Carbon Nanotubes as Nanoelectromechanical Systems

- Introduction: NEMS
- Model for suspended carbon nanotubes
- Displacement and eigenmodes
- Stability diagram
- Transport (preliminary)
- Outlook and conclusions

Delft Nanomechanics Team

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Quantum Transport

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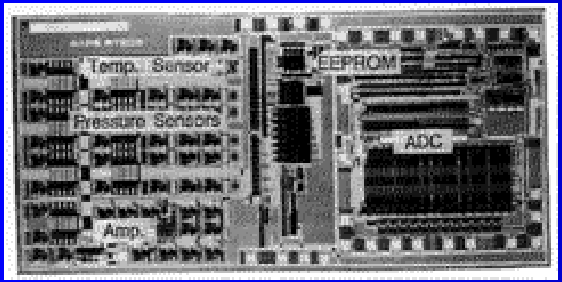
- Y.M.B.

Molecular biophysics

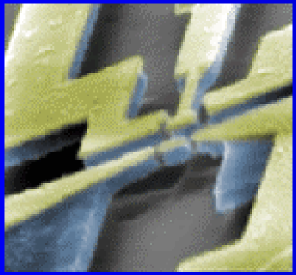
- Jing Kong
- Abdou Hassanien
- Cees Dekker

Nanoelectromechanical systems

Electromechanical systems



Microelectromechanical systems



Nanoelectromechanical systems

Nanoelectromechanical systems

NEMS – nanoscale devices which convert electrical current into mechanical energy or vice versa.

Experiments: precise measurements

attoNewtons of force (Stowe *et al* ‘97)

electrometry (Cleland and Roukes ’98)

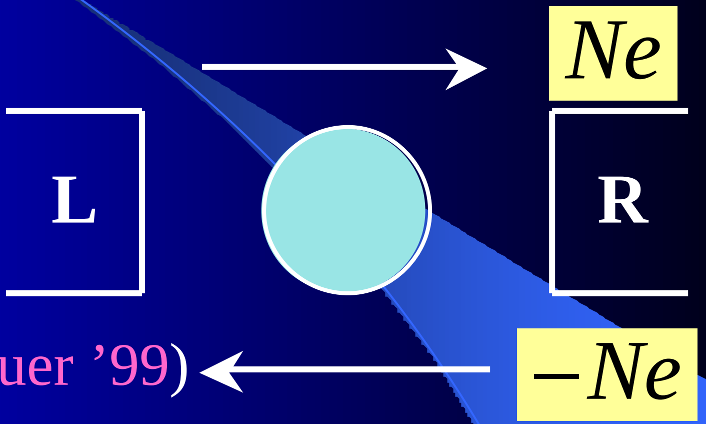
quantum of thermal conductance (Schwab *et al* ’00)

Casimir force (Chan *et al* ’01)

Possible applications: nanoscale sensors and actuators

NEMS research directions

➢Shuttling: First theoretical proposal by Gorelik *et al* '98

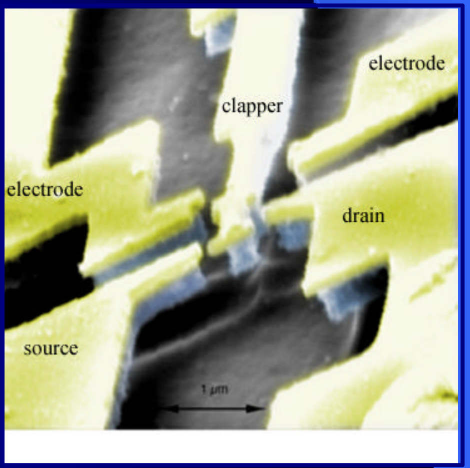


Experiments:

- Classical shuttle (Erbe *et al* '98)
- Silver grain (Tuominen, Krotkov, Breuer '99)
- Fullerene molecule (Park *et al* '00)
- ac driven cantilever (Erbe *et al* '01)

➢Suspended beams

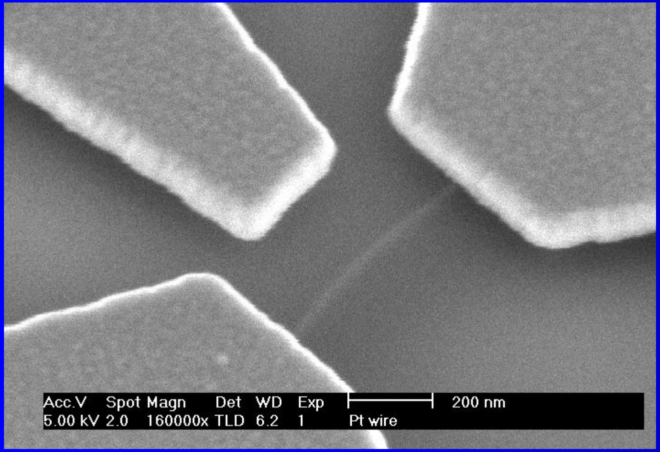
- Electromechanical noise
- Bistability and quantum effects
- Position and eigenmodes

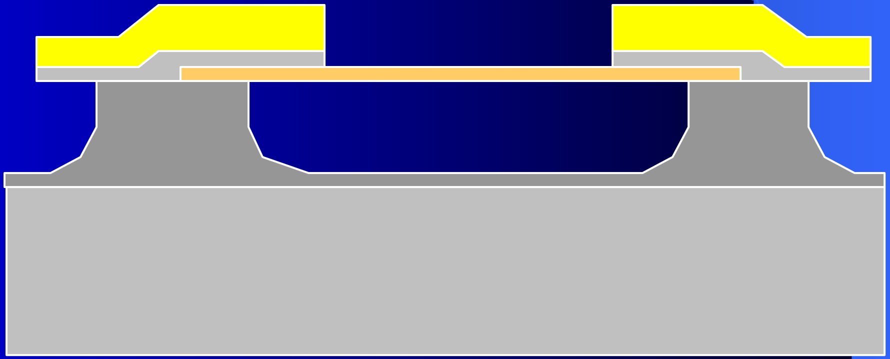


Suspended beams - experiments

❖ Silicon quantum dot embedded into a suspended beam
(Höhberger *et al* ’02)

