Phase diagram of surface superconductor in parallel magnetic field

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Content

- Rashba interaction and chiral states of electrons   Gorkov, Rashba PRL (2001)

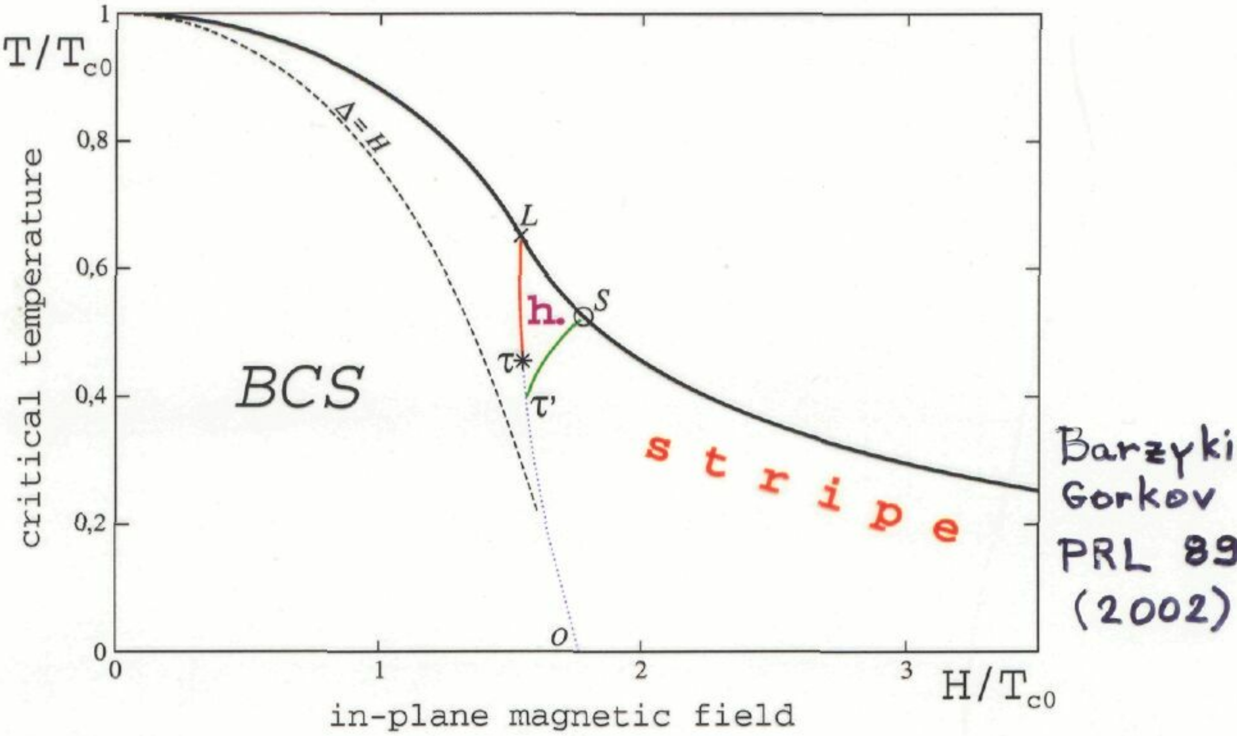
- p-pairing and inhomogeneous states

- Lifshitz line and vanishing superconducting density

- $SU(2)$-symmetric point and continuous vortex

- Non-magnetic impurities

Phase diagram



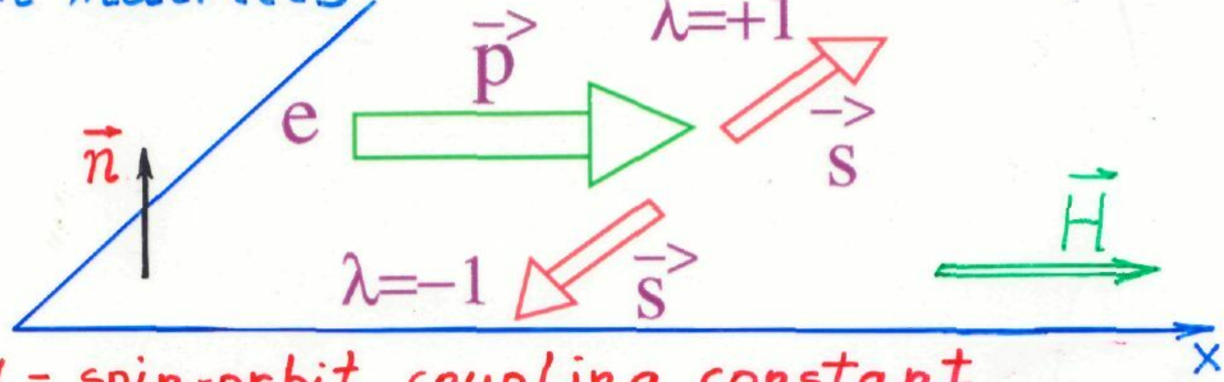
Superconducting phase transition line $T_c(H)$(solid) and two second order phase transition lines in the clean case: $\mathcal{LT}$ line between homogeneous and helical state and a $\mathcal{ST}'$ line of stability of helical state.

