Superconducting Tetrahedral Quantum Bits

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Outline

1. Tetrahedron: structure and symmetry
2. Low-energy spectrum
3. Perturbations and quantum manipulations
4. Measurement scheme
5. Conclusions

The goal:

Equivalent of quantum spin $\frac{1}{2}$
at zero magnetic field
made out of JJ

Non-abelian symmetry !

Will come from geometrical
symmetry of the device :

group T_d <=> S_4

tetrahedron — permutations of 4 elements

Contains 2-dimensional representations

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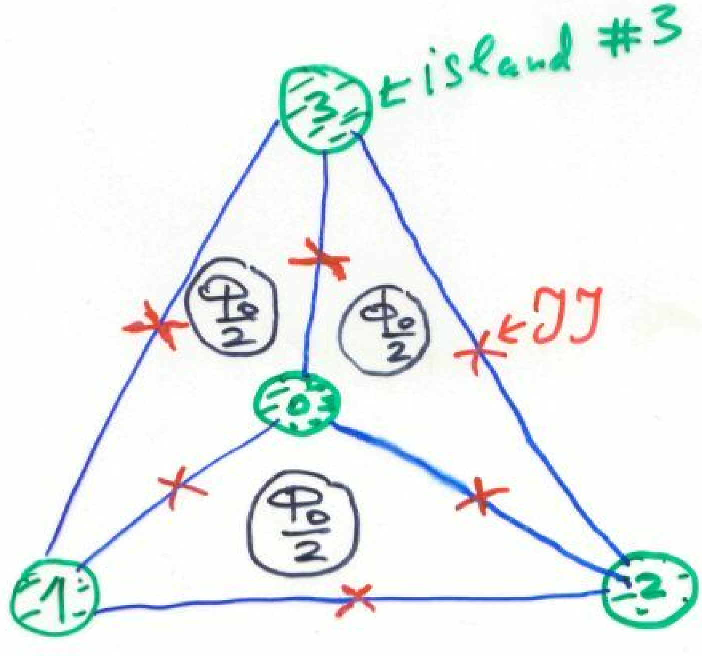
a) Easy manipulations
b) Stability to noise
c) Arraying



All triangles are frustrated by magnetic flux $\Phi_0/2$

$$V\{\varphi_i\} = E_J\Big[\cos\varphi_1 + \cos\varphi_2 + \cos\varphi_3 + \cos(\varphi_1-\varphi_2) + \cos(\varphi_2-\varphi_3) + \cos(\varphi_3-\varphi_1)\Big] =$$

$$= \frac{E_J}{2}\Big[\Big|\sum_j e^{i\varphi_j}\Big|^2 - 4\Big]$$

minimum at $\underbrace{\sum_{j=1,2,3} e^{i\varphi_j} = 0}$

Two equations for 3 variables!

Continuously degenerate minima of the potential energy $V\{\varphi_i\}$

Semiclassical analysis

$$\mathcal{L} = \frac{\hbar^2}{4e^2}\left[\sum_{i<j}\frac{C}{2}(\dot{\varphi}_i - \dot{\varphi}_j)^2 + \sum_j \frac{C_0}{2}\dot{\varphi}_j^2\right] -$$

$$- E_J \sum_{i<j} \cos(\varphi_i - \varphi_j)$$

New variables:

$$\chi_1 = \frac{1}{2}(\varphi_2 + \varphi_3 - \varphi_1)$$

$$\chi_2 = \frac{1}{2}(\varphi_1 + \varphi_3 - \varphi_2)$$

$$\chi_3 = \frac{1}{2}(\varphi_1 + \varphi_2 - \varphi_3)$$

Position of the line $O_1 - O_2$ is determined by χ_3 - slow variable

Two fast variables have frequencies:

$$\omega_{1,2} = \omega_J \sqrt{\frac{1 \mp \cos\chi_3}{2}} \qquad \omega_J = \frac{\sqrt{E_C E_J \cdot 8}}{\hbar}$$

