

# **ELECTRON ENERGY and PHASE RELAXATION in the PRESENCE of MAGNETIC IMPURITIES**

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**in collaboration with**

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**Thanks to:**

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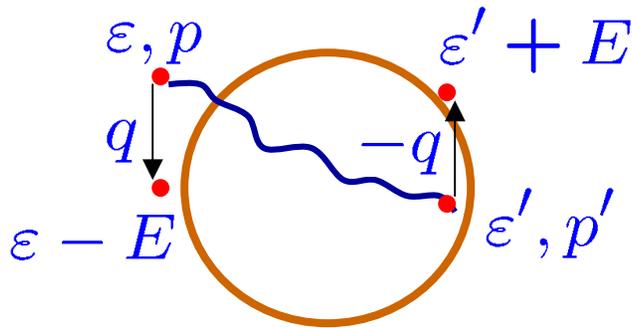
MESO 2003

# Outline

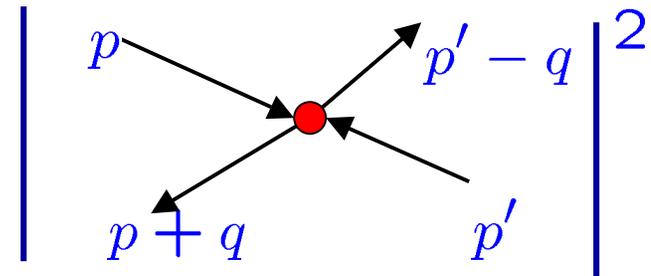
- Electron energy and phase relaxation in a Fermi liquid
- The effect of magnetic impurities on electron momentum and phase relaxation
- Electron energy relaxation facilitated by magnetic impurities
- Quenching the impurity dynamics by a magnetic field or RKKY interaction: consequences for conduction electrons

# Electron in a Fermi-liquid

## 1. Electron relaxation in a “clean” Fermi liquid



Rate of transitions:



Rate of transitions (near the Fermi surface) with energy transfer  $E$ :

$$K(E) = \int d^d q |V(q)|^2 \int d\mathbf{n}_p \delta(E - v_F \mathbf{q} \cdot \mathbf{n}_p) \int d\mathbf{n}_{p'} \delta(E - v_F \mathbf{q} \cdot \mathbf{n}_{p'}) \propto \text{const}$$

“form-factor” =  $\frac{1}{(qv_F)^2}$   
(3D metal)

Rate of electron **energy** relaxation:

$$\frac{1}{\tau_E} = \int_0^\epsilon dE \int_{-E}^0 d\epsilon' K(E) \propto \epsilon^2$$

$$K(E) \propto \text{const} \leftrightarrow \frac{1}{\tau_E} \propto \epsilon^2$$

# Electron in a Fermi-liquid

## 2. Electron **energy** relaxation in a “dirty” Fermi liquid

$$K(E) = \int d^d q |V(q)|^2 \frac{1}{(qv_F)^2}$$

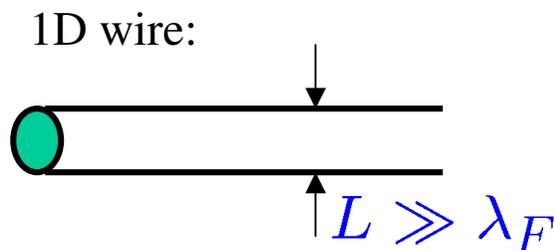
Particles **diffuse** instead of moving ballistically, stay together and interact for a **longer** time



1979 Schmid;  
Altshuler, Aronov

$$K(E) = \int d^d q |V(q)|^2 \left[ \frac{Dq^2}{E^2 + (Dq^2)^2} \right]^2$$

$$D = \frac{1}{3} v_F^2 \tau_{tr}$$



$$\frac{1}{\tau_\varepsilon} = \int E dE K(E) \propto \left( \frac{\lambda_F}{L} \right)^2 \sqrt{\frac{\varepsilon}{\hbar \tau_{tr}}} \propto \varepsilon^{1/2}$$

$$K(E) \sim \left( \frac{\lambda_F}{L} \right)^2 \frac{1}{\sqrt{\hbar \tau_{tr}}} E^{-3/2}$$

# Electron in a Fermi-liquid

## 3. Evolution of an electron distribution: kinetic equation

$$\frac{df}{dt} = I\{f\} \quad \text{collision integral}$$

$$I\{f\} = - \int d\varepsilon' \int dE K(E)$$


$$\times \{f(\varepsilon)f(\varepsilon')[1 - f(\varepsilon - E)][1 - f(\varepsilon' + E)] \\ - f(\varepsilon - E)f(\varepsilon' + E)[1 - f(\varepsilon)][1 - f(\varepsilon')]\}$$

$$\frac{1}{\tau_\varepsilon} \propto \varepsilon^2 \leftrightarrow K(E) \propto \text{const}$$

$$\frac{1}{\tau_\varepsilon} \propto \varepsilon^{1/2} \leftrightarrow K(E) \propto E^{-3/2}$$

# Electron in a Fermi-liquid

## 4. Electron **phase** relaxation in a “dirty” Fermi liquid

d=1 or 2: e-e collisions are quasi-elastic, **phase** is **lost faster** than **energy**

$$\frac{1}{\tau_\varphi(\varepsilon)} \sim \varepsilon^{1/3} \tau^{-2/3} \propto \varepsilon^{2/3} \quad (\text{Altshuler, Aronov, Khmelnitskii 1982})$$

$$1/\tau_\varphi \gg 1/\tau_\varepsilon \text{ at } \varepsilon \geq (\lambda_F/L)^4 \hbar \tau_{tr}^{-1}$$

Fermi Liquid:  $\tau_\varphi, \tau_\varepsilon \rightarrow \infty$  at  $\varepsilon \rightarrow 0$

How to measure  $\tau_\varphi, \tau_\varepsilon$ ?

$\tau_\varphi$  : Temperature dependence of magnetoresistance

$$\delta R(B/B^*); B^* \sim \Phi_0 / \sqrt{L^2 D \tau_\varphi(T)} \propto 1/\tau_\varphi^{1/2}$$

$\tau_\varepsilon$  : Relaxation of out-of-equilibrium electrons

# Experiments on Energy Relaxation: Cu

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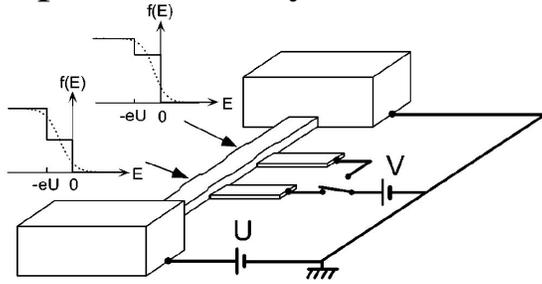
## Energy Distribution Function of Quasiparticles in Mesoscopic Wires

H. Pothier, S. Guéron, Norman O. Birge,\* D. Esteve, and M. H. Devoret

*Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay, F-91191 Gif-sur-Yvette, France*

(Received 25 April 1997)

Experimental layout:



Results:

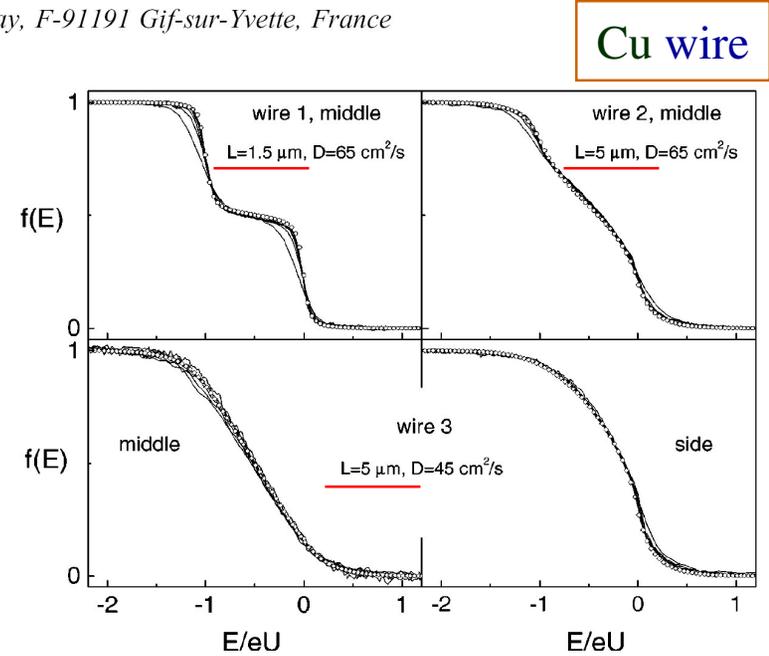


FIG. 3. Continuous lines in all four panels: distribution functions, for  $U$  ranging from 0.05 to 0.3 mV by steps of 0.05 mV, plotted as a function of the reduced energy  $E/eU$ , for the same positions as in Fig. 2. Open symbols are best fits of the data to the solution of the Boltzmann equation with an interaction kernel  $K(x, x', \varepsilon) = \tau_0^{-1} \delta(x - x') / \varepsilon^2$ ; in top panel, open circles correspond to the calculated distribution function in the middle of wires 1 and 2 ( $x = 0.5$ ), with  $\tau_0/\tau_D = 2.5$  and  $\tau_0/\tau_D = 0.3$ , respectively (both compatible with  $\tau_0 \sim 1$  ns). In bottom panels, open diamonds are computed at  $x = 0.5$  and  $x = 0.25$  with  $\tau_0/\tau_D = 0.08$  ( $\tau_0 \sim 0.5$  ns).