Model of spin-orbital superconductor

Rashba Zeeman

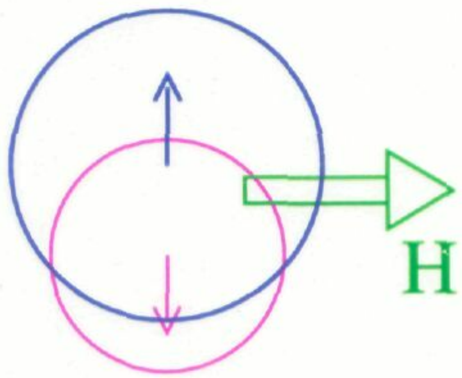
$$\hat{H} = \frac{\vec{p}^2}{2m} + \alpha\,[\vec{\sigma}\times\vec{p}]\cdot\vec{n} - \tfrac{1}{2} g\mu_B \vec{H}\cdot\vec{\sigma}$$

$\vec{\sigma}$ - Pauli matrices



α - spin-orbit coupling constant

$$\epsilon_\lambda(\vec{p}) = p^2/2m - \lambda\alpha|\mathbf{p}|$$



$H \ll \alpha p_F$

$T_c \ll \omega_D \ll \alpha p_F \ll \varepsilon_F$

$$\hat{H}_{int} = \frac{|\Delta_q|^2}{U} - \frac{\Delta_q}{2}\sum_{\lambda,\mathbf{p}} \lambda e^{-i\varphi_{\mathbf{p}}} a^\dagger_{\lambda,\mathbf{p}+\mathbf{q}/2} a^\dagger_{\lambda,-\mathbf{p}+\mathbf{q}/2} - h.c.$$

