Triplet proximity effect in FSF trilayers

Yakov Fominov [1,2], **Alexander Golubov** [2], **Mikhail Kupriyanov** [3]

[1] Landau Institute for Theoretical Physics, Russia
[2] University of Twente, The Netherlands
[3] Nuclear Physics Institute, Russia

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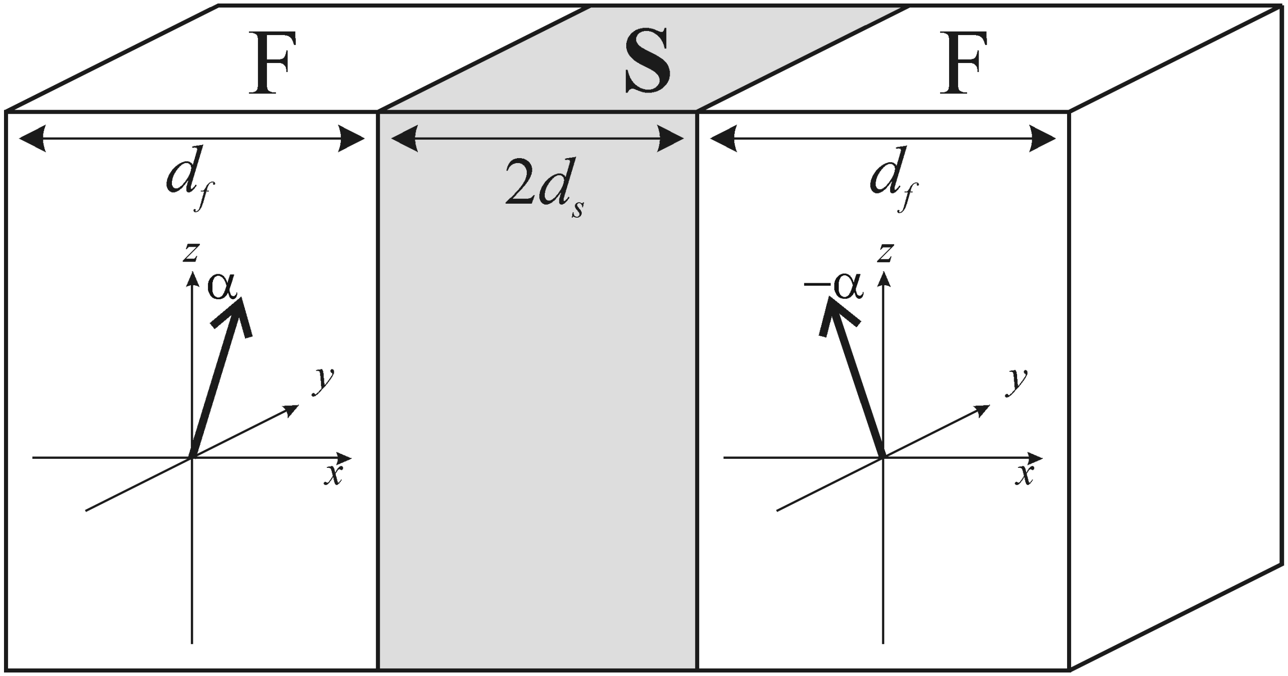
FSF trilayer

