

# Coherent Phenomena in Multiply Connected SNS Systems



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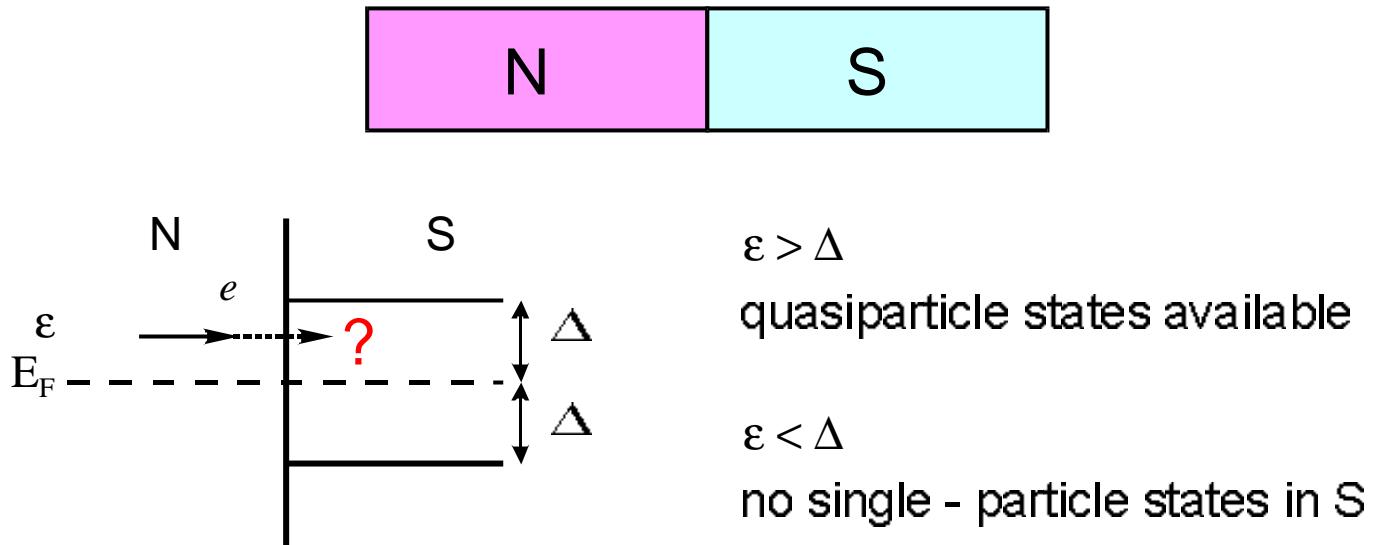
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## Outline

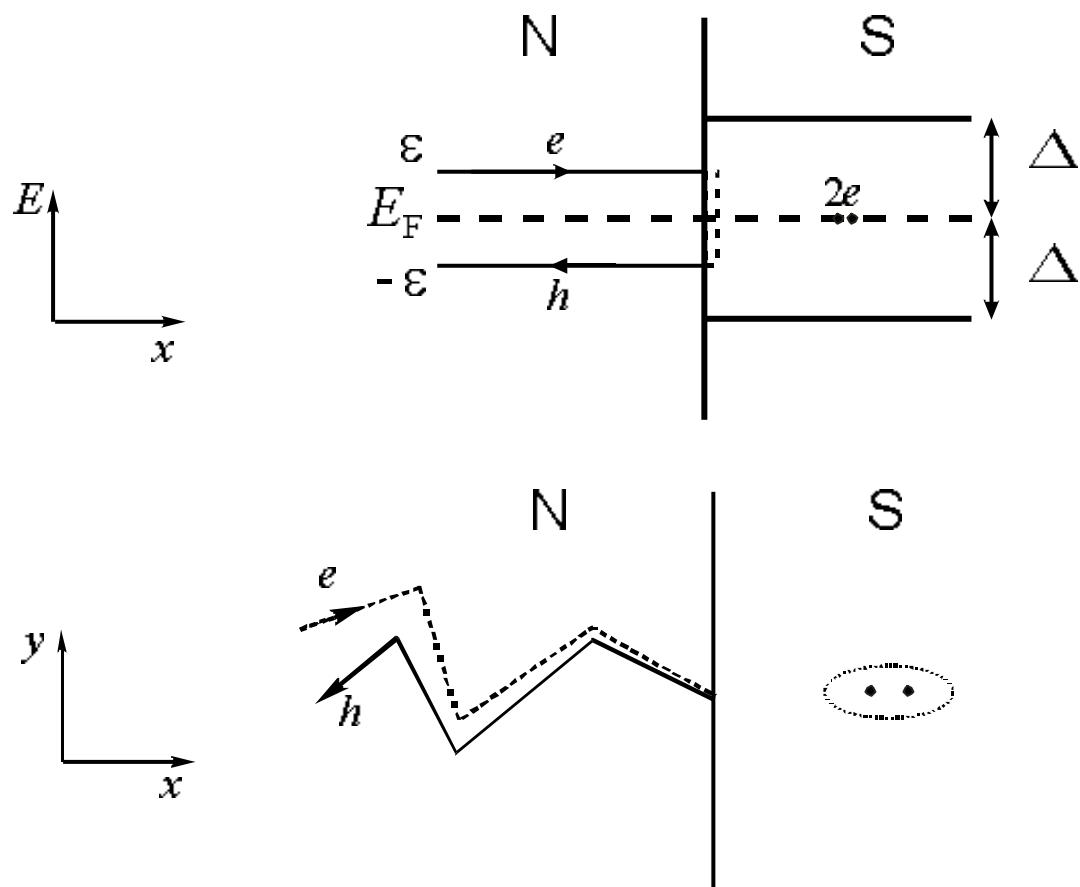
- ◆ Andreev reflection & proximity effect
- ◆ Experiment:
  - single SNS junctions
  - multiply connected SNS systems
    - two-dimensional array of SNS junctions
    - chains of SNS junctions
- ◆ Conclusions

# How do electrons pass from N to S?



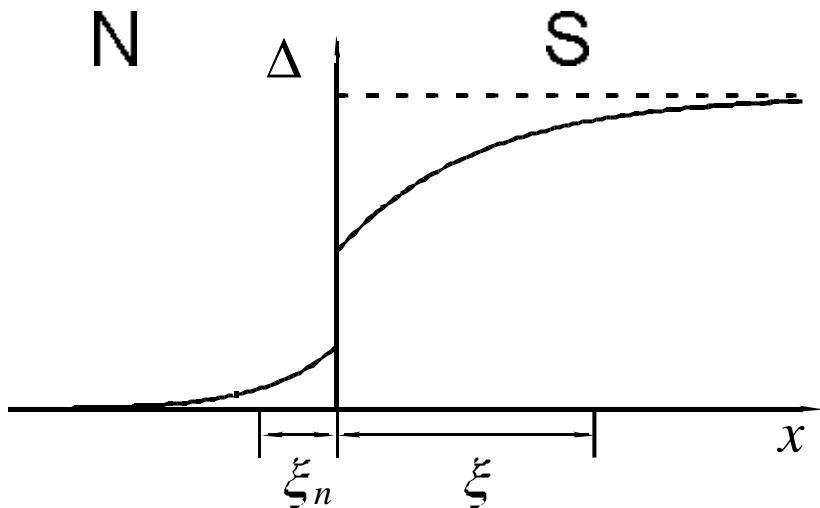
## Andreev Reflection:

an electron-like excitation is retro-reflected as a hole-like excitation, while a Cooper pair is transmitted into the superconductor.



At  $\epsilon = \Delta$ , reflected hole follows time-reversed path of incident electron

# Proximity Effect



Electron and reflected hole stay phase coherent for time:

$$\tau_\varepsilon = \frac{\hbar}{2\varepsilon}$$

Length scales

$$\left\{ \begin{array}{ll} \xi_n = \frac{\hbar v_F}{2\pi\varepsilon} & \text{ballistic} \\ \xi_n = l_T = \sqrt{\frac{\hbar D}{2\pi\varepsilon}} & \text{diffusive} \end{array} \right.$$

Transition from metallic to tunneling regimes in superconducting microconstrictions: Excess current, charge imbalance, and supercurrent conversion

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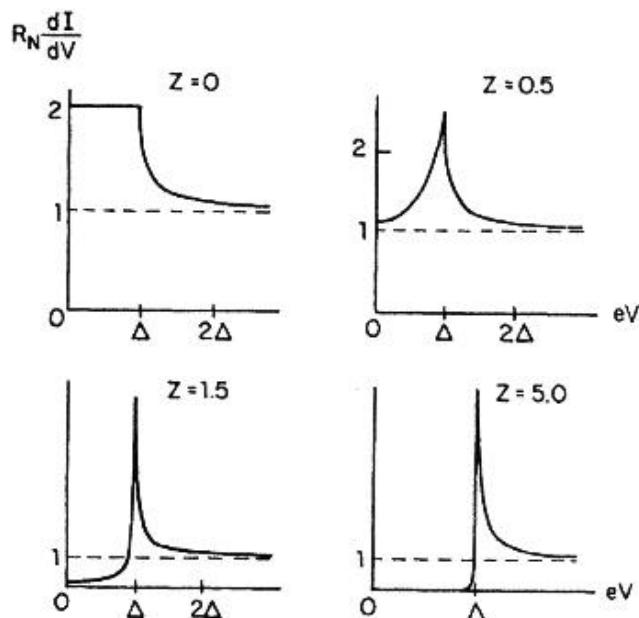
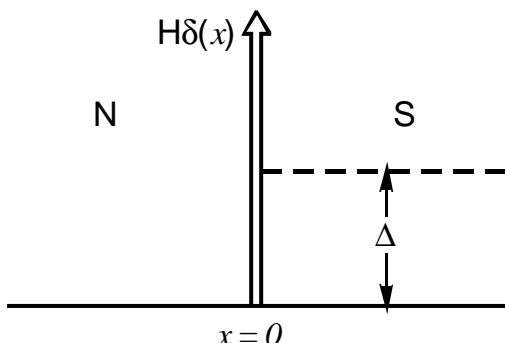


FIG. 7. Differential conductance vs voltage for various barrier strengths  $Z$  at  $T=0$ . This quantity is proportional to the transmission coefficient for electric current for particles at  $E=eV$ .