$$\Psi_{pair} = i\sigma_y\left(\sigma_x\, d_x(\vec{p})\right),$$

$$d_x(\vec{p}) = e^{-i\varphi_p} = \frac{p_x - ip_y}{|\vec{p}|}$$

p-pairing: $d_x(-p) = -d_x(p)$

Inhomogeneous states of the order parameter

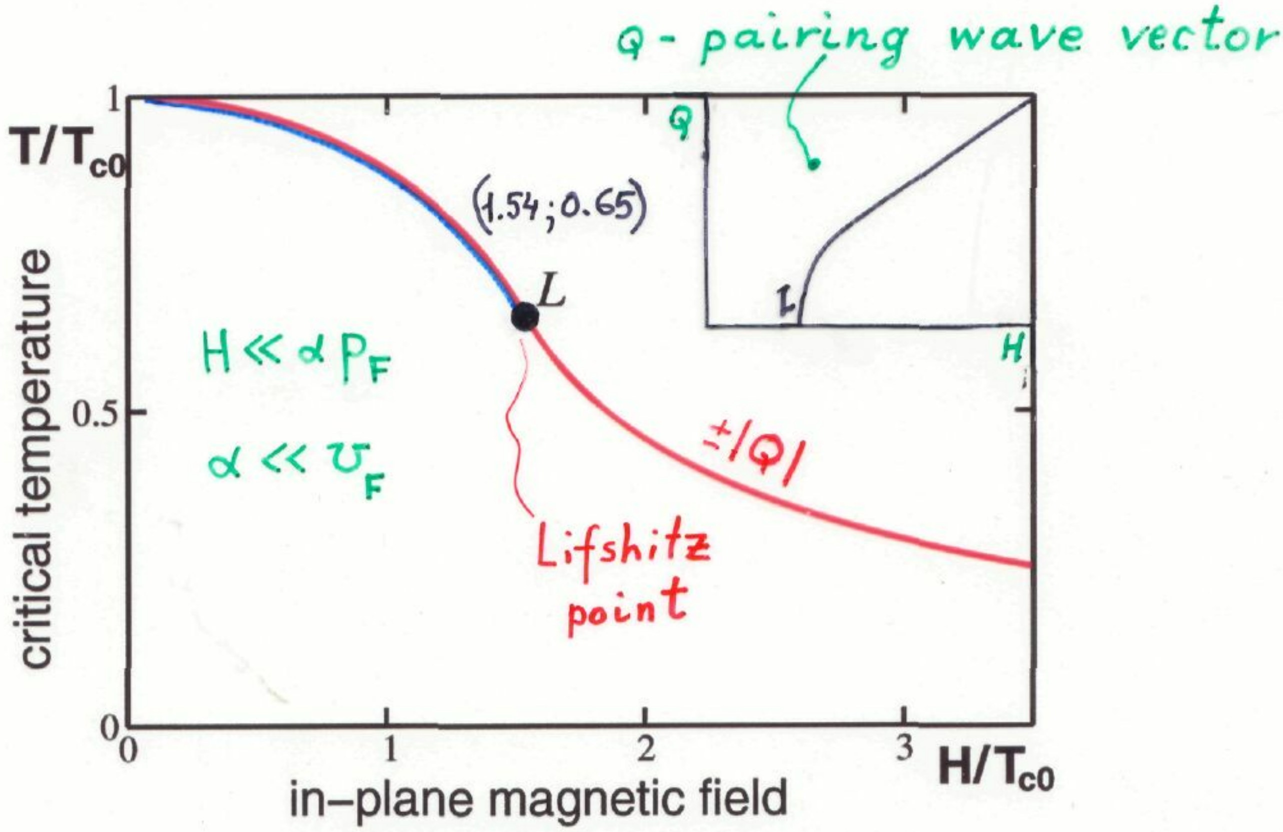
$$\Delta(\mathbf{r}) = \sum_i \Delta_i(\mathbf{r}) \exp\left(i\mathbf{Q}_i\mathbf{r}\right)$$

$\Delta_i(\vec{r})$ – slowly varying in space

GL: $$\Omega^{(2)} = \int \left(\alpha_i |\Delta_i|^2 + c_i \left| \left(-i\frac{\partial}{\partial \vec{r}} - \frac{2e}{c}\vec{A} \right) \Delta_i \right|^2 \right) d^2\vec{r}$$

$$\min_Q \alpha(\mathbf{Q}) = 0 \qquad \alpha(\vec{Q}) = \alpha(-\vec{Q})$$

gives $T_c(H)$ line



Cooper loop: $$\alpha(Q) = \frac{1}{U} - \pi\,\nu(\varepsilon_F)\,T \sum_{\omega>0,\ \lambda} \frac{1}{\sqrt{\omega^2 + (\lambda H + v_F Q/2)^2}}$$