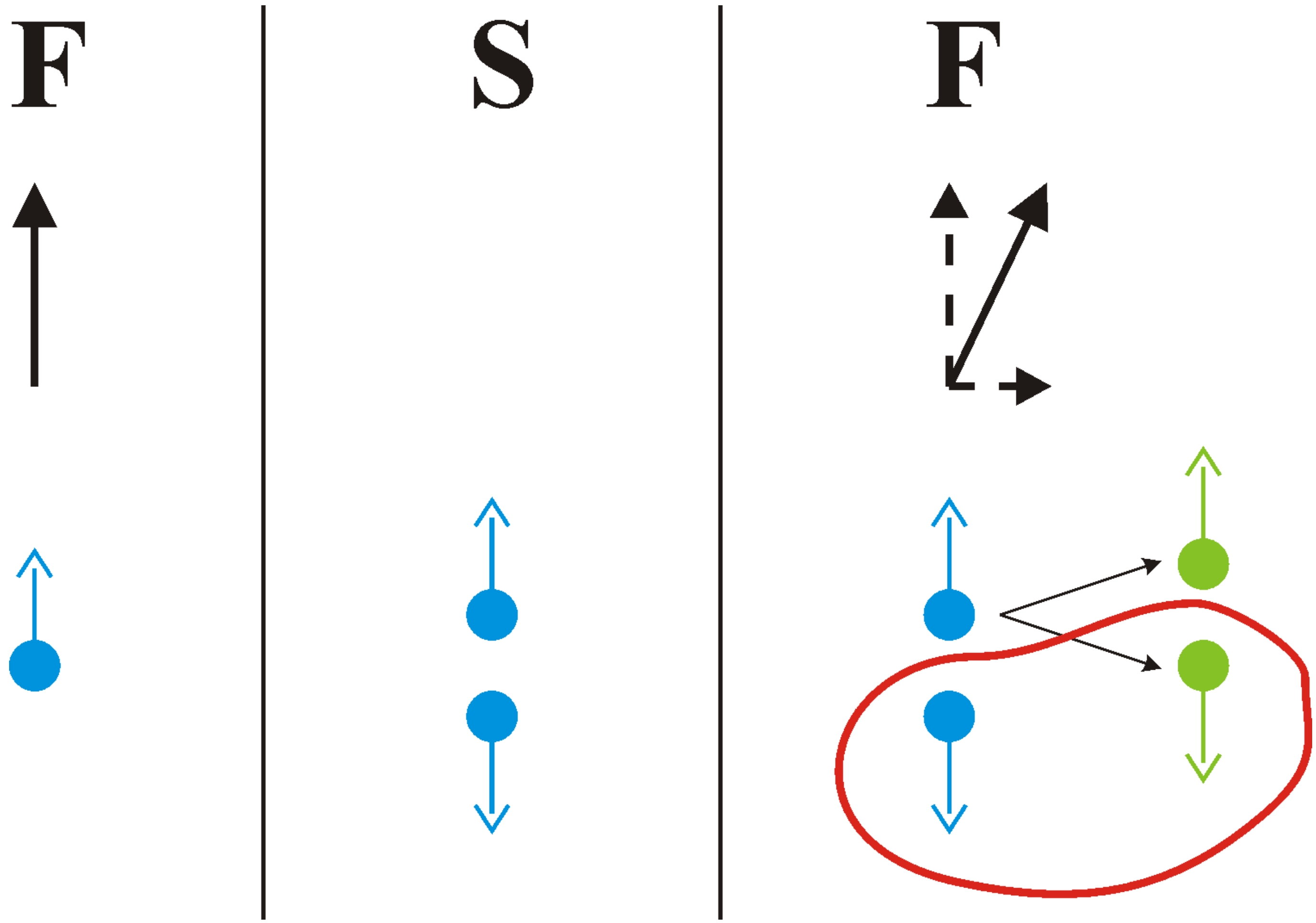


- Long-range triplet superconducting component at noncollinear magnetizations [Volkov, Bergeret, Efetov (2003)]

- Odd in energy, even in momentum [Berezinskii (1974)] ⇒ survives in the dirty limit

- $d_s \ll \xi_s = \sqrt{\dfrac{D_s}{2\pi T_{cs}}}$ — is there superconductivity at all ?

How the triplet component is generated



Quasiclassical equations in the dirty limit

The Usadel equation: $$\frac{D}{2}\frac{d^2\widehat{F}}{dx^2} - \omega_n \widehat{F} + \Delta\widehat{\sigma}_3 - \frac{i}{2}\left(\widehat{F}\widehat{H}^* + \widehat{H}\widehat{F}\right) = 0,$$

where $$\widehat{F} = \begin{pmatrix} f_{\uparrow\downarrow} & f_{\uparrow\uparrow} \\ f_{\downarrow\downarrow} & f_{\downarrow\uparrow} \end{pmatrix}, \qquad \widehat{H} = h\left(\widehat{\sigma}_2 \sin\alpha + \widehat{\sigma}_3 \cos\alpha\right)$$

The self-consistency equation: $$\Delta \ln \frac{T_{cs}}{T} = \pi T \sum_{\omega_n} \left(\frac{\Delta}{|\omega_n|} - f_{\uparrow\downarrow} \right)$$

