Extreme outages caused by polarization mode dispersion

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We study the dependence on fiber birefringence of the bit-error rate (BER) caused by amplifier noise in a linear optical fiber telecommunication system. We show that the probability-distribution function of the BER obtained by averaging over many realizations of birefringent disorder has an extended tail that corresponds to anomalously large values of BER. We specifically discuss the dependence of the tail on such details of pulse detection at the fiber output as setting the clock and filtering procedures.

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Transmission errors in modern optical telecommunication systems have various causes. In systems with a transmission rate of 40 Gbits/s or higher, polarization mode dispersion (PMD) is a major problem. PMD leads to splitting and broadening of an initially compact pulse.[1–4] The effect is usually characterized by the so-called PMD vector, which determines the leading PMD-related pulse distortion.[5–8] It is also recognized that the polarization vector does not provide a complete description of the PMD phenomenon, and some proposals to account for higher-order PMD effects were recently discussed.[9–12] Birefringent disorder does not vary, at least on the time scales related to optical signal propagation. Optical noise originating from amplified spontaneous emission is different: It is correlated on a short time scale of the signal width. In this Letter we discuss the joint effect of amplifier noise and birefringent disorder on the bit-error rate (BER). Our main goal is to estimate the probability of special rare configurations of the fiber birefringence that produce anomalously large values of BER and thus to determine the reliability of information transmission. Evaluation of the signal BER that is due to amplifier noise for a given instance of birefringent disorder is the first step in our theoretical analysis. We also intend to study the probability-density function (PDF; normalized histogram) of the BER, in which the statistics are collected for various fibers or, equivalently, for the states of a given fiber at different times, and focus on the probability of an anomalously large BER. We analyze the basic (no compensation) situation and compare it with the simplest compensation scheme, known as setting the clock.

The envelope of the optical field propagating in a given channel in the linear regime (i.e., at relatively low optical power), which is subject to PMD distortion and amplifier noise, satisfies the following equation[3–5]:

$$\partial_z \mathbf{\Psi} - i\hat{\Delta}(z)\mathbf{\Psi} - \hat{m}(z)\partial_t \mathbf{\Psi} - id(z)\partial_t{}^2\mathbf{\Psi} = \xi(z,t)\,. \tag{1}$$

Here z, t, ξ, and d are the position along the fiber, the retarded time, the amplifier noise, and the chromatic dispersion, respectively. Envelope $\mathbf{\Psi}$ is a two-component complex field; the two components represent two states of the optical signal polarization. The birefringent disorder is characterized by two random 2×2 traceless matrix fields related to the zero ($\hat{\Delta}$) and first ($\hat{m}$) orders in the frequency expansion with respect to the deviation from carrier frequency ω_0. Birefringence that affects the polarization of light is practically frozen (t-independence) on all propagation-related time scales. Matrix $\hat{\Delta}$ can be excluded from consideration by the transformations $\mathbf{\Psi} \to \hat{V}\mathbf{\Psi}$, $\xi \to \hat{V}\xi$, and $\hat{m} \to \hat{V}\hat{m}\hat{V}^{-1}$. Here, unitary matrix $\hat{V}(z) = T\exp[i\int_0^z \mathrm{d}z'\hat{\Delta}(z')]$ is the ordered exponential, defined as the formal solution of the equation $\partial_z \hat{V} = i\hat{\Delta}\hat{V}$ with $\hat{V}(0) = \hat{1}$. Hereafter, we always use renormalized quantities. We further represent the solution of Eq. (1) as $\mathbf{\Psi} = \varphi + \phi$, where

$$\varphi = \hat{W}(z)\mathbf{\Psi}_0(t),$$

$$\phi = \int_0^z \mathrm{d}z'\hat{W}(z)\hat{W}^{-1}(z')\xi(z',t)\,, \tag{2}$$

$$\hat{W}(z) = \exp\left[i\int_0^z \mathrm{d}z' d(z')\partial_t{}^2\right]T \times \exp\left[\int_0^z \mathrm{d}z'\hat{m}(z')\partial_t\right], \tag{3}$$

and $\mathbf{\Psi}_0(t)$ stands for the initial pulse shape.