Data for  $f(\varepsilon)$  scale

with  $E/eU \rightarrow K(E, U) \propto \frac{1}{E^2} g\left(\frac{eU}{E}\right)$

If  $K$  is  $U$ -independent, then

$$K(E) \approx \frac{1}{\tau_0 E^2}$$

with  $\tau_0 \approx 1$  ns;

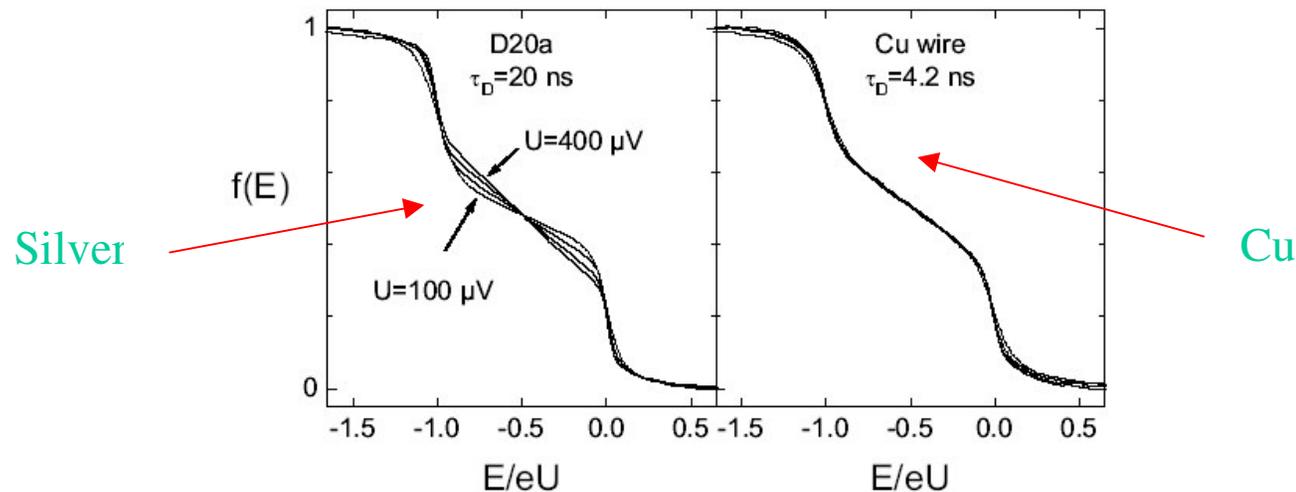
$1/\tau_\varepsilon$  does not go to zero at  $\varepsilon \rightarrow 0$  !

# Experiments on Energy Relaxation: Ag vs. Cu

Energy Redistribution Between Quasiparticles in Mesoscopic Silver Wires  
JLTP 118, p. 447 (2000)

F. Pierre, H. Pothier, D. Esteve, and M.H. Devoret

We have measured with a tunnel probe the energy distribution function of quasiparticles in silver diffusive wires connected to two large pads (“reservoirs”), between which a bias voltage was applied. From the dependence in energy and bias voltage of the distribution function we have inferred the energy exchange rate between quasiparticles. In contrast with previously obtained results on copper and gold wires, these data on silver wires can be well interpreted with the theory of diffusive conductors...



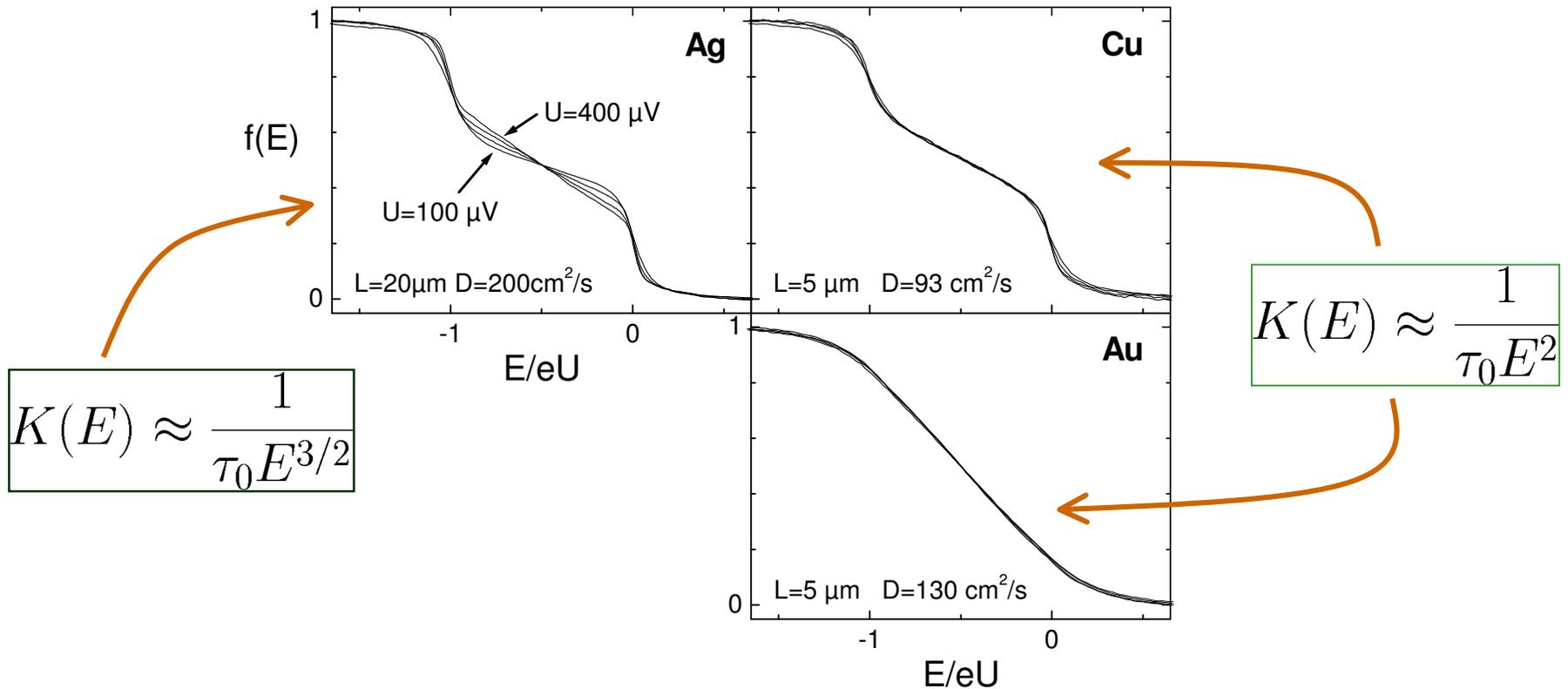
Distribution functions for  $U = 0.1, 0.2, 0.3,$  and  $0.4$  mV, plotted as a function of the reduced energy  $E/eU$ .

Left panel: Ag sample D20a; right panel: Cu sample,  $L = 5$   $\mu$ m.

“In silver samples we have assumed that the interaction kernel still obeys a power law  $K(\varepsilon) = \kappa_\alpha \varepsilon^{-\alpha}$ , with  $\kappa_\alpha$  and  $\alpha$  taken as fitting parameters... the best fits obtained with the exponent set at its predicted value  $\alpha = 3/2$ . ”

# Energy Relaxation in Ag, Cu, and Au wires

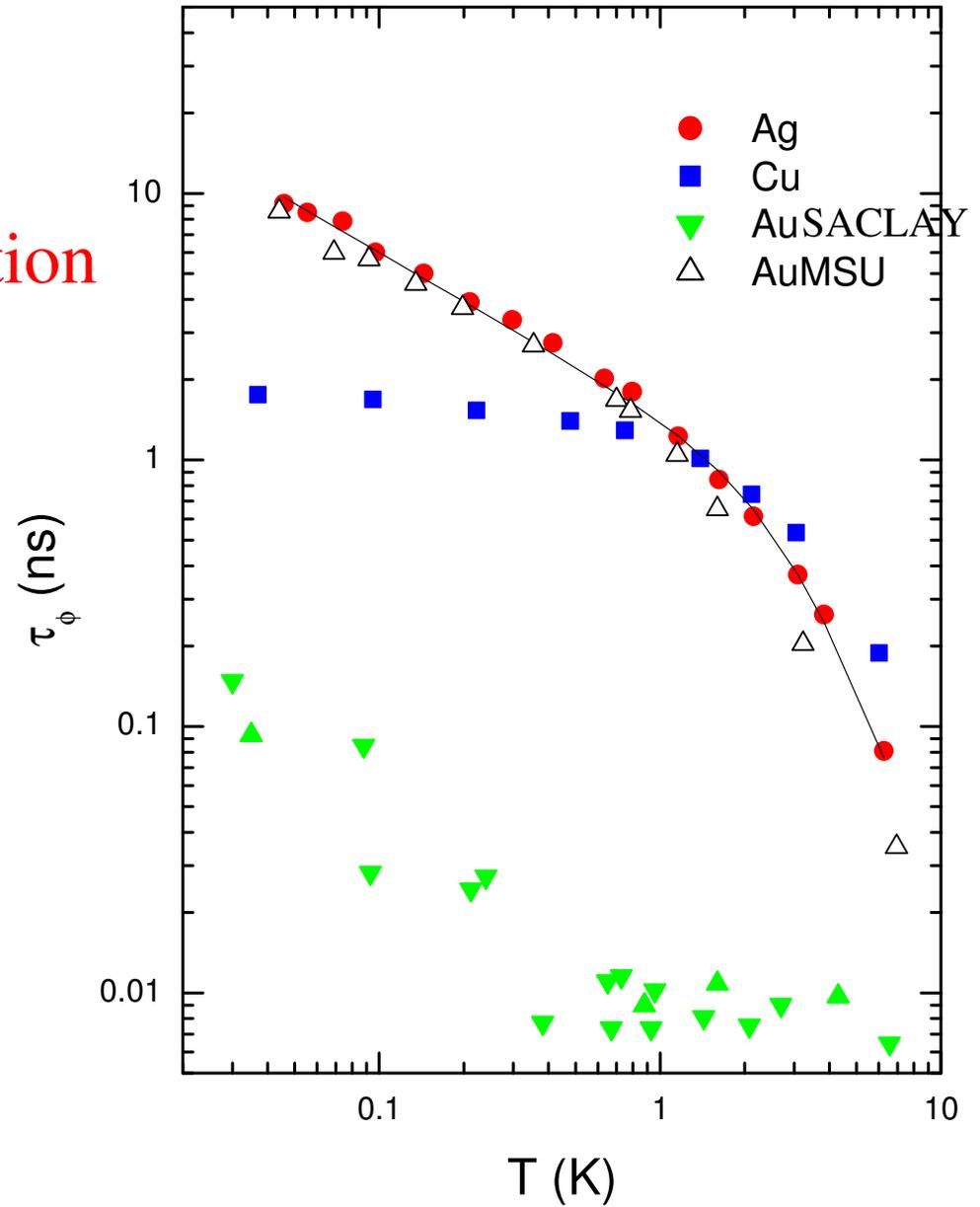
F. Pierre et al., JLTP 118, 437 (2000) and NATO Proceedings (cond-mat/0012038)



