# Scattering Theory of Mesoscopic Detectors

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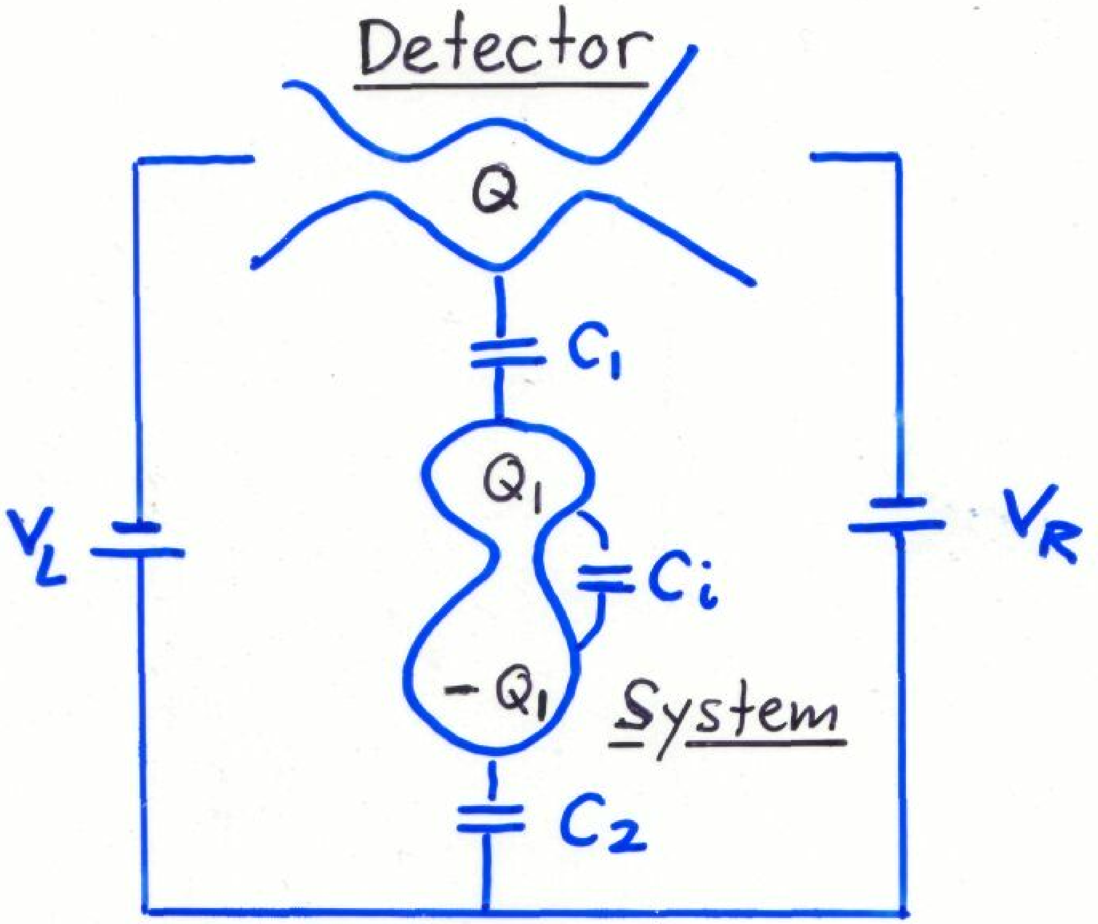
Efficiency of Mesoscopic Detectors
S. Pilgram and M. Büttiker
Phys. Rev. Lett. <u>89</u>, 11 Nov., (2002).

Quantum-Limited Measurement and
Information in Mesoscopic Detectors
A. A. Clerk, S.M. Girvin and A.D. Stone,
Phys. Rev. B <u>67</u>, 165324 (2003).