Quantum induced potential

$$V_{ind}(\chi_3) = \frac{\hbar}{2}\left[\omega_1(\chi_3) + \omega_2(\chi_3)\right]$$

minima at $\chi_3 = 0, \pi$

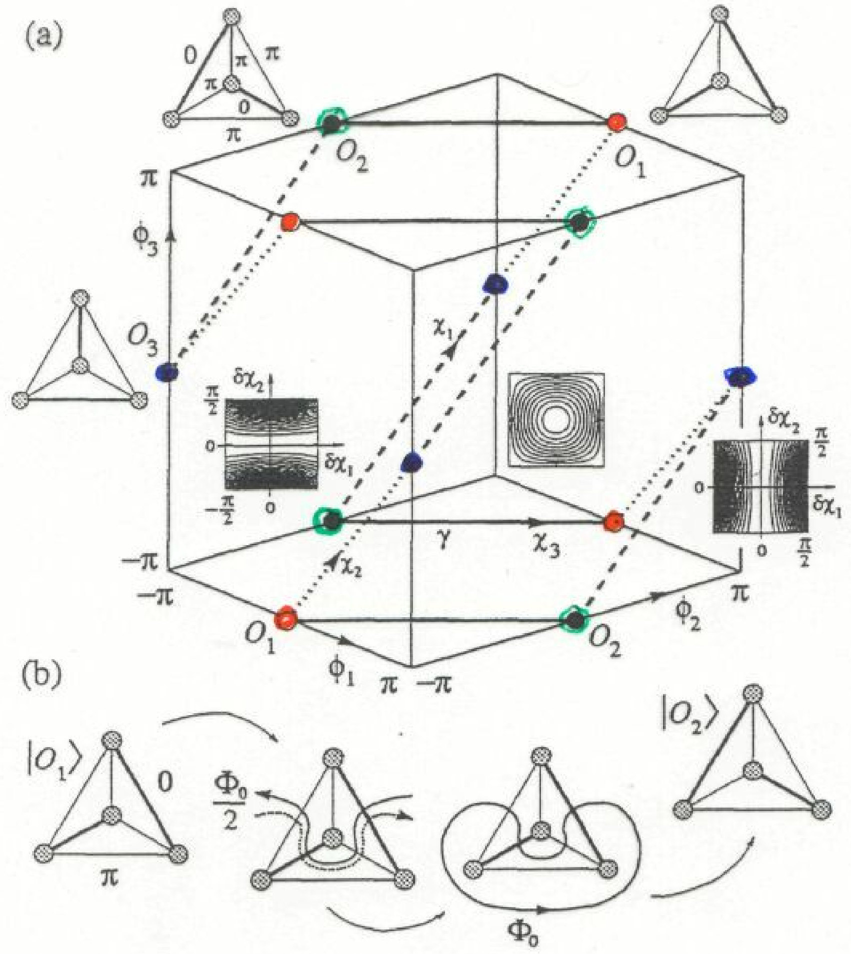


FIG. 2: (a) Continuous degeneracy of the classical minimal energy states. We identify 3 highly symmetric minimal states O_i, $i = 1, 2, 3$ (white, black, and grey dots) involving two opposite junctions with a phase difference $\phi_{ij} = 0$ and four remaining junctions with $\phi_{ij} = \pi$. Three families of lines (solid, dashed, dotted) connect these states around the cube $[-\pi, \pi]^3$ where the energy is continuously degenerate with a value $V_\pi = -2E_J$. The (non-orthogonal) coordinates χ_i are directed along the minima, e.g., $\chi_1 = 0$, $\chi_2 = -\pi$, $\chi_3 \in [0, \pi]$ along the reference line γ along which the change in the potential V_π is shown for the values $\chi_3 = 0, \pi/2, \pi$. Quantum fluctuations induce potential barriers along the minimal lines, separating the classical states O_i and transforming them into semi-classical states $|O_i\rangle$. (b) Tunneling between the states $|O_i\rangle$ establishes the low-energy spectrum of the tetrahedron. The tunneling process connecting the states $|O_1\rangle$ and $|O_2\rangle$ involves two trajectories where a fluxon $\Phi_0/2$ cuts through 4 junctions, flipping all of them by π. The phase difference between the two trajectories (the solid line corresponds to γ) produces the Aharonov-Bohm-Casher phase obtained when taking the fluxon Φ_0 around the islands '1' and '2'.

