Andreev Quantum for spin manipulation.

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QUANTUM COMPUTATION AND SPIN ELECTRONICS

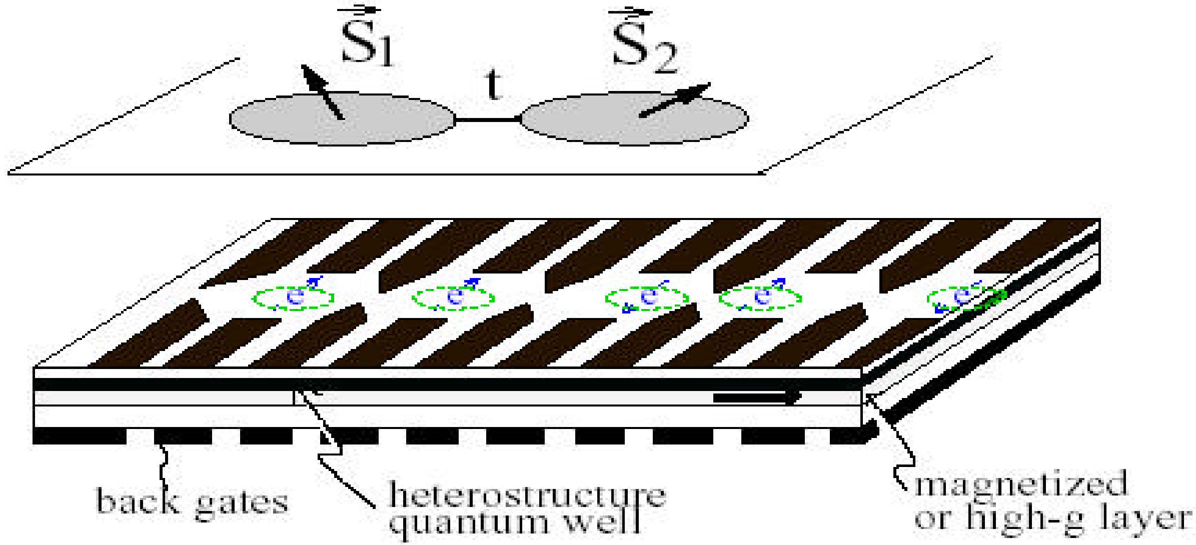


Figure 1. Above: Two coupled quantum dots, between which electrons can tunnel with amplitude t. This tunneling leads to an effective spin interaction $J \sim t^2/U$ (where U is the on-site Coulomb repulsion) between the excess spins S_1 and S_2 in the dots, which can be controlled by a number of external parameters. In principle, any type of tunnel-coupled confined structure is a candidate for the "spintronics" quantum computing proposal. E.g., the dots can be defined electrically in a two-dimensional electron gas (as suggested by this drawing), or they can be vertically coupled dots, or even atoms, *etc*. Below: Concept for quantum-dot array device.

The complexity of the spin-manipulation schemes proposed and severe difficulties with the read-out of these spin states drives one to think of alternatives…

- We concentrate on sufficiently resistive superconducting constrictions “X” where individual Bogolybov quasiparticles can be trapped in discrete Andreev bound states. We refer to such system as Andreev Quantum Dots (AQD). An AQD resembles a common quantum dot as long as discreetness of a (quaisi)particle number, spectrum and spin is concerned. Albeit in contrast to a common quantum dot the charge of the AQD is not fixed. This allows for superconducting current in the constriction and makes electron-electron interaction negligible. We propose to utilize spin states of the AQD's. We show that an AQD can be brought to the state with spin-1/2 that persists over a long time…

In general, Andreev levels are spin-dependent…

Bogoliubov eigenfunctions are made of two spinors [17] u^α, v^α [coefficients of the Bogoliubov transformation [16] $\Psi(r,\sigma) = \sum_n (u_n(r,\sigma)\gamma_n + g^{\sigma\mu} v_n^*(r,\mu)\gamma_n^\dagger)$] that satisfy

$$\begin{aligned} \varepsilon u^\alpha &= \hat{H}^\alpha{}_\beta u^\beta + \hat{\Delta} v^\alpha \\ \varepsilon v^\alpha &= -[\hat{H}^*]^\alpha{}_\beta v^\beta + \hat{\Delta}^* u^\alpha, \end{aligned} \tag{1}$$

Here "hat" denotes an operator over orbital degrees of freedom. We make explicit the spin structure of the single-particle Hamiltonian H and pair potential Δ, $g_{\alpha\beta} \equiv i\sigma^y$ being metric tensor in spinor space [17], $(\hat{H}^*)^\alpha{}_\beta \equiv g^{\nu\alpha}(\hat{H}^\nu{}_\mu)^* g_{\mu\beta}$. By virtue of Eq. (1) quasiparticle energy levels always come in pairs: each eigenstate with energy ε has a counterpart with energy $-\varepsilon$. This is due to a double-counting: there are two quasiparticle eigenfunctions per each state of H. Should $\hat{H}$ possess no spin structure, Andreev levels are *spin-degenerate*.

In general, Andreev levels are spin-dependent…

- From Eq. (1) follows:
- a) Andreev levels are symmetric with the respect to the Fermi energy, so in equilibrium at zero temperature there are as many empty Andreev levels above as occupied levels below;
- b) Andreev levels in superconducting junctions satisfy the relation:

$$\varepsilon_{n,\sigma}(\varphi, H) = \varepsilon_{n,-\sigma}(-\varphi, -H)$$

they are even functions of f if spin orbit interaction is disregarded.