We consider a situation in which the pulse's propagation distance substantially exceeds the interamplifier separation. Our approach allows us to treat discrete (erbium) and distributed (Raman) amplification schemes within the same framework. Additive noise ξ generated by optical amplifiers is zero on average. The statistics of ξ are Gaussian, with the spectral properties determined solely by the steady-state features of the amplifiers (gain and noise figure).[16] The noise correlation time is much shorter than the pulse's temporal width, and therefore ξ can be treated as δ correlated in time. Equations (2) and (3) imply that the contribution of noise to output signal ϕ is a zero-mean Gaussian field characterized by the following pair correlation function: $\langle \phi_\alpha(Z, t_1)\phi_\beta^*(Z, t_2)\rangle = D_\xi Z \delta_{\alpha\beta}\delta(t_1 - t_2)$. Therefore it is statistically independent of both $d(z)$ and $\hat{m}(z)$. Here Z is the total system length, where the product $D_\xi Z$ is the amplified spontaneous emission spectral density of the line. Coefficient D_ξ is introduced to reveal the linear growth of the amplified spontaneous emission factor with Z.[16] The matrix of birefringence $\hat{m}$ can be parameterized by a three-component real field h_j, $\hat{m} = \sum h_j \hat{\sigma}_j$, where $\hat{\sigma}_j$ is a set of three Pauli matrices. Field $\mathbf{h}$ is zero on average, and its correlation scale in z is short. Transformation $\hat{m} \to \hat{V}\hat{m}\hat{V}^{-1}$ guarantees that the statistics of h_j will be isotropic. As $\mathbf{h}$ enters the observables described by Eqs. (2) and (3) in an integral form, the central-limit theorem implies that field h_j can be treated as a Gaussian field with $\langle h_i(z_1)h_j(z_2)\rangle = D_m \delta_{ij}\delta(z_1 - z_2)$, where averaging goes over the instances of birefringent disorder. For weak birefringent disorder the integral $\mathbf{H} = \int_0^z \mathrm{d}z\mathbf{h}(z)$ represents the PMD vector. Thus $D_m = k^2/12$, where k is the so-called PMD coefficient.

We consider the return-to-zero modulation format when the pulses are well separated in time. The signal detection at the line output, $z = Z$, corresponds to measuring output pulse intensity I:

$$I = \int \mathrm{d}tG(t)|\mathcal{K}\varphi(Z,t) + \mathcal{K}\phi(Z,t)|^2, \qquad (4)$$

where $G(t)$ is a convolution of the electrical (current) filter function with the sampling window function. Linear operator $\mathcal{K}$ in Eq. (4) stands for an optical filter and a variety of engineering tricks applied to the output signal, $\mathbf{\Psi}(Z, t)$. Of a variety of such tricks, we discuss here only those that are related to optical filtering and to setting-the-clock compensation, the latter of which can be formalized as $\mathcal{K}_{\mathrm{cl}}\mathbf{\Psi} = \mathbf{\Psi}(t - t_{\mathrm{cl}})$, where t_{cl} is the optimal time delay. Ideally, I takes two distinct values that correspond to the 0 and 1. However, noise and disorder force deviation of I from ideal values. We detect the output signal by introducing a decision level I_d and declaring that the signal code is 1 if $I > I_d$ and is 0 otherwise. Sometimes information is lost, i.e., an initial 1 is detected as 0 at the output or vice versa. The BER is the rate of such events that is extracted from measurement of many pulses coming through a fiber with a given instance of the birefringent disorder, $h_j(z)$. For successful system performance the BER should be extremely small; i.e., typically both impairments can cause only a small distortion of a pulse or, stated differently, the optical signal-to-noise ratio (OSNR) and the ratio of the squared pulse width to the mean-square value of the PMD vector are both large. The OSNR can be estimated as $I_0/(D_\xi Z)$, where I_0 is the initial pulse intensity, $I_0 = \int \mathrm{d}t|\Psi_0(t)|^2$, and the integration goes over a single slot populated by an ideal (initial) pulse, encoding 1.