## BTK Theory

ballistic propagation of quasiclassical electrons through the normal metal region, accompanied by Andreev and normal reflections from NS interface



$$Z = \frac{mH}{\hbar^2 k_F}$$

$Z = 0$  – perfect contact

$Z \rightarrow \infty$  – low-transparency tunnel barrier

## Subharmonic energy-gap structure in superconducting constrictions

M. Octavio

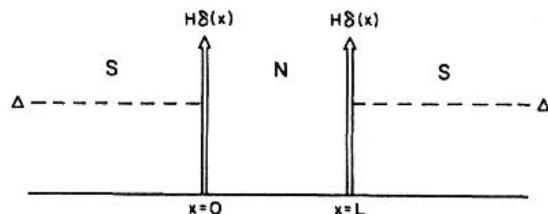
*Fundacion Instituto de Ingeniera, Apartado 1827, Caracas, Venezuela  
and Division of Applied Sciences, Harvard University, Cambridge, Massachusetts 02138*M. Tinkham, G. E. Blonder,\* and T. M. Klapwijk<sup>†</sup>*Physics Department, Harvard University, Cambridge, Massachusetts 02138*

## Subharmonic energy-gap structure in superconducting weak links

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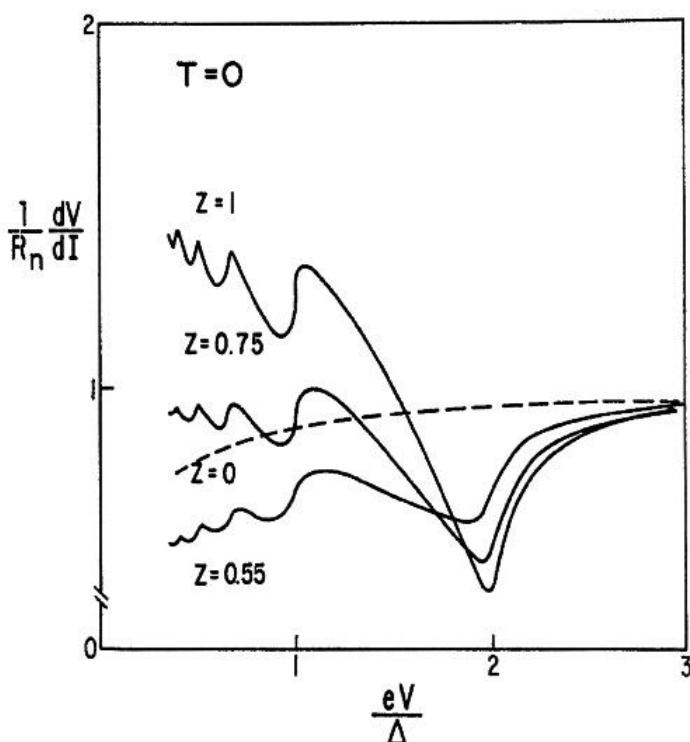
M. Octavio

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OTBK

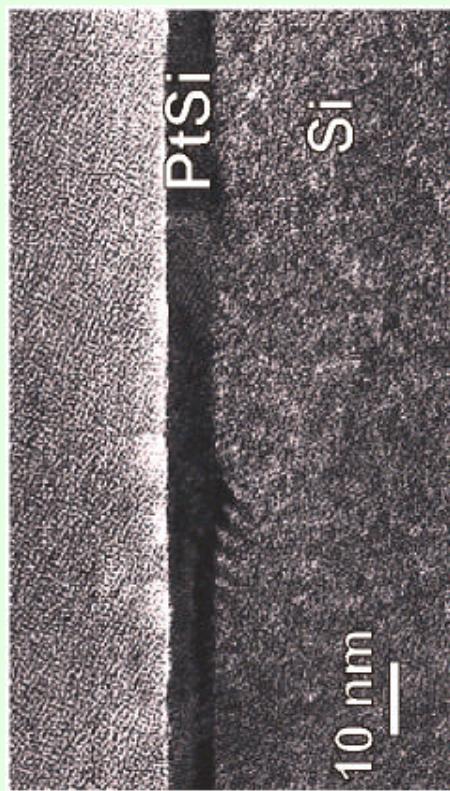
Theory

$$V_n = \frac{2\Delta}{en}$$

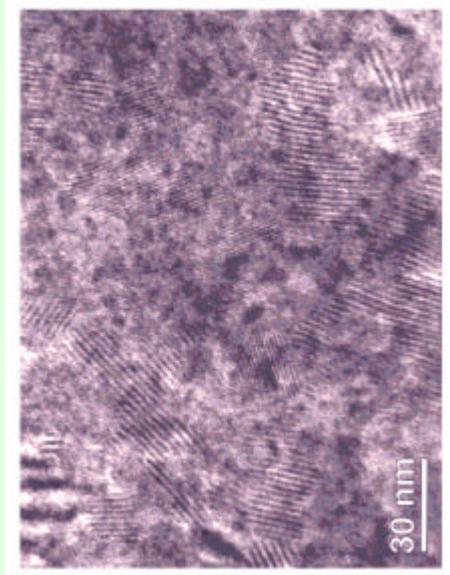
FIG. 2. Normalized differential resistance  $(1/R_n)(dV/dI)$  as a function of the normalized voltage  $eV/\Delta$  for  $T=0$ .

# PtSi film

## The structure & the main parameters



High-resolution cross-sectional TEM image of the PtSi layer



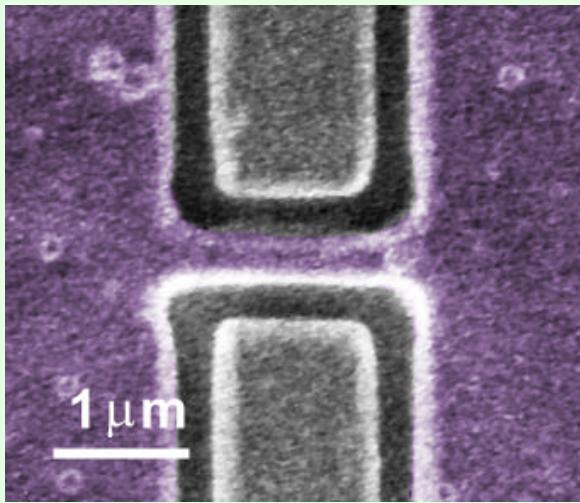
Plan-view TEM image of the PtSi layer  
An average grain size of  $\sim 30$  nm

TABLE. The basic parameters of the PtSi films

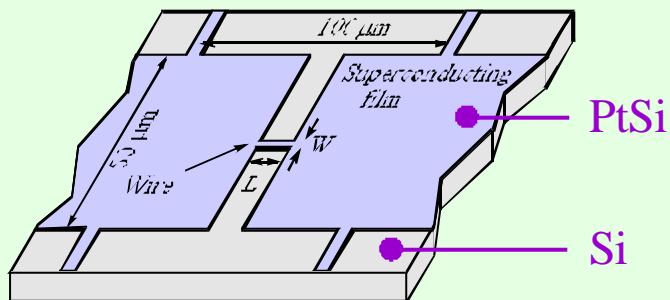
| $d$<br>nm | $T_c$<br>K | $R_{\square}^{\square}$<br>$\Omega$ | $n$<br>$10^{22} \text{ cm}^{-3}$ | $k_F$<br>$10^8 \text{ cm}^{-1}$ | $\tau$<br>$10^{-16} \text{ s}$ | $k_F l$ | $D$<br>$\text{cm}^2/\text{s}$ | $l_p$<br>nm |
|-----------|------------|-------------------------------------|----------------------------------|---------------------------------|--------------------------------|---------|-------------------------------|-------------|
| 6         | 0.56       | 104                                 | 7.0                              | 1.27                            | 8.2                            | 15.4    | 5.9                           | 1.2         |



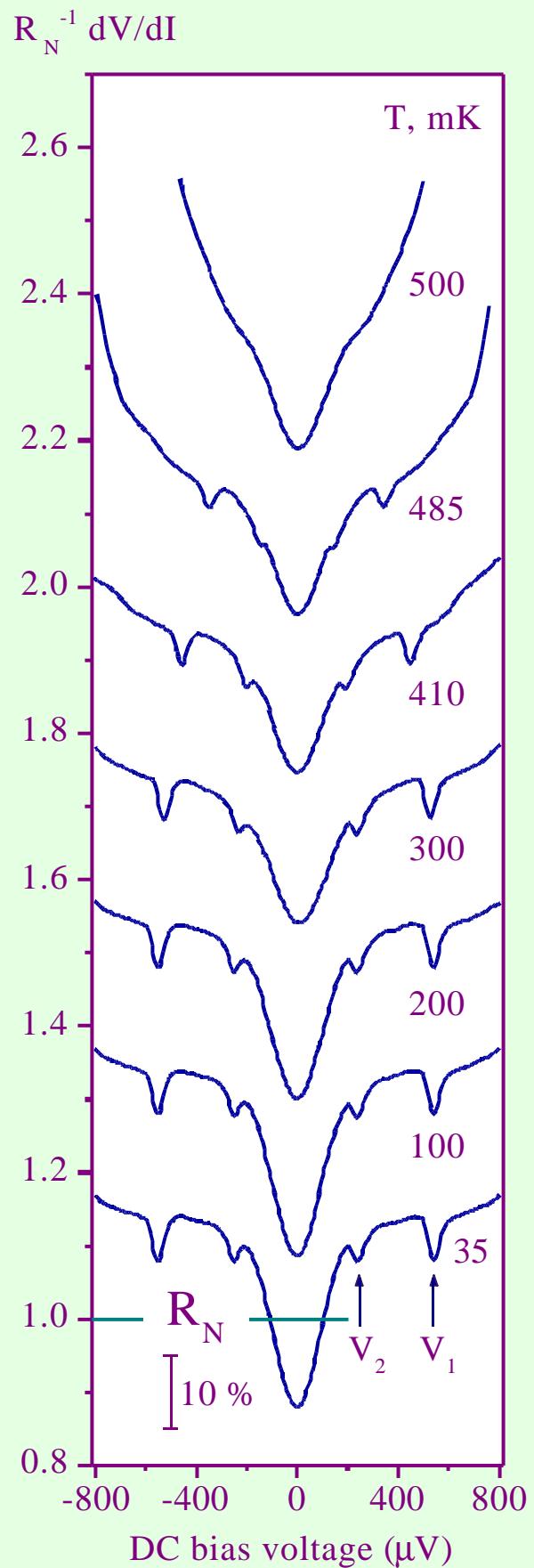
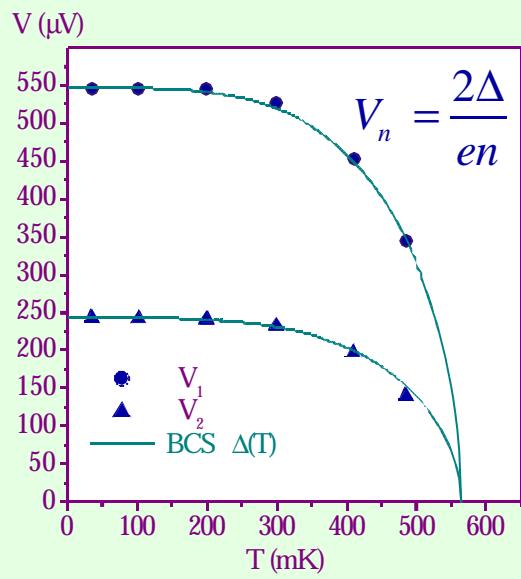
# Single SNS junction [1]



