Mean-Field Phase Diagram of Two-Dimensional Electrons with Disorder in a Weak Magnetic Field

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Outline

1. Introduction

2. The Landau expansion of free energy of the triangular and unidirectional charge density wave states

3. Mean-field phase diagram

4. Weak crystallization corrections to the mean-field results

5. Comparison with experiment

6. Conclusions

The system considered is

- 2D interacting electrons in a weak perpendicular magnetic field with the filling factor $\nu \gg 1$ and in the presence of the quenched disorder.

The problem discussed is

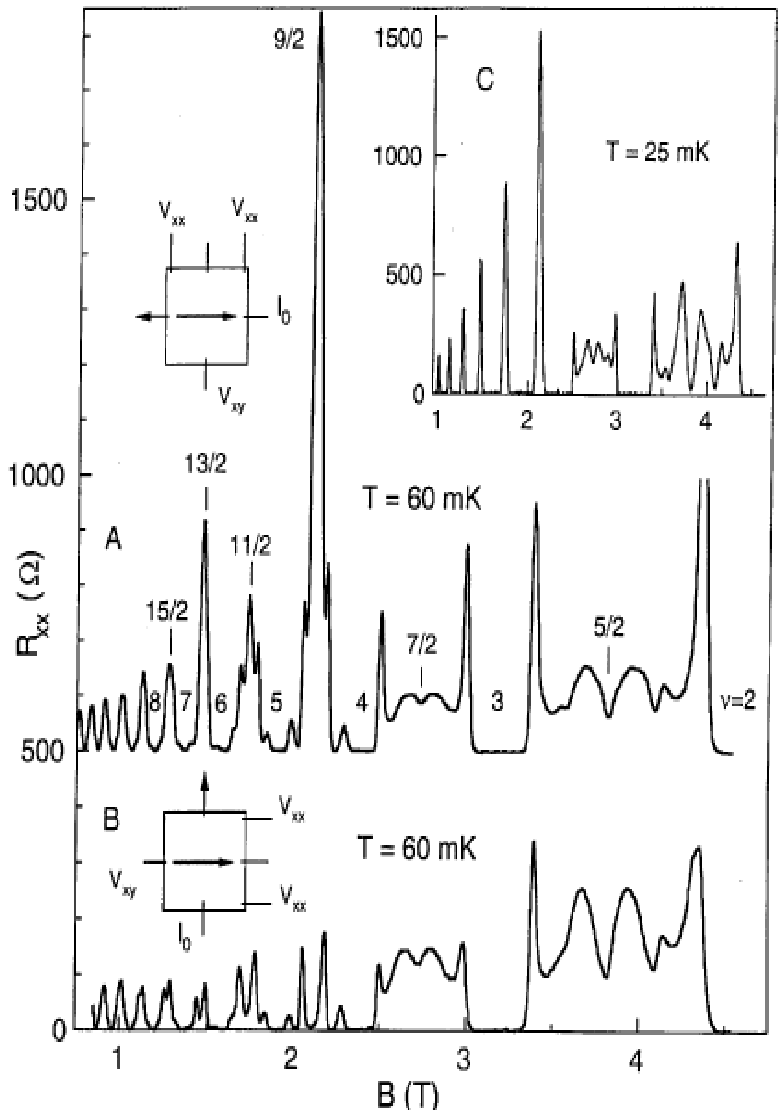
- the mean-field phase diagram for the partially filled highest N-th Landau level where $N = \left[\frac{\nu}{2}\right] \gg 1$.

A clean case was considered in

- A.A. Koulakov, M.M. Fogler, and B.I. Shklovskii, Phys. Rev. Lett. **76**, 499 (1996), Phys. Rev. B. **54**, 1853 (1996)
- R. Moessner and J.T. Chalker, Phys. Rev. B **54**, 5006 (1996)

A possible manifestation of CDW state was observed by

- M.P. Lilly et.al., Phys. Rev. Lett. **82**, 394 (1999)
- R.R. Du et.al., Solid State Commun. **109**, 389 (1999)



$$\frac{1}{\tau_0} \ll \omega_H$$

$$\frac{1}{\tau_0} \sim T$$

- We assume disorder potential to be short-range

$$\langle V_{dis}(\vec{r}_1) V_{dis}(\vec{r}_2)\rangle = \frac{1}{2\pi\rho\tau_0}\delta(\vec{r}_1 - \vec{r}_1)$$

Here ρ is thermodynamical density of states and τ_0 is the elastic collision time in the absence of magnetic field.