# The Model




- Conductance of detector sensitive to the state of the system
- Measurement implies decoherence of the system

- Role of screening in detector
- How does the geometry of the detector affect the speed and efficiency of the measurement process

# Related Works

## Experiments on Josephson Qubits

D. Vion et al. Science 296, 886 (2002)
Y. Yu et al. Science 296, 889 (2002)
Y. Nakamura et al., Nature 398, 357 (2002).

## Experiments on controlled dephasing

E. Buks et al., Nature 385, 417 (1997)
D. Sprinzak et al., Phys. Rev. Lett. 84, 5820 (2000)

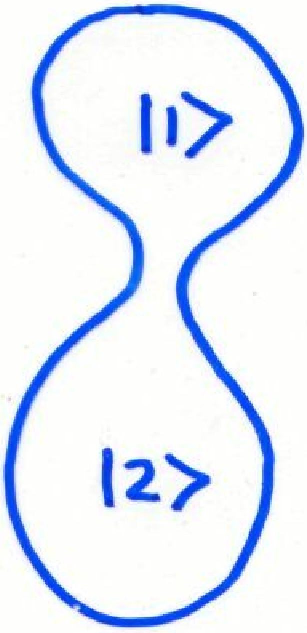
## Theory of Josephson Qubits

Y. Makhlin et al., Rev. Mod. Phys. 73, 357 (2001)

## Theory of controlled dephasing

R. A. Harris and Stodolsky, Phys. Lett. B116, 464 (1982)
I. L. Aleiner et al. Phys. Rev. Lett. 79, 3740 (1997).
S. A. Gurvitz, Phys. Rev. B 56, 15215 (1997)
Y. Levinson, Europhys. Lett. 39, 299 (1997)
M. Büttiker and A. Martin, Phys. Rev. B61, 2737 (2000)
D. V. Averin, quant-ph/000814; cond-mat/0004364
D. V. Averin and A. N. Korotkov, Phys. Rev. B64, 165310 (2002).

# The System




Density matrix

$$\hat{\rho} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

Hamiltonian

$$\hat{H} = \frac{1}{2} \begin{pmatrix} \varepsilon & \Delta \\ \Delta & -\varepsilon \end{pmatrix}$$

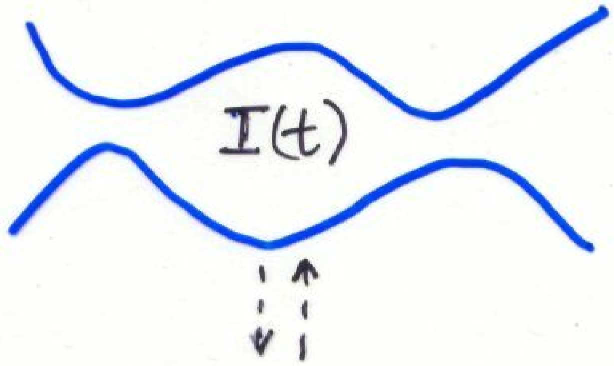
Master equation

$$\dot{\hat{\rho}} = -i\,[\hat{H}, \hat{\rho}] + L\hat{\rho}$$

Time evolution governed by three rates:

- Oscillation frequency $\Omega = \sqrt{\varepsilon^2 + \Delta^2}$
- Relaxation rate $\Gamma_{rel}$
- Decoherence rate $\Gamma_{dec}$