Scanning electron micrograph of the PtSi wire with length  $L = 1.5 \mu\text{m}$  and width  $W = 0.3 \mu\text{m}$  formed by electron beam lithography and subsequent plasma etching

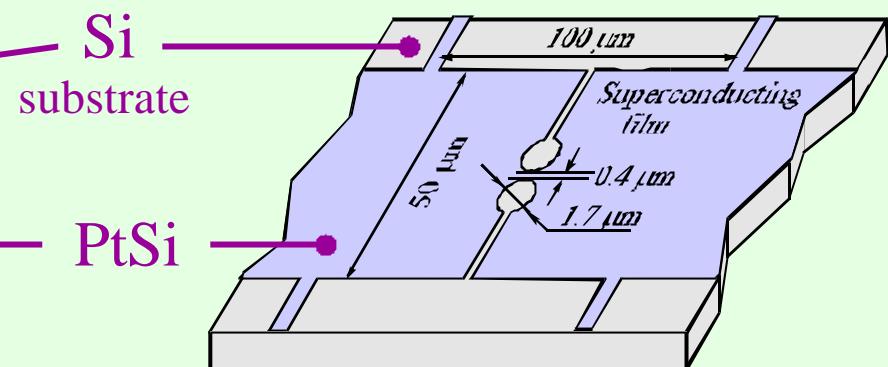
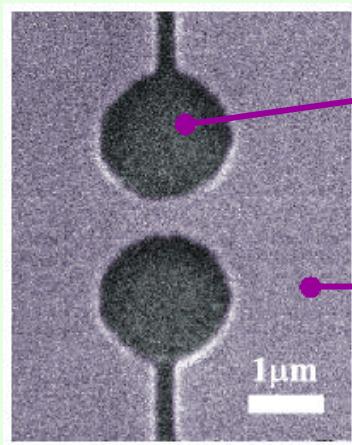


Schematic view of a junction (not to scale) showing the layout of the wire in the Hall bridge



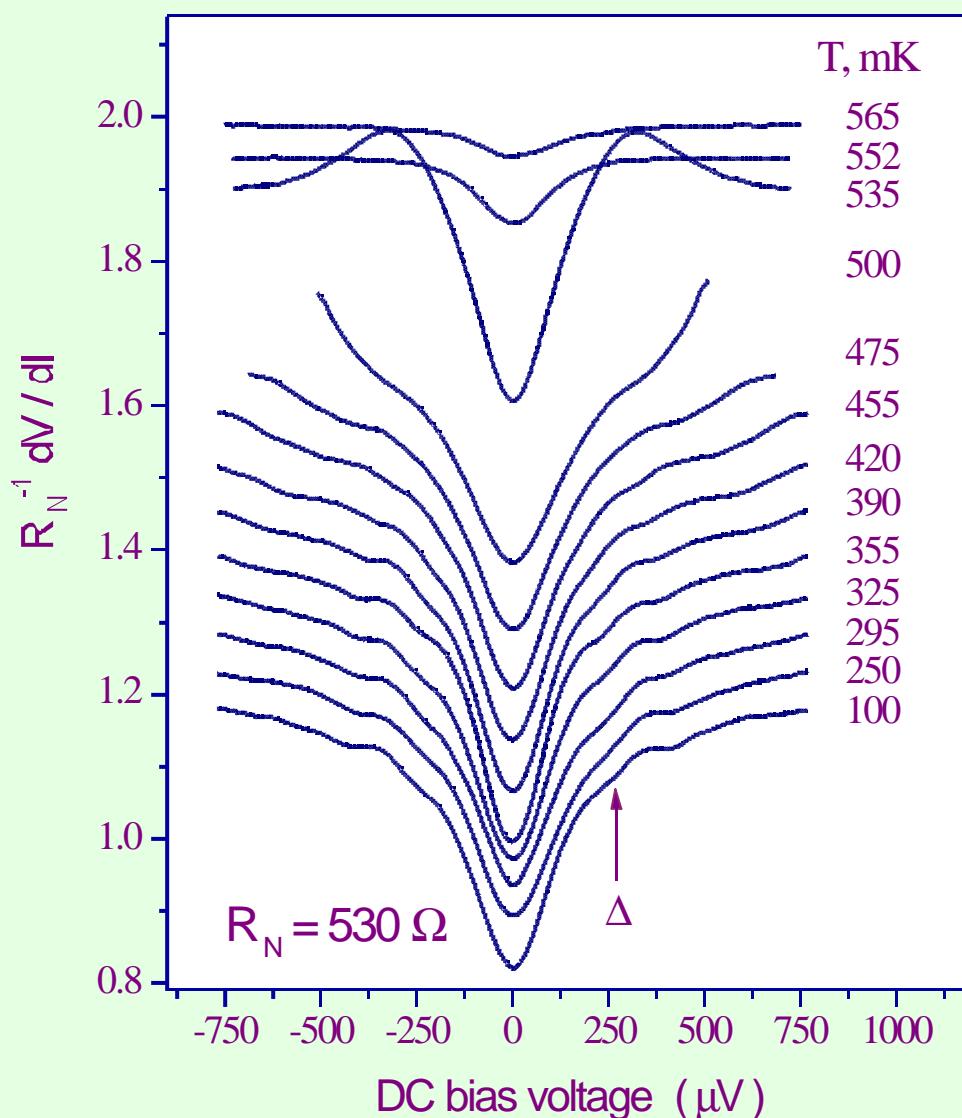


## Single SNS junction [2]



Scanning electron micrograph of the sample with a single constriction formed by electron beam lithography and subsequent plasma etching of the 6 nm PtSi film grown on a Si substrate

Schematic view of a junction (not to scale) showing the layout of the constriction in the Hall bridge



## Kinetic-equation approach to diffusive superconducting hybrid devices

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## Diffusive Conductors as Andreev Interferometers

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(Received 17 August 1995)

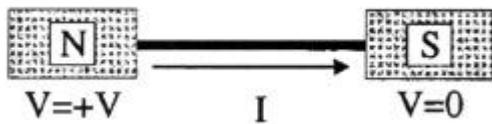


FIG. 2. (a) The simplest possible system, consisting of a diffusive normal metal wire of length  $L$ , that is connected on the left to a normal reservoir and to the right to a superconducting one.

$$x = l_T = \sqrt{\frac{\hbar D}{2p kT}}$$

$SN + NS$

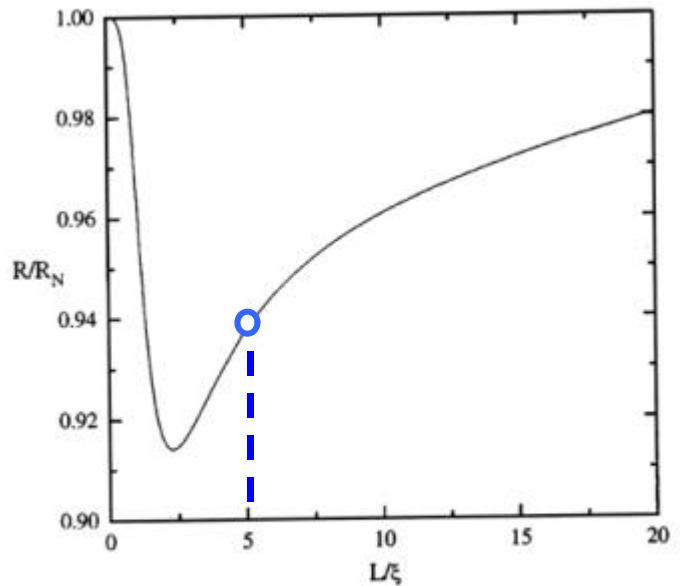


FIG. 7. Temperature dependence of the normalized resistance of the structure shown in Fig. 2(a). The temperature is proportional to  $(L/\xi)^2$ .