# Electron Phase Relaxation in Ag, Cu, and Au wires

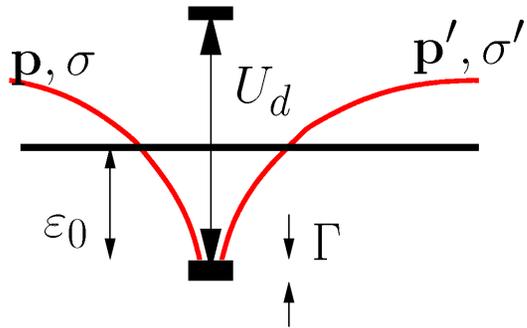
MSU-SACLAY collaboration

Gougam et al.,  
JLTP **118**, 447 (2000)

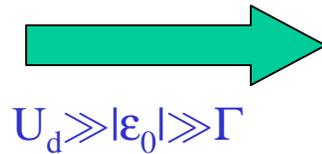


# Effect of magnetic impurities on the electron transport

## 1. Interaction of a conduction electron with a magnetic impurity



Anderson impurity model



$$U_d \gg |\epsilon_0| \gg \Gamma$$

$$\mathcal{H} = \frac{J_0}{2} \sum_{\mathbf{p}\mathbf{p}'\sigma\sigma'} \mathbf{S}\mathbf{S}_{\sigma\sigma'} c_{\mathbf{p},\sigma}^\dagger c_{\mathbf{p}',\sigma'}$$

Exchange interaction model,  $vJ_0 = \Gamma/|\epsilon_0|$

## 2. Scattering off a magnetic impurity (Born approximation)

$$|A_{\mathbf{p}\sigma \rightarrow \mathbf{p}'\sigma'}^{(1)}| \propto J_0^2 \delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'})$$

Scattering is elastic, but scrambles electron spin

Contribution to the momentum and phase relaxation rates:

$$\frac{1}{\tau_s} = 2\pi\nu n_s J_0^2 S(S+1) \quad (\text{saturates at low } T)$$

# Effect of magnetic impurities on the electron transport

## 3. Scattering in the leading logarithmic approximation

$$A_{\mathbf{p}\sigma \rightarrow \mathbf{p}'\sigma'}^{(2)} \propto 2 \int_{-D}^0 d\varepsilon'' \frac{\nu J_0^2}{\varepsilon - \varepsilon''} - \int_0^D d\varepsilon'' \frac{\nu J_0^2}{\varepsilon'' - \varepsilon}$$

$$\propto \nu J_0^2 \ln \frac{D}{|\varepsilon|} \quad [\text{Kondo (1964)}]$$

Sum of the leading log-terms (Abrikosov; Suhl, 1965):

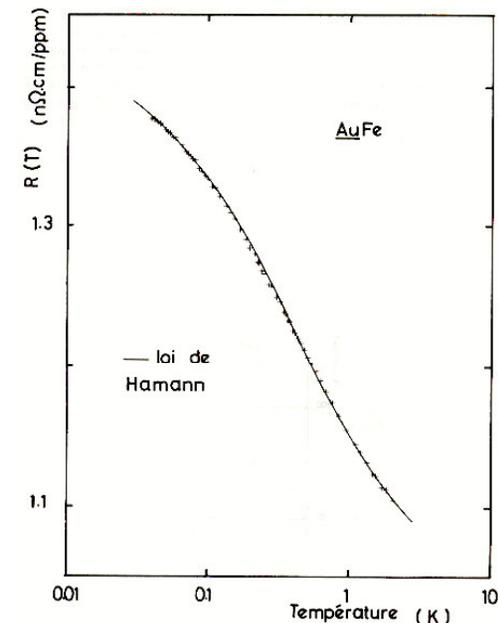
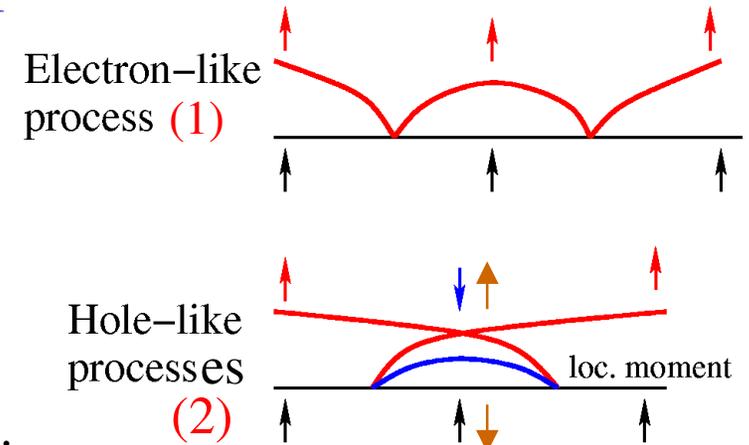
$$J_0 \rightarrow J = \frac{J_0}{1 - J_0 \nu \ln |D/\varepsilon|} = \frac{1}{\nu} \frac{1}{\ln |\varepsilon/T_K|}$$

Kondo temperature:  $T_K = D \exp\left(-\frac{1}{J_0 \nu}\right)$

Contribution to the resistivity and phase relaxation rate:

$$\delta\rho_K \propto \frac{1}{\tau_s} \propto \frac{n_s}{\nu} \frac{1}{\ln(T/T_K)}$$

Scattering is still elastic!



Laborde, SCom. 71'

# Inelastic scattering off a magnetic impurity

## 1. Simplest inelastic process in a toy model

$$\mathcal{H} = \mathcal{H}_0 + \hat{V}_{\text{toy}} \quad \hat{V}_{\text{toy}} = J_0 \sum_{\mathbf{p}_1 \mathbf{p}_2} \left( S^+ \sigma^- c_{\mathbf{p}_2 \downarrow}^\dagger c_{\mathbf{p}_1 \uparrow} + S^- \sigma^+ c_{\mathbf{p}_2 \uparrow}^\dagger c_{\mathbf{p}_1 \downarrow} \right)$$

$$\mathcal{H}_0 = \sum_{\mathbf{p}\sigma} \xi_{\mathbf{p}} c_{\mathbf{p}\sigma}^\dagger c_{\mathbf{p}\sigma}$$

el.1	el.2	imp
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only **two** electrons in the band,  $\Psi_{\text{in}} = |\mathbf{p}' \uparrow, \mathbf{p} \downarrow, \uparrow\rangle$

$$\Psi_{\text{out}} \equiv \hat{T} \Psi_{\text{in}} \quad \text{T-matrix: } \hat{T} = \hat{V} + \hat{V} \frac{1}{\varepsilon - \mathcal{H}_0} \hat{V} + \dots$$

Born

2<sup>nd</sup> order

$$\Psi_{\text{out}}^{(2)} \propto J_0^2 \sum_{\mathbf{p}_1 \mathbf{p}_2} S^+ \sigma^- c_{\mathbf{p}_1 \downarrow}^\dagger c_{\mathbf{p}_2 \uparrow} \frac{1}{\varepsilon - \mathcal{H}_0} \sum_{\mathbf{p}_3 \mathbf{p}_4} S^- \sigma^+ c_{\mathbf{p}_3 \uparrow}^\dagger c_{\mathbf{p}_4 \downarrow} |\mathbf{p}' \uparrow, \mathbf{p} \downarrow, \uparrow\rangle$$

$$= \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}} |\mathbf{p}_1 \downarrow, \mathbf{p}_3 \uparrow, \uparrow\rangle$$


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# Inelastic scattering off a magnetic impurity

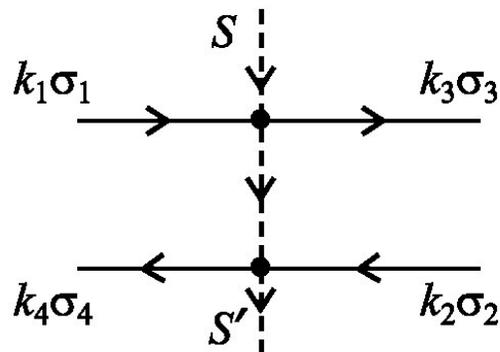
$$A_{\mathbf{p}, \mathbf{p}' \rightarrow \mathbf{p}_1, \mathbf{p}_3}^{(2)} \sim \frac{J_0^2}{\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3}}$$