$\omega_n = \pi T(2n+1)$

$\nu(\varepsilon_F) = \frac{m}{2\pi\hbar^2}$

$$T_{c0} = \frac{2\omega_D}{\pi} \exp\left(-\frac{1}{U\nu(\varepsilon_F)} + \gamma \right)$$

$$\Delta(\vec{r}) = \Delta_+ e^{iQy} + \Delta_- e^{-iQy}$$

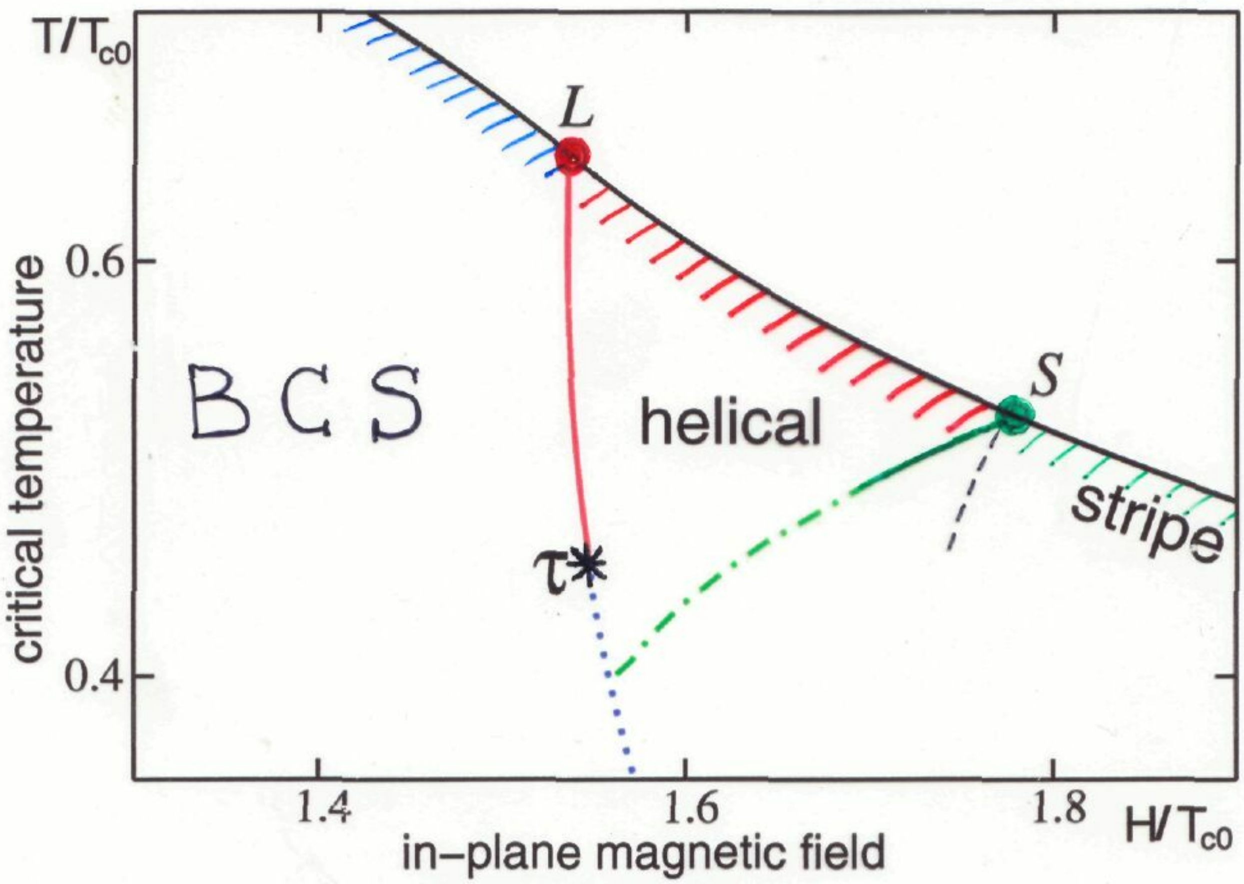
$$\langle|\Delta|^2\rangle = |\Delta_+|^2 + |\Delta_-|^2$$