The boundary conditions:

outer surfaces

$$\frac{d\widehat{F}_f}{dx} = 0$$

SF interfaces

$$\xi_s \frac{d\widehat{F}_s}{dx} = \gamma \xi_f \frac{d\widehat{F}_f}{dx}, \qquad \gamma = \frac{\rho_s \xi_s}{\rho_f \xi_f}, \qquad \xi_{s(f)} = \sqrt{\frac{D_{s(f)}}{2\pi T_{cs}}}$$

$$\xi_f \gamma_b \frac{d\widehat{F}_f}{dx} = \widehat{F}_s - \widehat{F}_f, \qquad \gamma_b = \frac{R_b}{\rho_f \xi_f / \mathcal{A}}$$

The solution can be represented in the form $\widehat{F} = f_0\widehat{\sigma}_0 + f_1\widehat{\sigma}_1 + f_3\widehat{\sigma}_3$

The Usadel equation in components:

$$\frac{D}{2}\frac{d^2 f_0}{dx^2} - \omega_n f_0 - ihf_3 \cos\alpha = 0,$$

$$\frac{D}{2}\frac{d^2 f_1}{dx^2} - \omega_n f_1 + hf_3 \sin\alpha = 0,$$

$$\frac{D}{2}\frac{d^2 f_3}{dx^2} - \omega_n f_3 - ihf_0 \cos\alpha - hf_1 \sin\alpha + \Delta(x) = 0$$

The wave vectors: $k_s = \sqrt{\frac{2\omega_n}{D_s}}, \quad k_f = \sqrt{\frac{2\omega_n}{D_f}}, \quad k_h = \sqrt{\frac{h}{D_f}}, \quad \widetilde{k}_h = \sqrt{k_f^2 + 2ik_h^2}$

The solution in the left F layer:

$$\widehat{F}_f = C_1\left(i\widehat{\sigma}_0 \sin\alpha + \widehat{\sigma}_1 \cos\alpha\right)\cosh\left[k_f\left(x + d_s + d_f\right)\right] +$$
$$+ C_2\left(\widehat{\sigma}_0 \cos\alpha + i\widehat{\sigma}_1 \sin\alpha + \widehat{\sigma}_3\right)\cosh\left[\widetilde{k}_h\left(x + d_s + d_f\right)\right] +$$
$$+ C_3\left(\widehat{\sigma}_0 \cos\alpha + i\widehat{\sigma}_1 \sin\alpha - \widehat{\sigma}_3\right)\cosh\left[\widetilde{k}_h^*\left(x + d_s + d_f\right)\right]$$

We can consider only one half of the system with the following boundary conditions in the center of the system:

$$\frac{df_0}{dx} = 0, \qquad f_1 = 0, \qquad \frac{df_3}{dx} = 0$$

Effectively, we obtain the following problem:

$$\Delta \ln \frac{T_{cs}}{T} = 2\pi T \sum_{\omega_n > 0} \left(\frac{\Delta}{\omega_n} - f_3 \right),$$

$$\frac{D_s}{2} \frac{d^2 f_3}{dx^2} - \omega_n f_3 + \Delta = 0,$$

$$\xi_s \frac{df_3(-d_s)}{dx} = W f_3(-d_s), \qquad \frac{df_3(0)}{dx} = 0$$

All information about the F layers is in the function W:

$$W = \mathrm{Re}\, V_h + \frac{(\mathrm{Im}\, V_h)^2}{k_s \xi_s A(\alpha) + \mathrm{Re}\, V_h},$$

$$A(\alpha) = \frac{k_s \xi_s \tanh(k_s d_s) + V_f \left[\sin^2\alpha + \tanh^2(k_s d_s)\cos^2\alpha\right]}{k_s \xi_s \left[\cos^2\alpha + \tanh^2(k_s d_s)\sin^2\alpha\right] + V_f \tanh(k_s d_s)},$$

$$V_f = \frac{\gamma k_f \xi_f \tanh(k_f d_f)}{1 + \gamma_b k_f \xi_f \tanh(k_f d_f)}, \qquad V_h = \frac{\gamma \widetilde{k}_h \xi_f \tanh(\widetilde{k}_h d_f)}{1 + \gamma_b \widetilde{k}_h \xi_f \tanh(\widetilde{k}_h d_f)}$$

