FQHE Jain's Rule and vortex lattices

S.V. Iordanski
Landau Institute

Integer QHE, Klitzing (1980)

Theoretical explanation

Laughlin (1981), one electron picture, large cyclotron energy $\hbar\omega_c \gg E_c$ (gap)

$E_c = e^2\sqrt{n_e}$, localization in the random field of impurities.

Fractional QHE

Tsui, Stormer, Gossard (1982)

no one particle gap, macroscopical degeneration of LL at these fillings

Partial explanation
(inverse odd fillings)

Laughlin (1983) by special trial wave functions of g. state and of excitations. Coulomb energy gap $\sim e^2\sqrt{n_e}\cdot const$
pure experimental confirmation.
Theoretical assumtion $\hbar\omega_c \gg E_c$
experimental $\hbar\omega_c \gtrsim E_c$.
Fenomenological Hierarchial schemes daughter states from basic one (inverse Odd fillings), Haldane (1983), Laughlin (1984), Halperin (1984)
Most successful phenomenological description
Jain (1989) by composite fermions

Electrons „dressed" by
by some number of magnetic flux quanta.

Formalization of the idea in a number of papers to „Chern-Simons" Hamiltonian

I follow close to Halperin, Lee Read (1993)

Canonical transformation of the basis of multiparticle wave functions

$$\Phi \rightarrow e^{i\hat{S}}\Phi \quad \text{with Hermitian}$$

$$\hat{S} = \int h(\xi,\xi')\,\psi^{+}(\xi)\,\psi^{+}(\xi')\,\psi(\xi')\,\psi(\xi)\,d\xi\,d\xi'$$

$$h(\xi,\xi') = h(\xi',\xi)$$

new electronic field operators

$$\chi(r) = e^{-i\hat{S}}\psi(r)e^{i\hat{S}}, \quad \chi^{+}(r) = e^{-i\hat{S}}\psi^{+}(r)e^{i\hat{S}}$$

result of commutation

$$\hat{S}\psi(r) - \psi(r)\hat{S} = \hat{\alpha}(r)\psi(r) =$$

$$= 2\int h(r,\xi)\psi^{+}(\xi)\psi(\xi)d\xi\,\psi(r)$$

$$\chi(r) = e^{i\hat{\alpha}(r)}\psi(r), \quad \chi^{+}(r) = \psi^{+}(r)e^{-i\hat{\alpha}(r)}$$

assuming all spins identical

„Dressing" by the flux

$$h(r-\xi) = K\,\mathrm{arctg}\,\frac{r_x - \xi_x}{r_y - \xi_y}$$

$$\hat{\alpha}(r) = 2K\int \mathrm{arctg}\,\frac{r_x - \xi_x}{r_y - \xi_y}\,\hat{\rho}_e(\xi)\,d^2\xi$$

Operator $e^{i\hat{S}}$ must be single valued K is integer

Chern-Simons Hamiltonian

$$H = \int \chi^{+}\frac{1}{2m}\left(-i\bar{\nabla} + \bar{A}_0 + \bar{a}\right)^2\chi\,d^2r +$$

$$+\int \frac{V(r-r')}{2}\chi^{+}(r)\chi^{+}(r')\chi(r')\chi(r)\,dr\,dr'$$

$$\mathrm{rot}\,\bar{a} = 4\pi K\hat{\rho}_e$$

Artifically introdused 6 fermion interaction-too complicate.

Really mean field aproximation

$$\mathrm{rot}\,\vec{a} = 2K\varphi_0 \langle n_e \rangle$$

$$B_{eff} = B_0 + 2K\varphi_0 n_e$$

The gap for integer fillings of Le in B_{eff} $n_e = q\,\frac{B_{eff}}{\Phi_0}$

$$n_e = \frac{q}{1-2qK}\,\frac{B_0}{\Phi_0},\quad \nu = \frac{q}{1-2qK}$$

The Choice $K=-1$ gives almost all observed fractions.

$q \to \infty$ $\nu \to 1/2$ Fermi-liquid

$B_{eff} = 0$ also have exp. support.