Three states on the phase transition line

$$\Omega^{(4)} = \int \left(\beta_s|\Delta|^4 + \beta_a(|\Delta_+|^2 - |\Delta_-|^2)^2\right) d^2\vec{r}$$

$\Delta_{hel}(\mathbf{r}) = \Delta \exp(iQy)$ $\quad$ $\Delta_{stripe}(\mathbf{r}) = \Delta \cos(Qy)$

helical state, $\beta_a < 0$ $\quad$ stripe state, $\beta_a > 0$



$$\frac{\partial\Omega_{hel}}{\partial\Delta} = 0 \qquad \frac{\partial\Omega_{hel}}{\partial Q} = 0$$

$\Omega_{hel}^{(H,T)}$ - SC energy of the helical state.

Helical state

$$\frac{1}{\nu(\epsilon_F)U} = \sum_{\lambda,\omega_n} \frac{2T\mathbf{K}(k)}{\sqrt{\omega_n^2 + \left(|\lambda H + \frac{Q}{2}| + \Delta\right)^2}}$$

$$\sum_{\omega>0,\lambda} f\left(\lambda H + \frac{Q}{2}, \omega\right) = 0$$

$$k = \sqrt{\frac{4|\lambda H + \frac{Q}{2}|\Delta}{\omega_n^2 + \left(|\lambda H + \frac{Q}{2}| + \Delta\right)^2}}$$

$$f(H) = \frac{2}{H}\left((\omega^2 + H^2 + \Delta^2)\frac{\mathbf{K}(k)}{r(H,\omega)} - r(H,\omega)\mathbf{E}(k)\right)$$

$$r\left(\lambda H + \frac{Q}{2}, \omega\right) = \sqrt{\omega^2 + (|\lambda H + \frac{Q}{2}| + \Delta)^2}$$

Properties of the helical state

Expansion near $\mathcal{LT}$ - line

$$\delta\Omega_{hel} = \frac{\hbar^2 n_{yy}}{8m} Q^2 + bQ^4 + cQ^6$$

$b = 0$ $\qquad \mathcal{T}$ - critical point

$$n_{yy} = \frac{2m}{\hbar^2} v_F^2 \nu(\epsilon_F) T \sum_{\lambda\omega} \frac{\partial f(H_\lambda, \omega)}{\partial H_\lambda} = 0$$

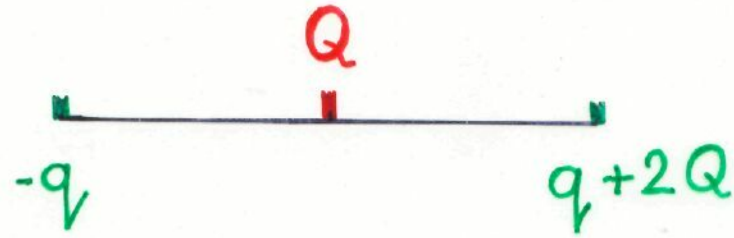
$$n_{xx} = \frac{4m}{\hbar^2} \left(\frac{v_F^2}{2} \Delta \frac{\partial}{\partial \Delta} \left(\frac{1}{\Delta} \frac{\partial \Omega_{hel}}{\partial \Delta} \right) - \frac{\partial^2 \Omega_{hel}}{\partial Q^2} \right) > 0$$

Superconducting current

Yip PRB, **65**, 144508 (2002)

$$\vec{j}_s = \frac{2e}{\hbar} \frac{\partial \Omega_{hel}}{\partial \vec{Q}} = 0$$

Stability of the helical state



(v_{-q}, v^*_{q+2Q}) - spinor of perturbation

$\delta\Omega = \vec{v}^{+}\hat{A}(q)\vec{v}$ - energy of perturbation

„Cooper loop" matrix

$$\hat{A}(q) = \frac{\hat{1}}{U} - \sum_{\omega>0,\lambda,\mathbf{p}} \begin{pmatrix} G_{\lambda p_-}G_{\lambda -p_+} & F_{\lambda p_-}F_{\lambda -p_+} \\ F^*_{\lambda p_-}F^*_{\lambda -p_+} & G_{\lambda p_+ +Q}G_{\lambda -p_- +Q} \end{pmatrix}$$