## Calculations Experiment

|          |                     |                            |  |     |
|----------|---------------------|----------------------------|--|-----|
| $SNS[1]$ | $T = 35 \text{ mK}$ | $\frac{L}{2l_T} \approx 5$ | $\frac{\Delta R_{SNS}}{R_N} \approx 6\%$ | 12% |
|----------|---------------------|----------------------------|--|-----|

|          |                      |                            |  |     |
|----------|----------------------|----------------------------|--|-----|
| $SNS[2]$ | $T = 100 \text{ mK}$ | $\frac{L}{2l_T} \approx 5$ | $\frac{\Delta R_{SNS}}{R_N} \approx 6\%$ | 18% |
|----------|----------------------|----------------------------|--|-----|

$(L \sim 1 \text{ mm})$

## Circuit theory of multiple Andreev reflections in diffusive SNS junctions: The incoherent case

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and NTT Basic Research Laboratories, Atsugi-shi, Kanagawa 243-0198, Japan*

E. N. Bratus<sup>\*</sup>

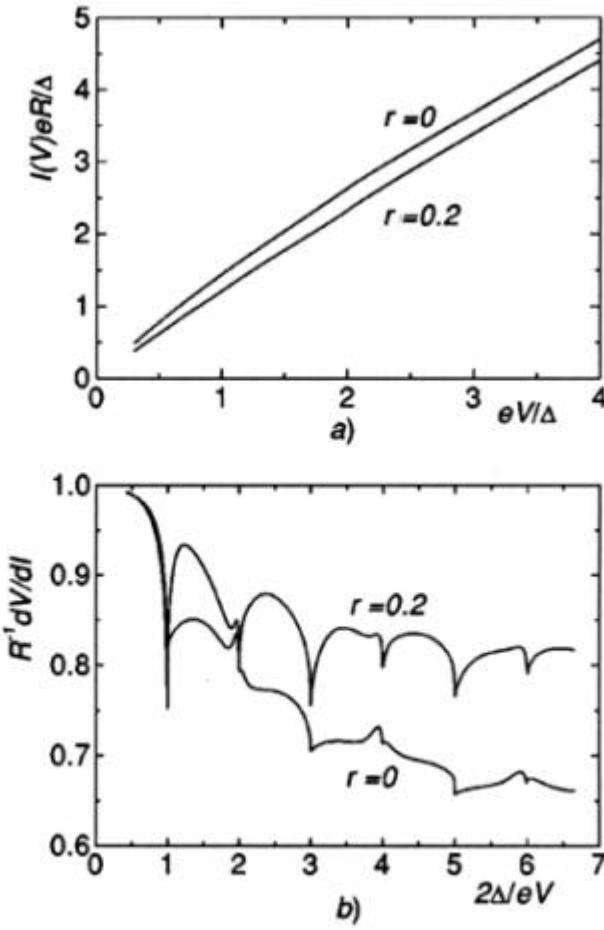
*B. Verkin Institute for Low Temperature Physics and Engineering, Kharkov 61164, Ukraine*

V. S. Shumeiko and G. Wendin

*Chalmers University of Technology, S-41296 Gothenburg, Sweden*

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*NTT Basic Research Laboratories, Atsugi-shi, Kanagawa 243-0198, Japan*



$$r = \frac{R_{SN}}{R_N}$$

$$\frac{dV}{dI}(0) = R_N \left( 1 - 2.64 \frac{\mathbf{x}_\Delta}{L} \right)$$

$$\mathbf{x}_\Delta = \sqrt{\frac{\hbar D}{\Delta}}$$

FIG. 9.  $I$ - $V$  characteristics (a) and differential resistances vs inverse voltage (b) of an SNS junction with high-transparent interfaces.

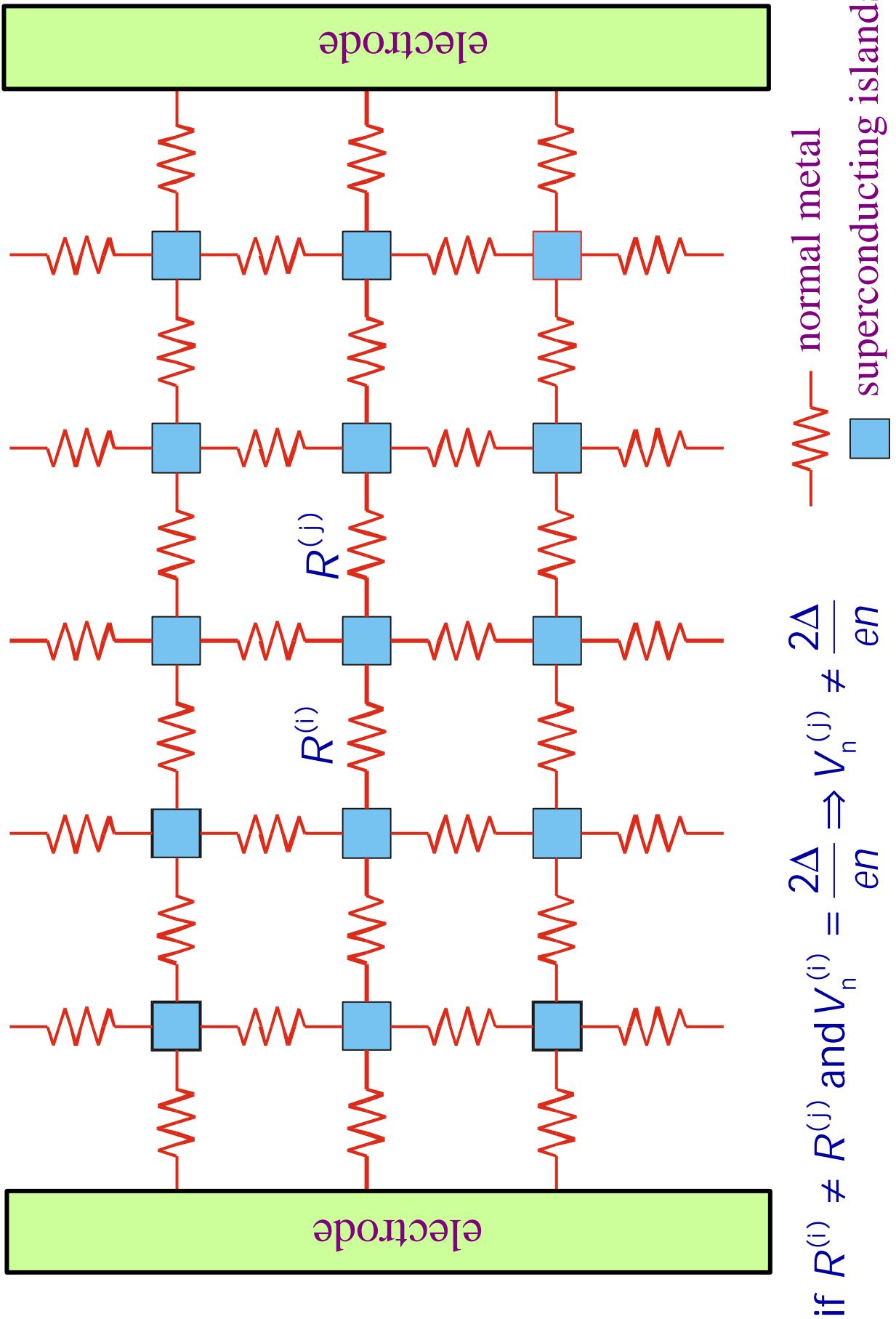
## Calculations      Experiment

$$SNS[1] \quad \frac{\Delta R_{SNS}}{R_N} \simeq 7\% \quad 12\%$$

$$SNS[2] \quad \frac{\Delta R_{SNS}}{R_N} \simeq 10\% \quad 18\%$$

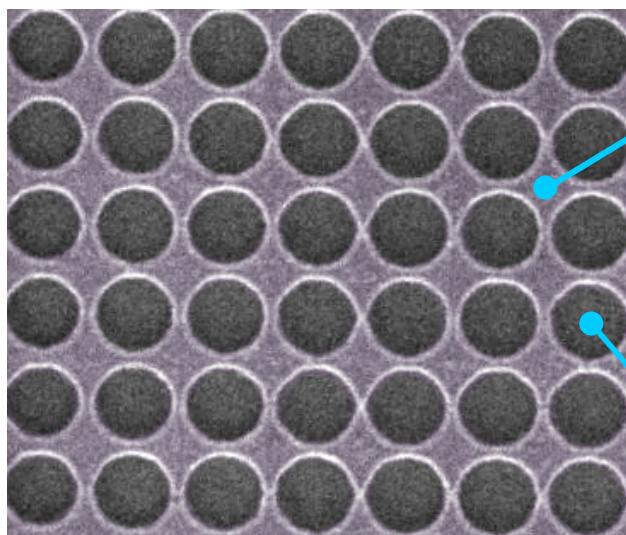
## Puzzle

### Two-dimensional array of SNS junctions

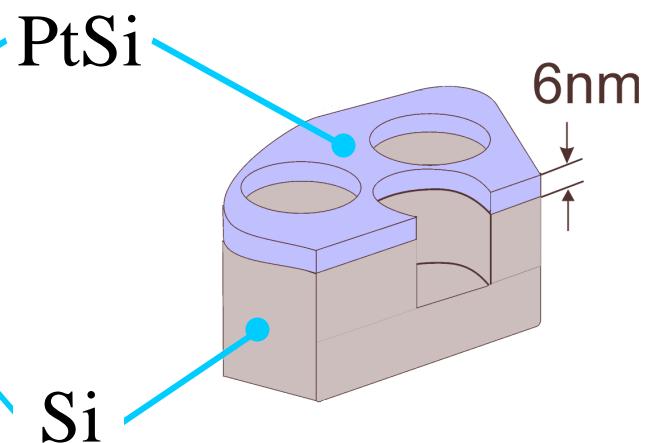


# Two-dimensional array of SNS junctions

## The structure

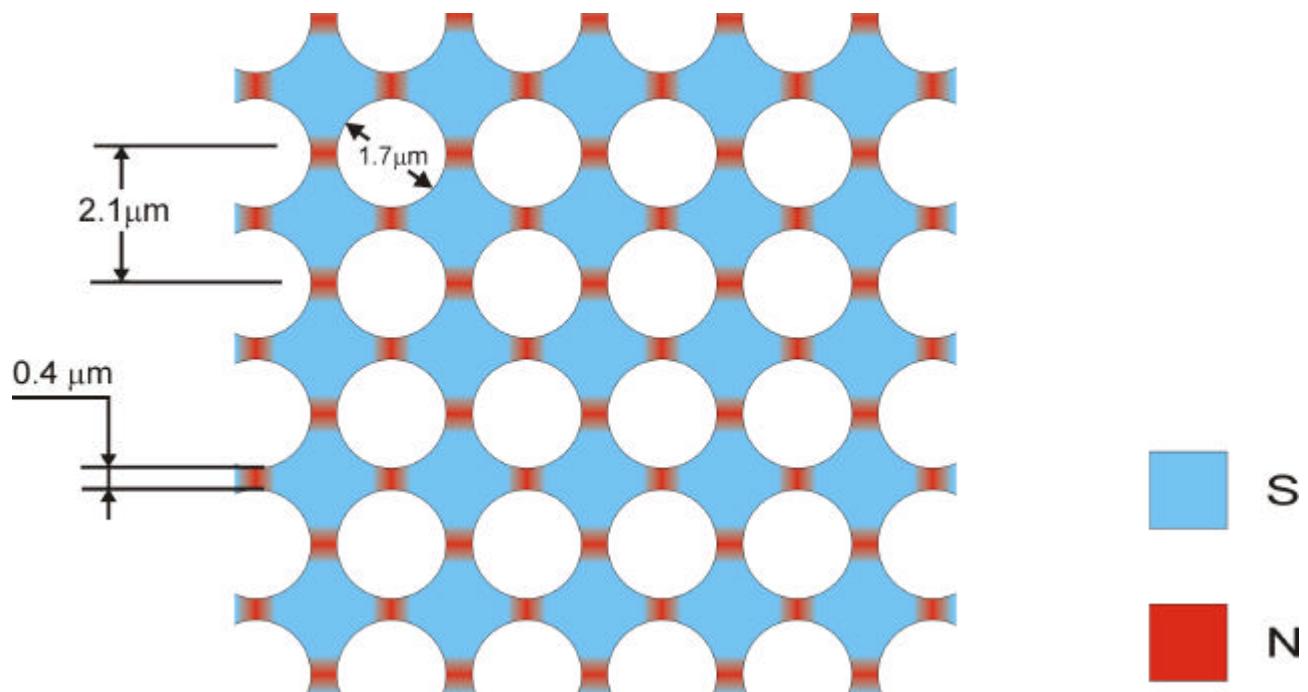


Scanning electron micrograph of the sample formed by electron beam lithography and subsequent plasma etching