# The Detector

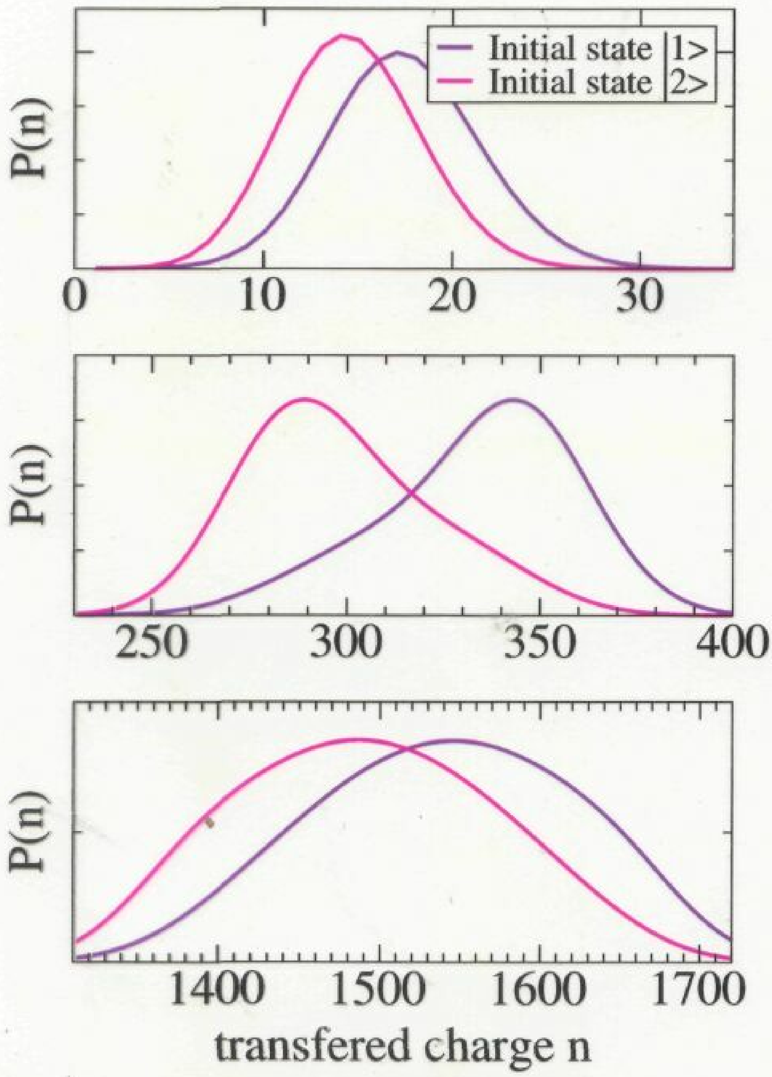



Probability
$P(n,t)$
that $n$ charges
passed in time $t$

Three time domains

- Noise dominated $\quad t < \tau_{meas}$
- Signal dominated $\quad \tau_{meas} < t < \Gamma_{rel}^{-1}$
- Relaxation dominated $\quad \Gamma_{rel}^{-1} < t$

# The Detector



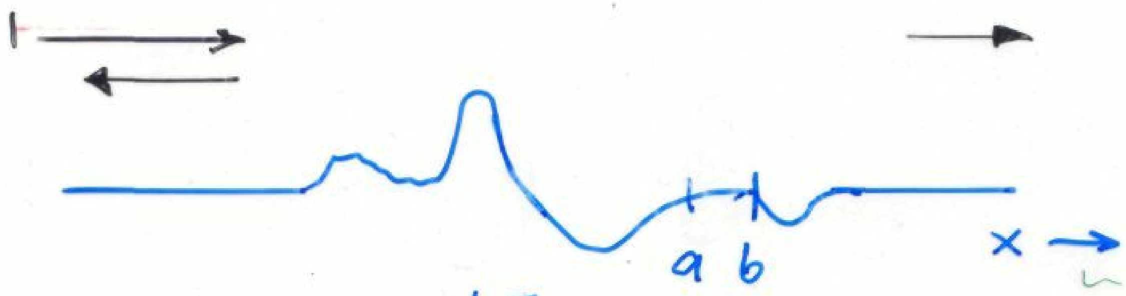

noise dominated

signal dominated

relaxation dominated

# Wave functions and the scattering matrix

A simple example:




$$eU(x) = \begin{cases} eU_0(x) + i\hbar\Gamma & x \in [a,b] \\ eU_0(x) & x \notin [a,b] \end{cases}$$

$$\frac{I_{abs}}{I} = \frac{\Gamma}{v} \int_a^b dx \, |\psi(x)|^2 = -\frac{\hbar\Gamma}{i} \int dx \sum_{\beta=1}^{2} S^{\dagger}_{\beta 1} \frac{\partial S_{\beta 1}}{\partial eU(x)}$$