In general, Andreev levels are spin-dependent…

- spin splitting of Andreev levels by magnetic/exchange field in a ballistic superconducting junction with few open orbital channels:

$$\cos[(\epsilon_{n\sigma} - \sigma g\mu_B H)2\tau_{\text{flight}}/\hbar - 2\arccos(\epsilon_{n\sigma}/\Delta)] = \cos(\varphi)$$

where $\sigma = \pm 1$ and $\tau_{\text{flight}} = L/v_F$

Short constriction

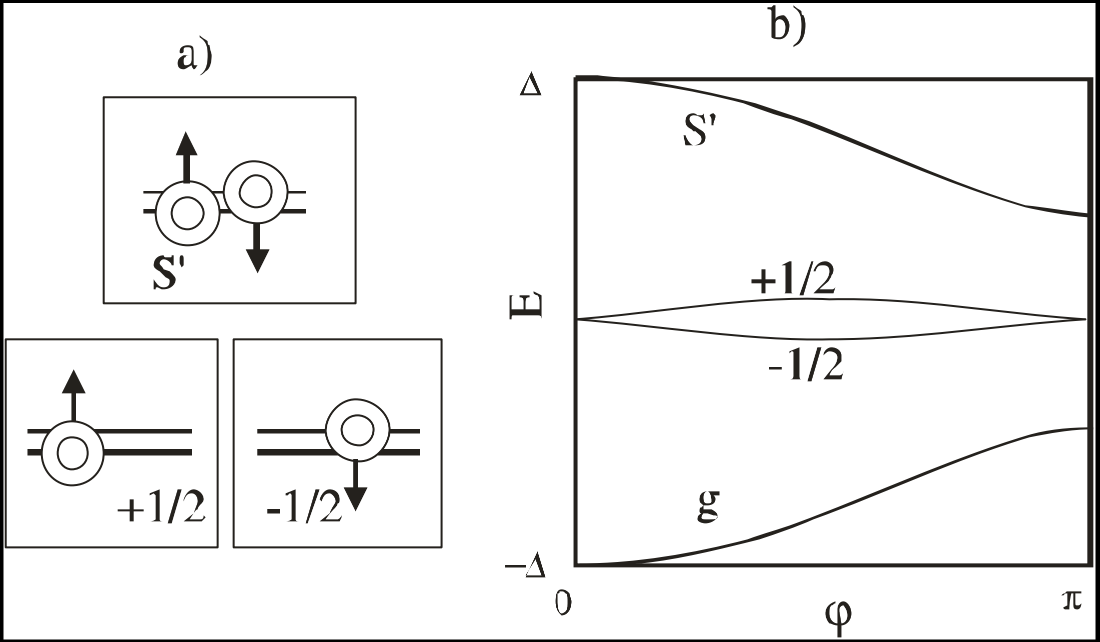
We concentrate on a short constriction, such that the typical time for an electron τ_{flight} to traverse the junction satisfies the condition $\tau_{\text{flight}} \ll \hbar/\Delta$. In the limit $\tau_{\text{flight}}\Delta/\hbar \to 0$, and in the absence of magnetic field Andreev levels are *spin-degenerate* and can be universally expressed [19] through eigenvalues T_n of the transmission matrix square, $\varepsilon_{n_1,n_2;\sigma} = \text{sign}(n_2)\Delta\sqrt{1 - T_{n_1}\sin^2(\varphi/2)}$. Here the integer index n_1 labels orbital channels, $n_2 = \pm 1$, $\sigma = \pm 1$ is spin-index and φ stands for the superconducting phase difference between the leads.

SXS: Andreev levels

- Andreev levels that are relevant for electron transport, and for manipulation of spin states, originate from $T_n=1$. These levels are distributed in energy strip $?\cos(f/2)<|e|<?$. Their typical spacing is given by $dE \sim G_Q R$, R being the normal state resistance of the constriction, $G_Q R$ being the conductance quantum. In the ground state of the dot, quasiparticles occupy Andreev levels with negative energy. The **f**-dependent part of the ground state energy reads $E_0=1/2?_{ns}\, e_{ns}\, ?(e_{ns})$.

Stability of the AQD ½-state

The spin-1/2 state of an AQD with the lowest energy (that corresponds to the most transparent transport channel) is of particular interest because it is very stable.



The transition to ground state require the ½ change of spin. This means that a quasiparticle must either leave or enter the AQD. The probabilities of these processes contain exponentially small factors exp(-?/T), this means that at zero temperature the AQD would remain in spin-1/2 forever.

How to ***set*** the AQD to spin-1/2 state?

- Possibilities include ***microwave irradiation*** and quasiparticle injection.

$$\Delta + e_0 < \hbar w < 2\Delta .$$

The irradiation quanta takes place in the constriction only.

i. Both quasiparticles appear in bound Andreev states

ii. ii. One quasipartilce appears in a bound state whereas another one acquires energy $>\Delta$ and gets to the extended state. The latter quasiparticle leaves the AQD almost immediately and never comes back.

The outcome of ii) is thus one extra quasiparticle in the AQD. Therefore, at a given moment of time there is either odd or even number of quasiparticles in the AQD. Let us now switch off the irradiation. If there is an even number of quasiparticles in the AQD, the subsequent energy relaxation will drive the system to the ground state. For an odd number of particles, the relaxation will result in a single quasipartice occupying the lowermost Andreev level. This means that with roughly 50% probability the system ends up in the spin-½ state.

How to ***detect*** the spin-½ state?

- The energy of a spin-1/2: $e_{ns}>0$.
- The current equals to:

$$I = \frac{e}{\hbar}\frac{\partial}{\partial j}e_{ns} + 2\frac{e}{\hbar}\frac{\partial}{\partial j}E_0$$

- The change from the ground to spin-1/2 state is manifested as a change of supeconducting current by a value of

$$\mathrm{d}I = \frac{e}{\hbar}\frac{\partial}{\partial j}e_{ns}$$

- The detection of such current jumps in superconducting constrictions would amount to the direct experimental observation of the spin-1/2 state.

How to detect ***spin*** in the spin-½ state?

$$E_n^{(SO)} = \Delta(\boldsymbol{\alpha}_n \cdot \boldsymbol{\sigma}) \sin(\varphi)(\tau_{\text{flight}} \Delta/\hbar),$$

$$|\boldsymbol{\alpha}| \simeq Z(e^2/\hbar c)$$

for $\varphi > 0$: $\boldsymbol{\alpha} \upuparrows \mathbf{p} \times \mathbf{E}$;

when $\varphi < 0$

$\boldsymbol{\alpha} \upuparrows -\mathbf{p} \times \mathbf{E}$

$$I_\sigma = e\sigma\partial_\varphi E^{(SO)}/\hbar$$

$$\tau_{\text{flight}} \simeq \Delta/\hbar \, , |\boldsymbol{\alpha}| \simeq 1$$

Spin manipulation in the AQD

Spin-orbit splitting

$$\sigma E_n^{(SO)}$$

Zeeman splitting

$$\sigma E^{(Z)}$$

$$H = E^{(Z)}\hat{\sigma}_y + E^{(SO)}\hat{\sigma}_z$$

$$\Psi = a|\uparrow\rangle + b|\downarrow\rangle$$

$$\Psi(t) = |\downarrow\rangle\left(\cos(\Omega t) + i\sin(\Omega t)(E^{(SO)}/\hbar\Omega)\right) - \\ -|\uparrow\rangle\sin(\Omega t)(E^{(Z)}/\hbar\Omega),$$