Based on Eq. (4), one can conclude that input 0 is converted into output 1 primarily as a result of noise ϕ and therefore that the probability of such an event is insensitive to the birefringent disorder. Therefore, anomalously large values of BER originate solely from the $1 \to 0$ transitions (note that corrections to the BER associated with interference with a pulse from neighboring slots are not considered here, as the corrections are small). Let B be the probability of such an event. Because the OSNR is large, B can be estimated as the probability of an optimal fluctuation ϕ leading to $I = I_d$. Then one concludes that the product $D_\xi Z \ln B$ depends on the disorder, the chromatic dispersion coefficient, and the method of measurement, whereas it is insensitive to the noise characteristics. Because the OSNR is large, even weakly birefringent disorder could produce a large increase in the value of B. This fact permits a perturbative evaluation of the dependence of B on disorder, starting with an expansion of the ordered exponential [Eq. (3)] in powers of $\mathbf{h}$. If no compensation is applied, the linear term prevails. It is convenient to introduce a dimensionless coefficient μ_1 in accordance with $(D_\xi Z/I_0)\ln(B/B_0) = \mu_1 H_3/b + O(H^2)$, where the initial pulse Ψ_0 is assumed to be linearly polarized and $\Psi_0 \propto (1, 0)$, where b and B_0 are the signal width and a typical value of B that corresponds to $h_j = 0$, respectively; $\ln B_0 \sim -I_0(D_\xi Z)$; and H_i $(i = 1, 2, 3)$ denote the components of vector $\mathbf{H}$. Setting-the-clock compensation cancels out the linear H contribution, i.e., $\mu_1 = 0$ if t_{cl} is chosen to be equal to H_3. In this case and also when the output signal is not chirped one gets $(D_\xi Z/I_0)\ln(B/B_0) = \mu_2(H_1{}^2 + H_2{}^2)/b^2 + O(H^3)$, where μ_2 is another dimensionless coefficient.

Aiming to analyze the dependence of the parameters $\Gamma_0 \equiv -(D_\xi Z/I_0)\ln B_0$, μ_1, and μ_2 on the measurement procedure, we present here the results of our calculations for a simple model. We assume a Lorentzian shape for the optical filter: $\mathcal{K}_f\mathbf{\Psi} = \int_0^\infty \mathrm{d}t' \exp(-t'/\tau)\mathbf{\Psi}(t - t')/\tau$, where τ is the optical filter's width. Thus the PDF, $\mathcal{P}(\mathcal{K}\phi)$, of the inhomogeneous contribution satisfies

$$\ln \mathcal{P} \approx -\frac{1}{D_\xi Z}\int \mathrm{d}t(|\mathcal{K}\phi|^2 + \tau^2|\partial_t\mathcal{K}\phi|^2). \qquad (5)$$

The large value of the OSNR justifies the saddle-point approximation for calculating B. The saddle-point equation, found by varying Eq. (5) with respect to ϕ, reads as

$$[\tau^2\partial_t{}^2 - 1 - uG(t)]\mathcal{K}\phi = uG(t)\mathcal{K}\varphi\,, \qquad (6)$$

where u is a parameter to be extracted from self-consistency condition (4). B can be estimated by $\mathcal{P}(\phi_0)$, with ϕ_0 being the solution of Eqs. (4) and (6) for $I = I_d$. We