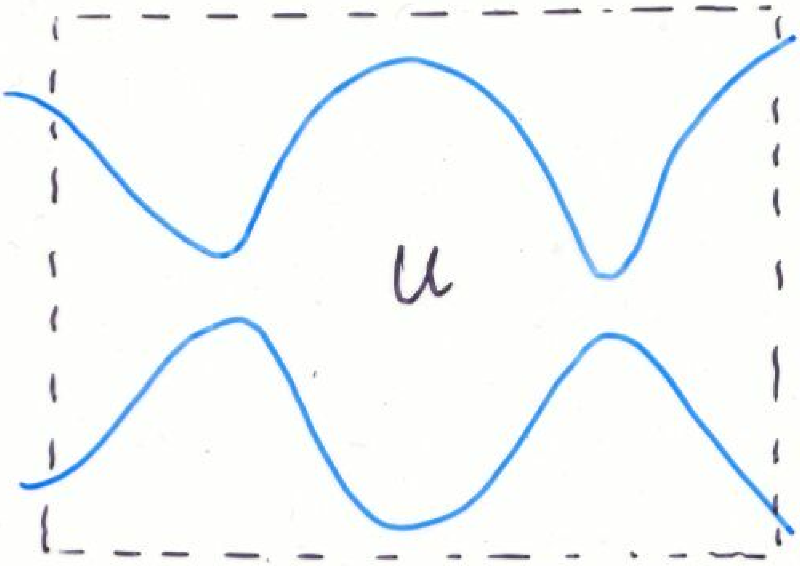
general connection

$$-\frac{1}{2\pi i} \sum_{\alpha} S^{\dagger}_{\alpha\gamma} \frac{\partial S_{\alpha\delta}}{\partial eU(r)} = \frac{1}{h(v_\delta v_\gamma)^{1/2}} \psi^*_\gamma(r) \psi_\delta(r)$$

"generalized" Wigner-Smith matrix

$$\boxed{N_{\gamma\delta}(r) = -\frac{1}{2\pi i} \sum_{\alpha} S^{\dagger}_{\alpha\gamma} \frac{\delta S_{\alpha\delta}}{\delta eU(r)}}$$

# Scattering Theory




$S_{\alpha\beta}(U)$

Example: block-diagonal matrix

$$S^{(n)} = \begin{pmatrix} -i\sqrt{R_n}\, e^{i(\phi_n+\phi_{A,n})} & \sqrt{T_n}\, e^{i(\phi_n-\phi_{B,n})} \\ \sqrt{T_n}\, e^{i(\phi_n+\phi_{B,n})} & -i\sqrt{R_n}\, e^{i(\phi_n-\phi_{A,n})} \end{pmatrix}$$

Transmission (and reflection) probabilities

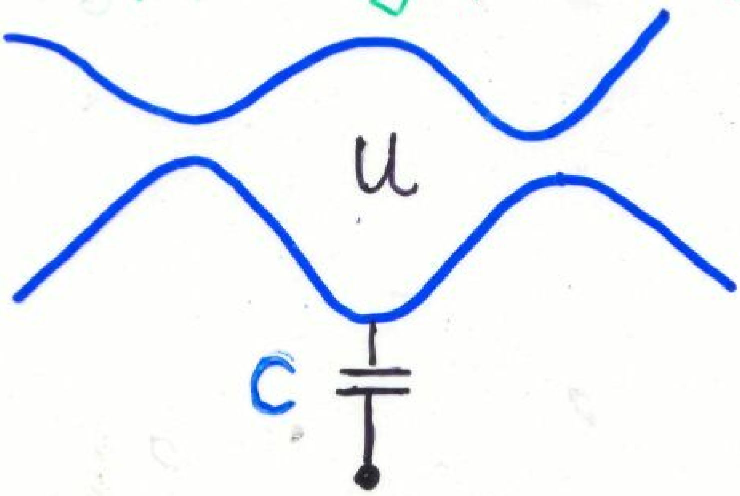
$$T_n(U) = 1 - R_n(U)$$

Scattering phases

$$\phi_n(U),\ \phi_{A,n}(U),\ \phi_{B,n}(U)$$

# Auxiliary Problem: Fluctuations at a Gate

Büttiker, Thomas, Prêtre, Phys. Lett. A180, 364 (1993)
Pedersen, van Langen, Büttiker, Phys. Rev. B57, 1838 (98)