Energy transferred in the collision:

$$\xi_{\mathbf{p}} - \xi_{\mathbf{p}_3} \equiv E$$

Scattering cross-section:  $\left| A_{\mathbf{p}, \mathbf{p}' \rightarrow \mathbf{p}_1, \mathbf{p}_3}^{(2)} \right|^2 \sim \frac{J_0^4}{E^2} \delta(\xi_{\mathbf{p}} + \xi_{\mathbf{p}'} - \xi_{\mathbf{p}_1} - \xi_{\mathbf{p}_3})$

## 2. Full 2<sup>nd</sup> order perturbation theory result

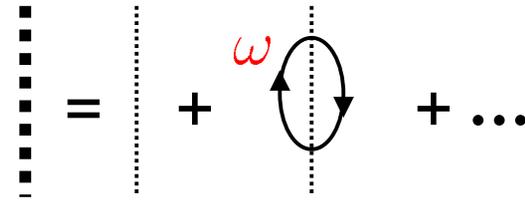
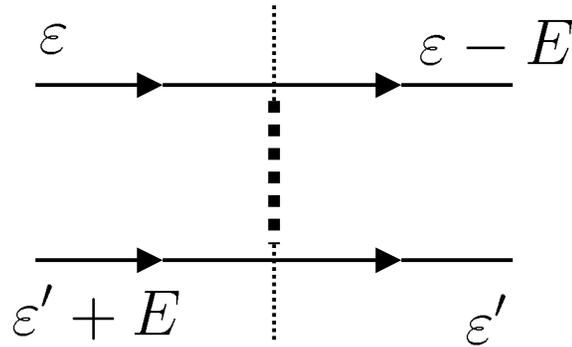


Total cross-section  $\varepsilon, \varepsilon' \rightarrow \varepsilon - E, \varepsilon' + E$   
averaged over  $S$ :

$$K(E) = \frac{\pi n_s}{2 \nu} (J\nu)^4 S(S+1) \frac{1}{E^2}$$

# Inelastic scattering off a magnetic impurity

## 3. Dressing the bare diagram: relaxation of the impurity spin



$$\langle \mathbf{S}(t)\mathbf{S}(0) \rangle \propto \exp\left(-\frac{t}{\tau_K}\right)$$

$$\frac{\hbar}{\tau_K} = (J_0\nu)^2 \times \begin{cases} \frac{2\pi}{3}T, & T \gg eU \\ \gamma eU, & eU \gg T \end{cases} \quad \gamma \sim 1, \text{ depends on the electron distribution}$$

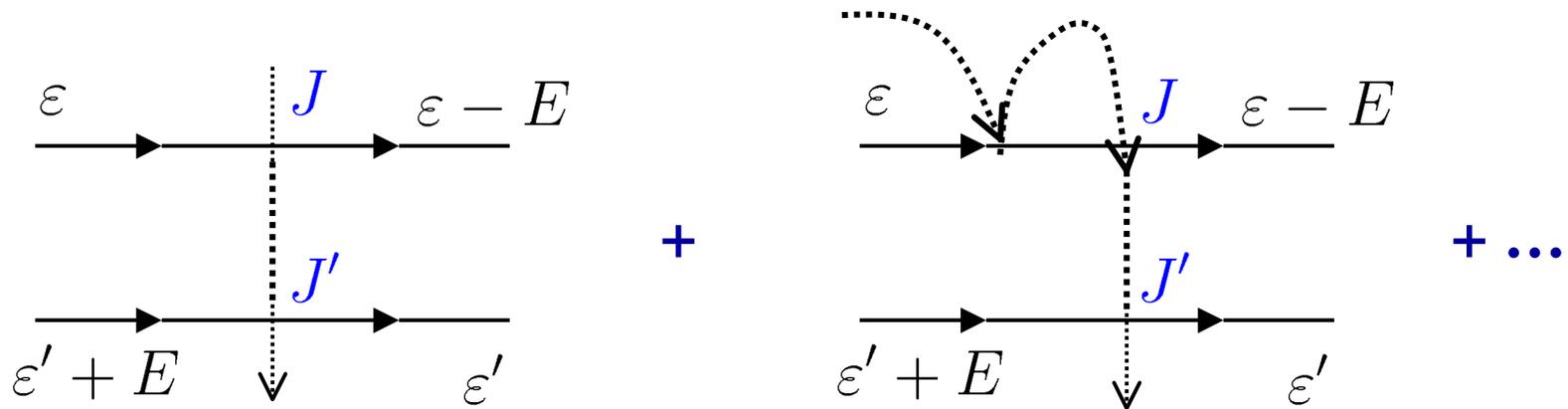
(generalized Korringa relation)

$$K(E) \propto \frac{1}{E^2 + (\hbar/\tau_K)^2} \quad \xrightarrow{\quad} \quad K(E) \propto \frac{1}{E^2} g\left(\frac{eU}{E}\right)$$

$\frac{\hbar}{\tau_K} \propto eU$       **Scaling is preserved**

# Inelastic scattering off a magnetic impurity

## 4. Kondo renormalization of the inelastic scattering



Leading log approximation:

$$\nu J \longrightarrow \left[ \ln \frac{|\varepsilon|}{T_K} + \ln \frac{|\varepsilon - E|}{T_K} \right]^{-1}, \quad \nu J' \longrightarrow \left[ \ln \frac{|\varepsilon'|}{T_K} + \ln \frac{|\varepsilon' + E|}{T_K} \right]^{-1}$$

$$K(E) \propto (J \cdot J')^2 \longrightarrow K(\varepsilon, \varepsilon', E)$$

The dependence on  $\varepsilon, \varepsilon'$  is **weak**, and the kernel can be simplified, if the electron distribution  $f(\varepsilon)$  is smooth

## Inelastic scattering off a magnetic impurity

$$\underline{K(E)} \approx \frac{\pi n_s}{2 \nu} S(S+1) \left[ \ln \frac{eU}{T_K} \right]^{-4} \frac{1}{\underline{E^2 + \left[ \frac{\hbar}{\tau_K(eU)} \right]^2}}$$

$$\frac{\hbar}{\tau_K(eU)} = \gamma \left[ \ln \frac{eU}{T_K} \right]^{-2} \cdot eU, \quad \gamma \sim 1$$

Scaling is intact at  $eU \gg T_K$ , where the  $\ln$ -factors are almost const

At  $E, eU \ll T_K$ : back to FL behavior,  $K(E) \sim \frac{n_s}{\nu} \frac{1}{T_K^2}$

(energy-independent)

$$\boxed{\text{Au:Fe} \rightarrow T_K \simeq 0.3\text{K}; U \lesssim 30 \mu\text{V}}$$

# Energy relaxation: experiment vs. theory

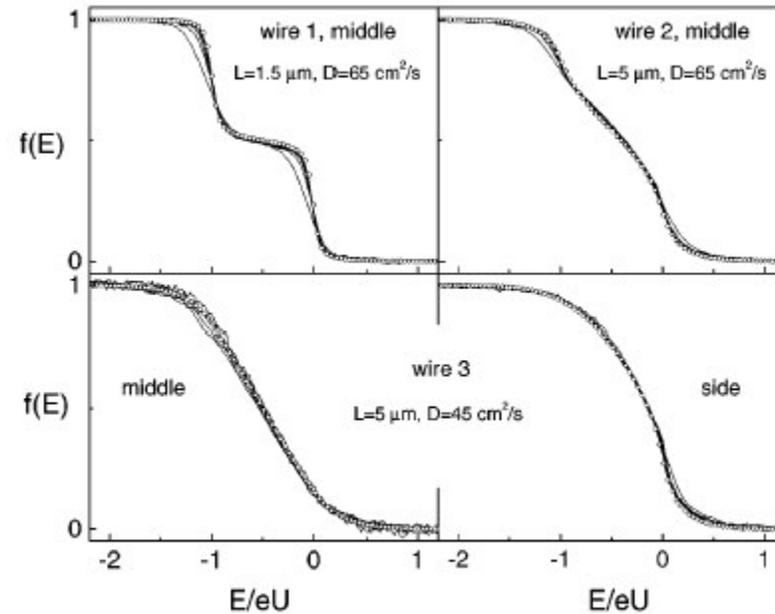
## Experiment:

Observed scaling of  $f(\epsilon)$   
suggests  $K(E) \sim 1/E^2$ .

## Theory:

Magnetic impurities lead to  
 $K(E) \sim 1/E^2$ .