$$\hbar\Omega = \sqrt{(E^{(SO)})^2 + (E^{(Z)})^2}$$

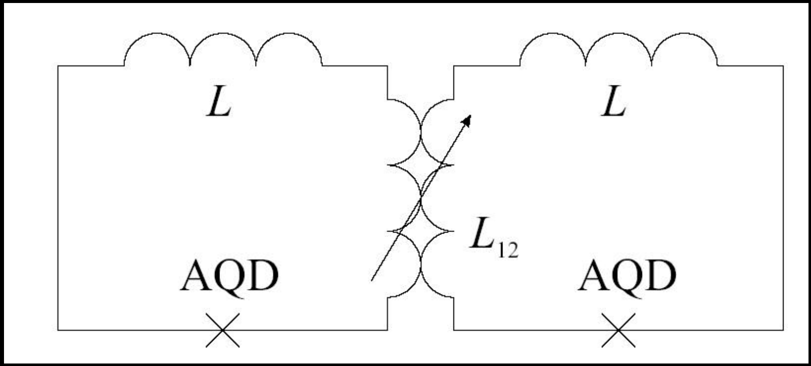
$$I_a(t) = 2\sin^2(\Omega t)\left(\frac{E^{(Z)}}{\hbar\Omega}\right)^2 e\partial_\varphi E^{(SO)}/\hbar$$

How the Andreev quantum dots can be utilized for universal quantum computations.

- An AQD in the spin-1/2 state would be a qubit.
- XOR operation.
- The XOR operation does the following: given two qubits in the states |x>, |y>, it leaves the |y> state unchanged if |x>=|?>, while flipping it when |x>=|?>.

How the Andreev quantum dots can be utilized for universal quantum computations.

- The basic idea is to organize the interaction between AQD's via inductive coupling between SQUID loops containing these AQD's.



$$H = L_{12} I_1 I_2$$

$$I_{1,2} = I_{1,2}^{(0)} + I_{1,2}^{s} s_{1,2}^{z}$$

How the Andreev quantum dots can be utilized for universal quantum computations.

This simple Ising-type form of Hamiltonian brings us to the old-fashioned but solid "optical" quantum computer [26]. In this approach, the one-bit operations are performed at $H_{12} = 0$ by pulses at resonant frequencies $H_1/\hbar$ or $H_2/\hbar$, the pulse duration being tuned to achieve the spin flip. The XOR operation is performed at $H_{12} \neq 0$ by the same pulse with frequency $(H_1 + H_2)/\hbar$. An alternative way is to use non-oscillating pulses of $H_{12}/\hbar$. Such pulses would shift phases of two states with antiparallel spins with respect to the phases of the states with parallel spins thus realizing "quantum phase shift gate" [27]. The XOR operation can be performed by combining two such phase shifts with two rotations of the target spin.

advantages

- This approach of organizing two-qubit interactions has two important practical advantages.
- First, in contrast to other spin-based proposals, the interaction does not have to be organized at microscopic level. To exaggerate, one can use inch-scale transformers to vary inductive coupling between the AQD's. To make a practical suggestion, one can use the well-developed techniques of SQUID circuitry to couple, array, bias, and measure many AQD qubits.
- Second advantage is the simple Ising form of the resulting interaction that prevents undesired phase shifts and simplifies design of complicated quantum circuits.

conclussions

- To conclude, we analyze prospectives of Andreev quantum dots for spin manipulation and quantum computing. Our theoretical results seem to be promising enough to launch detailed experimental investigations and design efforts in this direction.
- **N.M. Chtchelkatchev and Yu. Nazarov, Phys. Rev. Lett., 90, 226806 (2003)**

PRL referees about SXS AQD:

- Spin-orbit interaction breaks time-reversal symmetry and Cooper pairs. Thus, “for a material made of heavy nuclei” one may have pair breaking in the superconducting banks. This issue should be addressed, at least briefly.

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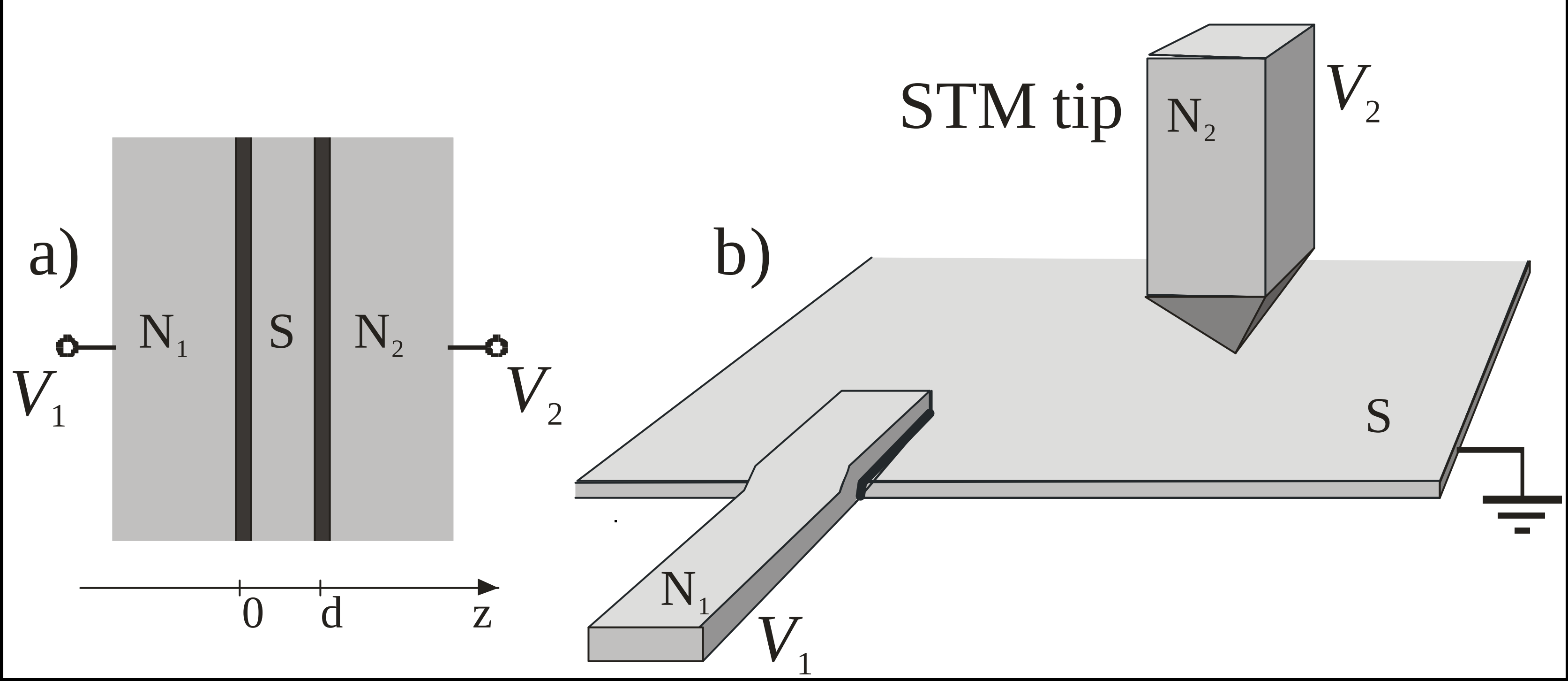
Subgap tunneling in a system of two coupled N-S (F-S) contacts.