$$S_{II}(\omega) = \omega^2 S_{QQ}(\omega) = \omega^2 C^2 S_{UU}(\omega)$$

$$= 2\omega^2 C_\mu^2 \left( R_q kT + R_V |eV| \right)$$

$$N_{\delta\gamma} = -\frac{1}{2\pi i} \sum_\alpha S^+_{\alpha\delta} \frac{dS_{\alpha\gamma}}{dU}$$

$$C_\mu^{-1} = C^{-1} + (e^2 \, Tr \, N)^{-1}$$

$$R_q = \frac{1}{2} \frac{h}{e^2} \frac{Tr \, N^2}{(Tr \, N)^2}$$

$$R_V = \frac{h}{e^2} \frac{Tr(N_{12} N_{21})}{(Tr \, N)^2}$$

- Three constants describe the detector

# Examples of Charge Relaxation Resistances

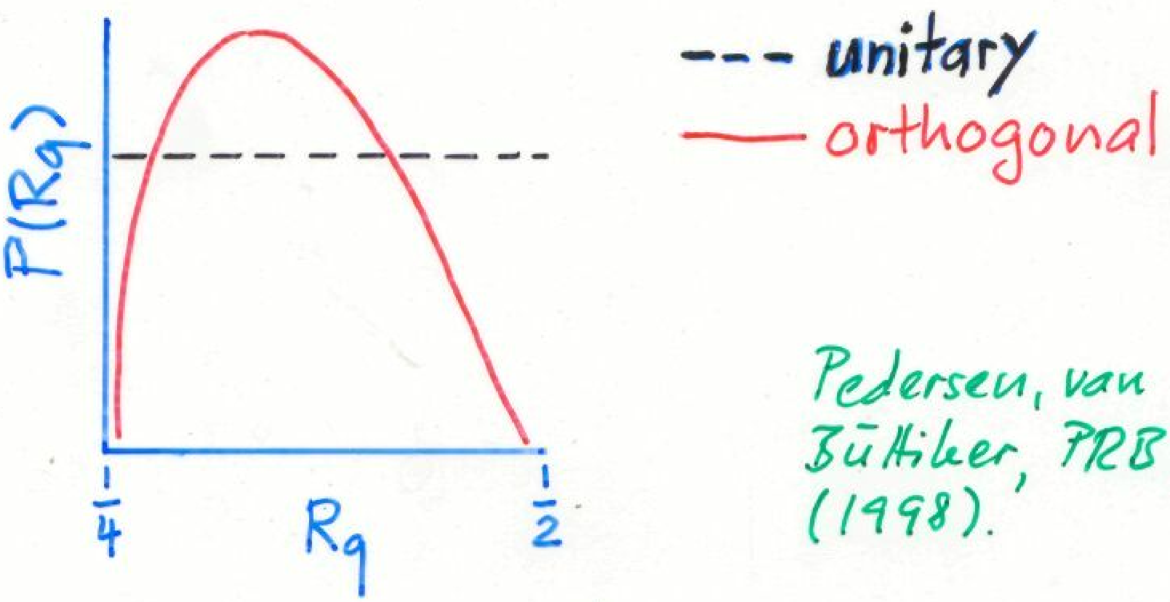
Ballistic spinless one-channel wire $h/4e^2$

Ballistic spinfull one-channel wire $h/8e^2$

Carbon Nanotube (metallic, perfect) $h/16e^2$

Chaotic cavity coupled via QPC's
$N_1 \gg 1$, $N_2 \gg 1$ $\frac{h}{e^2}\frac{1}{N_1+N_2}$

Chaotic cavity coupled via QPC's
$N_1 = N_2 = 1$




Pedersen, van Langen, Büttiker, PRB 57, 1838 (1998).