❖ Suspended doubly-clamped carbon nanotubes
(P. Jarillo-Herrero *et al*, in progress)

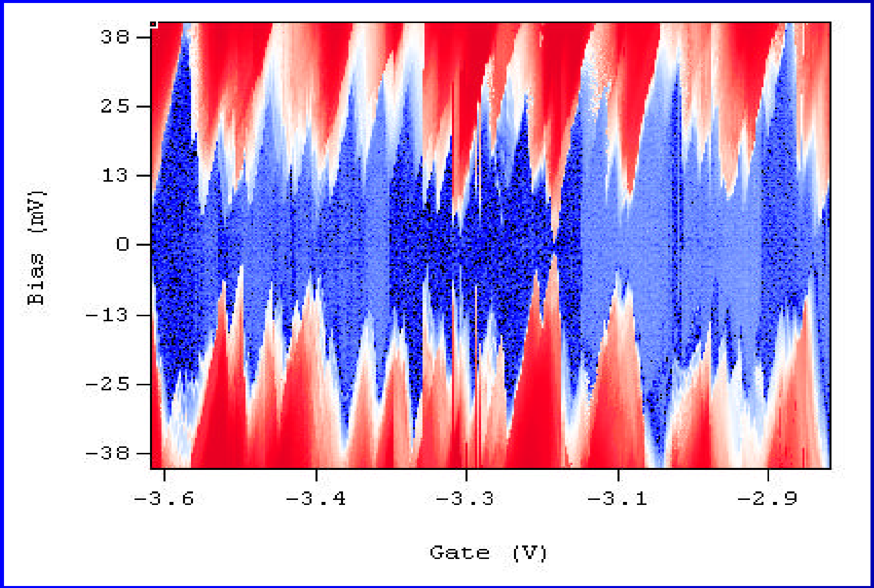




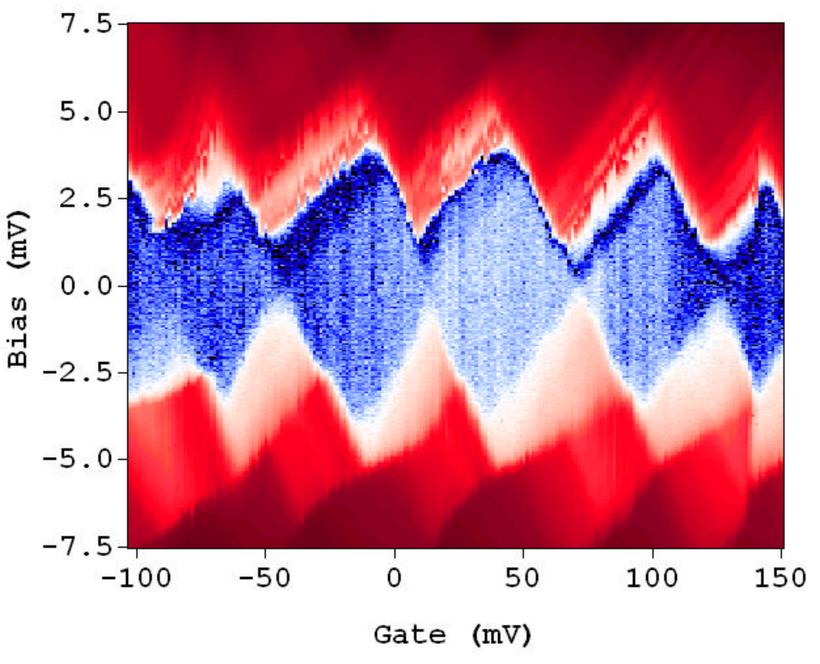
Stability diagram – Delft experiments

L=1200nm, T=300mK

Before under-etching

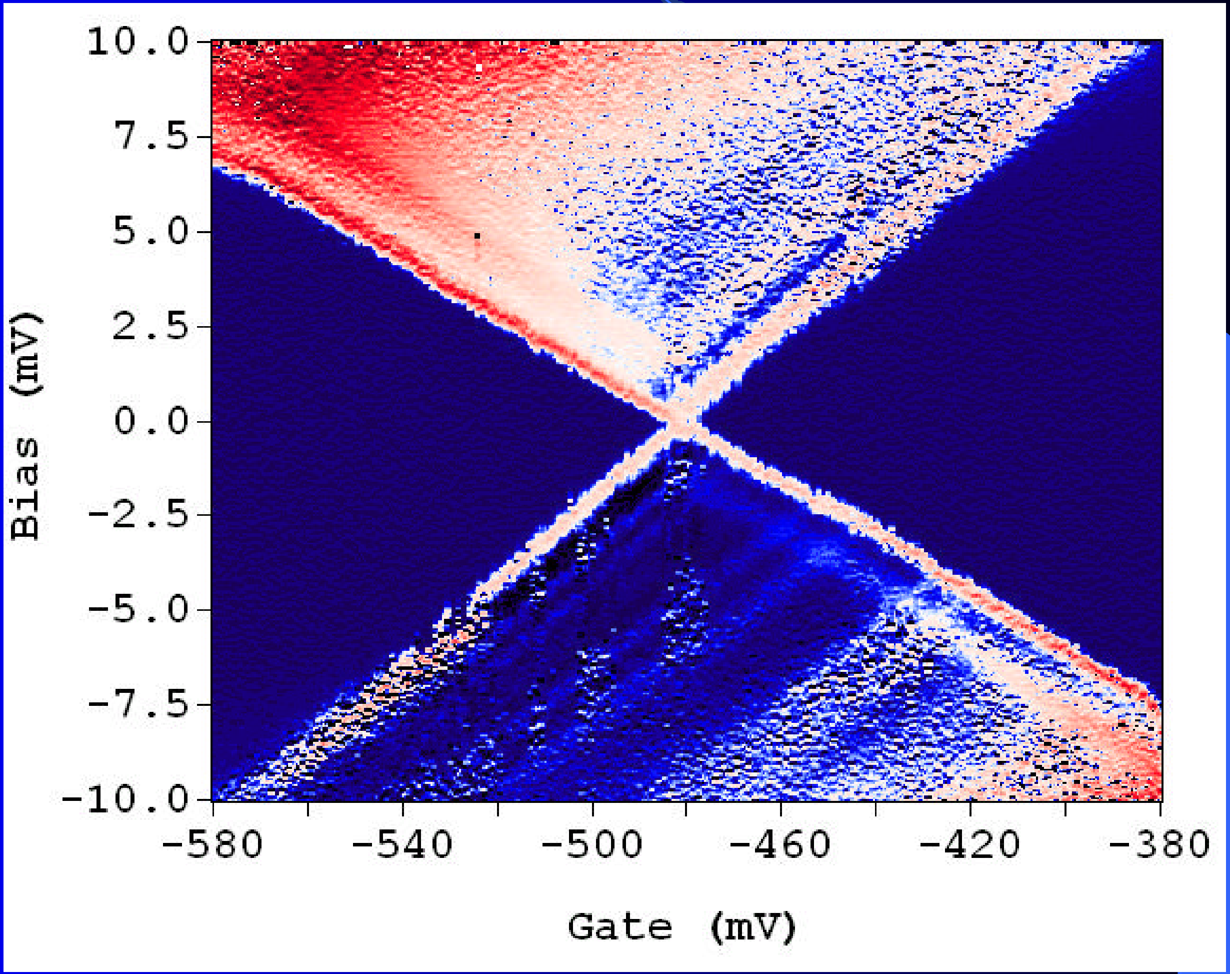


After under-etching