$$G_{12} \equiv \partial_{V_1} I_2(V_1, 0)|_{V_1=0} = 0$$

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Assumptions of Falci *et al*:

1) the bias is **much** smaller than the superconducting gap.

2) The superconductor is **clean.**

G. Deutscher, D. Feinberg, Applied Physics Letters **76**, 487 (2000).

G. Falci, D. Feinberg, F.W.J. Hekking, Europhysics Letters 54, 225 (2001); R. Mélin, D. Feinberg, cond-mat/0106329; R. Melin, J. Phys.: Condens. Matter 13, 6445 (2001);

Result of Falci *et al:* $G_{12} \equiv \partial_{V_1} I_2(V_1, 0)|_{V_1=0} = 0$

Our assumptions:

1) The bias is **smaller** than the superconducting gap.

We find *analytically* $I_{1,2}(V_1,V_2)$ more or less in any tree-terminal layout with two terminals weakly coupled to the superconductor.

in particular *:* $I_2(V_1, V_2 = 0) = 0 \text{ for } |V_1| < \Delta$

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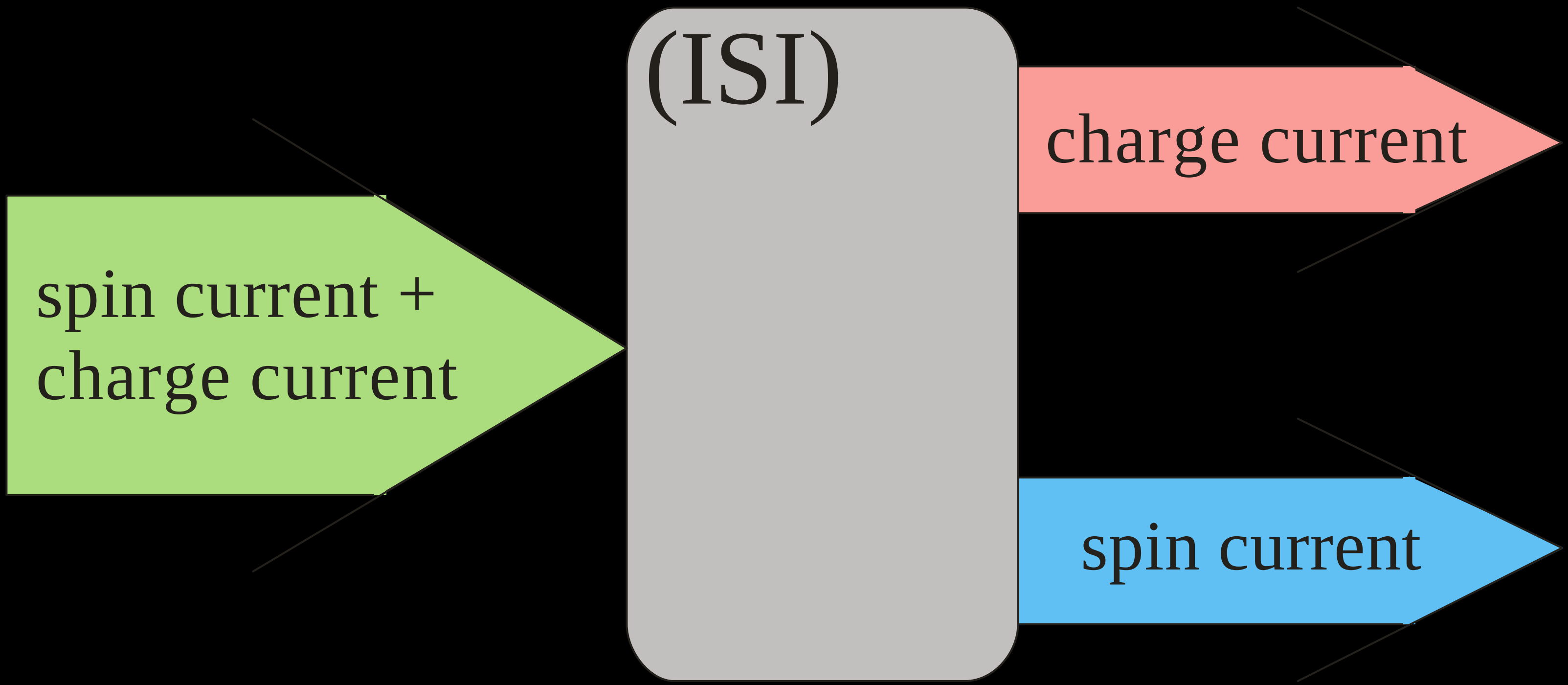
$$I_2^{(s)}(V_1, V_2 = 0) \neq 0$$

$$I_2(V_1, V_2 = 0) = 0 \text{ for } |V_1| < \Delta$$



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$$I_2(V_1, V_2 = 0) = \\ = \frac{e}{\hbar}\int_0^\infty dE \sum (T_{ee}^{(2\leftarrow 1)}(f^{(1)}(E) - f^{(2)}(E)) - \\ T_{he}^{(2\leftarrow 1)}(f^{(1)}(E) - [1 - f^{(2)}(-E)])) = \\ \frac{e}{\hbar}\int_0^\infty dE \sum (T_{ee}^{(2\leftarrow 1)} - T_{he}^{(2\leftarrow 1)})(f^{(1)}(E) - f^{(2)}(E)),$$

solution of 8×8 linear system

equations. Analytical progress can be made. It follows that if there is no barrier at NS boundaries (except Δ) $T_{eh}(\theta)/T_{ee}(\theta) \lesssim (\Delta/E_F)^2$ for any thickness d