# $R_q$ and $R_v$ from the scattering matrix

$$S^{(n)} = \begin{pmatrix} -i\sqrt{R_n}\, e^{i(\phi_n+\phi_{A,n})} & \sqrt{T_n}\, e^{i(\phi_n-\phi_{B,n})} \\ \sqrt{T_n}\, e^{i(\phi_n+\phi_{B,n})} & -i\sqrt{R_n}\, e^{i(\phi_n-\phi_{A,n})} \end{pmatrix}$$

$$R_q = \frac{h}{2e^2} \frac{\sum_n \left(\frac{d\phi_n}{dU}\right)^2}{\left(\sum_n \frac{d\phi_n}{dU}\right)^2}$$

$$R_v = \frac{h}{e^2} \frac{\sum_n \left( \frac{1}{4R_nT_n}\left(\frac{dT_n}{dU}\right)^2 + T_nR_n\left(\frac{d\phi_{A,n}}{dU} - \frac{d\phi_{B,n}}{dU}\right)^2 \right)}{\left(\sum_n \frac{d\phi_n}{dU}\right)^2}$$

$$R_m = \frac{h}{e^2} \frac{\left(\sum_n \frac{dT_n}{dU}\right)^2}{\left(\sum_n \frac{d\phi_n}{dU}\right)^2 \; 4\sum_n R_nT_n}$$

# Relaxation and Decoherence Rate

Scattering theory
Screening (RPA)
Bloch-Redfield equations
$\Longrightarrow$

$$\Gamma_{rel} = 2\pi \frac{\Delta^2}{\Omega^2} \left(\frac{C_\mu}{C_i}\right)^2 R_q \frac{\Omega}{2} \coth \frac{\Omega}{2kT}$$

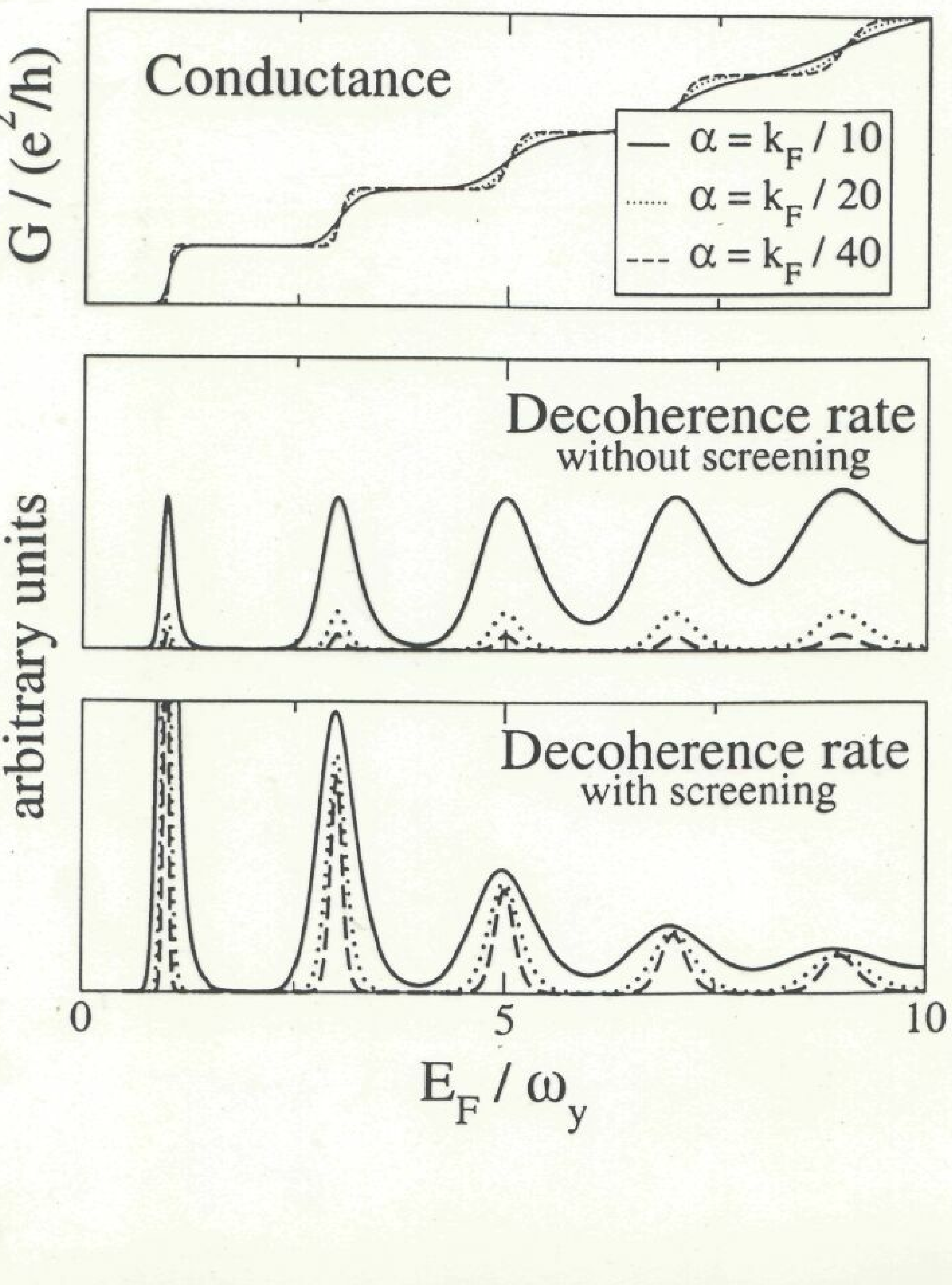
$$\Gamma_{dec} = \frac{1}{2} \Gamma_{rel} + 2\pi \frac{\varepsilon^2}{\Omega^2} \left(\frac{C_\mu}{C_i}\right)^2 \left(R_q kT + R_V e|V|\right)$$

