Comment on "Ginzburg-Landau Theory of the Phase Diagram of Superconducting UPt_3"

A model often discussed in connection with the H-T phase diagram of UPt_3 is based on the superconducting (SC) order parameter $\vec{\eta} = (\eta_x, \eta_y)$ in the 2D irreducible representation E_{1g} of the point group D_{6h}. A symmetry lowering ($D_{6h} \rightarrow D_{2h}$) due to a weak antiferromagnetic (AFM) order would lead to a double transition ($T_c \rightarrow T_{c+}, T_{c-}$) in zero magnetic field and an intersection of two branches of the upper critical field, H_+ and H_- which would yield a sharp kink in H_{c2}.

In his recent Letter Garg [1] has used this model to check the stability of this kink for the different field orientations. He concluded [statement (A) in [1]] that this kink can exist for both $\mathbf{H} \perp \mathbf{z}$ and $\mathbf{H} \parallel \mathbf{z}$. We show that his statement does not apply for $\mathbf{H} \parallel \mathbf{z}$. His argumentation is based on the classification of the SC states by special quantum numbers near H_{c2} [2]. The kink can exist only if the phases related to T_{c+} and T_{c-} have *different quantum numbers.* Otherwise it should be absent or at least smeared out. If $\mathbf{H}$ is directed along a 2D symmetry axis, the classification is reduced to two quantum numbers. (a) $\sigma = \pm 1$ parity with respect to reflection at the plane *perpendicular* to H. (b) $N = 0, 1$ is a "generalized Landau level" number [2] (it corresponds to $Q = 1, 0$ in the notations of Ref. [1]). This quantum number N has a "geometrical" origin. It corresponds to the symmetry of the Ginzburg-Landau (GL) functional with respect to magnetic translations and C_2 rotations about H [2]. Because of the AFM symmetry lowering, $D_{6h} \rightarrow D_2$, the C_2-axis classification applies for $\mathbf{H} \perp \mathbf{z}$ as well as for $\mathbf{H} \parallel \mathbf{z}$.

For $\mathbf{H} \perp \mathbf{z}$ the solutions of the linearized GL equations related to T_{c+} and T_{c-} have different σ so that a kink can exist. However, for $\mathbf{H} \parallel \mathbf{z}$, the branches $H_+(T), H_-(T)$ corresponding to η_y and η_x have the *same* quantum number $\sigma = +1$ and $N = 1$. Hence the two phases are combinations of even Landau-level wave functions [2]. The odd Landau-level eigenfunctions have smaller "critical fields," so that they do not correspond to H_{c2}, opposite to the suggestion in Ref. [1]. To show this we consider here two limiting cases. For $H << H^*$, near T_{c+} we can neglect the admixture of the η_x component. The exact solution of the appropriate one-component GL equation is $\eta_y \Phi_0(Ax, By)$ where $\Phi_0(x, y)$ is the Abrikosov lattice wave function based on the zeroth Landau level; $A, B = (1 \mp u)^{-1/2}$ for the AFM vector $\mathbf{n} \parallel \mathbf{y}$. $\Phi_0(Ax, By)$ is the combination of the even Landau-level wave functions $\Phi_{2n}(x, y)$. For $H >> H^*$ one can neglect the small T_c splitting. Depending on the parameters appearing in the GL functional, the solutions may be either $\alpha(\eta_x + i\eta_y)\Phi_0(x, y) + \beta(\eta_x - i\eta_y)\Phi_2(x, y)$ or $(\eta_x - i\eta_y)\Phi_0(x, y)$. Both are also formed from the even Landau levels. Hence, according to the argumentation based on the quantum numbers the kink should be smeared out for $\mathbf{H} \parallel \mathbf{z}$. From such symmetry argumentations we cannot exclude the possibility that a low lying "eigenfield" with different quantum number could cross $H_{c2}(T)$ twice in the intermediate field region. This can only be decided by explicit calculation of these levels as recently presented by Chen and Garg [3]. The resulting occurrence of two crossing points (kinks), however, would not agree with the situation found in the phase diagram of UPt_3.

In conclusion, the smearing of the kink (or the absence of a tetracritical point) for $\mathbf{H} \parallel \mathbf{z}$ is unavoidable in most theories based on the splitting of T_c due to a symmetry lowering vector field, like AFM order. One exception is, for example, found in the theory by Machida and co-workers [4]. However, note that it is rather difficult to decide from experimental data whether a tetracritical point really exists for $\mathbf{H} \parallel \mathbf{z}$, although its existence looks rather likely and cannot be excluded theoretically as pointed out by Garg [1]. Alternative scenarios to explain the H-T phase diagram are using the picture of two almost degenerate superconducting states of different symmetry. As shown by Joynt *et al.* [5] and more recently by Chen and Garg [3] such theories can produce a kink for both $\mathbf{H} \perp \mathbf{z}$ and $\mathbf{H} \parallel \mathbf{z}$. In fact, various such theories can be constructed. However, even in this case the symmetry analysis shows that for arbitrarily directed fields the kink is always smeared out either by quadratic or by higher order gradient terms in the GL theory.

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