- The broadening of the N-th Landau level in the presence of magnetic field is given by

$$\frac{1}{\tau} = \frac{1}{\tau_0}\sqrt{\frac{\omega_H \tau_0}{\pi}} \ll \omega_H, \qquad \omega_H \tau_0 \gg 1$$

where $\omega_H = \dfrac{eH}{m}$ is the cyclotron frequency.

Lengths in the problem

$a_B = \varepsilon / m e^2$ is Bohr radius

$l_H = 1/\sqrt{m\omega_H}$ is magnetic field length

$R_c = \sqrt{\nu}\, l_H$ is cyclotron radius

$l_{el} = R_c \omega_H \tau_0$ is the mean free path

we assume $a_B \ll l_H \ll R_c \ll l_{el}$

Energies in the problem

ω_H is cyclotron frequency

μ_N is chemical potential

$1/\tau$ is broadening of Landau level

e^2/R_c is characteristic scale of e-e interaction

T is temperature

we assume $e^2/R_c\,,\ 1/\tau\,,\ T\,,\ \mu_N \ll \omega_H$

• The screened electron-electron interaction on the N-th Landau level

$$U(q) = \frac{2\pi e^2}{\varepsilon q} \frac{1}{1 + \frac{2}{q a_B}\left(1 - \frac{\pi}{6\omega_H \tau}\right)\left(1 - \mathcal{J}_0^2(qR_c)\right)},$$

$$\frac{1}{\omega_H \tau} \ll 1, \qquad N^{-1} \ll r_s \ll 1$$

takes into account the effects of interactions with electrons on the other Landau levels. $\mathcal{J}_0(x)$ stands for the Bessel function of the first kind.

the clean case $[1/\tau = 0]$

I.V. Kukushkin, S.V. Meshkov, and V.B. Timofeev, Usp. Fiz. Nauk **155**, 219 (1988)
I.L. Aleiner and L.I. Glazman, Phys. Rev. B **52**, 11296 (1995)

the weakly disordered case $[\omega_H \tau \gg 1]$

I.S. Burmistrov, JETP **95**, 132 (2002)

• The action is given by

$$\mathcal{S} = - \int_{\mathbf{r}} \sum_{\alpha=1}^{N_r} \sum_{\omega_n} \Bigg\{ \overline{\psi^{\alpha}_{\omega_n}}(\mathbf{r}) \Big[i\,\omega_n + \mu - \mathcal{H}_0 - V_{dis}(\mathbf{r}) \Big] \psi^{\alpha}_{\omega_n}(\mathbf{r}) - \frac{T}{2} \sum_{\omega_m, \nu_l} \int_{\mathbf{r}'} \overline{\psi^{\alpha}_{\omega_n}}(\mathbf{r}) \psi^{\alpha}_{\omega_n - \nu_l}(\mathbf{r}) U(\mathbf{r}, \mathbf{r}') \overline{\psi^{\alpha}_{\omega_m}}(\mathbf{r}) \psi^{\alpha}_{\omega_m + \nu_l}(\mathbf{r}') \Bigg\}.$$

• The CDW ground state is characterized by the order parameter $\Delta(\mathbf{q})$ that is related to the electron density as

$$\rho(\mathbf{q}) = L_x L_y n_L F_N(q) \Delta(\mathbf{q}).$$

Here $n_L = 1/2\pi l_H^2$, and the form-factor $F_N(q)$ is

$$F_N(q) = L_N \left(\frac{q^2 l_H^2}{2} \right) \exp\left(-\frac{q^2 l_H^2}{4} \right) \approx \mathcal{J}_0(qR_c), \qquad N \gg 1$$

where $L_N(x)$ is the Laguerre polynomial.

•After the Hartree-Fock decoupling of e-e interaction term and the average over disorder, the thermodynamic potential is given by

$$\Omega = \Omega_\Delta - \frac{T}{N_r} \ln \int \mathcal{D}[\hat{Q}] \exp(-\mathcal{S}[\hat{Q}]),$$