Example:
$\Delta^2 \ll \varepsilon^2$, $\quad kT = 0 \quad \rightarrow \quad \Gamma_{dec} \propto R_V e|V|$

## Quantum point contact detector

S. Pilgram and M. Büttiker, Surface Science 532, 617 (2003).

$$V(y,z) = \frac{V_0}{\cosh^2 \alpha z} + \frac{1}{2} m \omega_y^2 y^2$$



# Measurement Time

Definition

- Signal $\quad \Delta Q = t\left(\langle I_{|1\rangle}\rangle - \langle I_{|2\rangle}\rangle\right) = t\,\Delta I$
- Noise $\quad \Delta Q = \sqrt{\left\langle \left(\int_0^t dt'\, \Delta\hat{I}(t')\right)^2 \right\rangle}$

$$\Gamma_{meas} = \tau_{meas}^{-1} = \frac{(\Delta I)^2}{4\,S_{II}} \propto \frac{\left(\sum_n \frac{dT_n}{dU}\right)^2}{4\sum_n R_n T_n}\, e|V|$$

# Efficiency

Definition

$$\eta = \frac{\Gamma_{meas}}{\Gamma_{dec}} \leq 1$$

When does $\eta = 1$ hold?

- time-reversal symmetry $(B = 0)$
- space-inversion symmetry
- statistical condition $(N > 1)$