- $T_c(\alpha)$ is a monotonically growing function; T_c is largest in the antiparallel case
- The dependence of T_c on α disappears if $d_s \gg \xi_s$
- T_c does not depend on d_f if $d_f \gg \xi_f$

Numerical method

The fundamental solution G satisfies the equations

$$\frac{D_s}{2}\frac{d^2G(x,y)}{dx^2}-\omega_n G(x,y)=-\delta(x-y),$$

$$\xi_s\frac{dG(-d_s,y)}{dx}=W(\omega_n)G(-d_s,y),\qquad \frac{dG(0,y)}{dx}=0$$

and can be readily found:

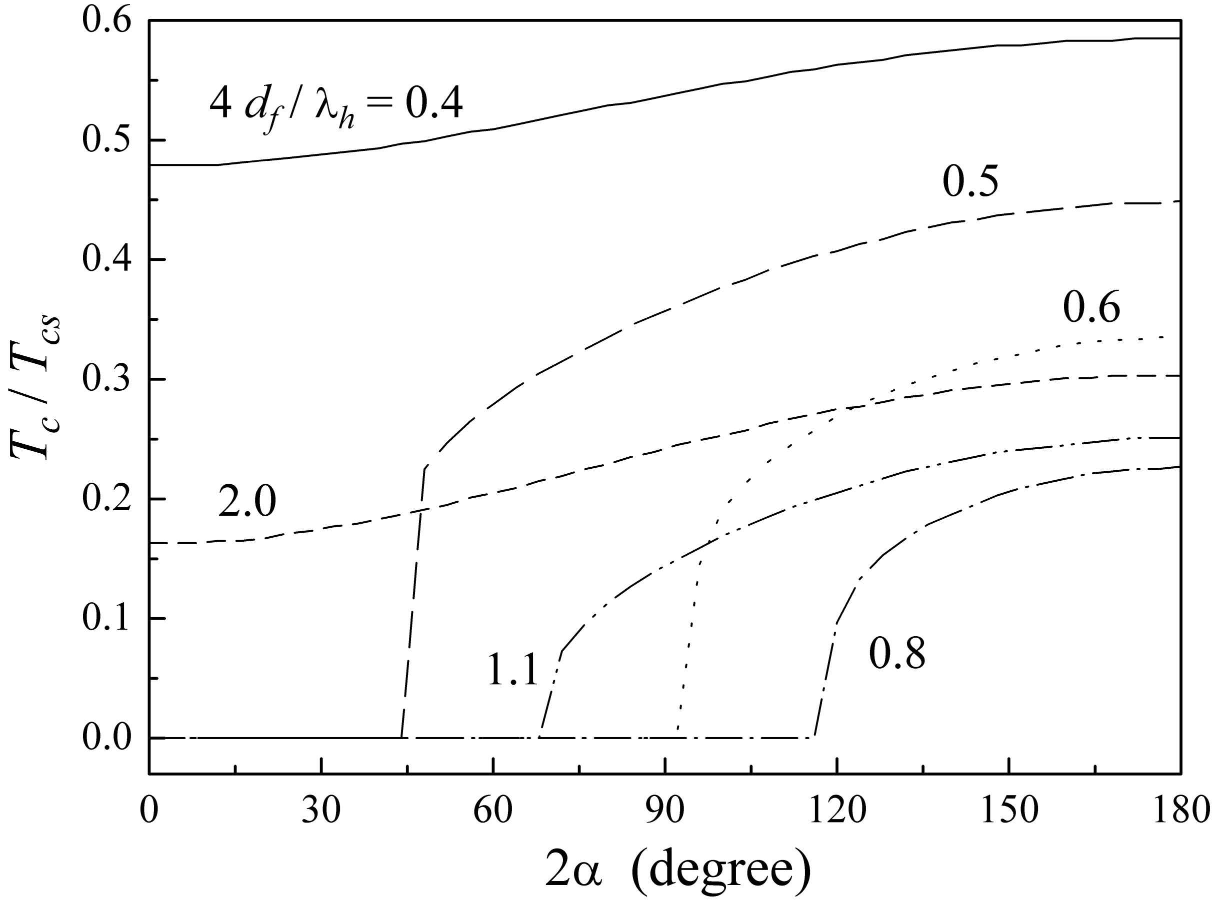
$$G(x,y;\omega_n)=\frac{k_s/\omega_n}{\sinh(k_s d_s)+(W/k_s\xi_s)\cosh(k_s d_s)}\times\begin{cases}v_1(x)v_2(y), & x\leqslant y\\ v_2(x)v_1(y), & y\leqslant x\end{cases}$$

where $v_1(x)=\cosh(k_s(x+d_s))+(W/k_s\xi_s)\sinh(k_s(x+d_s)),\quad v_2(x)=\cosh(k_s x).$