3 minima of the induced potential:

Tunnelling under the induced potential:

$$\hat{H} = \begin{pmatrix} 0 & t & t \\ t & 0 & t \\ t & t & 0 \end{pmatrix} \qquad \begin{aligned} E_s &= 2t \\ E_d &= -t \end{aligned}$$

$$t \propto \underbrace{\omega_J \exp\left[-1.88\left(\frac{E_J}{E_c}\right)^{1/4}\right]}_{a}$$

sign ?

!!!

instead of 1/2 !

$$\boxed{t = -2a\cos\left(\frac{\pi}{2}N\right)}$$

N – total number of Cooper pairs

Aharonov-Casher interference factor

$N = 4k$ $\qquad$ $N = 4k+2$

Doublet ground state

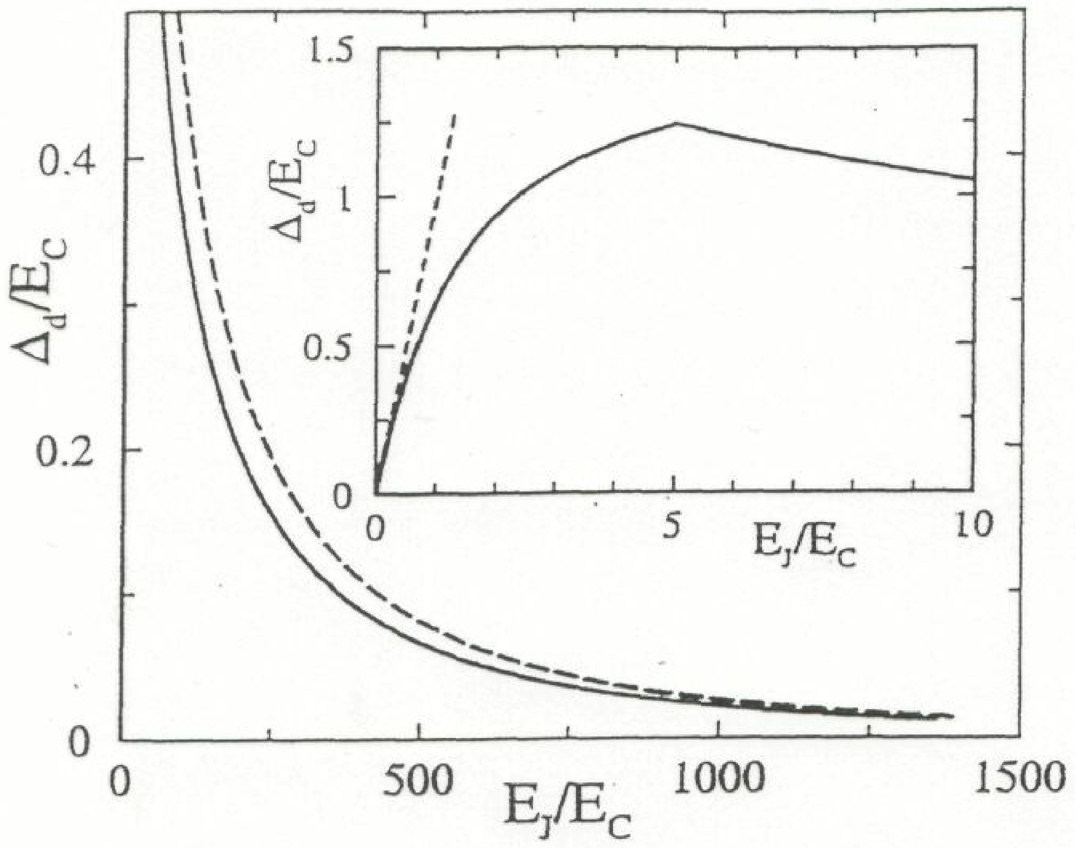
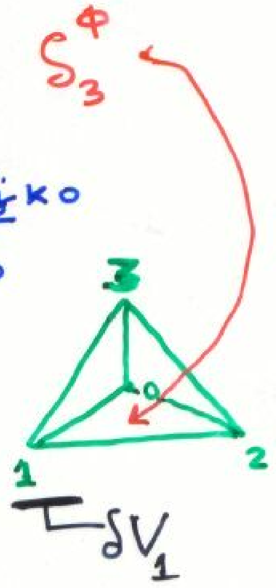


FIG. 3: Excitation gap Δ_d protecting the qubit doublet against higher excitations as a function of E_J/E_C; the dashed line is the semi-classic result based on (17). The inset shows an expanded view illustrating the crossing of the singlet and triplet excited states as the system enters the charge dominated regime at low values of E_J/E_C; the dashed line is the analytic result (20) valid in the charge dominated regime $E_C \gg E_J$. Data obtained with help of a Lanczos algorithm operating in the charge representation with 27 charge states between $q = \pm 13$.