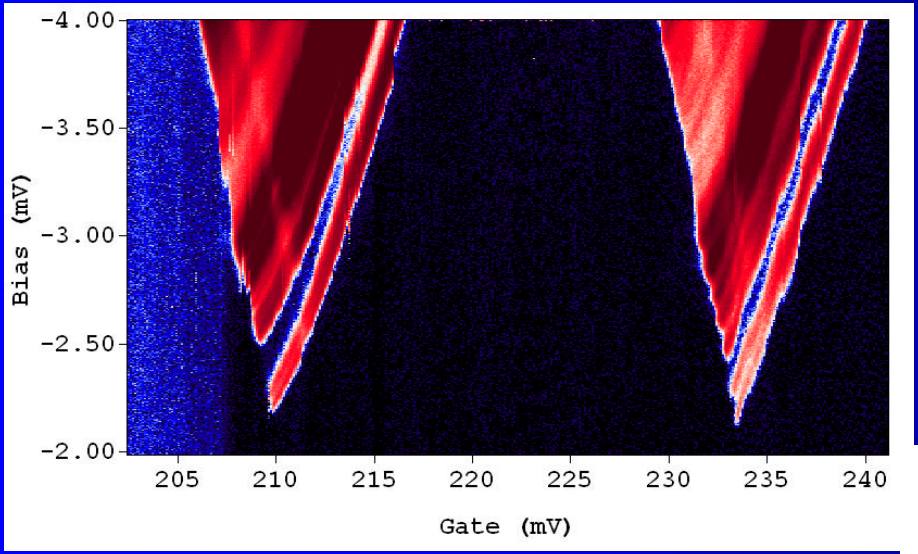
Stability diagram – Delft experiments

P. Jarillo-Herrero *et al*, in progress   L=140nm, T=300mK



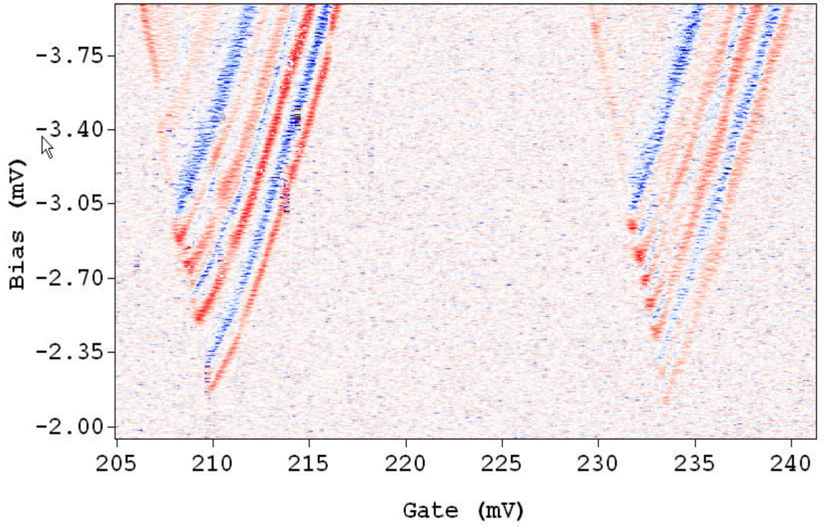
Stability diagram – Delft experiments

P. Jarillo-Herrero *et al*, in progress



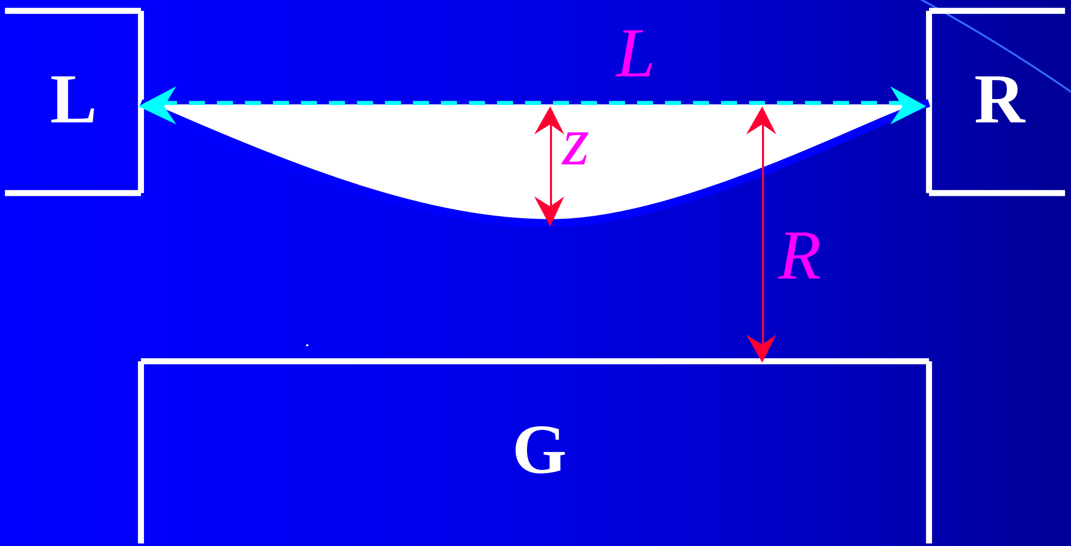
ln *I*

dI / *dV*



Inelastic tunneling?