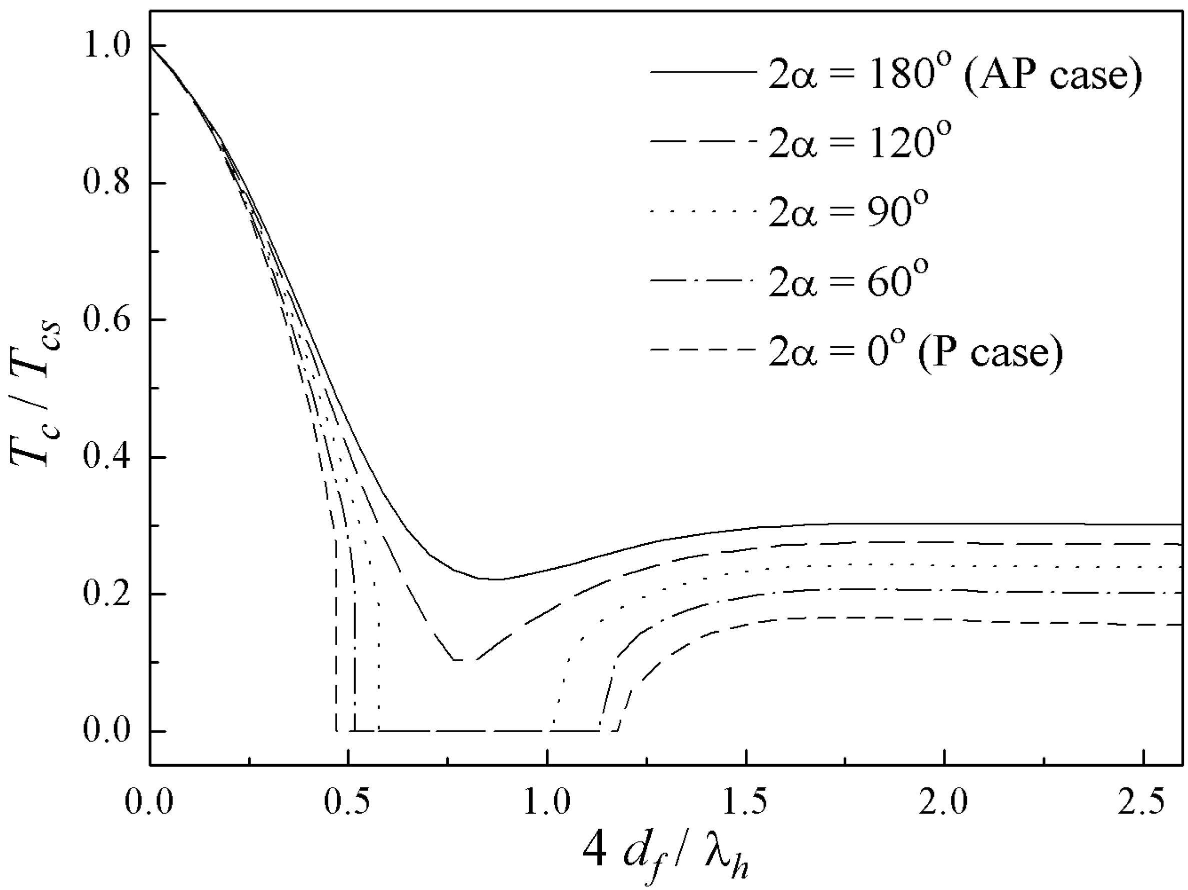
The solution of the Usadel equation: $\quad f_3(x;\omega_n)=\int_{-d_s}^{0}G(x,y;\omega_n)\Delta(y)dy$