For Fe impurities in Au, it is  
sufficient to have  $n \sim 10\text{ppm}$

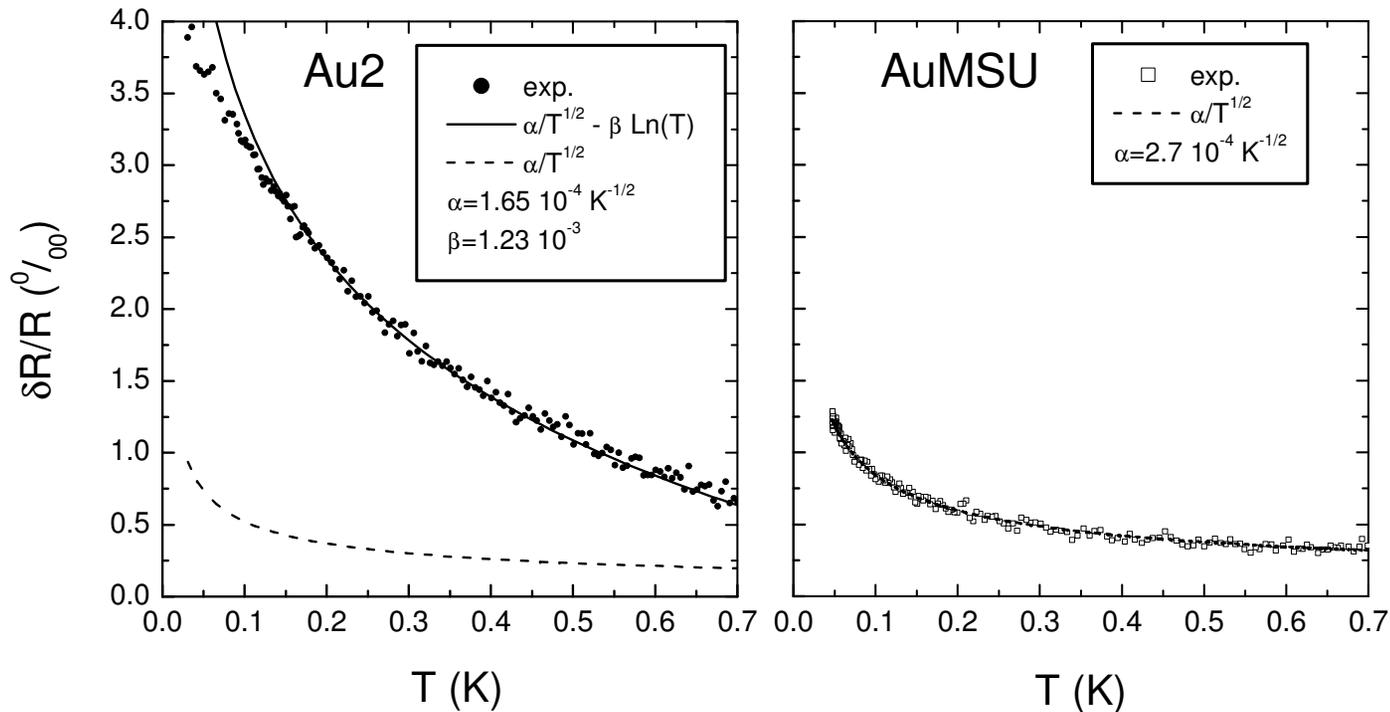


## Experiment:

SACLAY samples did contain Fe impurities with  $n$  up to  $\sim 30\text{ppm}$

## SACLAY vs MSU Au samples

Pierre et al., in Kondo Effect and Dephasing in Low-Dim. Metallic Systems (cond-mat/0012038)



Kondo effect:  $R(T) = A - B \log(T) \Rightarrow n_{\text{Fe}} \approx 50 \text{ ppm}$  (courtesy N. Birge)

# Energy relaxation: experiment vs. theory

**Theory:**

Prediction: Magnetic field should suppress electron energy relaxation at  $E \leq g\mu_B B$ .

# Effect of spin polarization on the energy relaxation

1. Suppression of  $K(E)$  at  $\varepsilon \ll g\mu_B B$

$$K(E) \sim \frac{n_s}{\nu} \frac{1}{\ln^4(g\mu_B B/T_K)} \left( \frac{1}{g\mu_B B} \right)^2$$

(A.K., L.G., 2001)

2. Enhancement of  $1/\tau_\varepsilon$  at  $\varepsilon > g\mu_B B$

Kinetic equation in the presence of  $n_s$  and  $SO$  interaction

– Göppert, Galperin, Altshuler, Grabert, 2002

3. Step-like dependence of  $1/\tau_\varepsilon$  vs.  $\varepsilon$  at  $\varepsilon \sim g\mu_B B$

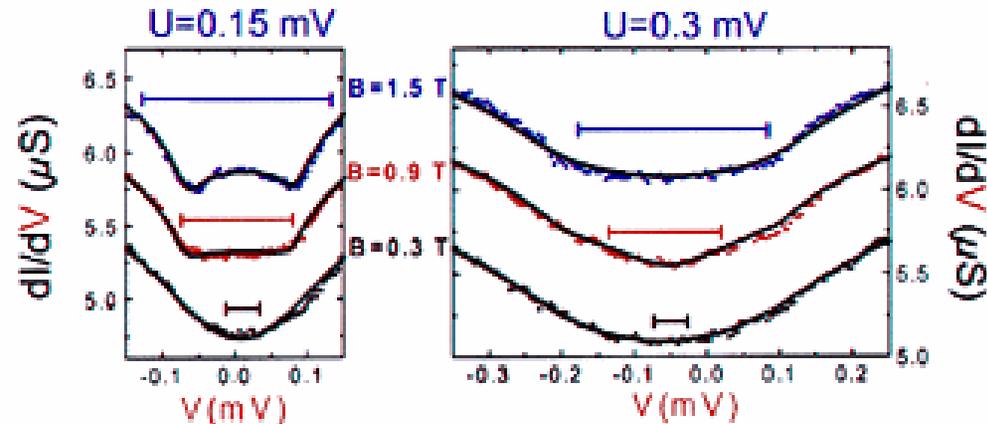
$\frac{\hbar}{\tau_\varepsilon} \sim \frac{n_s}{\nu} \frac{1}{\ln^4(g\mu_B B/T_K)}, \quad \varepsilon \lesssim g\mu_B B$

$\frac{\hbar}{\tau_\varepsilon} \sim \frac{n_s}{\nu} \frac{1}{\ln^2(g\mu_B B/T_K)}, \quad \varepsilon \gtrsim g\mu_B B$

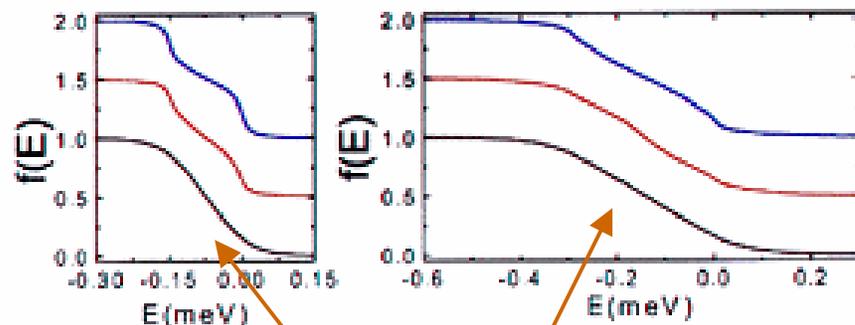
# Effect of spin polarization on the energy relaxation: Ag[Mn]

Experiment: Ag[Mn], Anthore et al. (SACLAY), cond-mat/0301070

Zero-bias anomaly at fixed  $U$  and three different  $B$



Electron distribution function at fixed  $U$  and different  $B$

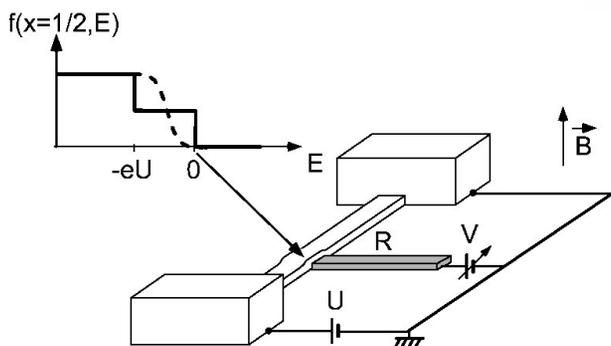


spin 1/2  
 $T_K=40$  mK  
 $g=2.9$   
 $c=1.8$  ppm  
 $vJ=0.3$

$U > g\mu_B B$

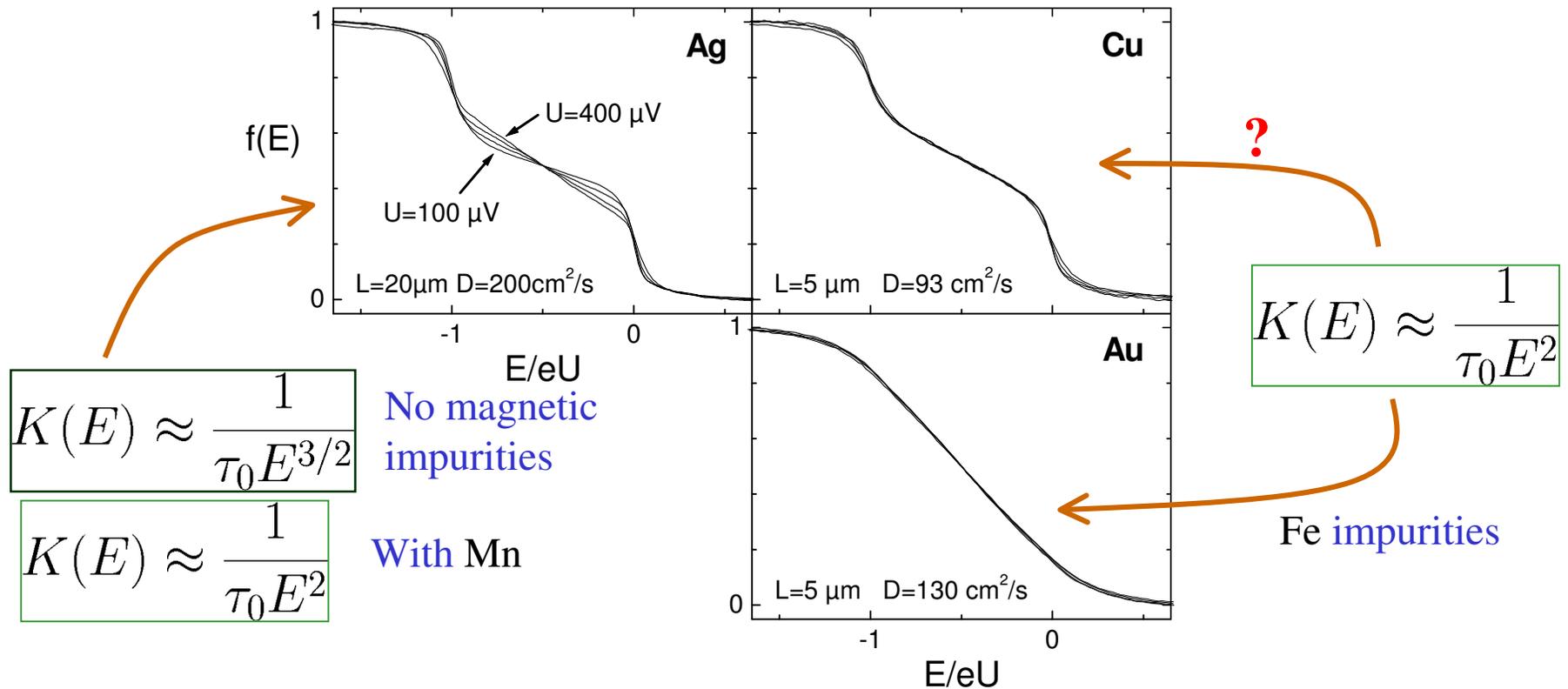
Remaining questions:

1. Why  $g \neq 2$
2. Relaxation is too fast at  $n_s=1.8$  ppm



# Energy Relaxation in Ag, Cu, and Au wires

F. Pierre et al., JLTP 118, 437 (2000) and NATO Proceedings (cond-mat/0012038)



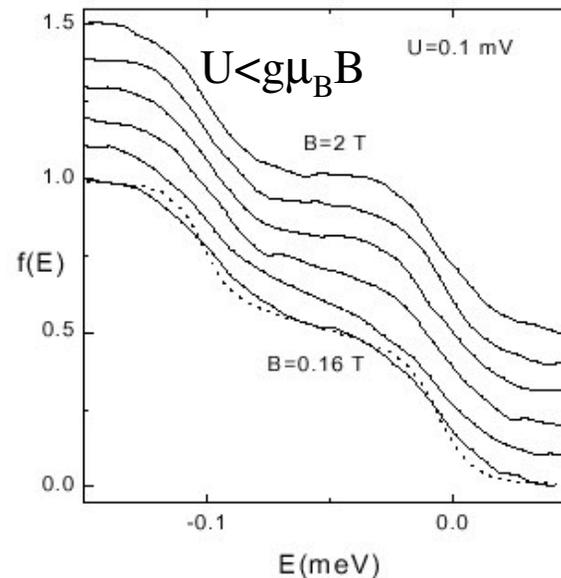
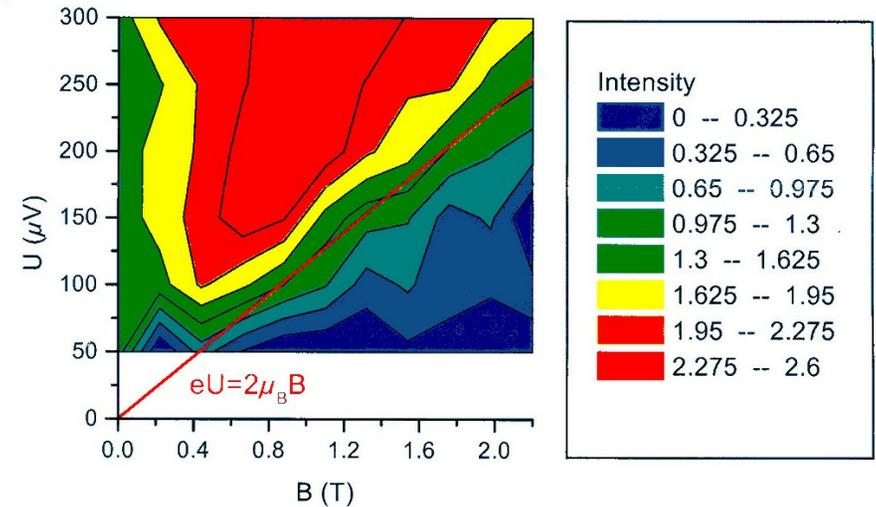
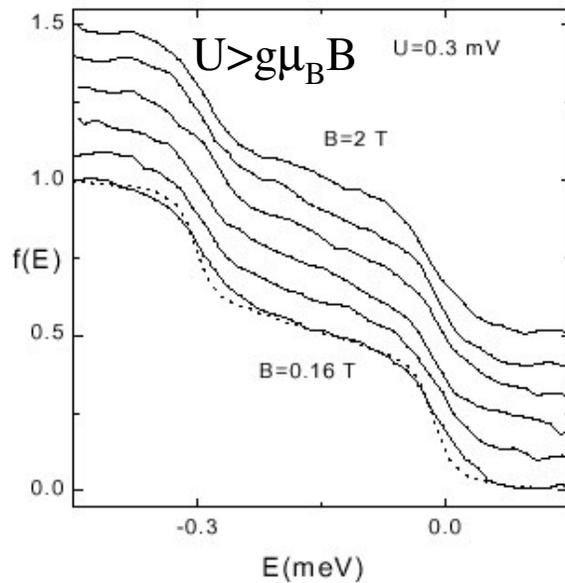
# Effect of spin polarization on the energy relaxation: Cu

## INFLUENCE OF MAGNETIC FIELD ON EFFECTIVE ELECTRON-ELECTRON INTERACTIONS IN A COPPER WIRE

A. Anthore, F. Pierre\*, H. Pothier, D. Esteve, and M. H. Devoret

Cond-mat/0109279

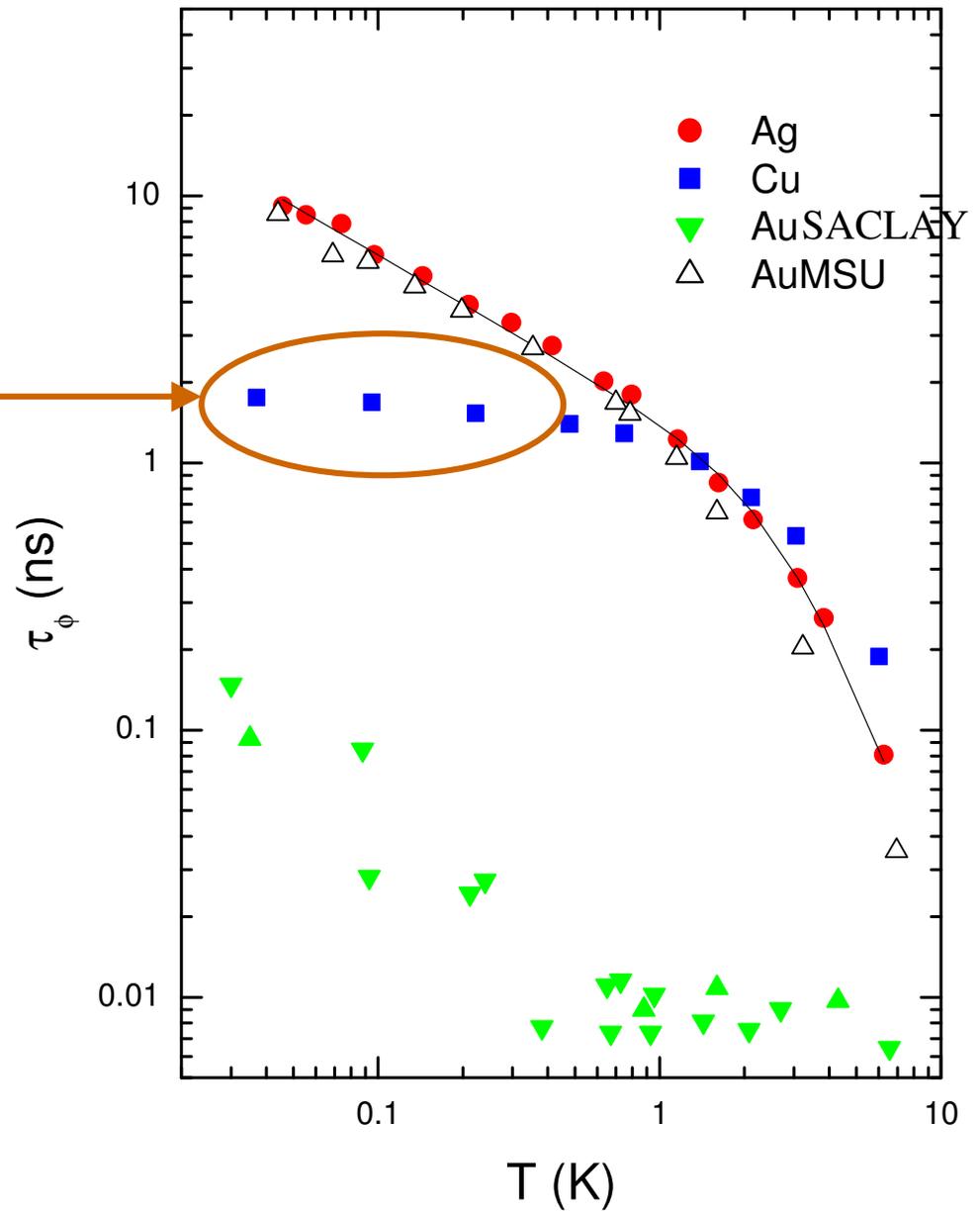
*Service de Physique de l'Etat Condensé, Commissariat à l'Energie Atomique, Saclay,  
F-91191 Gif-sur-Yvette, France*



Getting corroborating evidence...

# Effect of spin polarization on the phase relaxation

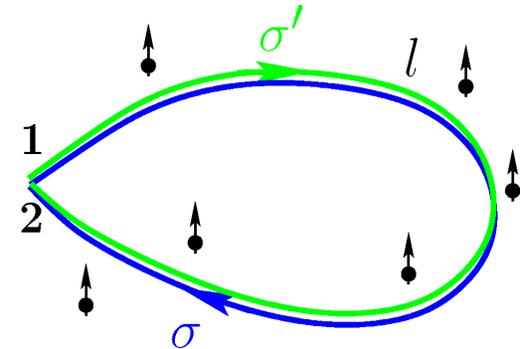
Can saturation be removed by a polarization of localized spins?