where

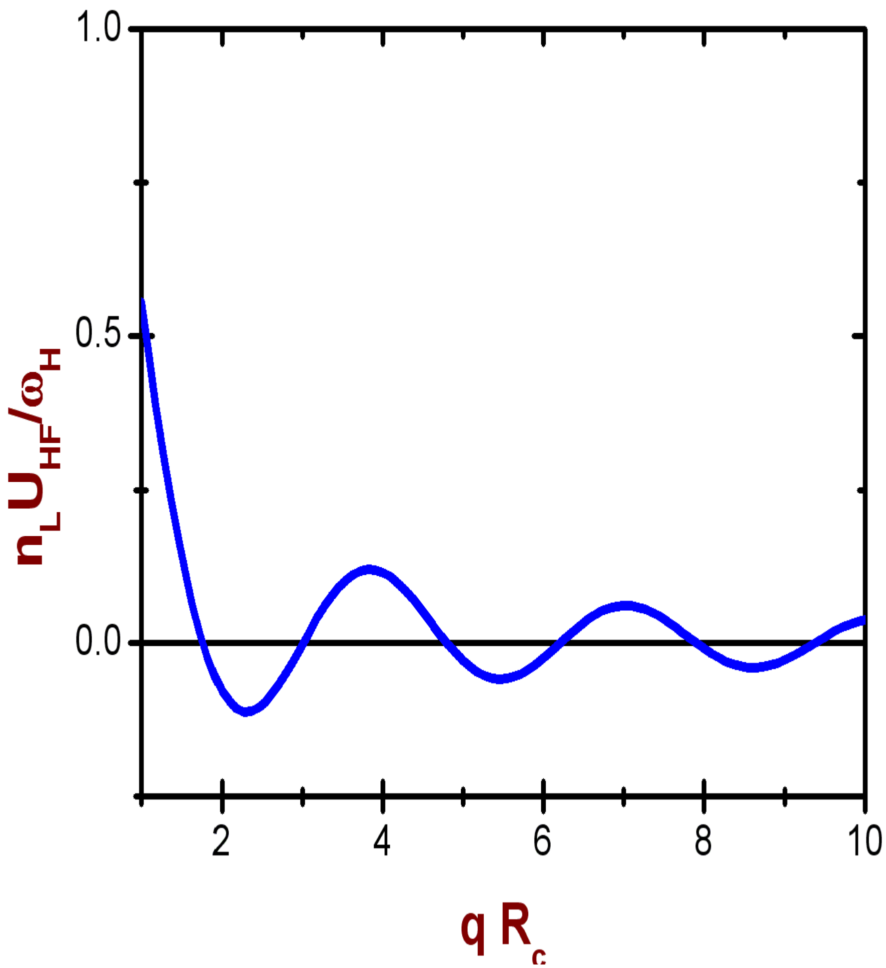
$$\Omega_\Delta = -\frac{(L_x L_y)^2 n_L^2}{2} \int_q U_{HF}(q) \boldsymbol{\Delta}(\mathbf{q}) \boldsymbol{\Delta}(-\mathbf{q}),$$

$$\mathcal{S} = -\frac{\pi \rho \tau_0}{2} \int_r \mathrm{tr}\, \hat{Q}^2 + \int_r \mathrm{tr} \ln \left(i\hat{\omega} + \mu - \mathcal{H}_0 + \lambda + i\hat{Q} \right),$$

$$\lambda(\mathbf{q}) = -L_x L_y n_L U_{HF}(q) F_N^{-1}(q) \boldsymbol{\Delta}(\mathbf{q}).$$

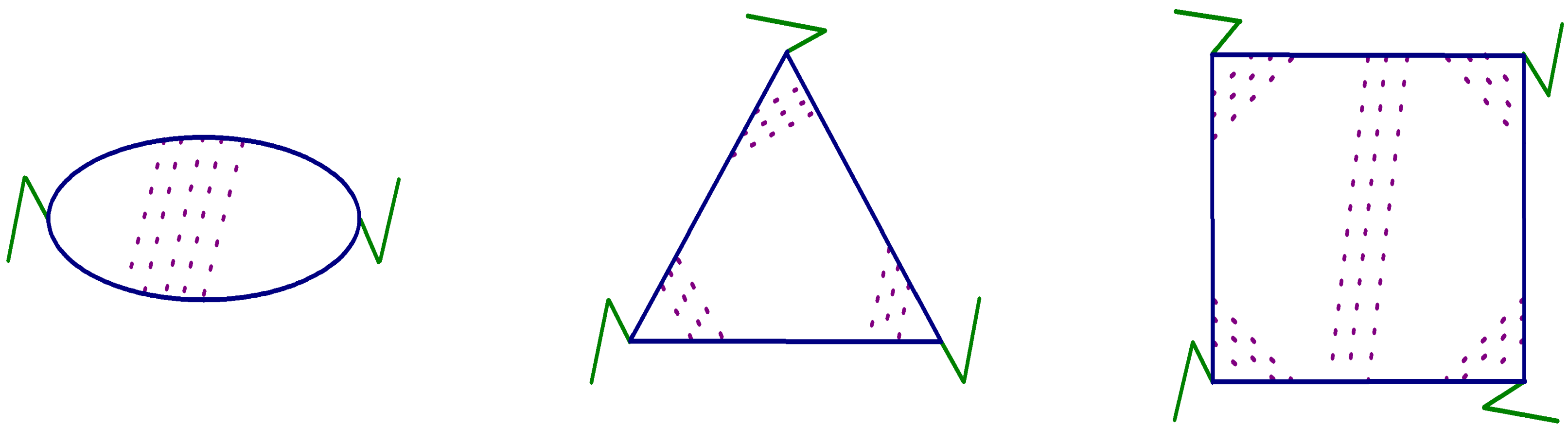
The Hartree-Fock potential is as follows