Modeling suspended nanotubes

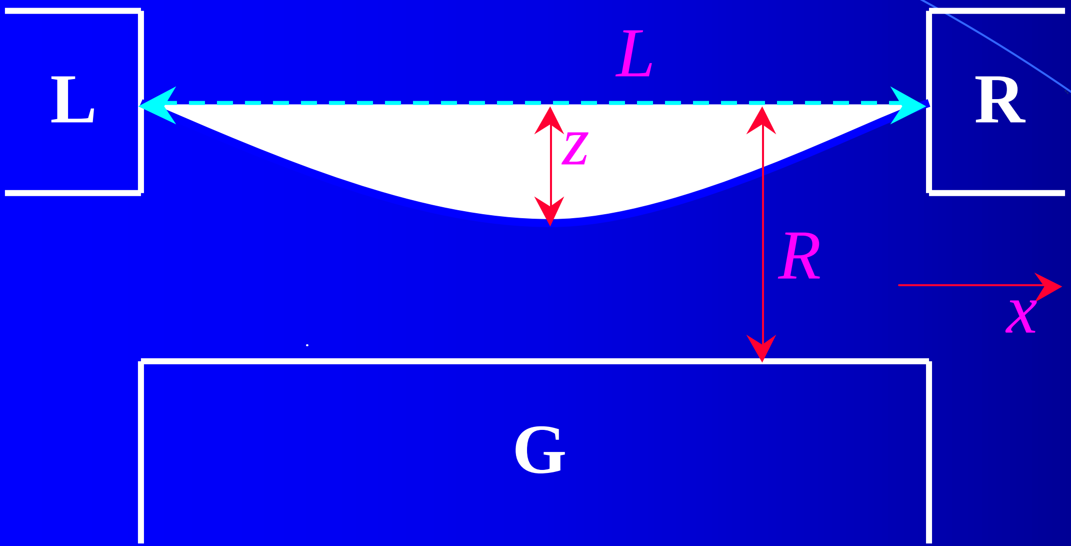


Sapmaz, Y.M.B., Gurevich, van der Zant '02

Features of the model:

- Interaction effects taken into account via charging energy;
- Mechanical degrees of freedom via classical theory of elasticity; nanotube modeled as an elastic rod

Elastic energy



Elastic modulus

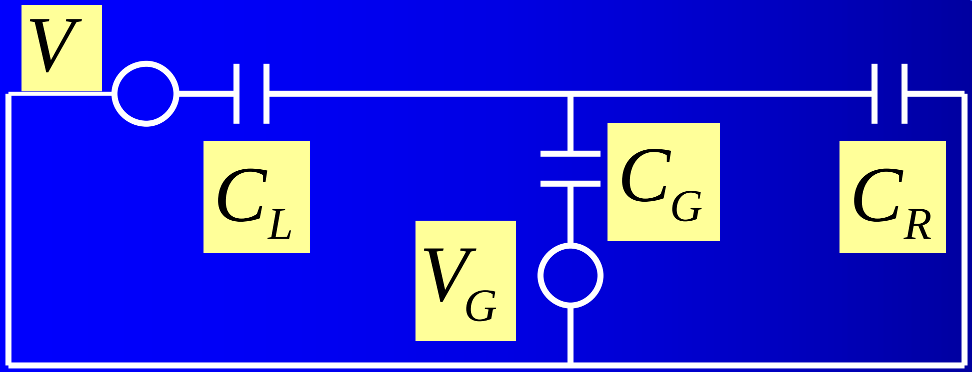
$$W_{el}\{z(x)\} = \int_0^L dx \left\{ \frac{EI}{2} z''^2 + \frac{T}{2} z'^2 \right\}$$

Residual stress

$$T = T_0 + \frac{ES}{2L} \int z'^2 dx$$

Stress induced by bending

Electrostatic energy



$$C_L, C_R \ll C_G$$

$$C_G = \int \frac{dx}{2\ln\dfrac{2(R - z[x])}{r}}$$

$$W_{e-st}\{z[x]\} = \frac{(ne)^2}{2C_G\{z\}} - neV_G \approx \frac{(ne)^2}{2C_0} - neV_G$$