$$\frac{dT_n/dU}{R_n T_n} = C \Longleftarrow$$

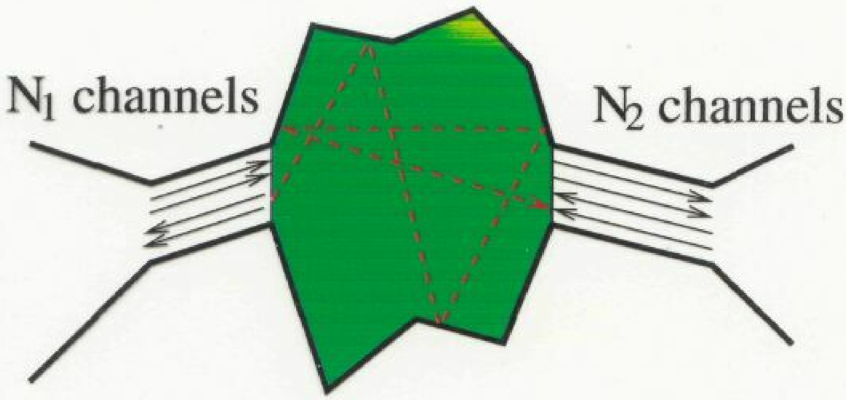
$$V(x,y,z) = Z(z) + W(x,y)$$

$\Longrightarrow$

$$T_n = \left[1 + e^{-[F(E) - F(E_n)]}\right]^{-1} \;;\; dF/dE = C'$$

# Example

## chaotic cavity




- Circular ensemble describes distribution of scattering matrix

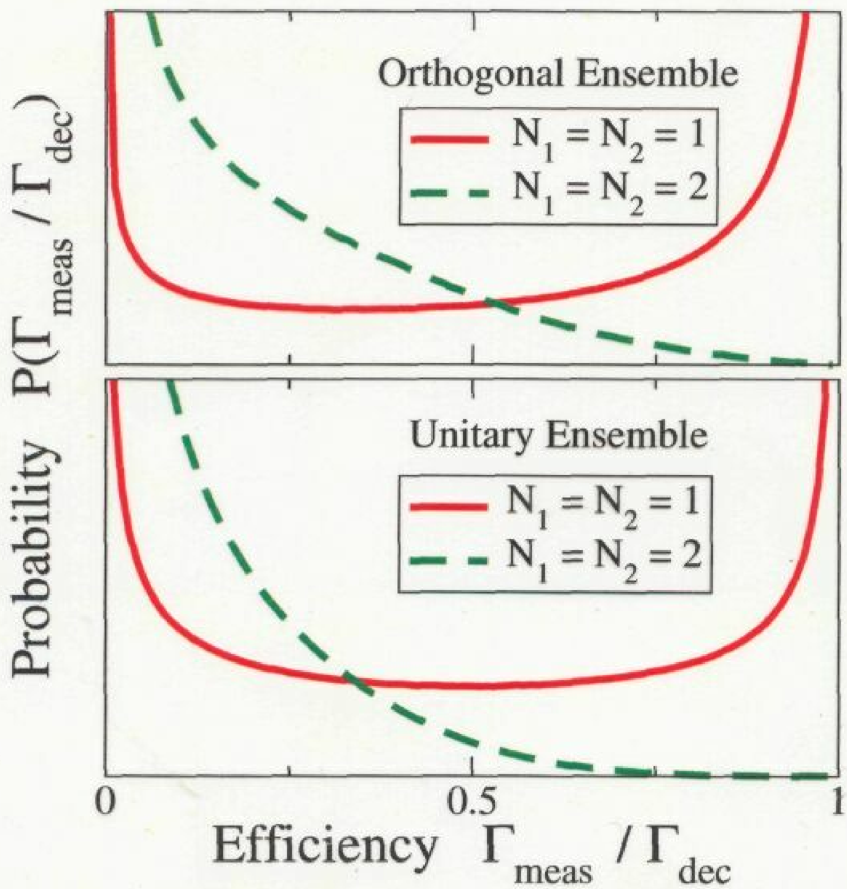
$$\hat{s} = \begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}$$

- Laguerre ensemble describes distribution of Wigner-Smith matrix

$$\hat{N}_E = \frac{1}{2\pi i} \Sigma_\alpha \left(s^\dagger_{\beta\alpha}\right)^{1/2} \frac{ds_{\gamma\alpha}}{dE} \left(s^\dagger_{\beta\alpha}\right)^{1/2}$$

# Example

Distribution of efficiencies of chaotic cavities ($\Gamma_{meas} = \tau_{meas}^{-1}$)




$$P(x) = \frac{1}{(x(1-x))^{1/2}}$$

# Conclusions

Detector characterized by three parameteres

$C_\mu$ electrochemical capacitance
$R_q$ equilibrium charge fluctuations
$R_v$ non-equilibrium charge fluctuations

determine
$\Gamma_{rel}$ relaxation rate
$\Gamma_{dec}$ decoherence rate

Theory without screening predicts
$\Gamma_{dec} \sim N$
theory with screening predicts
$\Gamma_{dec} \sim 1/N$

Maximum efficiency $\eta = 1$ requires

- time-reversal symmetry
- space-inversion symmetry
- separable potentials ( $N > 1$ )