As a result, the self-consistency equation takes the form

$$\Delta(x)\ln\frac{T_{cs}}{T_c}=2\pi T_c\sum_{\omega_n>0}\left[\frac{\Delta(x)}{\omega_n}-\int_{-d_s}^{0}G(x,y;\omega_n)\Delta(y)dy\right]$$



$$\lambda_h = 2\pi/k_h$$



1.0
0.8
0.6
0.4
0.2
0.0
T_c / T_{cs}
0.0
0.5
1.0
1.5
2.0
2.5
$4\, d_f / \lambda_h$
$2\alpha = 180^o$ (AP case)
$2\alpha = 120^o$
$2\alpha = 90^o$
$2\alpha = 60^o$
$2\alpha = 0^o$ (P case)

Thin S layer ($d_s \ll \xi_s$), collinear magnetizations

$\Delta(x) = \text{const} \quad \Rightarrow \quad$ the Usadel equation is readily solved

In the parallel and antiparallel cases at $h \gg \pi T_{cs}$ we obtain:

$$\ln \frac{T_{cs}}{T_c^P} = \operatorname{Re} \psi \left(\frac{1}{2} + \frac{V_h}{2} \frac{\xi_s}{d_s} \frac{T_{cs}}{T_c^P} \right) - \psi \left(\frac{1}{2} \right),$$
$$\ln \frac{T_{cs}}{T_c^{AP}} = \psi \left(\frac{1}{2} + \frac{W}{2} \frac{\xi_s}{d_s} \frac{T_{cs}}{T_c^{AP}} \right) - \psi \left(\frac{1}{2} \right)$$

The critical thickness:

$$\frac{d_{sc}^P}{\xi_s} = 2e^C \left| V_h \right|, \qquad \frac{d_{sc}^{AP}}{\xi_s} = 2e^C W, \qquad C \approx 0.577$$

$d_s \ll \xi_s$, $h \gg \pi T_{cs}$, $k_h d_f \gg 1$, $\gamma_b = 0$, **arbitrary angle**

The superconductivity exists if $2e^C \gamma k_h \xi_f < \dfrac{d_s}{\xi_s} \ll 1$

The equation for T_c:

$$\ln \frac{T_{cs}}{T_c} = Q\,\psi\left(\frac{1}{2} + \frac{\Omega_1}{2\pi T_c}\right) + R\,\psi\left(\frac{1}{2} + \frac{\Omega_2}{2\pi T_c}\right) - \psi\left(\frac{1}{2}\right),$$

where

$$Q = \frac{1}{2} + \frac{\sin^2\alpha}{2\sqrt{\sin^4\alpha - 4\cos^2\alpha}}, \qquad R = 1 - Q,$$

$$\Omega_{1,2} = \frac{d_0}{d_s}\pi T_{cs}\left(1 + \cos^2\alpha \pm \sqrt{\sin^4\alpha - 4\cos^2\alpha}\right), \qquad d_0 = \gamma k_h \xi_f \xi_s / 2$$

The critical thickness:

$$\frac{d_{sc}(\alpha)}{d_0} = 4\sqrt{2}e^C \cos\alpha \left(\frac{1 + \cos^2\alpha + \sqrt{\sin^4\alpha - 4\cos^2\alpha}}{1 + \cos^2\alpha - \sqrt{\sin^4\alpha - 4\cos^2\alpha}}\right)^{\frac{\sin^2\alpha}{2\sqrt{\sin^4\alpha - 4\cos^2\alpha}}}$$

Odd triplet superconductivity in SF multilayers [Volkov, Bergeret, Efetov, PRL 90, 117006 (2003)]

The Josephson current is entirely due to the long-range triplet component if

$$d_s \ll \xi_s, \qquad k_h^{-1} \ll \xi_f < d_f$$

Under these conditions the superconductivity exists if

$$4e^C \gamma k_h \xi_f \frac{1 + 2\gamma_b k_h \xi_f}{(1 + 2\gamma_b k_h \xi_f)^2 + 1} < \frac{d_s}{\xi_s} \ll 1$$

- At $\gamma_b = 0$ (perfect interface) this yields: $2e^C \gamma k_h \xi_f < \dfrac{d_s}{\xi_s} \ll 1$
- At $\gamma_b \gtrsim 1$: $2e^C \dfrac{\gamma}{\gamma_b} < \dfrac{d_s}{\xi_s} \ll 1$