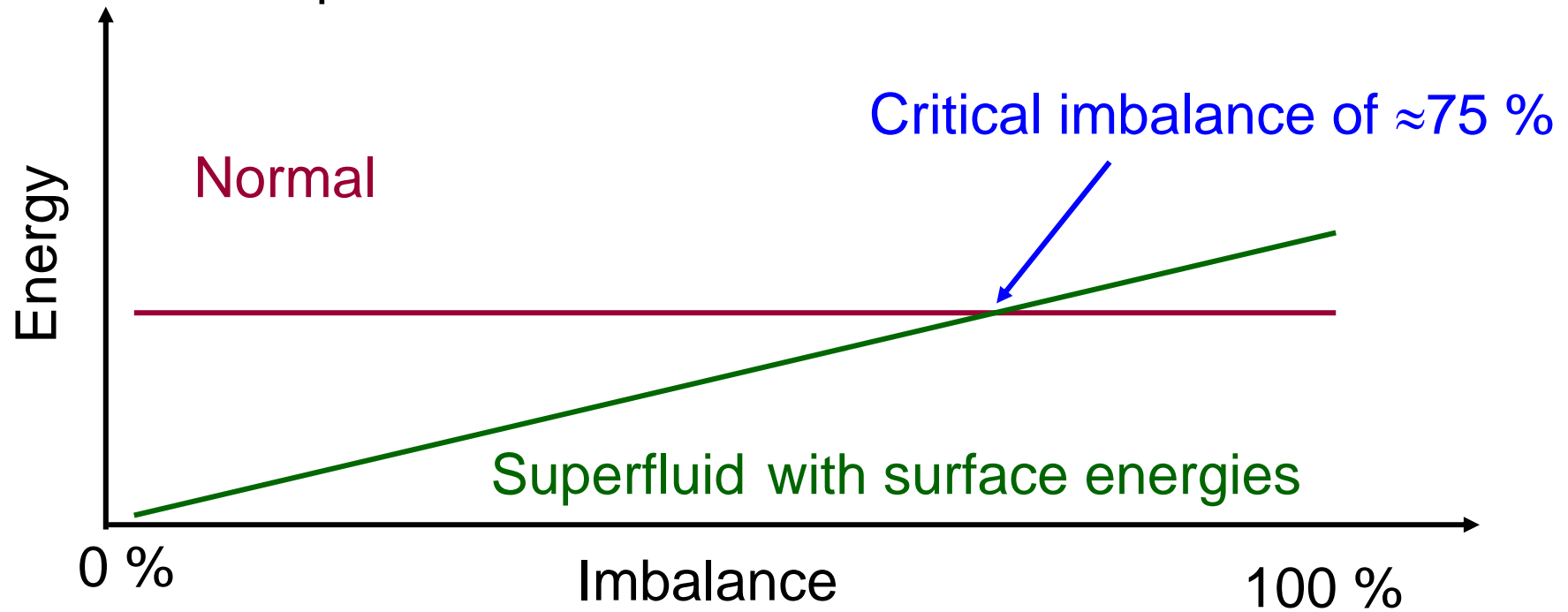
T=0

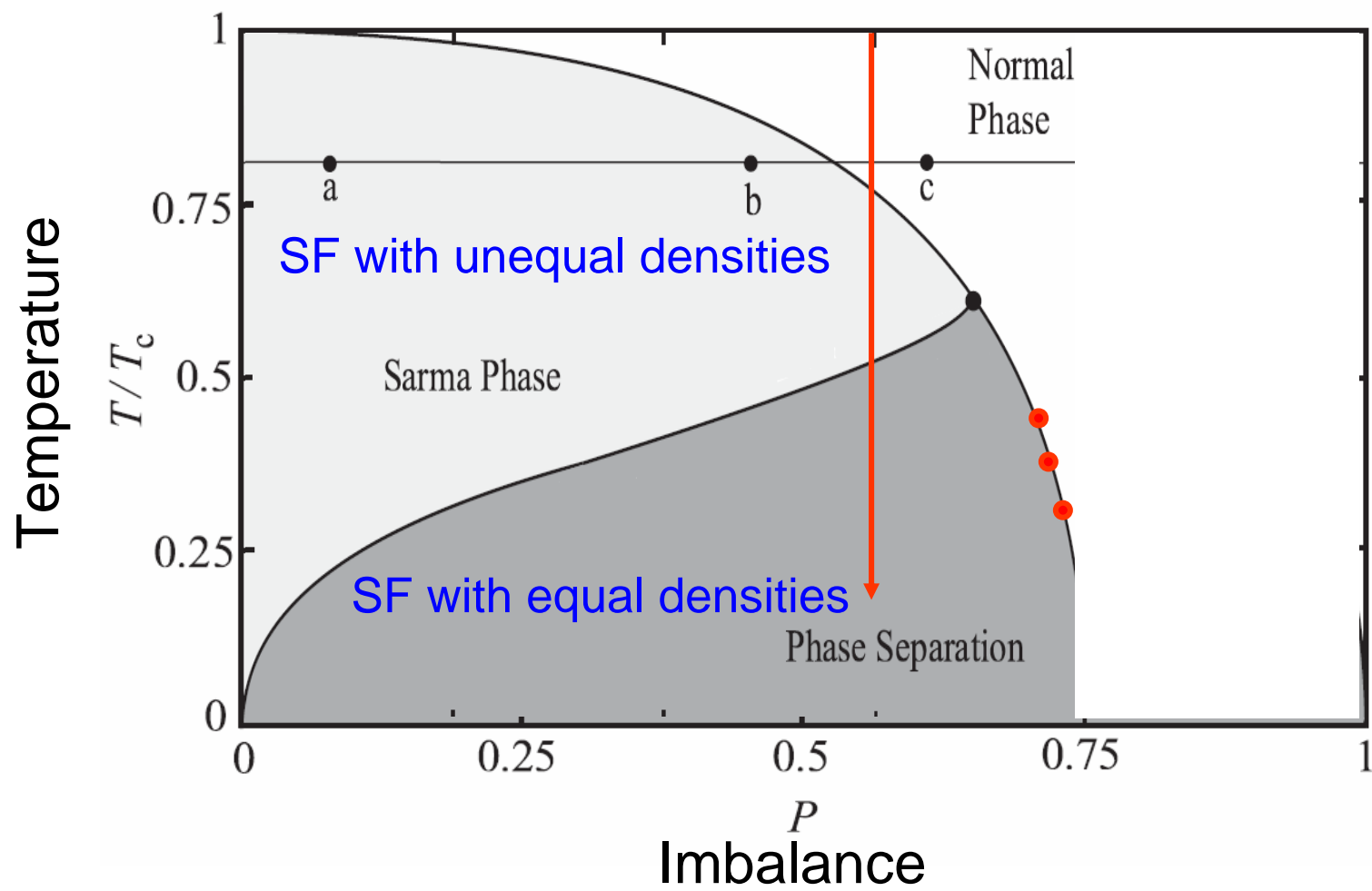


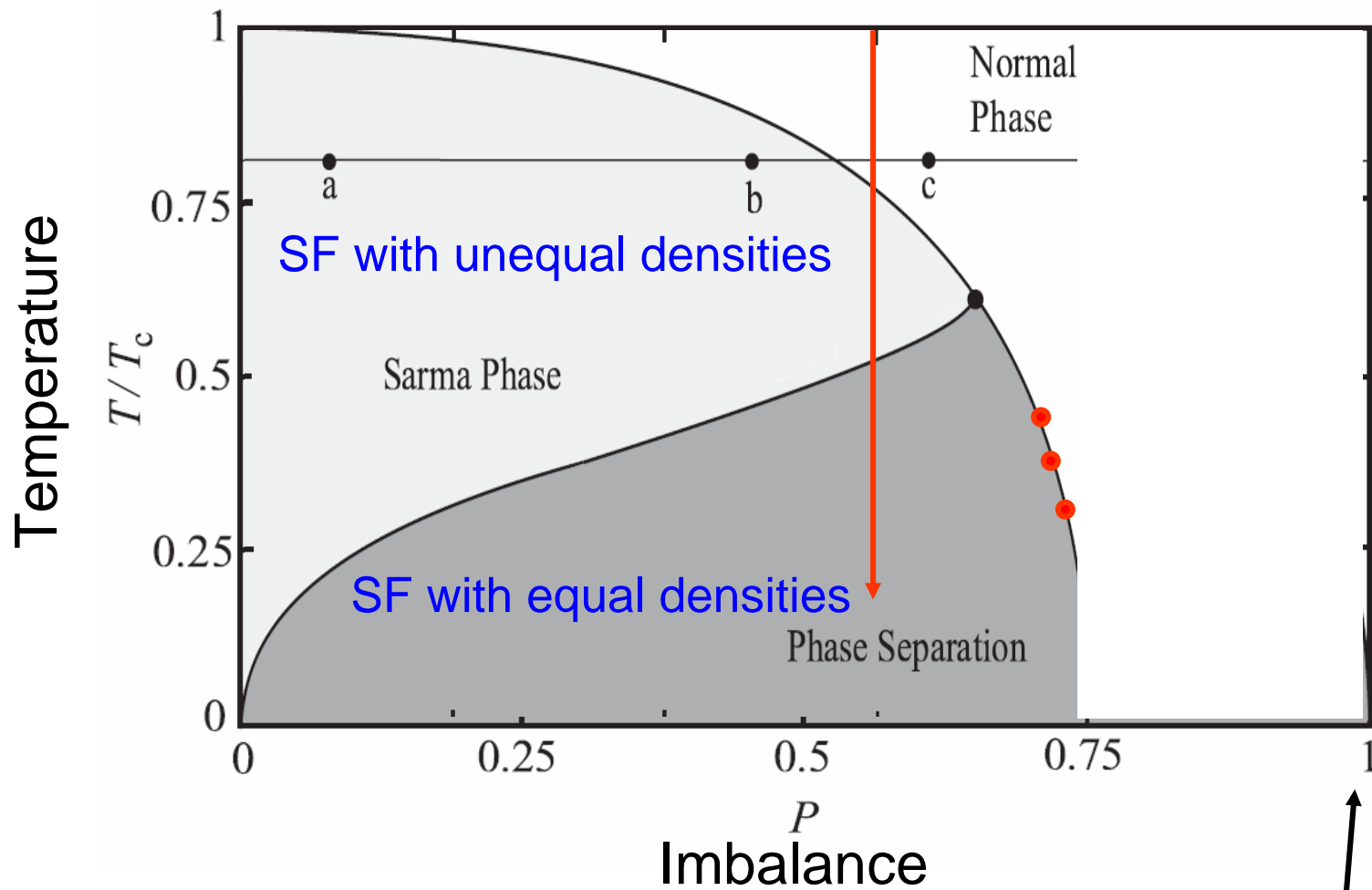
Superfluid state



Normal state





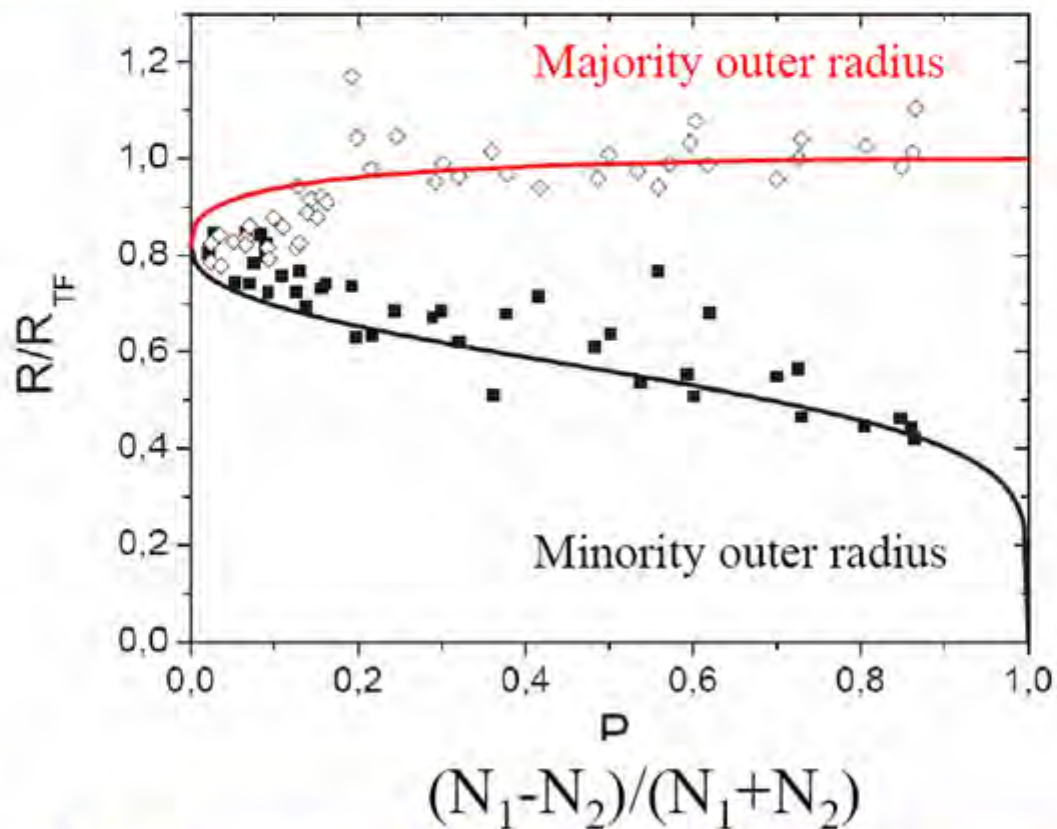


Rice: no Clogston limit
 Superfluidity at $> 90\%$ imbalance

Unresolved mystery: Rice expt

Partridge et al. Science **311**, 503 (2006)

Experiment by Rice's group: fully compatible with 2 phase scenario (no intermediate phase+LDA). No adjustable parameters.



Slide of the Paris group
(Chevy/Salomon)

For $P=0.7$, $q \sim 0.16$ differs from MIT and contradicts theoretical bounds: $q > 0.31$ set by $\eta_\alpha > -0.1$ and $\eta_\beta < 0.60$