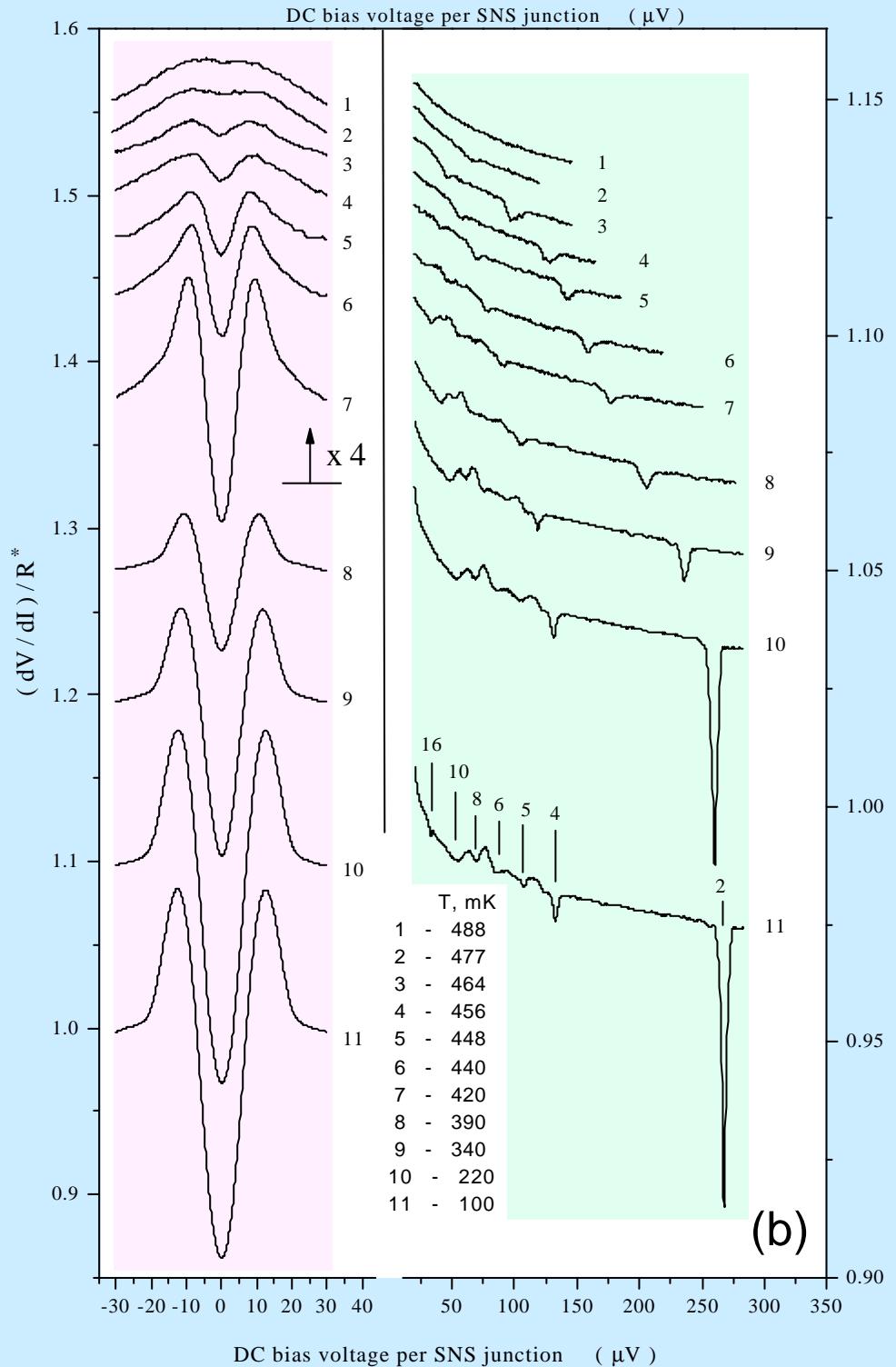
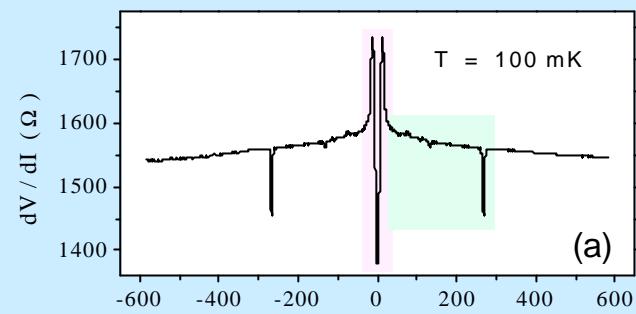
Schematic cutaway view

### Schematic view of the two-dimensional array of SNS junctions



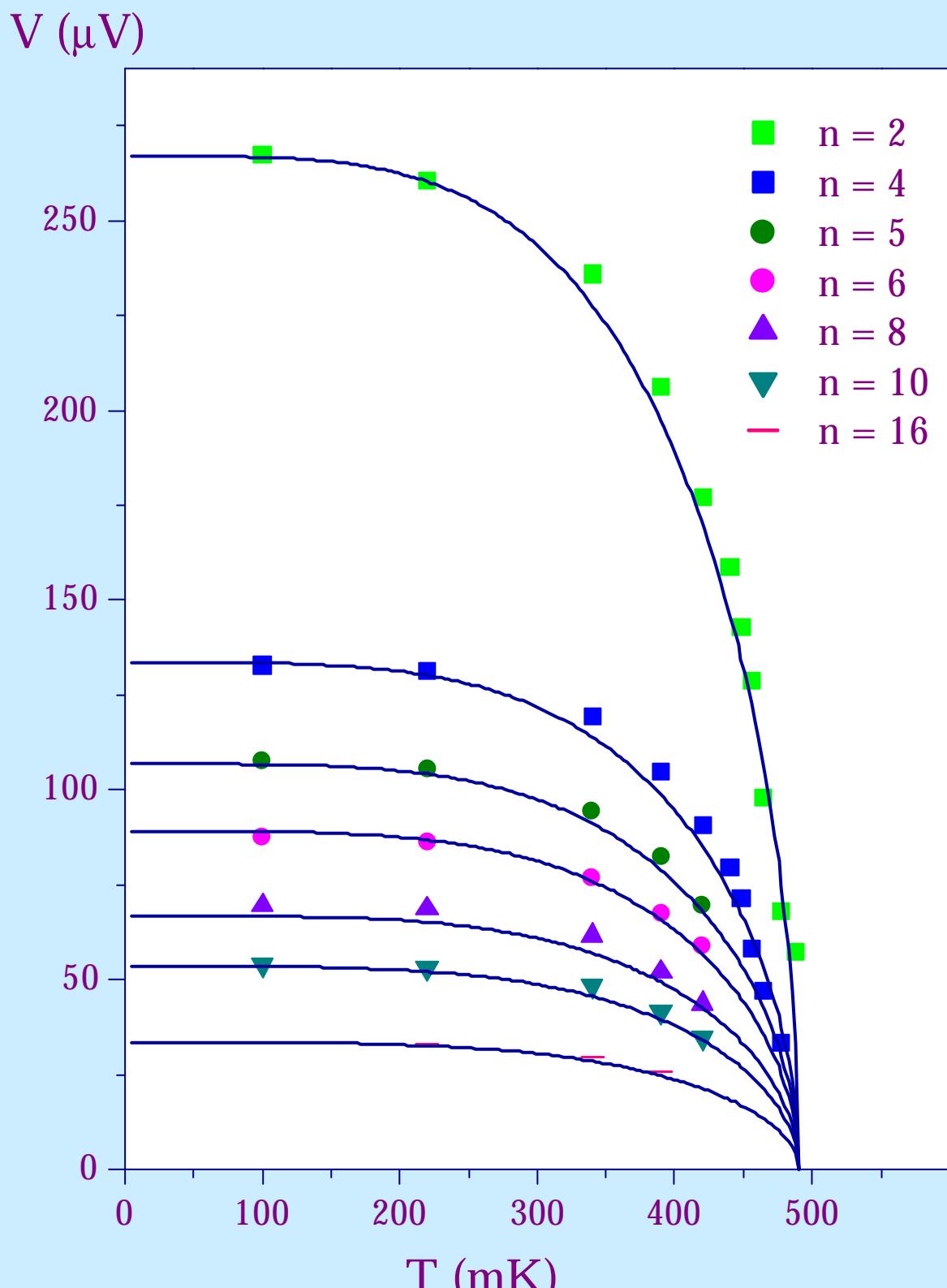


# Two-dimensional array of SNS junctions Experiment





# Two-dimensional array of SNS junctions Experiment



$$V_n = \frac{2\Delta}{en} \quad - \Delta(T) - \text{BCS}$$

## Dynamics of conversion of supercurrents into normal currents and vice versa

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(Received 14 March 2001; published 23 August 2001)

The generation and destruction of the supercurrent in a superconductor ( $S$ ) between two resistive normal ( $N$ ) current leads connected to a current source is computed from the source equation for the supercurrent density. This equation relates the gradient of the pair potential's phase to electron and hole wave packets that create and destroy Cooper pairs in the  $N/S$  interfaces. Total Andreev reflection and supercurrent transmission of electrons and holes are coupled together by the phase rigidity of the nonbosonic Cooper-pair condensate. The calculations are illustrated by snapshots from a computer movie.