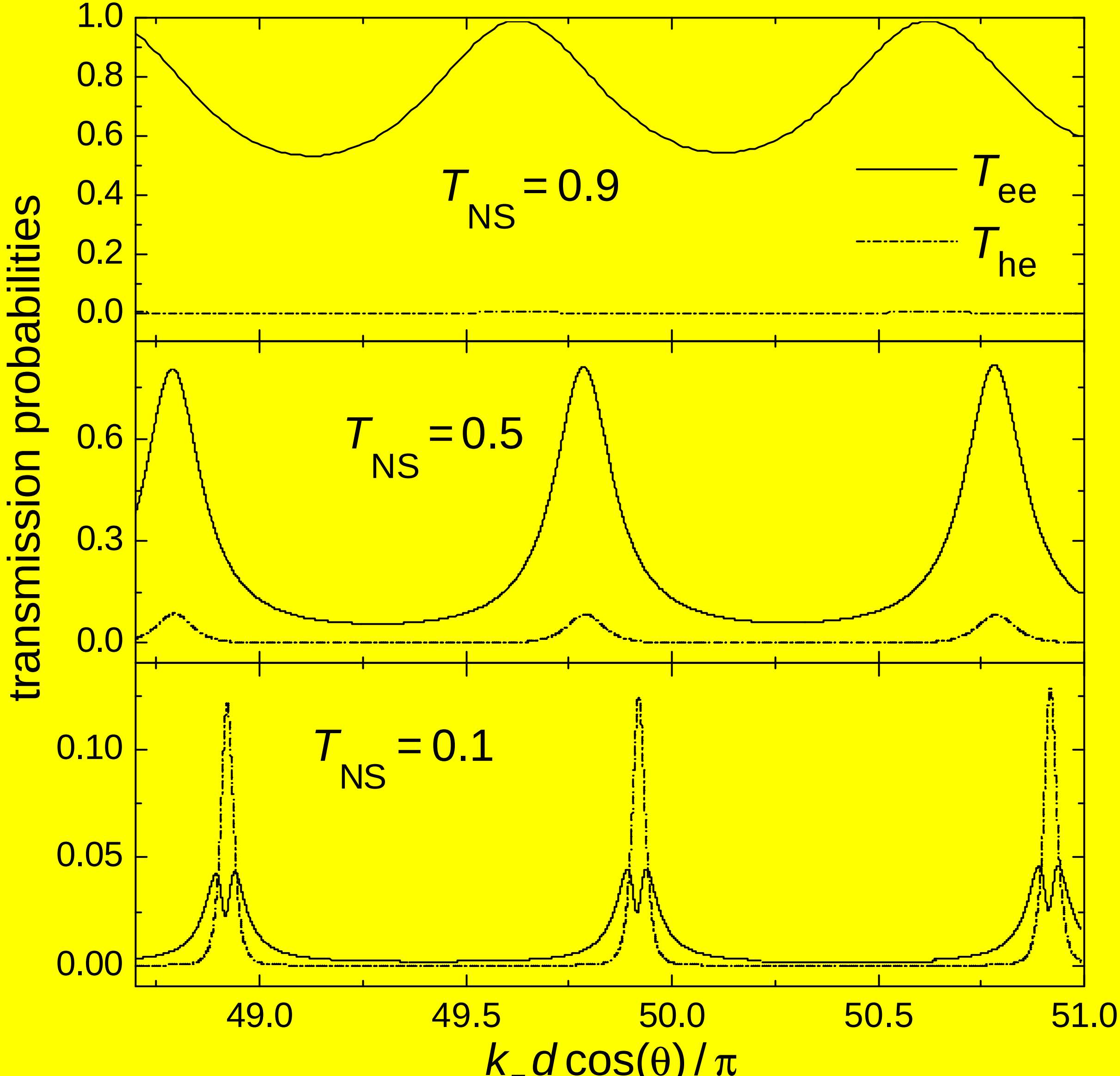
$$k_F d \cos(\theta_n) = \pi n,$$

However if there are barriers at NS boundaries in addition to Δ (i.e., insulating layers) then the situation changes completely: at certain choice of θ, when quasiparticle energies are much below Δ: $T_{eh} \sim T_{ee}$, and

$$\langle T_{he}\rangle \approx \langle T_{ee}\rangle, \qquad (2)$$

where $\langle \ldots \rangle = \sum_{\text{channels}}(\ldots)/N_{\text{channels}} \approx \int_0^1 (\ldots) d\cos^2\theta$.