$$U_{HF}(q) = U(q) F_N^2(q) - l_H^2 \int \frac{d^2 p}{2\pi} U(\mathbf{p}) F_N^2(\mathbf{p}) \exp[-i\, \mathbf{q}\, \mathbf{p}\, l_H^2].$$



1.0
0.5
0.0
2
4
6
8
10
$n_L U_{HF}/\omega_H$
$q R_c$

• Expansion of Ω to the forth order in $\Delta(\mathbf{q})$ and integration over $\hat{Q}$ correspond to the following diagrams in the standard diagrammatic technique



Here solid blue line denotes averaged electron Green function, red dashes are impurity lines and solid green vertex stands for $\lambda(\mathbf{q})$.

• Crossed impurity lines $\Rightarrow \ln N/N \ll 1$

Free energy of the triangular CDW state

$$\mathcal{F}_t = \mathcal{F}_0 + 4L_xL_yn_LT_0(Q)\left[a_t\Delta^2(Q) + b_t\Delta^3(Q) + c_t\Delta^4(Q)\right]$$

where $T_0(Q) = -n_LU_{HF}(Q)/4$, and

$$a_t = 3\left[1 - \frac{T_0(Q)}{\pi^2 T}\sum_n \frac{1}{\xi_n^2 + \gamma^2(Q)}\right],\ b_t = i\,8\frac{T_0^2(Q)}{\pi^3T^2}\cos\left(\frac{\sqrt{3}Q^2}{4}\right)\sum_n \frac{\xi_n^3}{\left[\xi_n^2+\gamma^2(Q)\right]^3},$$

$$\begin{aligned} c_t &= \frac{24T_0^3(Q)}{\pi^4T^3}\left\{\frac{1}{2}\sum_n \frac{\xi_n^4}{\left[\xi_n^2+\gamma^2(Q)\right]^4}\left[3D_n(0) + \left(1+\cos\frac{\sqrt{3}Q^2}{2}\right)\left(D_n(Q)\right.\right.\right. \\ &+ \left.\left. D_n(\sqrt{3}Q)\right) + \frac{1}{2}D_n(2Q)\right] + 3\left[\sum_n \frac{\xi_n}{\left[\xi_n^2+\gamma^2(Q)\right]^2}\right]^2\left.\left[\sum_n \xi_n^{-2}\right]^{-1}\right\}, \end{aligned}$$

$$\xi_n = n + \frac{1}{2} + \frac{1}{4\pi T\tau} - i\,\frac{\mu_N}{2\pi T}, \quad \gamma(Q) = \frac{F_N(Q)}{4\pi T\tau}, \quad D_n(Q) = \frac{\xi_n^2 - \gamma^2(Q)}{\xi_n^2+\gamma^2(Q)}.$$

Free energy of the unidirectional CDW state

$$\mathcal{F}_u = \mathcal{F}_0 + 4L_xL_yn_LT_0(Q)\left[a_u\Delta^2(Q) + c_u\Delta^4(Q)\right]$$