$\lambda_1(q) < \lambda_2(q)$ - eigen values of $\hat{A}(q)$

—·—·—·— border line of stability of helical state (starting from point S) $\longmapsto$

4 equations:

$$\begin{cases} \dfrac{\partial \Omega_{hel}}{\partial Q} = 0 \; ; \; \dfrac{\partial \Omega_{hel}}{\partial \Delta} = 0 \\ \lambda_1(q) = 0 \; ; \; \dfrac{\partial \lambda_1(q)}{\partial q} = 0 \end{cases}$$

SU(2) symmetric point and continuous vortex

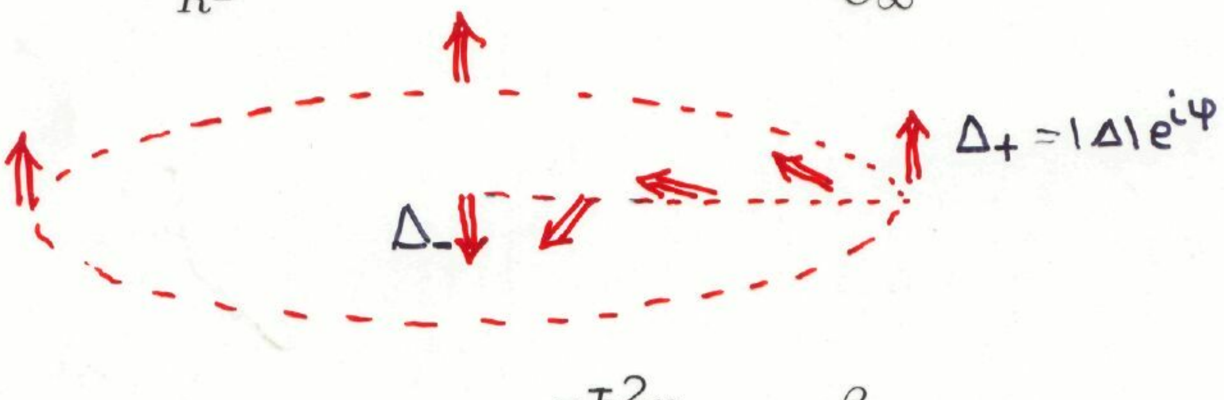
$$z = (z_1, z_2) = \left(\frac{\Delta_+}{|\Delta|}, \frac{\Delta_-}{|\Delta|}\right), \qquad |z|^2 = 1 \qquad S^3$$

Hopf projection $z^\dagger \vec{\sigma} z = \vec{n}$; $\quad \mathcal{A}_\mu = -i(z^\dagger \partial_\mu z - z \partial_\mu z^\dagger)$

$$E = \frac{\hbar^2}{8m} n_s \int \partial_\mu z^\dagger \partial_\mu z \, d^2\vec{r} = \frac{\hbar^2}{8m} n_s \int \left[\frac{1}{4}(\partial_\mu \vec{n})^2 + \mathcal{A}_\mu^2\right] d^2r$$