$$
\begin{pmatrix}
-1 & 0 & 0 & 0 & u\cdot\exp(-1i\cdot p_e) & u\cdot\exp(1i\cdot p_e) & v\cdot\exp(-1i\cdot p_h) & v\cdot\exp(1i\cdot p_h) \\
k_{ef} + 2\cdot 1i\cdot Z_1 & 0 & 0 & 0 & p_{ef}\cdot u\cdot\exp(-1i\cdot p_e) & -p_{ef}\cdot u\cdot\exp(1i\cdot p_e) & p_{hf}\cdot v\cdot\exp(-1i\cdot p_h) & -p_{hf}\cdot v\cdot\exp(1i\cdot p_h) \\
0 & -1 & 0 & 0 & v\cdot\exp(-1i\cdot p_e) & v\cdot\exp(1i\cdot p_e) & u\cdot\exp(-1i\cdot p_h) & u\cdot\exp(1i\cdot p_h) \\
0 & -k_{hf} + 2\cdot 1i\cdot Z_1 & 0 & 0 & p_{ef}\cdot v\cdot\exp(-1i\cdot p_e) & -p_{ef}\cdot v\cdot\exp(1i\cdot p_e) & p_{hf}\cdot u\cdot\exp(-1i\cdot p_h) & -p_{hf}\cdot u\cdot\exp(1i\cdot p_h) \\
0 & 0 & -1 & 0 & u\cdot\exp(1i\cdot p_e) & u\cdot\exp(-1i\cdot p_e) & v\cdot\exp(1i\cdot p_h) & v\cdot\exp(-1i\cdot p_h) \\
0 & 0 & -k_{ef} - 2\cdot 1i\cdot Z_2 & 0 & p_{ef}\cdot u\cdot\exp(1i\cdot p_e) & -p_{ef}\cdot u\cdot\exp(-1i\cdot p_e) & p_{hf}\cdot v\cdot\exp(1i\cdot p_h) & -p_{hf}\cdot v\cdot\exp(-1i\cdot p_h) \\
0 & 0 & 0 & -1 & v\cdot\exp(1i\cdot p_e) & v\cdot\exp(-1i\cdot p_e) & u\cdot\exp(1i\cdot p_h) & u\cdot\exp(-1i\cdot p_h) \\
0 & 0 & 0 & k_{hf} - 2\cdot 1i\cdot Z_2 & p_{ef}\cdot v\cdot\exp(1i\cdot p_e) & -p_{ef}\cdot v\cdot\exp(-1i\cdot p_e) & p_{hf}\cdot u\cdot\exp(1i\cdot p_h) & -p_{hf}\cdot u\cdot\exp(-1i\cdot p_h)
\end{pmatrix}
$$



transmission probabilities
$T_{NS} = 0.9$
T_{ee}
T_{he}
$T_{NS} = 0.5$
$T_{NS} = 0.1$
1.0
0.8
0.6
0.4
0.2
0.0
0.6
0.3
0.0
0.10
0.05
0.00
49.0
49.5
50.0
50.5
51.0
$k_F d \cos(\theta) / \pi$

$$
\begin{gathered}
I_2(V_1, V_2 = 0) = \\
= \frac{e}{\hbar} \int_0^\infty dE \sum (T_{ee}^{(2 \leftarrow 1)} (f^{(1)}(E) - f^{(2)}(E)) - \\
T_{he}^{(2 \leftarrow 1)} (f^{(1)}(E) - [1 - f^{(2)}(-E)])) = \\
\frac{e}{\hbar} \int_0^\infty dE \sum (T_{ee}^{(2 \leftarrow 1)} - T_{he}^{(2 \leftarrow 1)})(f^{(1)}(E) - f^{(2)}(E)),
\end{gathered}
$$

$$
\begin{gathered}
I_2^{(s)}(V_1, V_2 = 0) = \\
= \frac{1}{2} \frac{e}{\hbar} \int_0^\infty dE \sum (T_{ee}^{(2 \leftarrow 1)} (f^{(1)}(E) - f^{(2)}(E)) + \\
T_{he}^{(2 \leftarrow 1)} (f^{(1)}(E) - [1 - f^{(2)}(-E)])) = \\
\frac{e}{2\hbar} \int_0^\infty dE \sum (T_{ee}^{(2 \leftarrow 1)} + T_{he}^{(2 \leftarrow 1)})(f^{(1)}(E) - f^{(2)}(E)).
\end{gathered}
$$

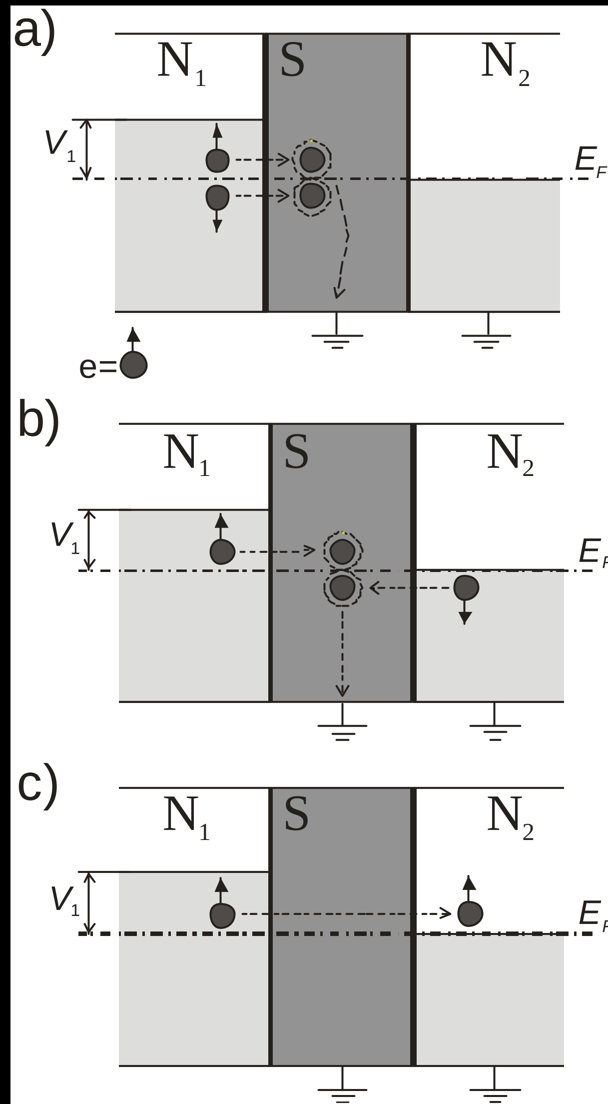
the hamiltonian: $\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{H}_S + \hat{H}_T$, where $\hat{H}_{1,2}$ refer to the electrodes N_1 and N_2, and $\hat{H}_S$ to the superconductor. The tunnel Hamiltonian $\hat{H}_T$, which we consider as a perturbation, is given by two terms $\hat{H}_T = \hat{H}_T^{(1)} + \hat{H}_T^{(2)}$ corresponding to one-particle tunneling through each tunnel junction. These terms are given by:

$$\hat{H}_T^{(i)} = \sum_{k,p} \left\{ \hat{a}_k^{(i)\dagger} t_{kp}^{(i)} \hat{b}_p + h.c. \right\}, \tag{4}$$

where indices $i = 1, 2$ refer to normal (ferro) electrodes; $t_{kp}^{(i)}$ is the matrix element for tunneling from the state $k = (\mathbf{k}, \sigma)$ in normal lead N_i to the state $p = (\mathbf{p}, \sigma')$ in the superconductor. The operators $\hat{a}_k^{(i)}$ and $\hat{b}_p$ correspond to quasiparticles in the leads and in the superconductor, respectively.