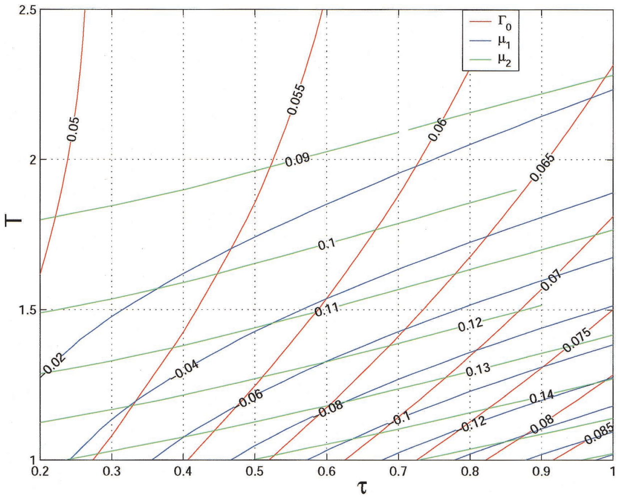

Fig. 1. Dependence of Γ_0 and of $\mu_{1,2}$ on T/b and τ/b, i.e., the information slot width and the optical filter width measured in the units set by rescaling of the pulse width to unity.

next assume that $G(t) = 1$ at $|t| < T$ and $G(t) = 0$ otherwise, where T is the information slot width, $T > b$. Then, for a given value of u, the solution of Eq. (6) can be found explicitly. The value of parameter u is fixed implicitly by Eq. (4). Thus u (and then B) can be found perturbatively in h_j, i.e., as $u \approx u_0 + \delta u$ and $\delta u \ll u_0$, where u_0 is the solution of the system of Eqs. (4) and (6) at $h_j = 0$. For the Gaussian shape of the initial pulse, where $\Psi_0 \propto \exp[-t^2/(2b^2)]$ and I_d is half of the ideal output intensity [corresponding to $\Psi(Z) = \Psi_0$], the dependence of Γ_0 and of $\mu_{1,2}$ on τ/b and T/b, found numerically, is shown in Fig. 1. We can find the PDF of $S(B)$ of B by recalculating the statistics of H_j and substituting the result into the corresponding expression that relates B to H_j. Our prime interest is in describing the PDF tail that corresponds to H_j that substantially exceeds its typical value $\sqrt{D_m Z}$ but remains, however, much smaller than signal width b. In this range one gets the following estimate of differential probability $S(B)\mathrm{d}B$:

$$\exp\left[-\frac{D_\xi{}^2 Z b^2}{2D_m \mu_1{}^2 I_0{}^2}\ln^2\left(\frac{B}{B_0}\right)\right]\frac{\mathrm{d}B}{B}, \tag{7a}$$

$$\frac{B_0{}^\alpha \mathrm{d}B}{B^{1+\alpha}}, \tag{7b}$$

where expression (7a) marks the no-compensation case, expression (7b) stands for the optimal setting-the-clock case, and $\alpha \equiv D_\xi b^2/(2\mu_2 D_m I_0)$. Note that the result that corresponds to expression (7b) shows a steeper decay than that for expression (7a), which is a natural consequence of the compensation procedure that has been applied.

Summarizing, our major result [expressions (7)] shows the emergence of the extremely long tail in the PDF of the BER that is a result of a complex interplay between noise and disorder. To illustrate this focal point of our analysis we consider a fiber line with parameters $\Gamma_0 = 0.06$, $\mu_1 = 0.06$, and $\mu_2 = 0.12$, which is also characterized by typical bit-error probability $B_0 = 10^{-12}$, which corresponds to $I_0/(D_\xi Z) \approx 460$. Let us also assume that the PMD coefficient, $k = \sqrt{12D_m}$, is $0.14\ \mathrm{ps}/\sqrt{\mathrm{km}}$; that the pulse width is $b = 25$ ps; and that the fiber length is $Z = 2500$ km, i.e., that $D_m Z/b^2 \approx 6 \times 10^{-3}$. Then we can find a probability for B to exceed, say, 10^{-8}, that is, to become at least four orders larger than B_0. Our results [expressions (7a) and (7b)] give for this probability 10^{-4} and 10^{-6}, respectively, which essentially exceed any naïve Gaussian or exponential estimate of the PDF tail. Also note that some numerical results that are consistent with expressions (7) are already available: Fig. 2(a) of Ref. 17 replotted in log–log variables shows that the relation between $\ln S$ and $\ln B$ is close to the linear relation given by expression (7b).

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