# Effect of spin polarization on the phase relaxation

## 1. Weak localization in a wire

$$\Delta\sigma_{\text{wl}}^{\sigma\sigma'} \propto \langle A_{\sigma'} A_{\sigma}^* \rangle \propto \langle e^{i\Phi_{\sigma'}} e^{-i\Phi_{\sigma}} \rangle$$



Case of  $B=0$ :

$$\Delta\sigma_{\text{WL}} = -\frac{e^2}{2\pi\hbar} \sqrt{D} \left( \frac{2}{\sqrt{2/3\tau_s + 4/3\tau_{\text{so}}}} + \frac{1}{\sqrt{2/3\tau_s + 4/3\tau_{\text{so}}}} - \frac{1}{\sqrt{2/\tau_s}} \right).$$

Cooperon spin:  $S=1, S_z=\pm 1$        $S=1, S_z=0$        $S=0$

(Hikami, Larkin, Nagaoka, 1980)

spin relaxation rate:  $\frac{1}{\tau_s} = 2\pi \frac{n_s}{\nu} \frac{S(S+1)}{\ln^2 |T/T_K|},$

SO scattering rate in metals (Li ÷ Ag):  $\frac{1}{\tau_{\text{so}}} \simeq 10^{-6} Z^4 \frac{1}{\tau_{\text{tr}}}$

(Gershenson, Sharvin, ~1980)

# Effect of spin polarization on the weak localization

Affected by polarization ( $\sigma = \sigma'$ )

Not affected by polarization

$$\Delta\sigma_{\text{WL}} = -\frac{e^2}{2\pi\hbar}\sqrt{D}\left(\frac{2}{\cancel{\sqrt{2/3\tau_s}} + 4/3\tau_{\text{so}}} + \frac{1}{\sqrt{2/3\tau_s} + 4/3\tau_{\text{so}}} - \frac{1}{\sqrt{2/\tau_s}}\right).$$

Cooperon spin:

$S=1, S_z=\pm 1$

$S=1, S_z=0$

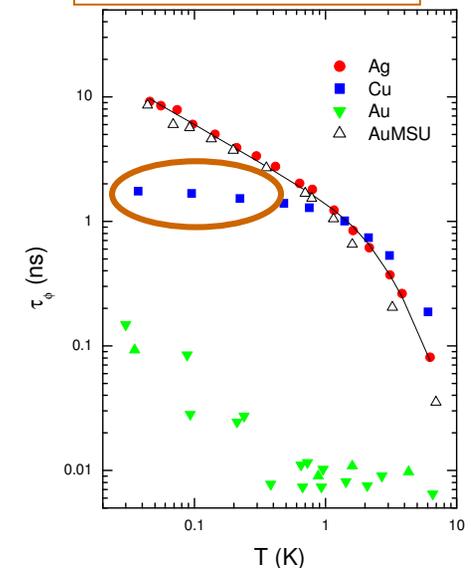
$S=0$

Vanish at  $\tau_{\text{so}}=0$   
(no use for Cu)

Complete crossover function for  $\Delta\sigma_{\text{WL}}$  vs.  $B/T$   
in the absence of SO scattering (“light” metals):

$$\sigma_{\text{wl}}(B, T) = -\frac{e^2}{\pi\hbar}\sqrt{D}\int\frac{d\varepsilon}{4T\cosh^2\varepsilon/2T}\frac{1}{\sqrt{\Gamma(\varepsilon, B/T)}}.$$

$\Gamma(\varepsilon, B/T)$  depends only on the ratio  $B/T$ ;  
exponentially small in the limit  $B/T \rightarrow \infty$   
(Bobkov, Falko, Khmelnitskii 1990).



# Effect of spin polarization on the phase relaxation

## 2. Effect of the magnetic impurities on mesoscopic conductance fluctuations

Landauer formula for DC conductance:  $G(t) = \frac{e^2}{\pi\hbar} \sum_{\alpha\beta} |T_{\alpha\beta}(t)|^2$

WL: spin polarization may change the statistics of random transmission amplitudes  $T_{\alpha\beta}$ .

$1/\tau_{SO} = 0$ : the change does occur,

$$\Delta G = \langle G \rangle_{\text{GUE}} - \langle G \rangle_{\text{GOE}} \neq 0$$

$\tau_{SO} = 0$ : the change does not occur,

$$\Delta G = \langle G \rangle_{\text{GUE}} - \langle G \rangle_{\text{GUE}} = 0$$

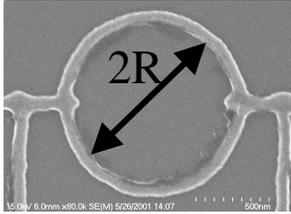
Mesoscopic Conductance Fluctuations: spin dynamics  $\rightarrow$  imp. configuration changes with time,  $S(t') \neq S(t)$  at  $|t-t'| \gg \tau_K$ . The difference between samples vanishes:

$$R(\Delta B) \equiv \langle \bar{G}(t, B) \bar{G}(t', B + \Delta B) \rangle - \langle \bar{G}(t, B) \rangle \langle \bar{G}(t', B + \Delta B) \rangle = 0$$

Polarization  $\rightarrow$  no spin dynamics  $\rightarrow$  mesoscopic fluctuations restored (at any  $\tau_{SO}$ )

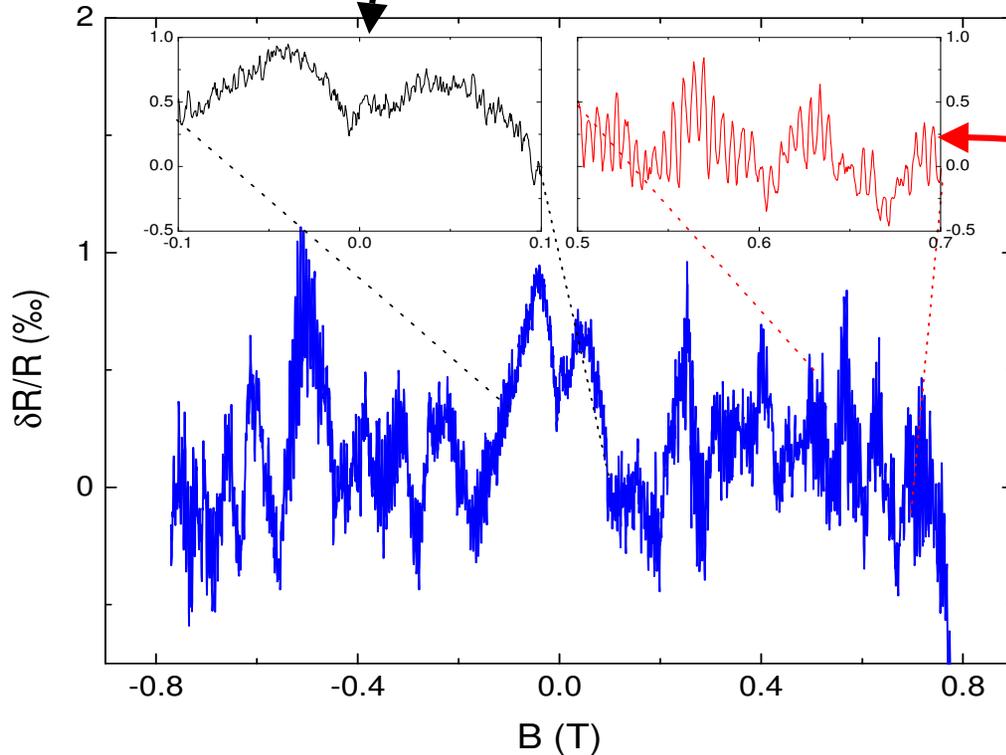
$$R(\Delta B) \equiv \langle \bar{G}(t, B) \bar{G}(t', B + \Delta B) \rangle - \langle \bar{G}(t, B) \rangle \langle \bar{G}(t', B + \Delta B) \rangle \neq 0$$

# $h/e$ Aharonov-Bohm oscillations in a Cu ring



At  $B=0$  the spin orientation varies faster than the dc measurement — averaging over spin orientation:

$$\frac{\text{var}G(n_s)}{\text{var}G(n_s = 0)} = \exp \left\{ -\frac{2\pi R}{\sqrt{D\tau_s}} \right\}$$



Data: Pierre&Birge, PRL2002

Strong magnetic field aligns all impurity spins — no spin fluctuations.

AB oscillations at  $g\mu_B B \gg T$  are restored (no effect of magnetic impurities).