Fig.5 a) Direct Andreev tunneling (Andreev reflection), b) Crossed Andreev tunneling (Andreev transmission), and c) Elastic Cotunneling (normal transmission).

$$\Gamma^{S\leftarrow 1(2)}_{(DA)}$$

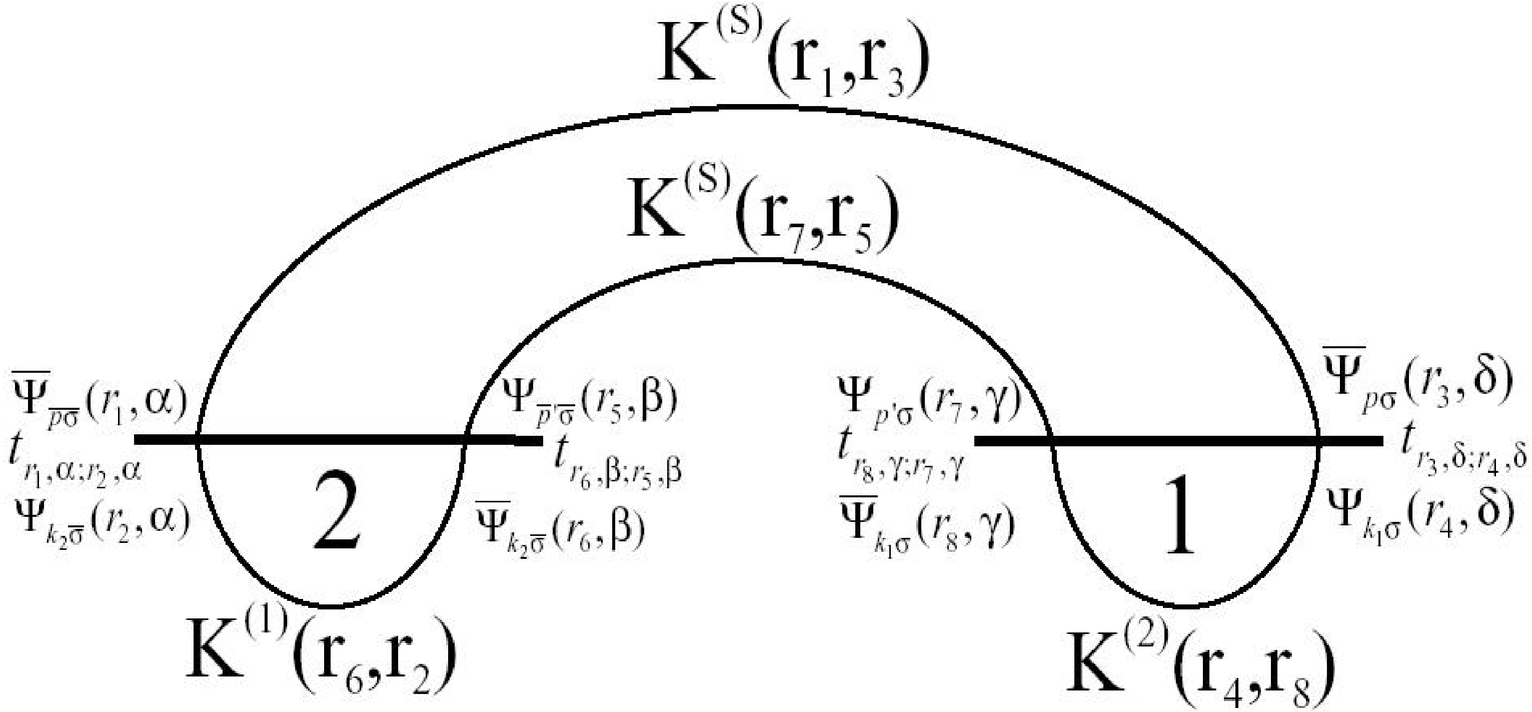


$$\Gamma^{S\leftarrow}_{CA}(V_1, V_2)$$

$$T_{he}^{(2\leftarrow 1)}$$

$$\Gamma^{2\leftarrow 1}_{(EC)}$$

$$T_{ee}^{(2\leftarrow 1)}$$



Calculation of Ξ_{CA}:

$$\Xi_{CA}(\zeta,\zeta';\xi_1,\xi_2) = \sum_{p,p';\sigma;k^{(1)}k^{(2)}} t^{(2)}_{\bar{p}\bar{\sigma};k^{(2)}\bar{\sigma}} t^{(1)}_{p\sigma;k^{(1)}\sigma} \overline{t^{(2)}_{\bar{p}'\bar{\sigma};k^{(2)}\bar{\sigma}} t^{(1)}_{p'\sigma;k^{(1)}\sigma}} \delta(\zeta-\varepsilon_p)\delta(\zeta'-\varepsilon_{p'})\delta(\xi-\xi_{k^{(1)}\sigma})\delta(\xi'-\xi_{k^{(2)}\bar{\sigma}}) = \quad (48)$$

$$= \sum_{p,p';\sigma;k^{(1)}k^{(2)}} \bar{\psi}_{\bar{p}\bar{\sigma}}(r_1,\bar{\sigma}) t^{(S2)}_{r_1,\bar{\sigma};r_2,\bar{\sigma}} \psi_{k^{(2)}\bar{\sigma}}(r_2,\bar{\sigma}) \bar{\psi}_{p\sigma}(r_3,\sigma) t^{(S1)}_{r_3,\sigma;r_4,\sigma} \psi_{k^{(1)}\sigma}(r_4,\sigma) \psi_{\bar{p}'\bar{\sigma}}(r_5,\bar{\sigma}) \overline{t^{(S2)}_{r_5,\bar{\sigma};r_6,\bar{\sigma}}} \bar{\psi}_{k^{(2)}\bar{\sigma}}(r_6,\bar{\sigma}) \times \quad (49)$$