$\frac{t}{E_J} \simeq 2 \cdot 10^{-3}$ for $\frac{E_J}{E_C} = 100$

Perturbations



magnetic flux: $\delta_i^{\Phi} = 2\pi \frac{\delta\Phi_{jk0}}{\Phi_0}$

electric potential: $\delta_i^{Q} \propto \delta V_i$

Doublet states
in the σ_z basis:

$\zeta = e^{\frac{2\pi i}{3}}$

$$|+\rangle = \left[|1\rangle + \zeta|2\rangle + \zeta^*|3\rangle\right]\frac{1}{\sqrt{3}}$$

$$|-\rangle = \left[|1\rangle + \zeta^*|2\rangle + \zeta|3\rangle\right]\frac{1}{\sqrt{3}}$$

Singlet: $|0\rangle = \frac{1}{\sqrt{3}}\left[|1\rangle + |2\rangle + |3\rangle\right]$

Projecting perturbations to the doublet subspace:

$$\hat{H} = \vec{h}\,\vec{\sigma}$$

$$h_z = a\cdot\left(\frac{E_J}{E_c}\right)^{3/4}\Big[(\delta_1^{\Phi}+\delta_2^{\Phi})(\delta_1^{Q}+\delta_2^{Q}) + (\delta_1^{\Phi}+\delta_3^{\Phi})(\delta_1^{Q}+\delta_3^{Q}) + (\delta_2^{\Phi}+\delta_3^{\Phi})(\delta_2^{Q}+\delta_3^{Q})\Big]$$

$$h_x = h_1 - \frac{h_2+h_3}{2} \qquad h_y = \frac{\sqrt{3}}{2}(h_2-h_3)$$

$$h_i = \frac{E_J}{3}\left(\frac{E_J}{E_c}\right)^{1/3}(\delta_j^{\Phi}+\delta_k^{\Phi})^2 \qquad \text{e.g. } h_1 \propto (\delta_2^{\Phi}+\delta_3^{\Phi})^2$$

Why 1^{st}-order perturbations are absent?

- physical answer: ground state does not carry current or charge

- math. answer: original symmetry = T_d

perturbation at single island or single loop reduces symmetry down to C_{3v} - that one still contains 2-Dim. representatio

Two good messages:

1. effects of e.-m. noise are weak
2. effective "magnetic field" $\vec{h}$ can be easily manipulated by fluxes and local gates

