Methods of the theory of one-dimensional quantum systems

Abstract

The purpose of the course is to acquaint students with methods for obtaining exact results in the theory of one-dimensional quantum systems and some of there applications.

1. $O(2)$-model and Berezinskii–Kosterlitz–Thouless transition.

   Vortices in the $O(2)$-models and the Coulomb gas. Expression in terms of a non-local field and equivalence to the sine-Gordon model. Plasma — gas transition. The scaling dimension of the perturbation operator and the exact value of the transition point.

2. Bosonization of the Thirring model.

   Representation of fermions in terms of bosonic fields (bosonization). Cancellation of divergent parts in the Lagrangian and the exact relation between the coupling constants.

3. Renormalization group for the Berezinskii–Kosterlitz–Thouless transition

   Renormalization of coupling constants in the sine-Gordon model in the second order of the perturbation theory. Renormalization group and renormalization group flows.

4. $O(3)$-model: mass generation by instantons.

   Topological properties of the $O(3)$-model, topologically nontrivial solutions in the Euclidean plane. Qualitative description of the mass generation by instantons.

5. $O(N)$-model: $1/N$-expansion.

   $1/N$ perturbation theory for the $O(N)$ -models. Mass generation. Kinematic scattering conditions and calculation of the S-matrix via the perturbation theory.

6. $O(N)$-model: integrability and exact S-matrix.

   Higher integrals of motion and factorization S-matrices. Young–Baxter equation. Calculation of the S-matrices of the $O(N)$-models from factorization conditions and comparison with the perturbative result.

7. Thirring model: Bethe Ansatz solution.

   Pseudovacuum and wave functions of the Thirring model in Bethe Ansatz. Bethe equations and their thermodynamic limit. Renormalization of charge. Spectrum and S-model matrix.

8. Heisenberg spin chain and its scaling limit.

   XYZ-model. Jordan–Wigner transformation and XY-model. Scaling limit and the relation to the Thirring/Sine-Gordon model.

9. Young–Baxter equation and Bethe Ansatz.

   XXZ-model and six-vertex model. Young equation–Baxter and commuting transfer-matrices. Pseudovacuum and eigenstates in the algebraic Bethe Ansatz. Bethe equations and their solution in the thermodynamic limit. Calculation of the free energy of the six-vertex model.

10. Kondo problem: derivation of the Bethe Ansatz.

    Kondo effect. Reducing to a one-dimensional problem. Primary and secondary Bethe Ansatz. The system of Bethe equations for the Kondo problem.

11. Kondo problem: solving Bethe equations.

    Ground state in the zero magnetic field. Speculation on the derivation of magnetization for a system in an external magnetic field. A brief discussion of the final temperature case.