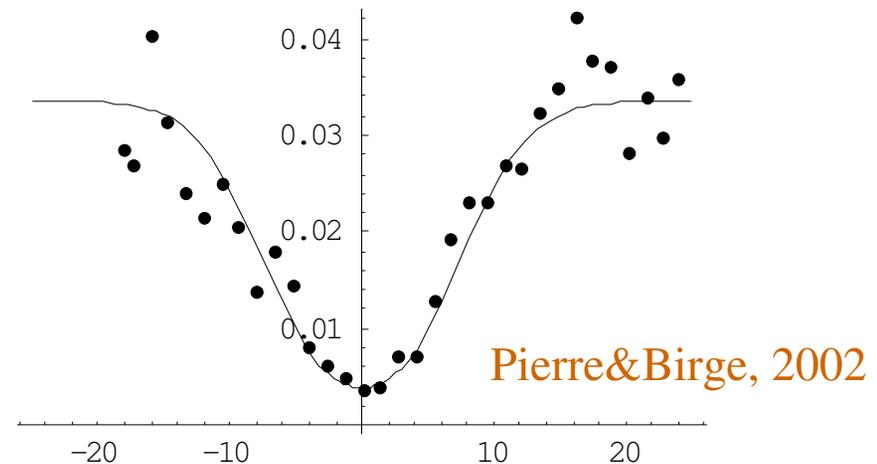
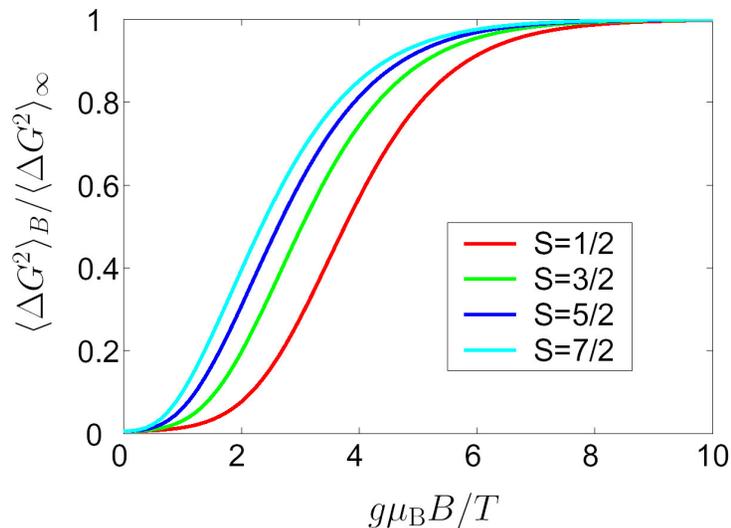
[Bobkov, Falko, Khmelnitskii, 1990; Falko, 1992]

# h/e Aharonov-Bohm oscillations – full crossover function

$$\text{var}G(B, T) \propto \frac{e^4}{\pi^2 \hbar^2} \frac{D^{3/2}}{T} \int \frac{d\varepsilon}{4T \cosh^4 \varepsilon/2T} \frac{\exp(-2\pi R \sqrt{\Gamma(\varepsilon, B/T)/D})}{\sqrt{\Gamma(\varepsilon, B/T)}}$$

$$\Gamma(\varepsilon, B/T) = \left[ 1 - \frac{\langle \hat{S}_z \rangle^2 + \langle \hat{S}_z \rangle \tanh(\varepsilon + g\mu_B B)/2T}{S(S+1)} \right] \frac{1}{\tau_s}$$

Relaxation rate  $\Gamma$  depends on  $B/T$  only; valid for arbitrary spin.

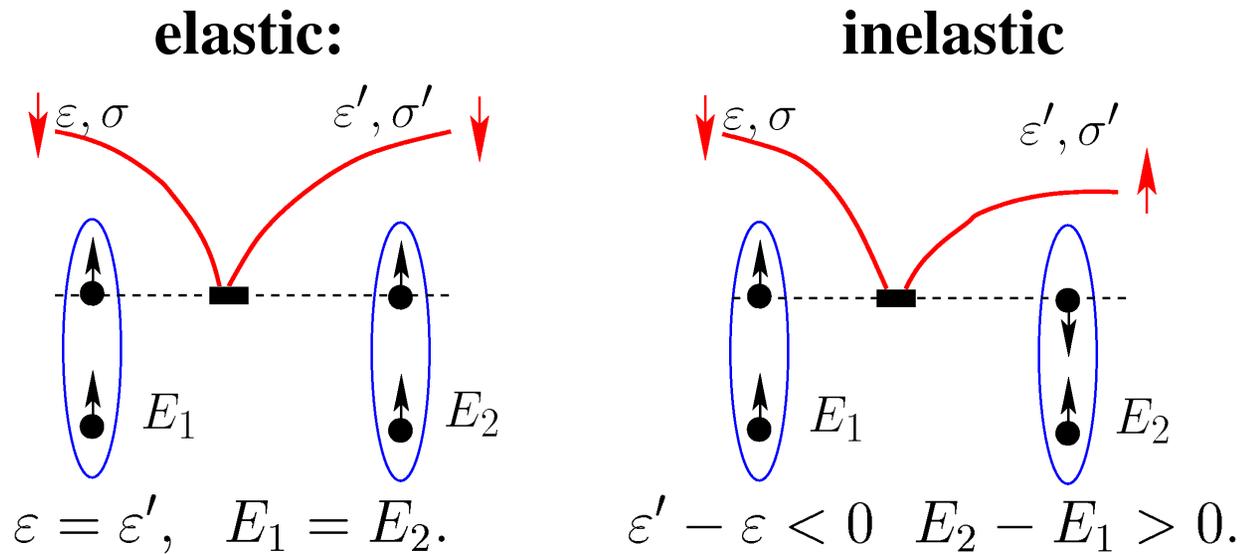


Remaining questions:

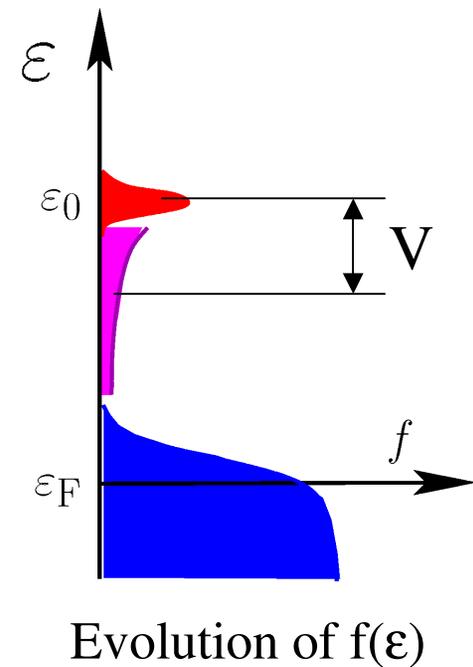
1. Why  $g=0.44$ ? What is  $S$ ?
2. Rate  $\tau_\phi$  disagree with  $\tau_\varepsilon$

# Effects of interaction between magnetic impurities

## 1. Inelastic scattering off impurity pairs



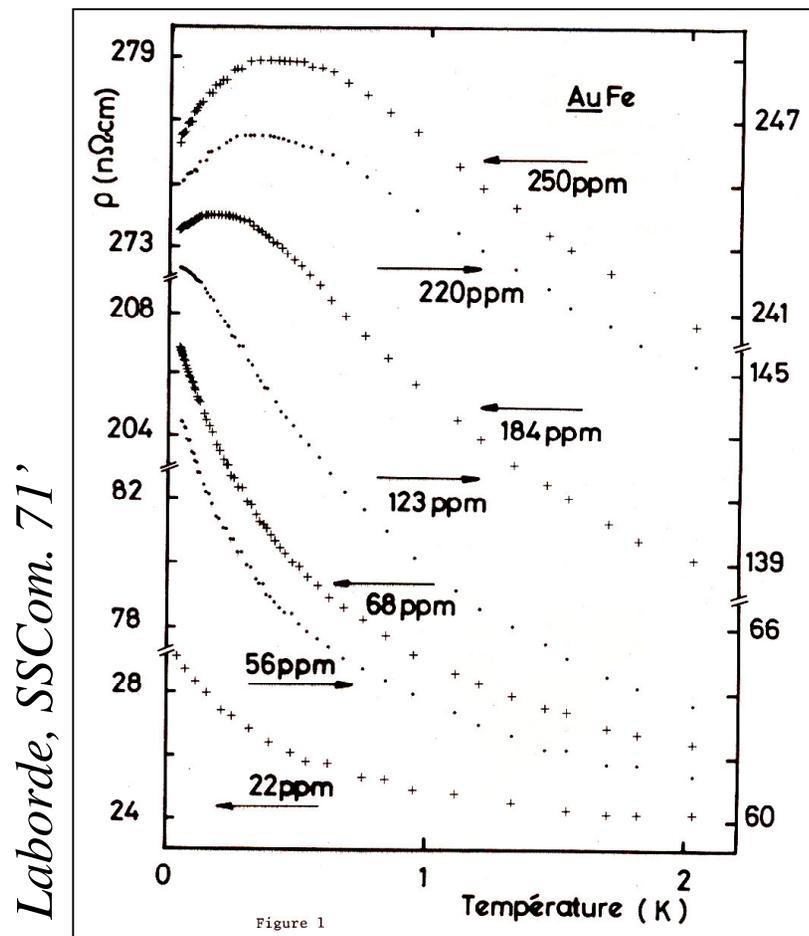
Tail below  $\varepsilon_0$ :  $f(\varepsilon_0 - V) \propto \frac{T_{sg}}{\tau_s V^2}$



# Effects of interaction between magnetic impurities

## 2. Resistivity and $1/\tau_\phi$ vs. $T$

$R(T)$  and  $\tau_\phi^{-1}(T)$  have maxima **only** if  $n_s > n_{cr}$ , allowing for a spin-glass state.



At  $T_{sg} \gg T_K$ ,

the maxima are at  $T \sim T_{sg} \ln \frac{T_{sg}}{T_K}$ .

(virial expansion, valid at  $T \gg T_{sg}$ )

Contradictory data at  $\tau_\phi$ , R for  
 $n_s \propto 3 \div 60$  ppm:

F. Schopfer, C. Bäuerle et al. PRL (2003).

# Conclusions

- Magnetic impurities help to re-distribute energy between the conduction electrons
- Small energy transfers are favored by this mechanism,  $K(E) \propto 1/E^2$
- Energy relaxation at  $\epsilon < g\mu_B B$  is suppressed by Zeeman splitting
- Spin polarization affects the WL correction only at small  $1/\tau_{SO}$
- Spin polarization enhances the mesoscopic conductance fluctuations
- The discrepancy between the data on energy and phase relaxation is not resolved yet