$$\times \psi_{p'\sigma}(r_7,\sigma) \overline{t^{(S1)}_{r_7,\sigma;r_8,\sigma}} \bar{\psi}_{k^{(1)}\sigma}(r_8,\sigma) \delta(\zeta-\varepsilon_p)\delta(\zeta'-\varepsilon_{p'})\delta(\xi-\xi_{k^{(1)}\sigma})\delta(\xi'-\xi_{k^{(2)}\bar{\sigma}}), \quad (50)$$

$$I_2(V_1, V_2) = I_2^{(I)}(V_1, V_2) + I_2^{(D)}(V_2)$$

$$I_2^{(I)}(V_1, V_2) = \Gamma_{(EC)} - \Gamma_{(CA)}, \tag{5a}$$

$$\Gamma^{(EC)} = \Gamma_{(EC)}^{1\leftarrow 2} - \Gamma_{(EC)}^{2\leftarrow 1}, \tag{5b}$$

$$\Gamma^{(CA)} = \Gamma_{(CA)}^{S\leftarrow 1,2} - \Gamma_{(CA)}^{S\rightarrow 1,2}, \tag{5c}$$

$$I_2^{(D)}(V_2) = \Gamma_{(DA)}^{S\leftarrow 2} - \Gamma_{(DA)}^{S\rightarrow 2}. \tag{5d}$$

$$\Gamma^{1\rightarrow 2}_{(EC)} = 4\pi^3 \int d\xi \sum_{\sigma} n^{(1)}_{\sigma}(\xi - V_1)(1 - n^{(2)}_{\sigma}(\xi - V_2)) \times \frac{\Delta^2}{[\Delta^2 - \xi^2]} \tilde{\Xi}^{(EC)}_{\sigma}(2\sqrt{\Delta^2 - \xi^2}), \quad (9)$$

$$\Gamma^{S\leftarrow 12}_{CA}(V_1, V_2) = 4\pi^3 \int d\xi \sum_{\sigma} n^{(1)}_{\sigma}(\xi - V_1) n^{(2)}_{-\sigma}(-\xi - V_2) \times \frac{\Delta^2}{[\Delta^2 - \xi^2]} \tilde{\Xi}^{(CA)}_{\sigma}(2\sqrt{\Delta^2 - \xi^2}), \quad (6)$$

$$\Xi_\sigma^{(CA)}(t) = \\ = \frac{1}{8\pi^3 e^4 \nu_S} \int d\hat{p}_{1,2} \int dX_{1,2} P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) \times \\ \left\{ G^{(1)}(X_1, \hat{p}_1, \sigma) G^{(2)}(X_2, \hat{p}_2, \sigma) \sin^2\left(\frac{\theta(X_1, X_2)}{2}\right) + \\ + G^{(1)}(X_1, \hat{p}_1, \sigma) G^{(2)}(X_2, \hat{p}_2, -\sigma) \cos^2\left(\frac{\theta(X_1, X_2)}{2}\right) \right\}$$

$$\Xi_\sigma^{(EC)}(t) = \\ = \frac{1}{8\pi^3 e^4 \nu_S} \int d\hat{p}_{1,2} \int dX_{1,2} P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) \times \\ \left\{ G^{(1)}(X_1, \hat{p}_1, \sigma) G^{(2)}(X_2, \hat{p}_2, -\sigma) \sin^2\left(\frac{\theta(X_1, X_2)}{2}\right) + \\ + G^{(1)}(X_1, \hat{p}_1, \sigma) G^{(2)}(X_2, \hat{p}_2, \sigma) \cos^2\left(\frac{\theta(X_1, X_2)}{2}\right) \right\}.$$

Spin current

$$I_N^{(\mathrm{spin})}(V_F,0) = 4\pi^3 \int d\xi \sum_\sigma [n_\sigma^{(1)}(\xi - V_F) - n^{(2)}(\xi)]$$

$$\times \frac{\Delta^2}{[\Delta^2 - \xi^2]} \frac{1}{8\pi^3 e^4 \nu_S} \int d\hat{p}_{1,2} \int dX_{1,2} \times$$

$$P(X_1, \hat{p}_1; X_2, \hat{p}_2, t) G^{(1)}(X_1, \hat{p}_1, \sigma) G^{(2)}(X_2, \hat{p}_2). \quad (11)$$

FISIF

Finally we consider a FISIF junction. It was shown in [5] that in this junction $I_2(V_1,0) \neq 0$ and $I_2(V_1,0)$ changes its sign when the ferromagnetic terminals change their orientation from parallel to antiparallel. It was also noted in [5] that the cross conductance $G_{12} \equiv \partial_{V_1} I_2(V_1,0)|_{V_1=0}$ is suppressed as $1/(k_F r)^2$ when the characteristic distance between the ferromagnets $r < \xi$. In dirty regime there is no conductance suppression at λ_F-scale. Consider, for instance, the layout sketched in Fig. 1b; the width d

5. G. Falci, D. Feinberg, F.W.J. Hekking, Europhysics Letters 54, 225 (2001); R. Mélin, D. Feinberg, cond-mat/0106329; R. Melin, J. Phys.: Condens. Matter 13, 6445 (2001);

FISIF

Thus it is practically more convenient to measure finite effects related to subgap current injection from one terminal to the other through the superconductor when it is dirty rather than clean. In dirty case the terminals are not restricted to be as close as λ_F like in clean case but closer then $\xi \gg \lambda_F$.

Conclusions (XSX)

Consider two normal leads coupled to a superconductor; the first lead is biased while the second one and the superconductor are grounded. In general, a finite current $I_2(V_1, 0)$ is induced in the grounded lead 2; it magnitude depends on the competition between processes of Andreev and normal quasiparticle transmission from the lead 1 to the lead 2. It is known that in the tunneling limit, when normal leads are weakly coupled to the superconductor, $I_2(V_1, 0) = 0$, if $|V|/\Delta \to 0$ and the system is in the clean limit. In other words, Andreev and normal tunneling processes compensate each-other. We consider the general case: the voltages are below the gap, the system is either dirty or clean. It is shown that $I_2(V_1, 0) = 0$ for any distance between the normal leads; if the first lead injects spin polarized current then $I_2 = 0$, but spin current in the lead-2 is finite. Thus XISIN structure, where X is a source of the spin polarized current, can be applied in spintronics as a filter separating spin current from charge current. We do an analytical progress calculating the current-voltage characteristics: $I_1(V_1, V_2)$, $I_2(V_1, V_2)$.