$$- \frac{(ne)^2}{L^2 R}\int z[x]\,dx$$

Electrostatic force

Displacement

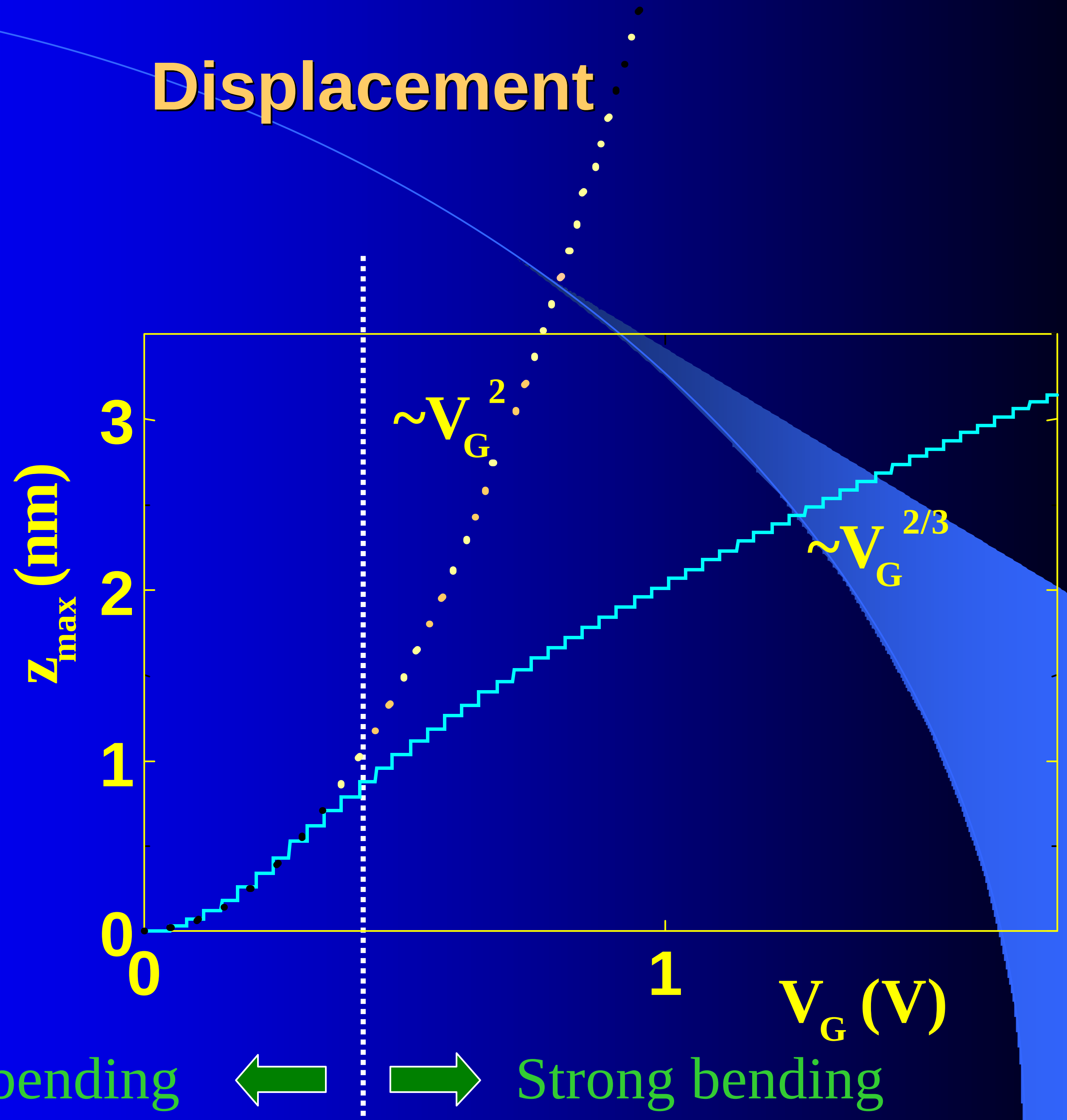
Equations of motion: $$IEz'''' - Tz'' = K_0 \equiv \frac{(ne)^2}{L^2 R}$$

1. To be solved for the displacement;
2. Stress to be found self-consistently

Results: Two regimes

- Weak bending: $z_{\max} < r,\ z_{\max} \propto V_G^2$
- Strong bending: $z_{\max} > r,\ z_{\max} \propto V_G^{2/3}$

Displacement



$L = 500\text{nm}$

$R = 100\text{nm}$

$C_L = C_R = 0$

Weak bending ⇦ ⇨ Strong bending

Eigenmodes

Equation:

$$\cosh y_1 \cos y_2 - \frac{1}{2}\frac{y_1^2 - y_2^2}{y_1 y_2}\sinh y_1 \sin y_2 = 1$$

$$y_{1,2} = \frac{L}{\sqrt{2}}\sqrt{\sqrt{x^2 + l^2} \pm x^2}\,;\, x = \sqrt{\frac{T}{EI}}\,;\, l = \sqrt{\frac{4rS}{EI}w}$$