$$\pi_2(S^3, S^1) = Z \qquad \pi_2(S^3, S^1 \times S^1) = Z \times Z$$

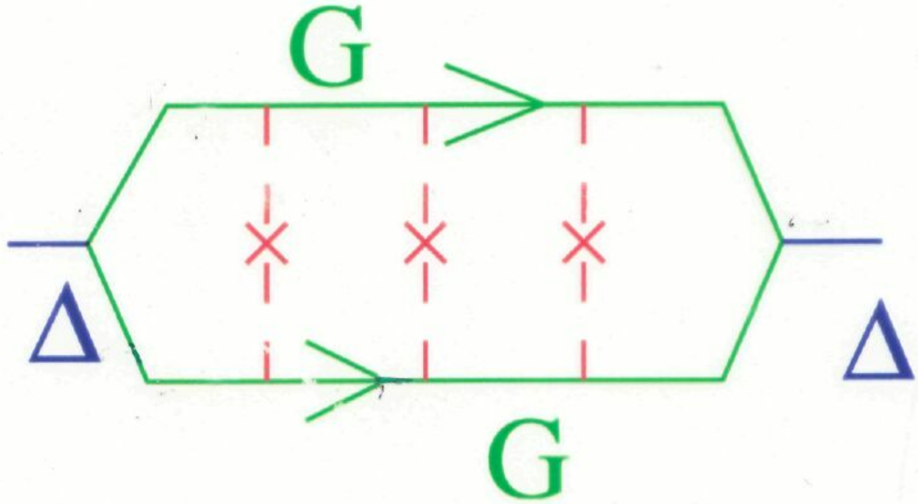
$$Q = \frac{1}{4\pi} \int_{R^2} \vec{n}\, [\partial_x \vec{n} \times \partial_y \vec{n}] \, d^2\vec{r} = \frac{1}{2\pi} \oint_{C_\infty} \vec{A} \cdot d\vec{l},$$



$$E_{Abr} - E_{cont} \propto \frac{\pi \hbar^2 n_s}{8m} \log \frac{\beta_s}{|\beta_a|}$$

Role of non-magnetic impurities on the phase diagram

$$\hat{H} = \sum_i \int \psi_\alpha^+(\mathbf{r})\psi_\alpha(\mathbf{r})u(\mathbf{r}-\mathbf{R}_i)d^2\mathbf{r}$$



$$\frac{1}{\nu(\epsilon_F)U} = \pi T \max_Q \sum_{\omega>0} 4\tau K\left(\omega, H_+, H_-, \frac{1}{2\tau}\right)$$

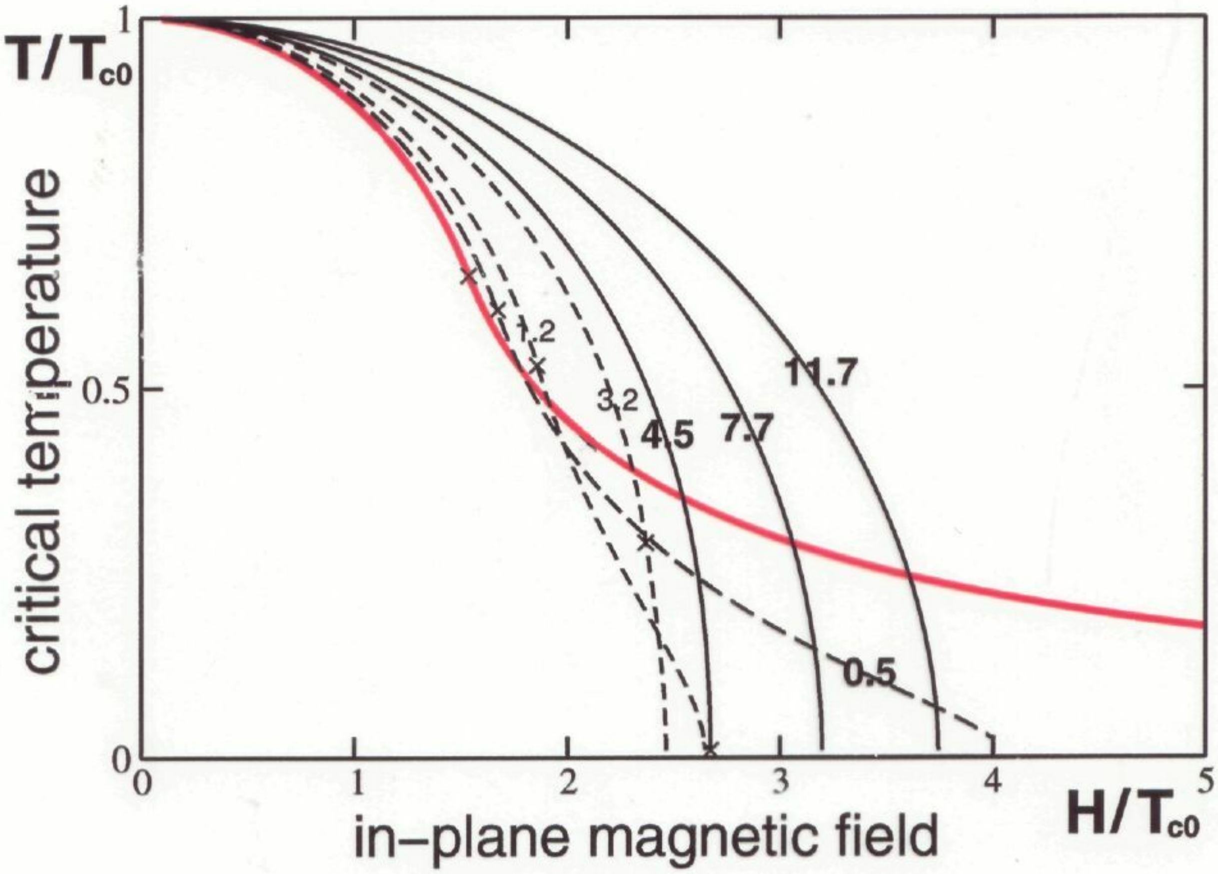
$$K = \frac{(I_+^0 + I_-^0)\left[1-(I_+^2+I_-^2)\right] + (I_+^1 - I_-^1)^2}{\left(1-(I_+^0+I_-^0)\right)\left[1-(I_+^2+I_-^2)\right] - (I_+^1-I_-^1)^2}$$