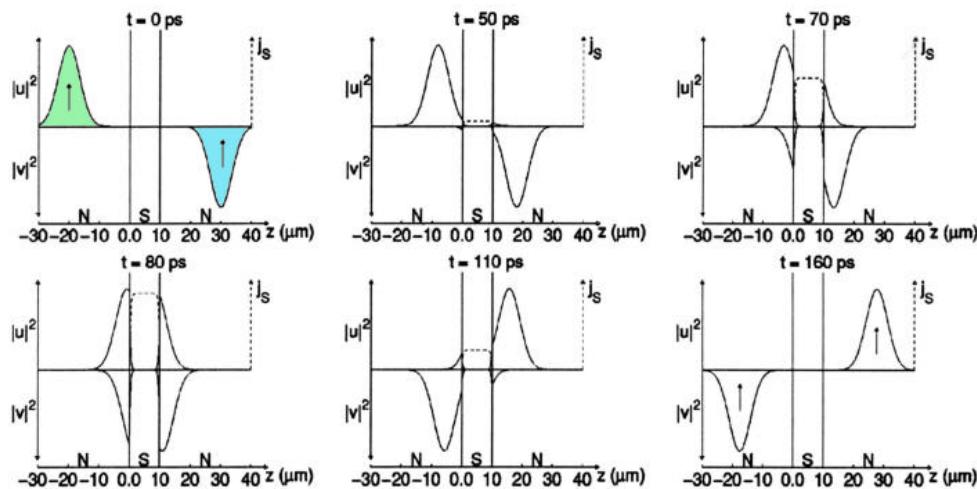
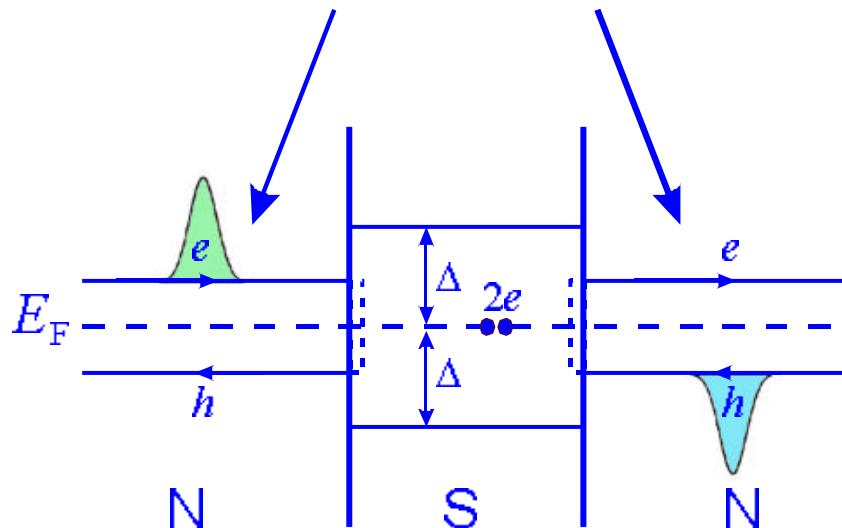


FIG. 1. Propagation and Andreev scattering of the probability densities  $|u|^2$  and  $|v|^2$  of a representative spin-up electron and hole wave packet configuration (solid lines), and the induced supercurrent density  $j_s$  (dashed line), in a current-carrying closed  $N/S/N$  circuit. For the sake of clear graphical representation the initial conditions for the electron wave packet, incident from the left, have been chosen as: energy in the center  $E_i = \Delta/2 = 0.15$  meV, spatial spread  $a_z = 5$   $\mu\text{m}$ , and  $k_{zF} = 0.9k_F = 2.06$   $\text{nm}^{-1}$ . Via charge conservation by supercurrent induction these conditions determine the parameters and the motion of the resulting hole wave packet incident from the right; the retardation time for AS,  $\tau_I = \hbar/[\Delta^2 - E_I^2]^{1/2}$ , is about two picoseconds (ps). More computer movies on electron  $\rightarrow$  hole and electron  $\rightarrow$  electron scattering in one  $N/S$  interface, also for energies above the gap, can be viewed in the Internet at URL <http://theorie.physik.uni-wuerzburg.de/TP1/kuemmel/films/filmse.html>

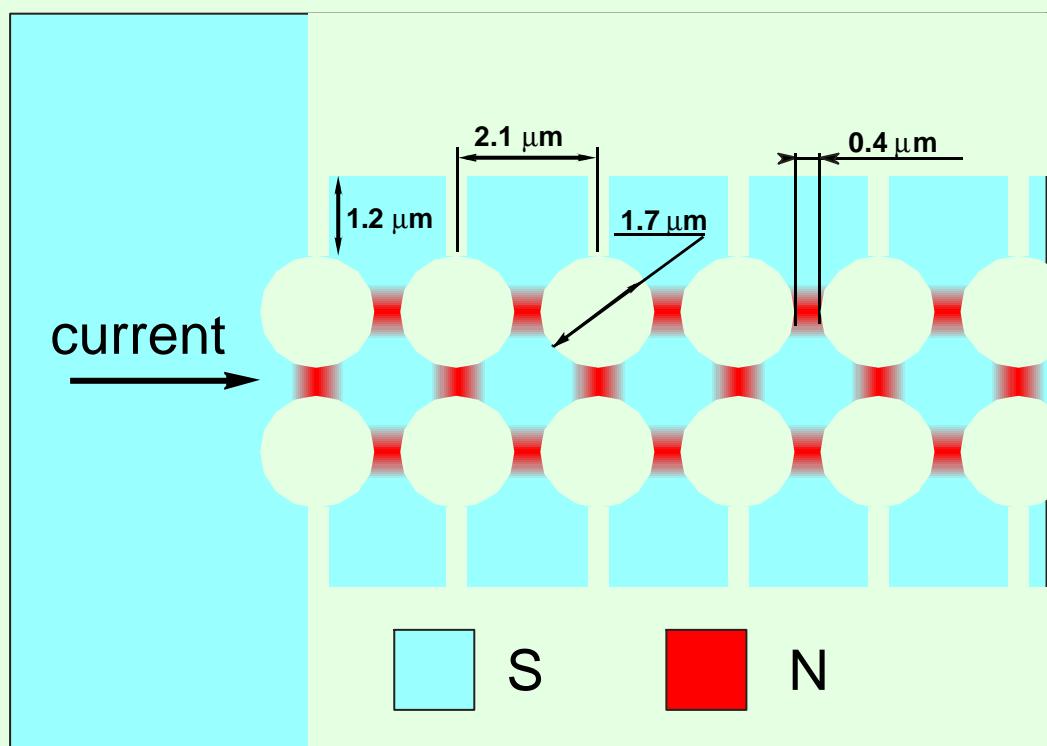
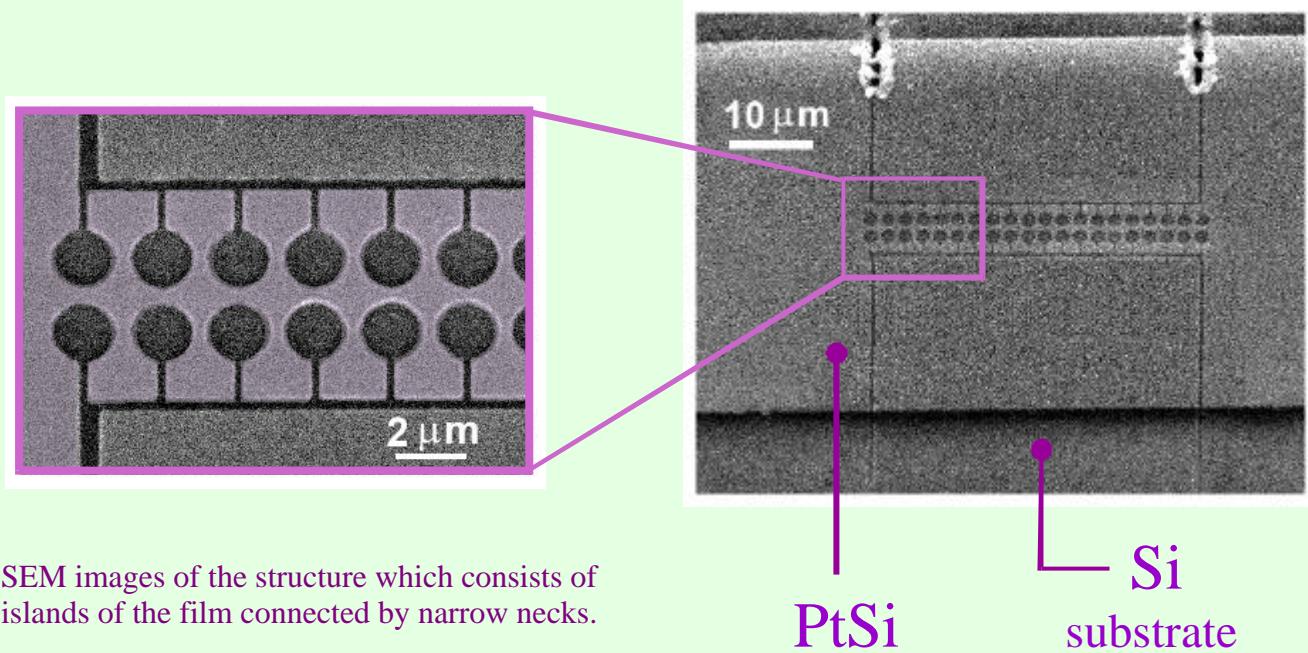
Andreev reflections occur simultaneously!