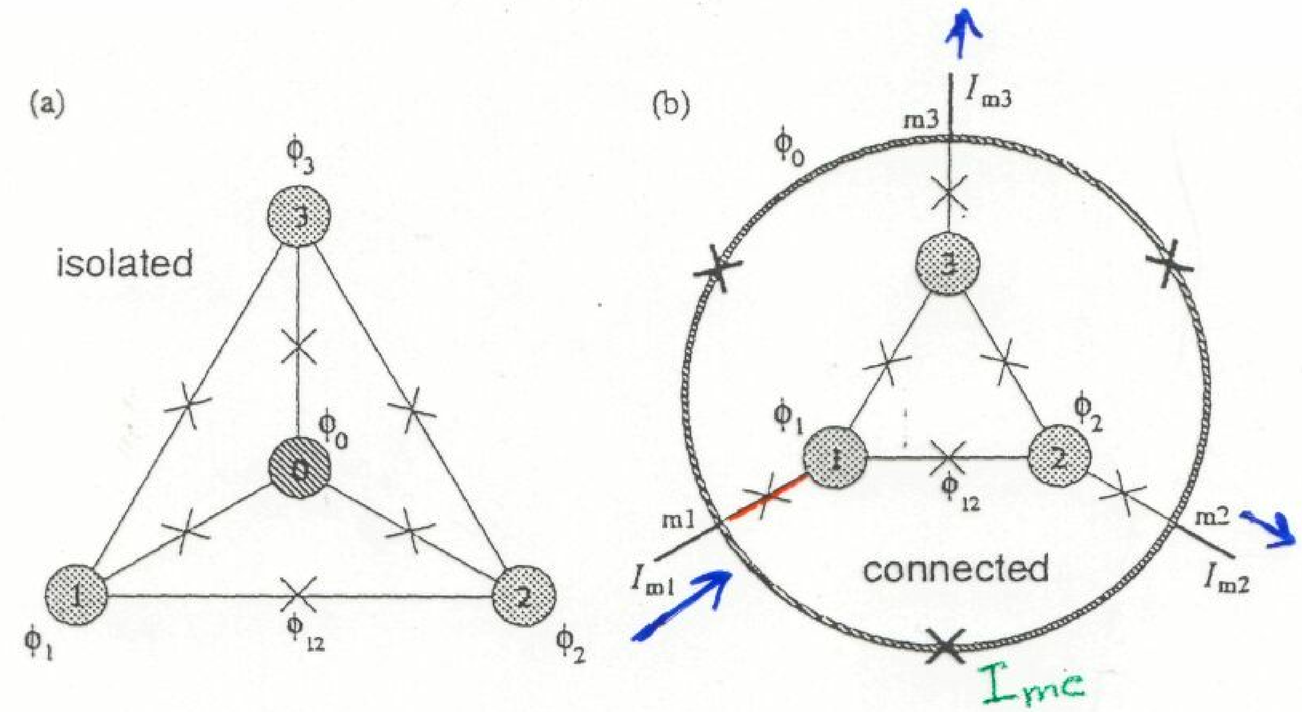


FIG. 1: (a) 'Tetrahedral' superconducting qubit involving four islands and six junctions; all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron; the inner dark-grey island in (a) is then transformed into the outer ring in (b). The additional junctions on the outer ring are used in the measurement process probing the qubit state via the external currents $I_{\mathrm{m}i}$; their coupling E_{m} is large compared to the Josephson energy E_J of the six qubit junctions, hence effectively binding the ring into one island.

Measurement scheme

1. Induce current by charge bias:

$$\hat{I}^{(m1-1)} = \frac{2e}{\hbar}\left(\frac{\partial \hat{H}^{(0)}}{\partial \delta_2^{\Phi}} - \frac{\partial \hat{H}^{(0)}}{\partial \delta_3^{\Phi}}\right) =$$

$$= i\frac{2es}{\hbar}\, a \sin(\pi\delta^{Q}) \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad \left(|O_i\rangle \text{ basis}\right)$$

where $\delta_1^{Q} = 0$, $\delta_2^{Q} = -\delta_3^{Q} \equiv \delta^{Q}$

Charge bias modify tunnelling matrix:

$$t \to t_{ij}^{(s)} + i\, t_{ij}^{(a)} \qquad t_{ij}^{(s)} = t\cos(\pi\delta_{ij}^{Q})$$

$$t_{ij}^{(a)} = s\, t \cdot \delta_{ij}^{\Phi} \sin(\pi\delta_{ij}^{Q})$$

Adiabatic evolution from $\delta^{Q} = 0$ to $\delta^{Q} = \frac{1}{2}$ produces $\hat{I}^{(m1-1)}$ operator <u>diagonal</u> in the $\hat{\sigma}_z$ basis, with no admixture <u>of singlet state</u>

$$\boxed{\hat{I}^{(m1-1)} = \frac{2\sqrt{2}es}{\hbar}\, a\, \hat{\sigma}_z}$$

② Detect current by means of Vion et al method :

External drive current I_{m1} "in"

equal currents $I_{m2} = I_{m3} = \frac{I_{m1}}{2}$ "out"

Choose $\boxed{I_{m1} \simeq 2 I_{mc}}$ putting large junctions into nearly-critical state

$$I_{m1-m2} = \frac{I_{m1}}{2} + \text{const}\ \hat{\sigma}_z$$

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voltage pulse for $|+\rangle$ state

no voltage for $|-\rangle$ state

Similar scheme can be used for measurement in $\hat{\sigma}_x$ basis

Conclusions

1. Trivial idle state

2. weak sensitivity (2^{nd} order) to electric and magnetic noise

3. relieved constraints on junction fabrication

4. full freedom of manipulations

5. convinient measurement schemes

6. Array ?