$$I_\lambda^0 = \frac{1}{\sqrt{(\omega+\frac{1}{2\tau})^2 + H_\lambda^2}} \cdot \frac{1}{4\tau},$$

$$H_\lambda = \lambda H + Q/2$$

$\vec{H} = 0$: $K = \frac{2}{|\omega|}$ agreeing with the Anderson theorem

Phase transition lines for different impurity scattering times τ : $\frac{1}{2\tau T_{c0}} = 0.5, ..., 11.7$



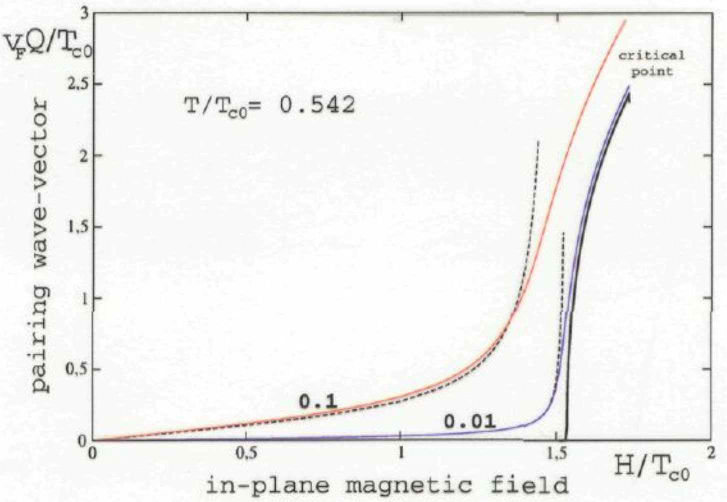
in the dirty limit $1/\tau \ll T_{c0}$ $\qquad H_{c2} \propto \sqrt{\frac{T_{c0}}{\tau}}$

Long-wavelength helical phase instead BCS

effects of $\frac{\alpha}{v_F}$

$$\delta\Omega_{hel} = \eta Q + \frac{\hbar^2 n_{yy}}{8m} Q^2$$

$$\eta = -\alpha\nu(\epsilon_F)\sum_\omega f(H,\omega)$$



$$Q_{hel} \approx 2\frac{\alpha H}{v_F^2}$$