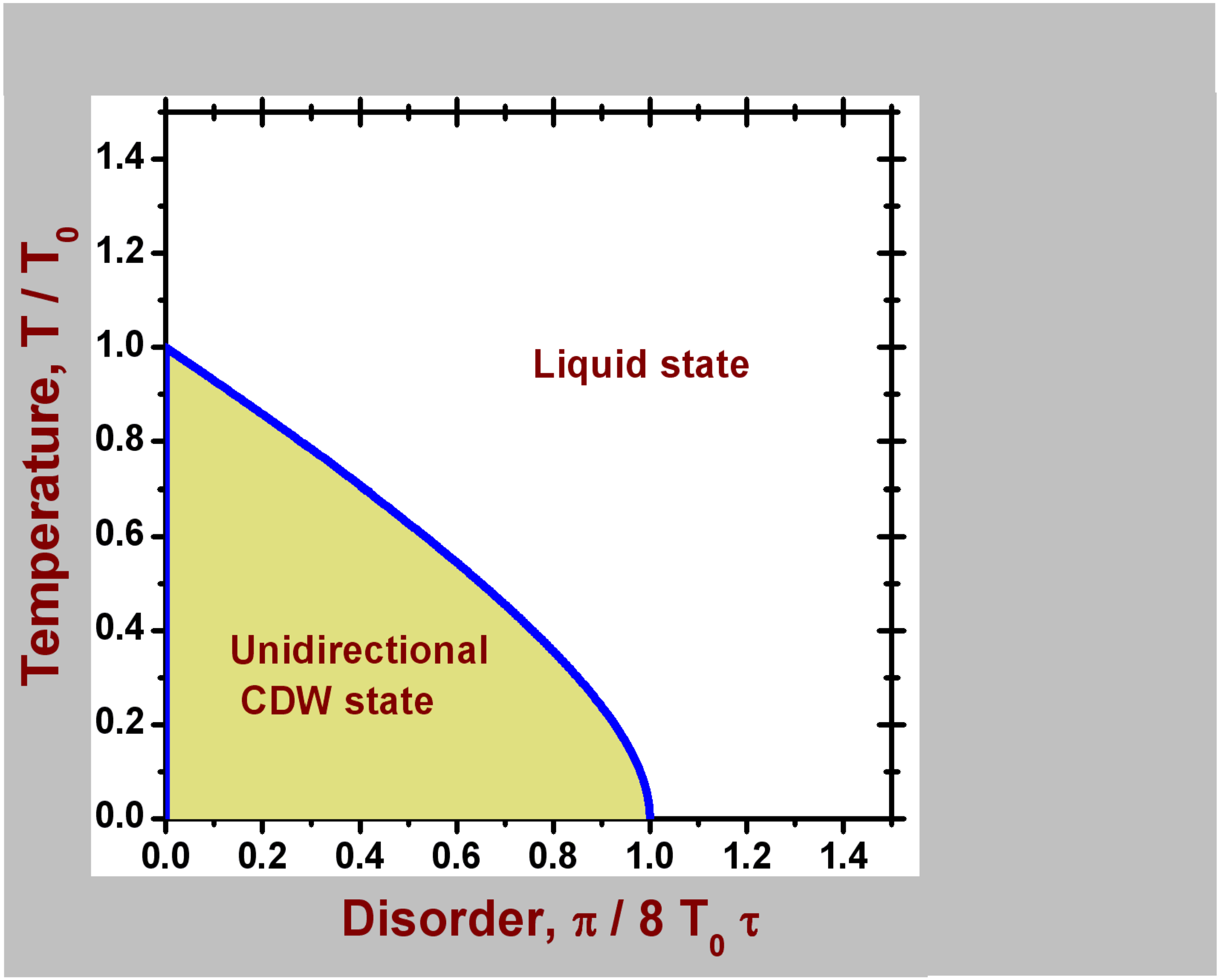
where $T_0(Q) = -n_LU_{HF}(Q)/4$, and

$$a_u = \left[1 - \frac{T_0(Q)}{\pi^2T}\sum_n \frac{1}{\xi_n^2 + \gamma^2(Q)}\right],$$

$$c_u = \frac{2T_0^3(Q)}{\pi^4T^3}\left\{\sum_n \frac{\xi_n^4\left[2D_n(0) + D_n(2Q)\right]}{\left[\xi_n^2 + \gamma^2(Q)\right]^4} + 4\left[\sum_n \frac{\xi_n}{\left[\xi_n^2 + \gamma^2(Q)\right]^2}\right]^2\left[\sum_n \xi_n^{-2}\right]^{-1}\right\}.$$

$$\xi_n = n + \frac{1}{2} + \frac{1}{4\pi T\tau} - i\,\frac{\mu_N}{2\pi T}, \quad \gamma(Q) = \frac{F_N(Q)}{4\pi T\tau}, \quad D_n(Q) = \frac{\xi_n^2 - \gamma^2(Q)}{\xi_n^2 + \gamma^2(Q)}$$

Phase diagram at half-filled Landau level ($\mu_N = 0$)



Phase diagram at half-filled Landau level ($\mu_N = 0$)

- The unidirectional CDW is created at the wave vector

$$Q_0 \approx 2.4 R_c^{-1}$$

which is independent on $1/\tau$.