# One-dimensional array of SNS junctions

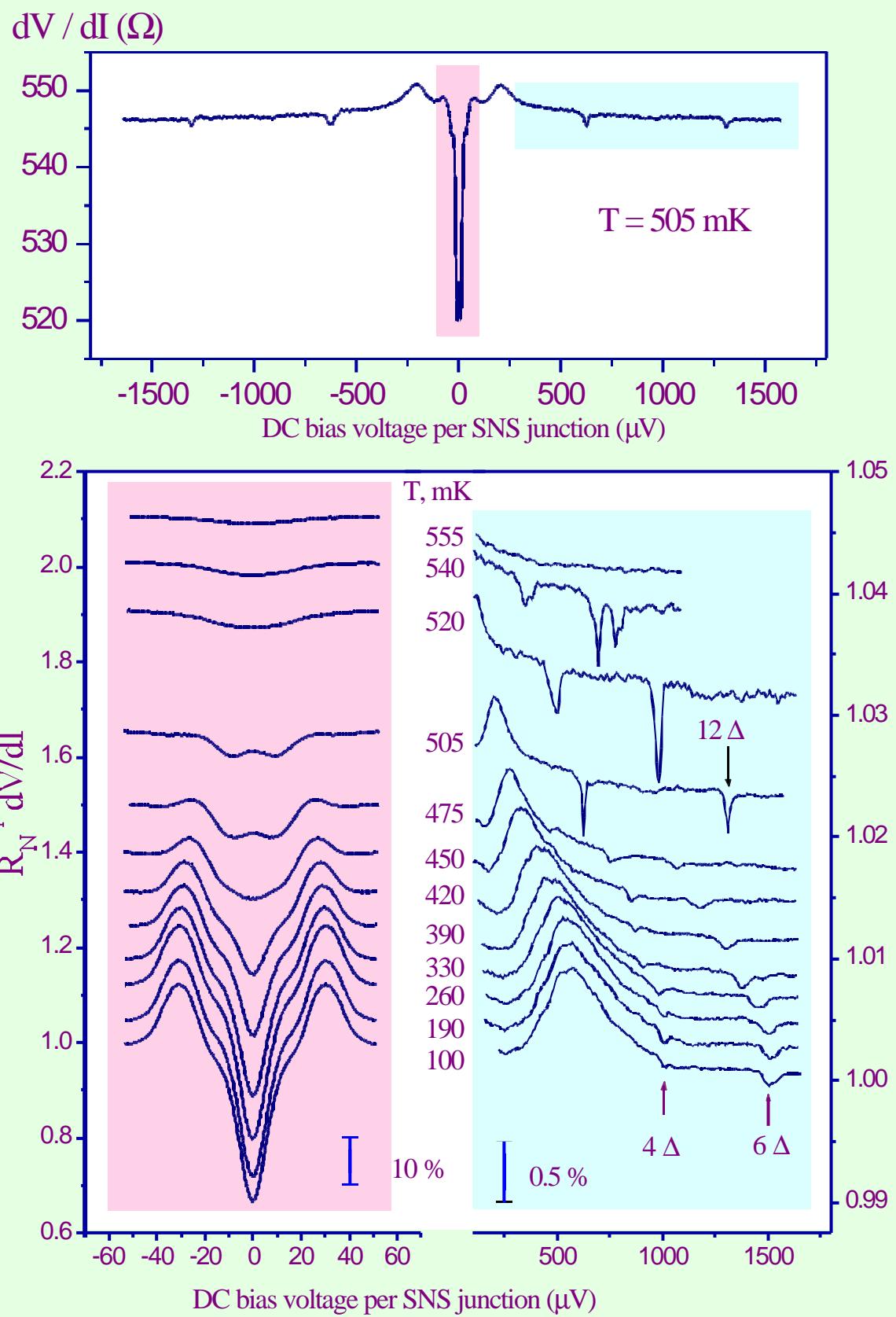
## The structure



The layout of the one-dimensional array of SNS junctions showing the dimensions of the structure.

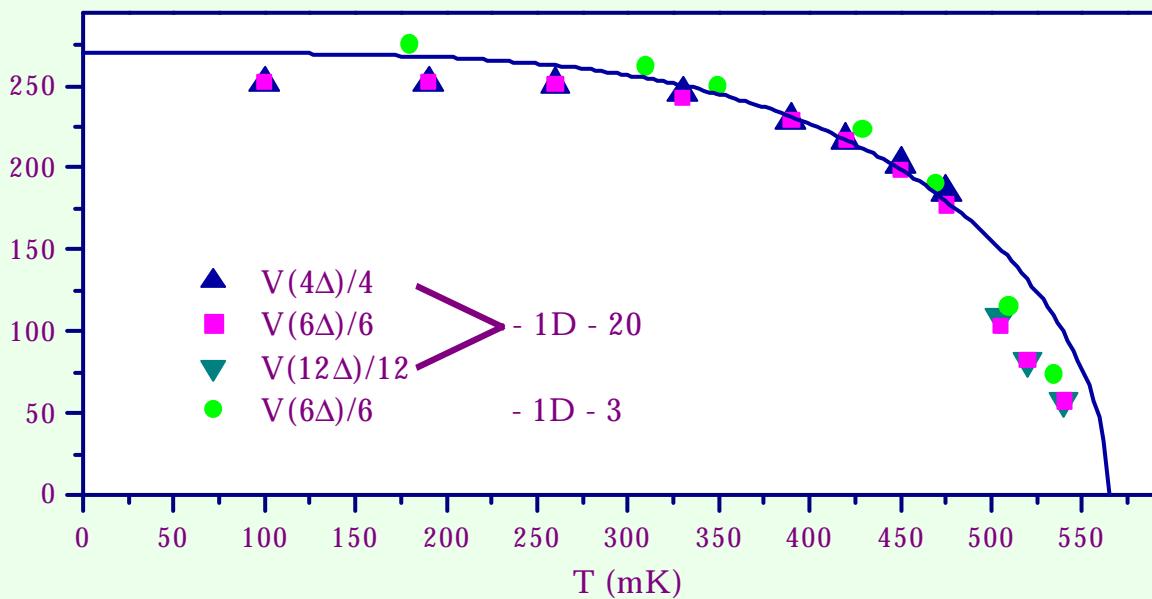
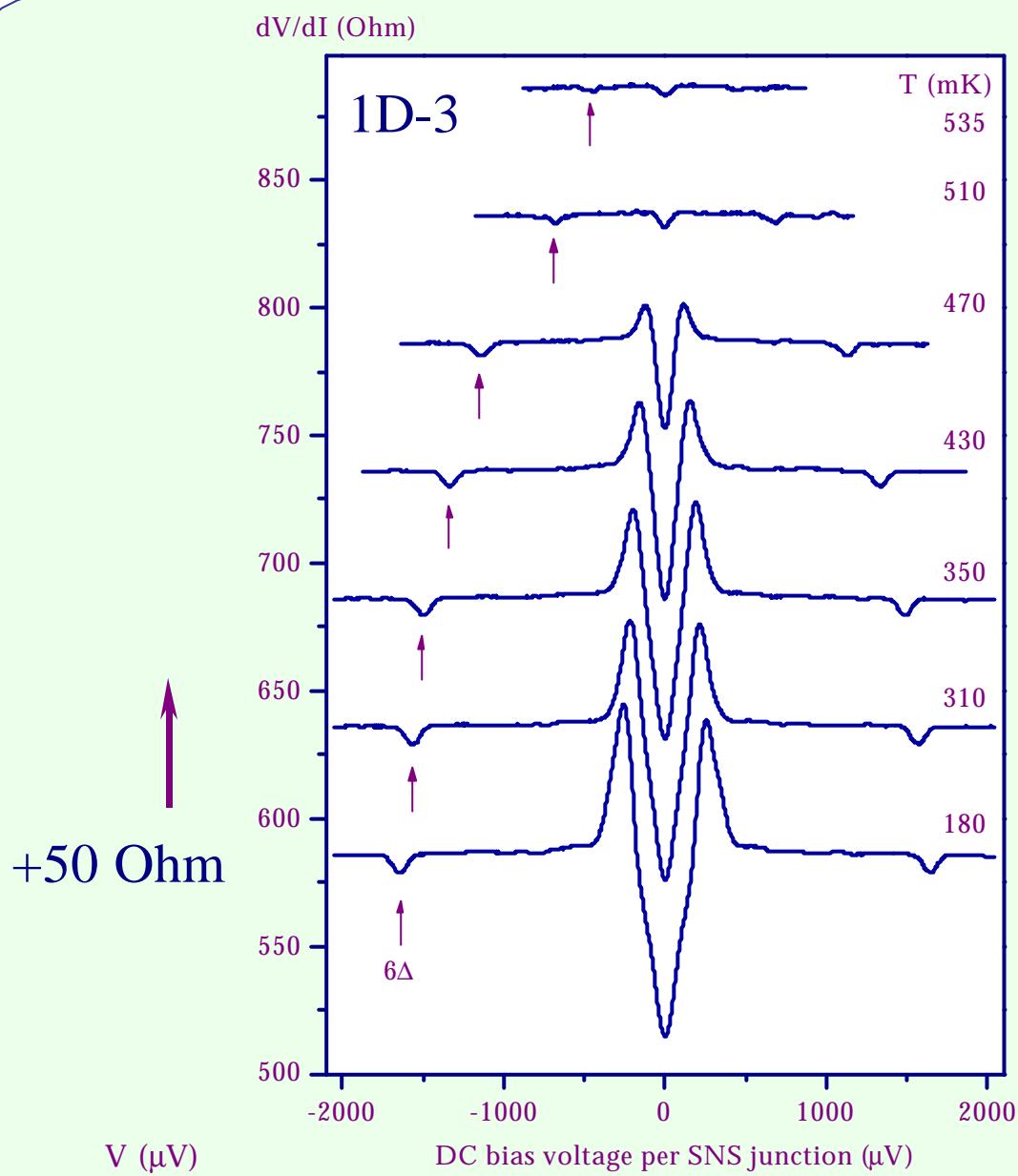