Results for the fundamental mode

Weak bending: $w_0 = \sqrt{\frac{EI}{rS}}\frac{22.38}{L^2} + (\ldots)V_G^4$

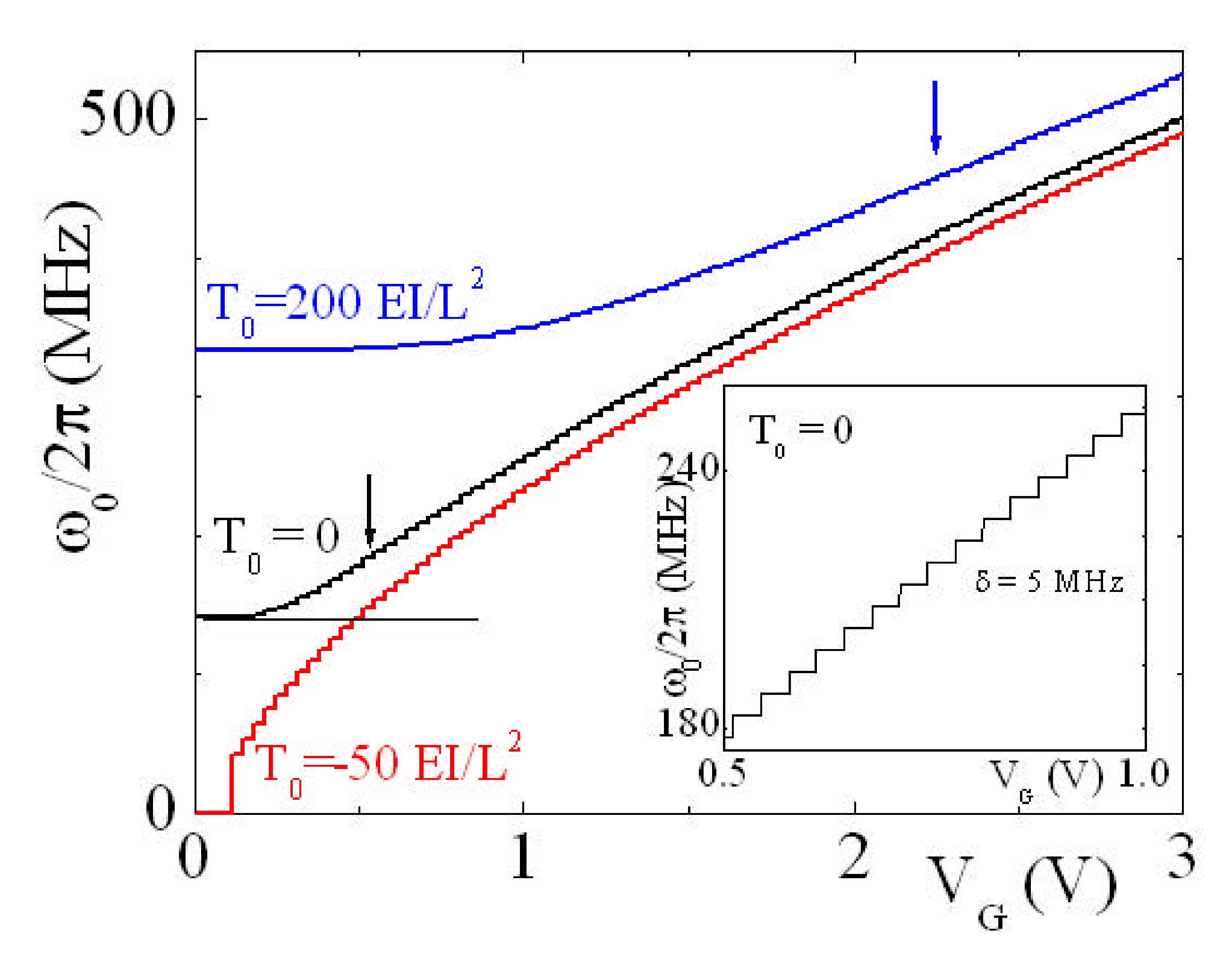
Strong bending: $w_0 \propto V_G^{2/3}$

Eigenmodes

$L = 500\text{nm}$

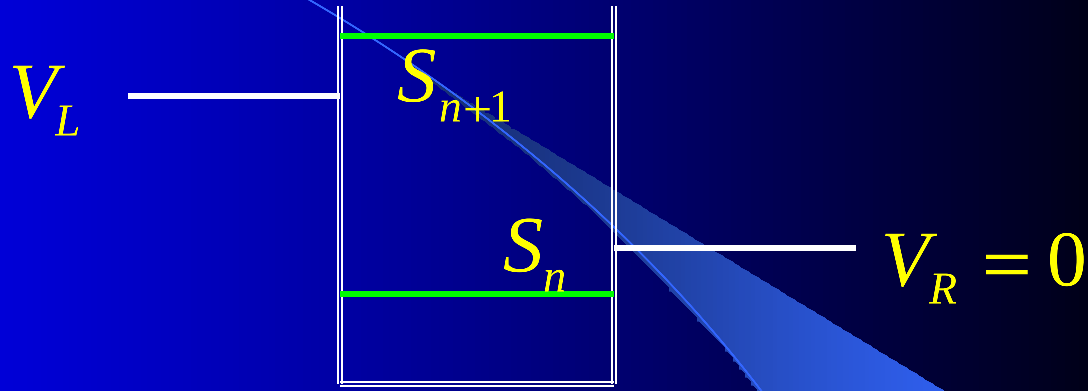
$R = 100\text{nm}$

$C_L = C_R = 0$



Stability diagram for quantum dots

$$S_n = W_{n+1} - W_n$$
$$\approx (n + \frac{1}{2}) \frac{e^2}{C_G} - eV_G$$



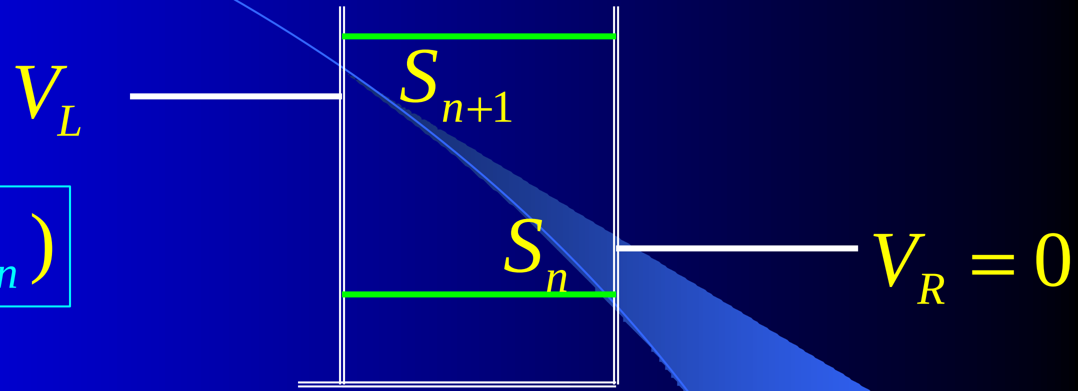
Conditions that current is not flowing:

(a) $S_{n+1} > eV_L$ (b) $S_n < eV_L$

(c) $S_{n+1} > 0$ (d) $S_n < 0$

Linear dependence ⟹ Coulomb diamonds

Stability diagram for suspended nanotubes



$$S_n = W_{n+1}(z_{n+1}) - W_n(z_n)$$

Conditions that current is not flowing: Non-linear

Consequences:

- “Curvilinear diamonds”
- Diamonds become smaller with increasing the charge

Magnitude of the effect: small

Probably not observable experimentally

Transport

Suspended nanotubes: damped oscillators

Quality factor: $Q = g / w_0 \square 10^2 \div 10^3$ (Reulet et al '00)

Weakly damped oscillators!