- The temperature T of the second order phase transition (spinodal line) can be found from the equation

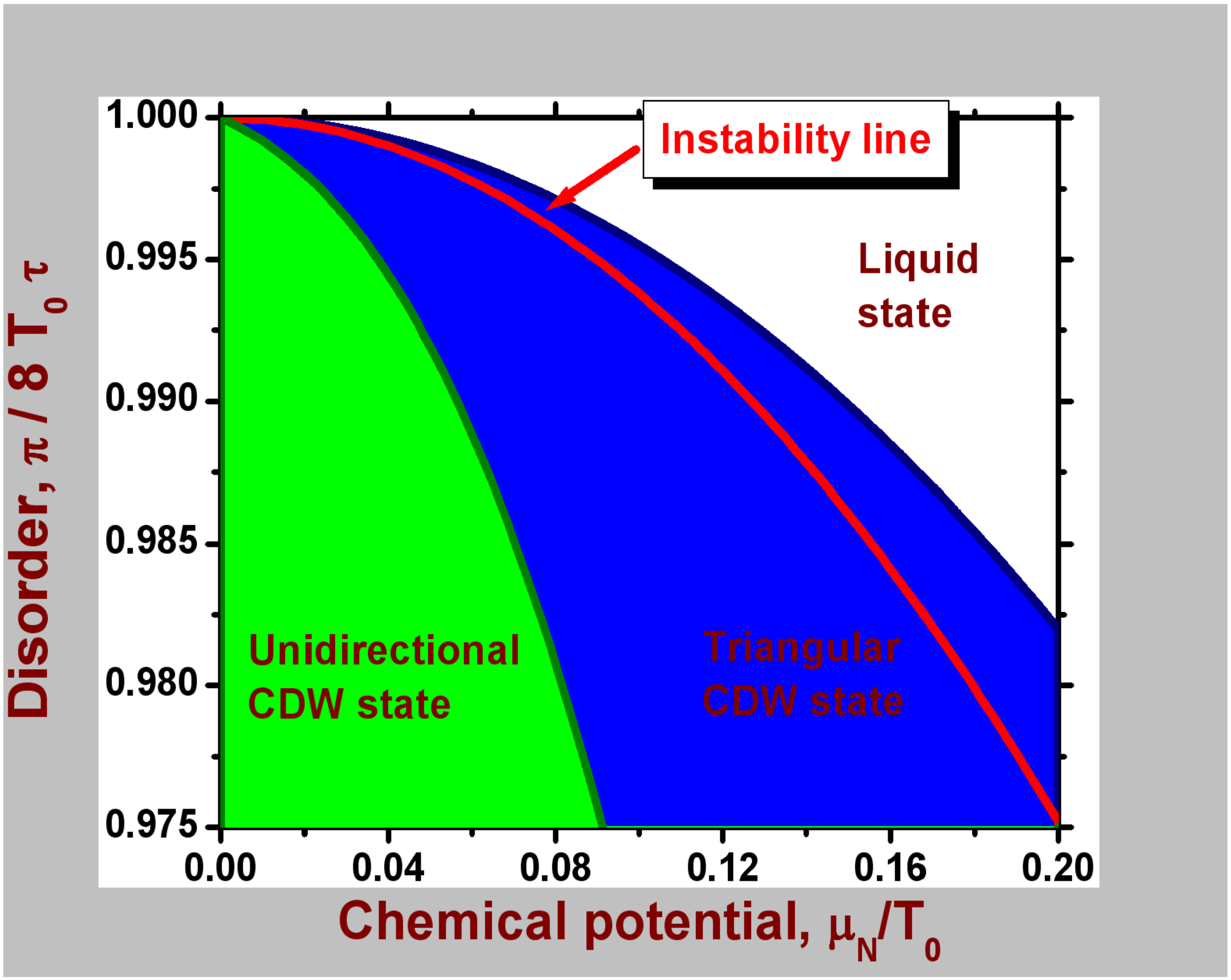
$$\frac{T}{T_0} = \frac{2}{\pi^2}\zeta\left(2, \frac{1}{2} + \frac{1}{4\pi T\tau}\right),$$

where $T_0 = T_0(Q_0)$ is the transition temperature in the clean case $1/\tau = 0$ and $\zeta(a, z)$ is generalized Riemann zeta function.