# One-dimensional array of SNS junctions Experiment [1D - 20 SNS]





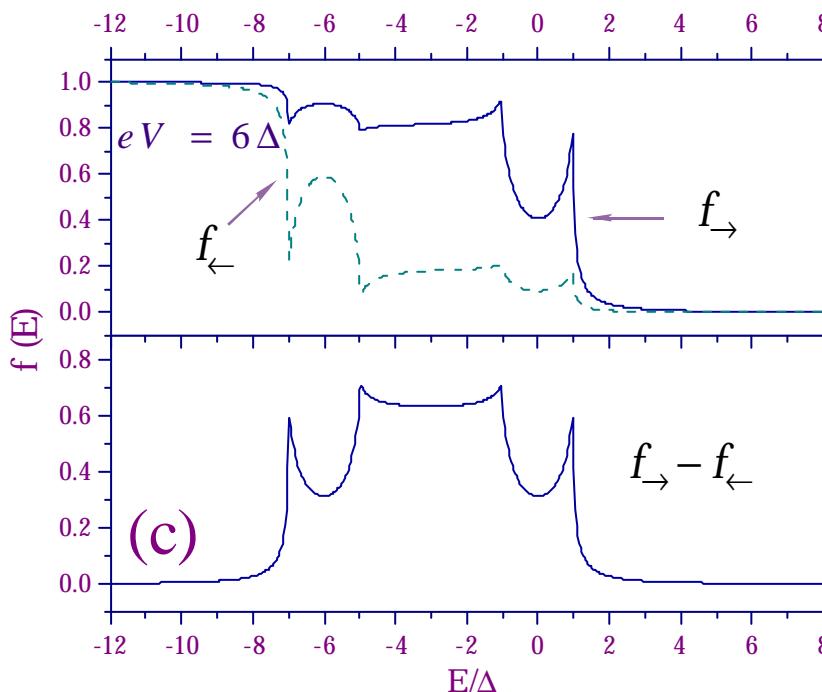
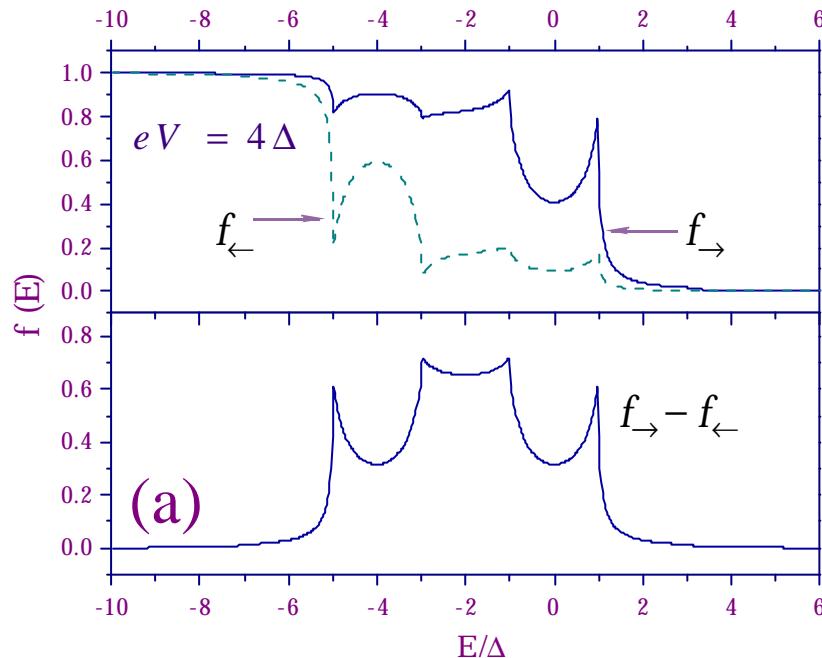
# One-dimensional array of SNS junctions Experiment





# One-dimensional array of SNS junctions Above-energy-gap structure

Nonequilibrium distributions in normal region for SNS junction calculated on a basis OTBK theory  
with  $Z = 0.55$ : (a,b) at  $eV = 4\Delta$ ; (c,d) at  $eV = 6\Delta$  and  $T = 0$ .



$$I \propto \int_{-\infty}^{+\infty} [f_{\rightarrow}(E) - f_{\leftarrow}(E)] dE$$

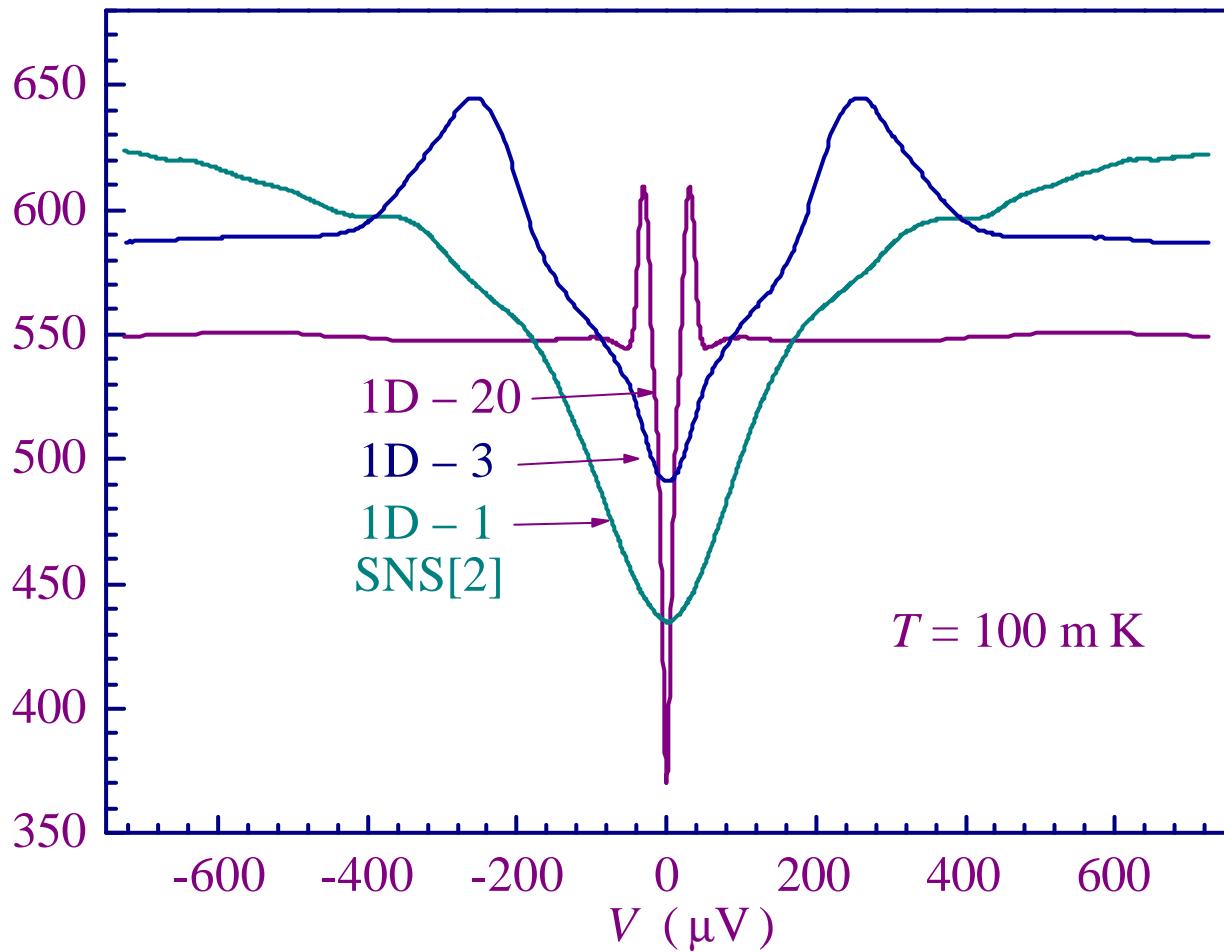


The decrease of the effective suppression voltage for the excess conductivity

$$V_{\text{ZBA}}(1D-1) > V_{\text{ZBA}}(1D-3) > V_{\text{ZBA}}(1D-20)$$

per SNS junction

$dV/dI$  ( $\Omega$ )



$$V_{\text{ZBA}}(1D-1) < V_{\text{ZBA}}(1D-3) < V_{\text{ZBA}}(1D-20)$$

total value



# Conclusions:

Multiply Connected SNS Systems  $\neq$   $(SNS)_1 + (SNS)_2 + (SNS)_3\dots$

Coherent phenomena governed by Andreev reflection manifest novel pronounced effects in Multiply Connected SNS systems:

- ◆ the gradual decrease of the  $V_{ZBA}$  as the quantity of the SNS junctions increases
- ◆ the strengthening of subharmonic energy gap structure
- ◆ above energy gap structure

**It is necessary to take into account:**

- parameters of the NS interfaces
- the superposition of multiple coherent scattering at the NS interfaces in the presence of disorder
- electron-electron interaction
- the coherent Andreev reflection on both NS interfaces of the superconducting island
- effect of quasiparticle injection in both the superconducting and normal regions of Multiply Connected SNS systems

## Publications

### Diffusive single and multiply connected SNS systems with high-transparent interfaces

1. T.I. Baturina, Z.D. Kvon, R.A. Donaton, M.R. Baklanov, E.B. Olshanetsky, K. Maex, A.E. Plotnikov, J.C. Portal, “Mesoscopic S-N-S Junctions on the Basis of Superconducting PtSi Films”, *Physica B* 284-8/2002, 1860 (**2000**). (LT22)
2. Z.D. Kvon, T.I. Baturina, R.A. Donaton, M.R. Baklanov, K. Maex, E.B. Olshanetsky, A.E. Plotnikov, J.C. Portal, “Proximity effects and Andreev reflection in a mesoscopic SNS junction with perfect NS interfaces”, *Physical Review B* 61, 11340 (**2000**). [cond-mat/9907247]
3. T.I. Baturina, Z.D. Kvon, and A.E. Plotnikov. “Two-dimensional array of diffusive SNS junctions with high-transparent interfaces”. *Physical Review B* 63, 180503(R) (**2001**). [cond-mat/0008402]
4. T.I. Baturina, Z. D. Kvon, A. E. Plotnikov, R. Donaton, M.R. Baklanov, “Diffusive single and multiply connected SNS systems with high-transparent interfaces”. *Usp. Fiz. Nauk (Suppl.)* **171**, 91 (**2001**).
5. T.I. Baturina, D.R. Islamov, Z.D. Kvon, “Subgap anomaly and above-energy-gap structure in chains of diffusive SNS junctions”. *Pis'ma JETP* **75**, issue 7, 397 (**2002**). *JETP Letters*, Vol. 75, No. 7, 326 (**2002**) [cond-mat/0202467]