"Stationary regime": an external force induces oscillations with the frequency of the fundamental mode.

$$q = W \sin w_0 t$$

Transport

Usmani & Y.M.B., in preparation

$$C_L = C_R = 0$$

Two charge states: n=0 & n=1

Time-dependent tunnel rates

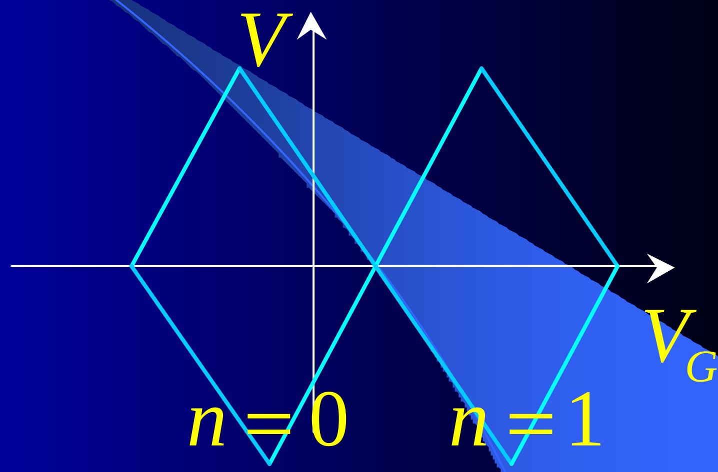
Time-dependent occupation probabilities

Stochastic force due to switching between the states

Energy dissipation over the period

Amplitude found self-cons.

Current



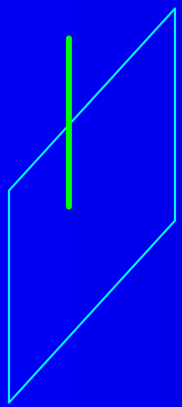
Tunnel rates

$$C_L = C_R = 0$$

$$\Gamma_L^{0\to 1}(t) = \frac{1}{e^2 R}\left[\frac{e^2}{2C_G} + eV_G - aW\sin w_0 t\right] q\left[\frac{e^2}{2C_G} + eV_G - aW\sin w_0 t\right]$$

$$\Gamma_R^{1\to 0}(t) = \frac{1}{e^2 R}\left[-\frac{e^2}{2C_G} + e(V - V_G) + aW\sin w_0 t\right] q\left[-\frac{e^2}{2C_G} + e(V - V_G) + aW\sin w_0 t\right]$$

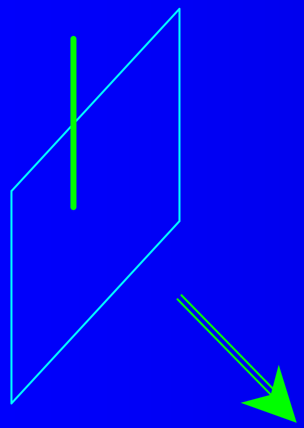
$$\Gamma_L^{1\to 0}(t) = \Gamma_R^{0\to 1}(t) = 0$$



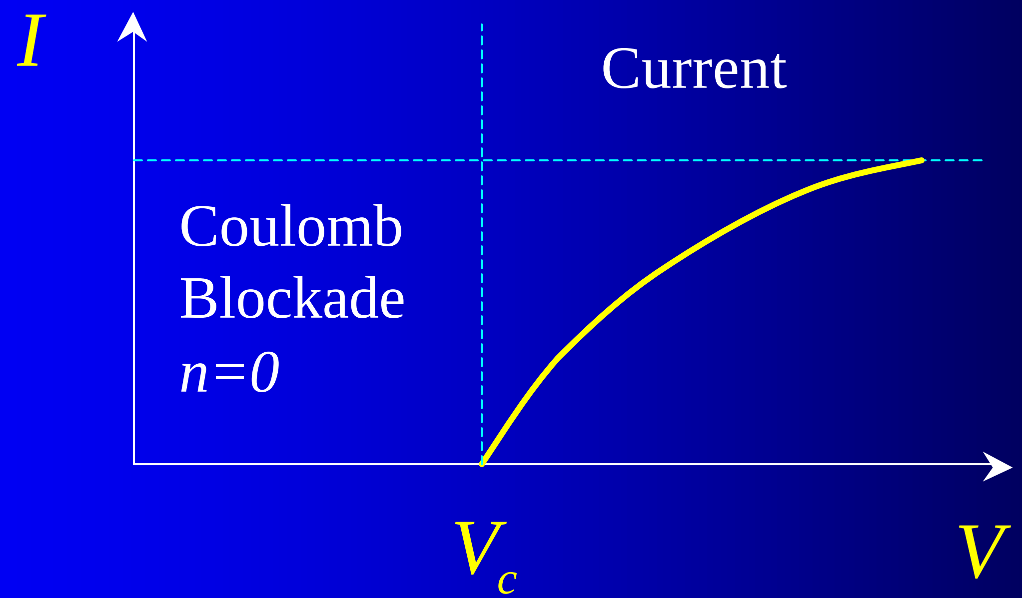
$$a = \frac{e^2}{LR}$$ - force for *n=1*

Tunnel rates

$$a = \frac{e^2}{LR}$$ - force for *n=1*

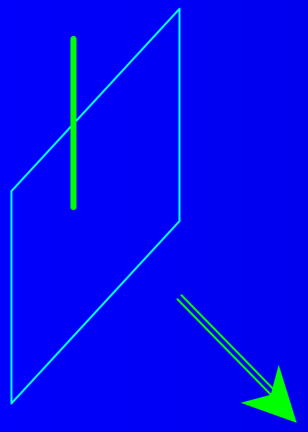


Without mechanical oscillations:

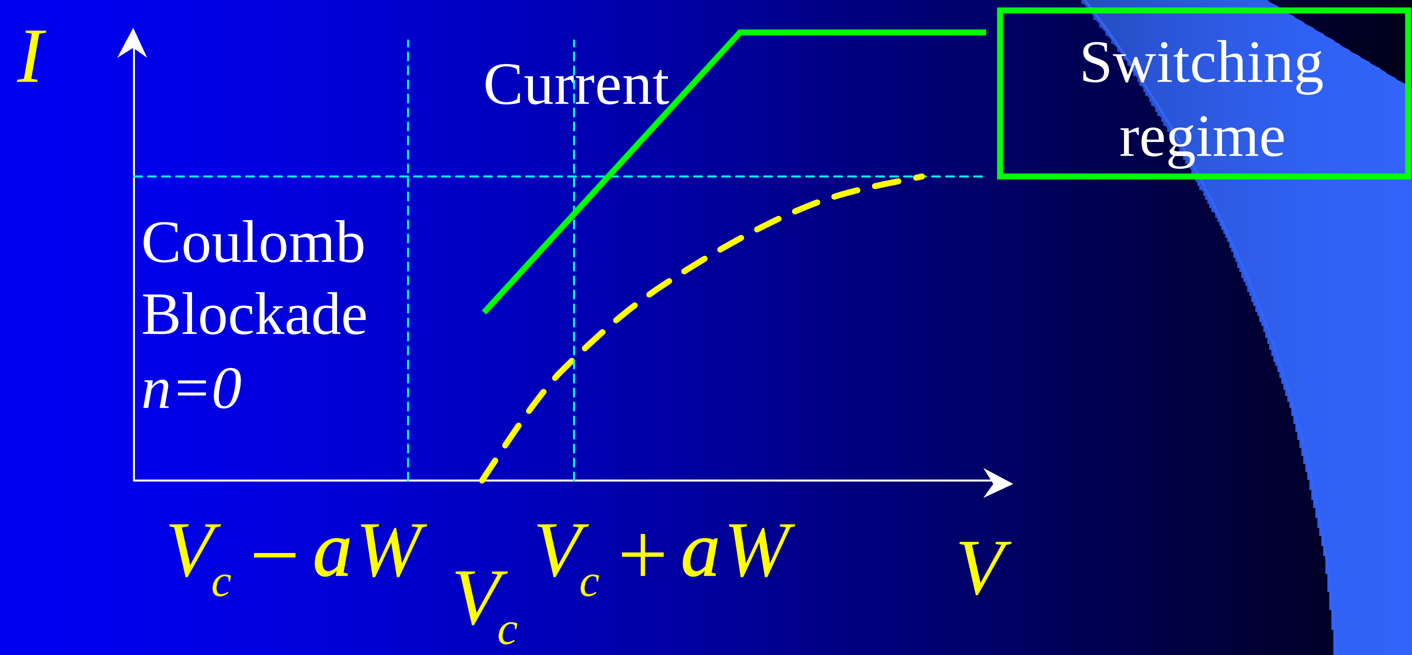


Tunnel rates

$$a = \frac{e^2}{LR}$$ - force for $n=1$



With mechanical oscillations:



Occupation probabilities

$P_1(t)$ - probability to have charge *e*

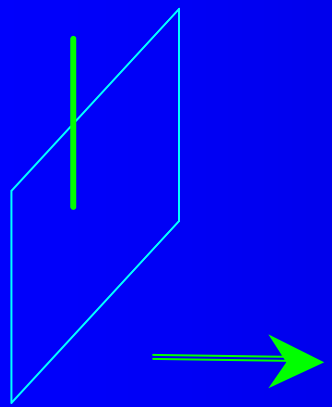
$P_0(t) = 1 - P_1(t)$ - probability to have charge *0*

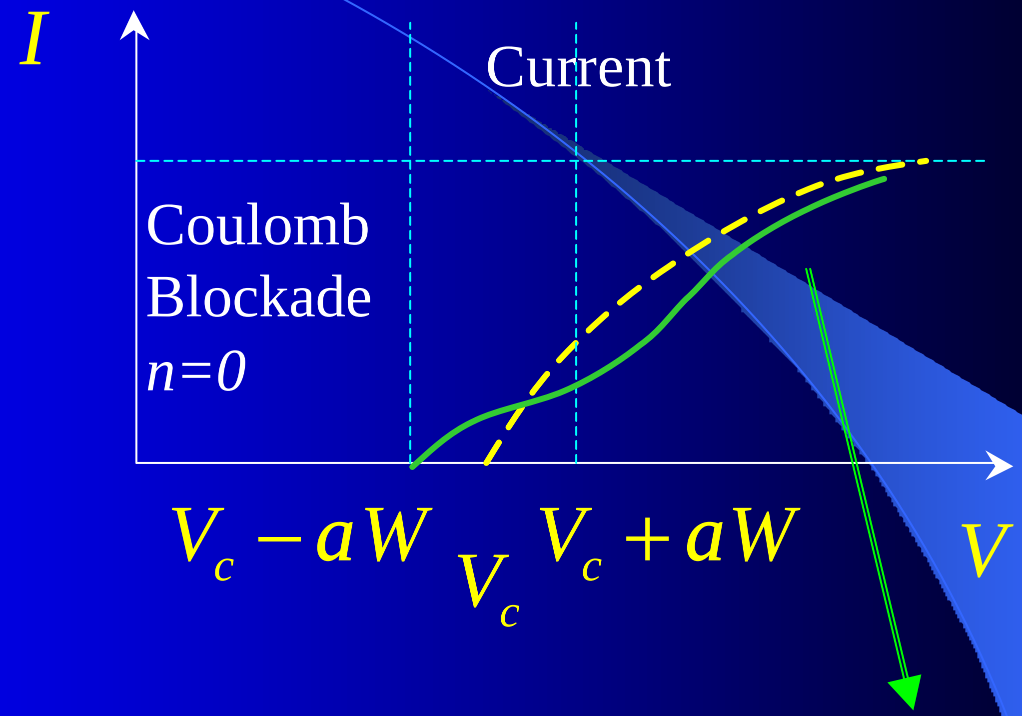
$$\frac{dP_1}{dt} = \Gamma_L^{0\to 1}(t) P_0(t) - \Gamma_R^{1\to 0}(t) P_1(t)$$

Current:

$$I = \frac{e}{2}\left[\Gamma_R^{1\to 0} P_1 + \Gamma_L^{0\to 1} P_0\right]$$

Current





$$dI \propto -\frac{Q}{w_0^3}\left(\frac{\Gamma}{\Gamma^2+w^2}\right)^2;\ \Gamma=\frac{V}{eR}$$

Conclusions

- Displacement: gate-voltage dependent; proceeds in steps
- Eigenmodes: modulated by the gate; steps; sensitive to the residual stress
- Insignificant effects of the mechanical degrees of freedom on the ground state energy (Coulomb diamonds) *vs* considerable influence on the current in non-equilibrium situation.