- The unidirectional CDW state exists only if disorder is rather weak

$$\frac{1}{\tau} \leq \frac{1}{\tau_c} = \frac{8T_0}{\pi}$$

Phase diagram at $T = 0$ near half-filling $\mu_N = 0$



Phase diagram at $T = 0$ near half-filling $\mu_N = 0$

The first order phase transition from the liquid state to the triangular CDW state occurs on the line

$$\frac{\pi}{8T_0\tau} = 1 - 0.45\frac{\mu_N^2}{T_0^2}, \qquad \mu_N \ll T_0,$$

the instability line is given by

$$\frac{\pi}{8T_0\tau} = 1 - 0.62\frac{\mu_N^2}{T_0^2}, \qquad \mu_N \ll T_0,$$

the first order transition from the triangular CDW state to the unidirectional CDW state occurs on the line

$$\frac{\pi}{8T_0\tau} = 1 - 2.84\frac{\mu_N^2}{T_0^2}, \qquad \mu_N \ll T_0.$$

NOTE: The triangular CDW state is created at the shifted wave vector

$$Q = Q_0 - 0.02(\mu_N\tau)^2 R_c^{-1}.$$

Weak crystallization corrections

Following S.A. Brazovskii [JETP **41**, 85 (1975)], we perform the following shift

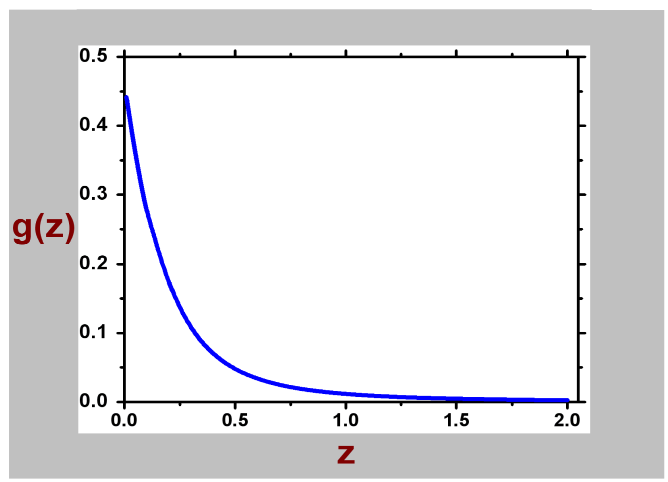
$$\Delta \to \Delta + \delta$$

and integrate out the fluctuations δ. This leads to the change in coefficients a_t, b_t, c_t and a_u, c_u. Now the transition temperature T at half-filling $\mu_N = 0$ can be found from the equation

$$\frac{T}{T_0} = \frac{2}{\pi^2}\zeta\left(2, \frac{1}{2} + \frac{1}{4\pi T\tau}\right)\left[1 - g\left(\frac{1}{4\pi T\tau}\right) N^{-2/3}\right],$$

Therefore, shift of the transition temperature

$$\frac{\delta T}{T} \propto \left(\frac{1}{N}\right)^{2/3} \ll 1$$



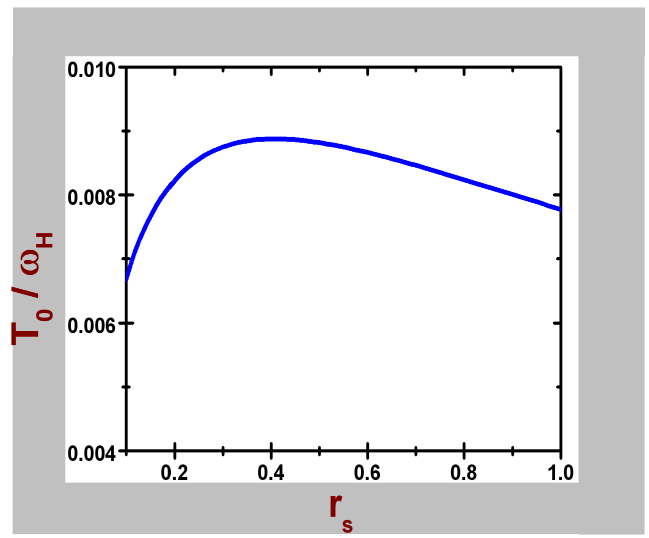
Comparison with experiment

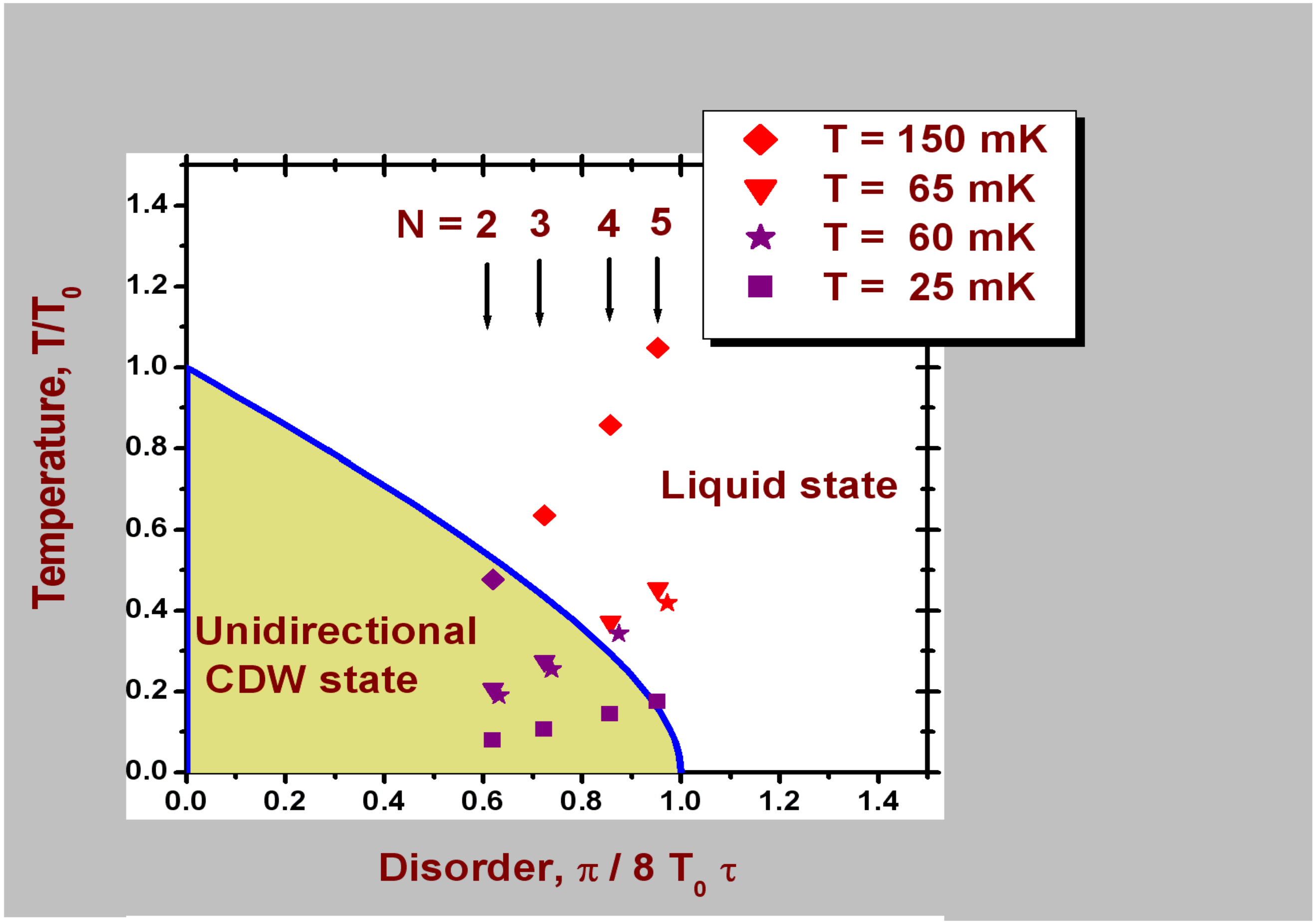
According to Koulakov, Fogler, and Shklovskii, temperature of instability in the clean case

$$T_0 = \frac{r_s \omega_H}{4\pi\sqrt{2}} \left[\ln\left(1 + \frac{0.3}{r_s}\right) - \frac{0.3}{\sqrt{2} + r_s} \right], \qquad N^{-1} \ll r_s \ll 1,$$

where $r_s = \sqrt{2}e^2/R_c\omega_H$. We can estimate T_0 and $1/\tau$ through mobility μ_0 and electron density n_e as follows

$$T_0 \simeq 0.008\,\omega_H, \ \frac{1}{\tau} \simeq \frac{\sqrt{2N}}{\pi}\sqrt{\frac{e}{\mu_0 n_e}}\,\omega_H.$$





The triangles, rhombi, and squares are extracted from the experimental data of M.P. Lilly et.al., where as stars after R.R. Du et.al.

Conclusions

- The mean-field CDW instability exists if the Landau level broadening $1/\tau \leq 1/\tau_c = 8T_0/\pi$.

- At half-filling $\mu_N = 0$ the unidirectional CDW state appears, and the presence of disorder does not change the vector of the CDW state.

- Near half-filling $\mu_N = 0$, the unidirectional CDW state is energetically more favorable than the triangular one.

- The weak crystallization corrections to the mean-field result are of the order of $(1/N)^{2/3} \ll 1$.