Intrinsic difficulties going outside mean field approximation

Alternative-to use more simple canonical transformation.

$$\hat{S} = \int \psi^{+}_{\alpha}(\xi) V_{\alpha\beta}(\xi) \psi_{\beta}(\xi)\, d^{2}\xi.$$

$\psi^{+}_{\alpha}, \psi_{\alpha}$ spinors, transformed spinors

$$\chi_{\alpha}(r) = U_{\alpha\beta}(r)\psi_{\beta}(r), \quad \chi^{+}_{\alpha}(r) = \psi^{+}_{\beta}(r) U^{+}_{\beta\alpha}(r)$$

$\hat{U} = e^{i\hat{V}}$, $\hat{U}^{+}U = 1$ rotation matrix

$$\hat{U} = \hat{U}_{z}(\alpha)\hat{U}_{y}(\beta)\hat{U}_{z}(\alpha)$$

a sequence of rotations with 2 Euler angles identical instead of artificial interaction

Transformed Hamiltonian

$$H = \int \frac{1}{2m} \chi^{+}(-i\nabla - A_{0} + \hat{\Omega})^{2} \chi \, d^{2}r +$$

$$+ \frac{1}{2} \int V(r-r') \chi^{+}(r) \chi^{+}(r') \chi(r') \chi(r)\, dr\, dr'$$

$\hat{\Omega} = \vec{\Omega}^{c} \sigma_{c} = -iU^{+}\nabla U$, σ_{c} Pauli matrices

$$\vec{\Omega}^{x} = \frac{1}{2}(1 + \cos\beta)\nabla\alpha$$

$$\vec{\Omega}^{y} = 1/2\,(\sin\beta\cos\alpha\,\nabla\alpha - \sin\alpha\,\nabla\beta)$$

$$\vec{\Omega}^{z} = 1/2\,(\sin\beta\sin\alpha\,\nabla\alpha + \cos\alpha\,\nabla\beta)$$

Topological invariant

$$K = \frac{1}{2\pi} \int \mathrm{rot}\, \Omega^z \, d^2 r$$

conserved at finite deformation $\hat{U}$

Electron problem must be solved for Hamiltonian H at fixed $\hat{U}$.

If $\nabla \hat{U}$ is small compared to $\hat{U}$ – perturbation theory in $\hat{\Omega}$.

$\mathrm{rot}\, \Omega^z$ localized,

ferromagnetic state at $r \to \infty$ at filling $\nu = 1$ gives topological texture – skyrmion

Sondhi et all (1993)

$$K = \frac{1}{2\pi} \oint_C \vec{\Omega}^z d\vec{r} = \frac{1}{2\pi} \oint_C \nabla \alpha \, d\vec{r}$$

because $\beta \to 0$ at $r \to \infty$. C is large contour. K is winding number for α, $\beta = \pi$ at the singular point of α.

It is possible to perform the calculation of energy in any order of $\hat{\Omega}$, Iordanski et all (1997, 2003)

The transformation performs flux „dressing" because at large r $\Psi(r) \approx e^{i\alpha(r)} \psi(r)$.

The energy is defined by interaction, vanishing in it's absence.

Periodic generalization with finite density of K:

periodic $\beta(r)$

$\beta(r) = 0$ on the sides of elementary cell's

$\beta = \pi$ at the points of singularity of α in each cell

$\alpha = \sum \alpha(r - r_i)$ sum over all cells

The abstraction for the sample is th thorus with sample dimensions

$$K = \int_{\sigma_c} \mathrm{rot}\,\Omega^2\, d^2 r = \oint \bar{\nabla} \alpha\, d\bar{r}$$

is arbitrary integer.

Regular band spectrum for electrons in periodic potential and magnetic field requires rational number of flux quanta through every cell

$\frac{p}{q}\Phi_0$ (Landau-Lifshitz IX)

For large p, q the spectrum is quite irregular, with no distinct gaps at irrational numbers. Assume in the spirit of Fermi liquid that it is valid for interaction case also.

Effective flux per cell

$$B_0 \sigma_c + K\Phi_0 = \frac{p}{q}\,\varphi_0$$

$$\frac{1}{\sigma_c} = \frac{B_0}{\varphi_0}\,\frac{q}{p-Kq}$$

At electron densities

$$n_e = 1/\sigma_c\,, \quad \nu = \frac{q}{p-Kq}$$

all states obtained from the lowest band without magnetic field will be filled and should be a gap to the next band. There are q magnetic subbands which are q fold (odd q) or $\frac{q}{2}$ fold (even q) degenerate with the fraction of the total number of states $1/q^2$ or $\frac{2}{q^2}$ respectively.

Total number of filled states

S/σ_c

It is difficult to calculate ground state energy and the gaps - no perturbation theory in $\hat{\Omega}$

all observed fractions are defined by

	$K=-2,\ p=1$								
q	1	2	3	4	-2	-3	-4	-5	$q=\infty$
ν	$1/3$	$2/5$	$3/7$	$4/9$	$2/3$	$3/5$	$4/7$	$5/9$	$1/2$

Jains rule

new

$K=-1,\ p=1$				
q	-4	4	2	
ν	$4/3$	$4/5$	$2/3$	double

$K=-1,\ p=2$					
q	-7	-5	5	2	
ν	$7/5$	$5/3$	$5/7$	$1/2$	

double not observed

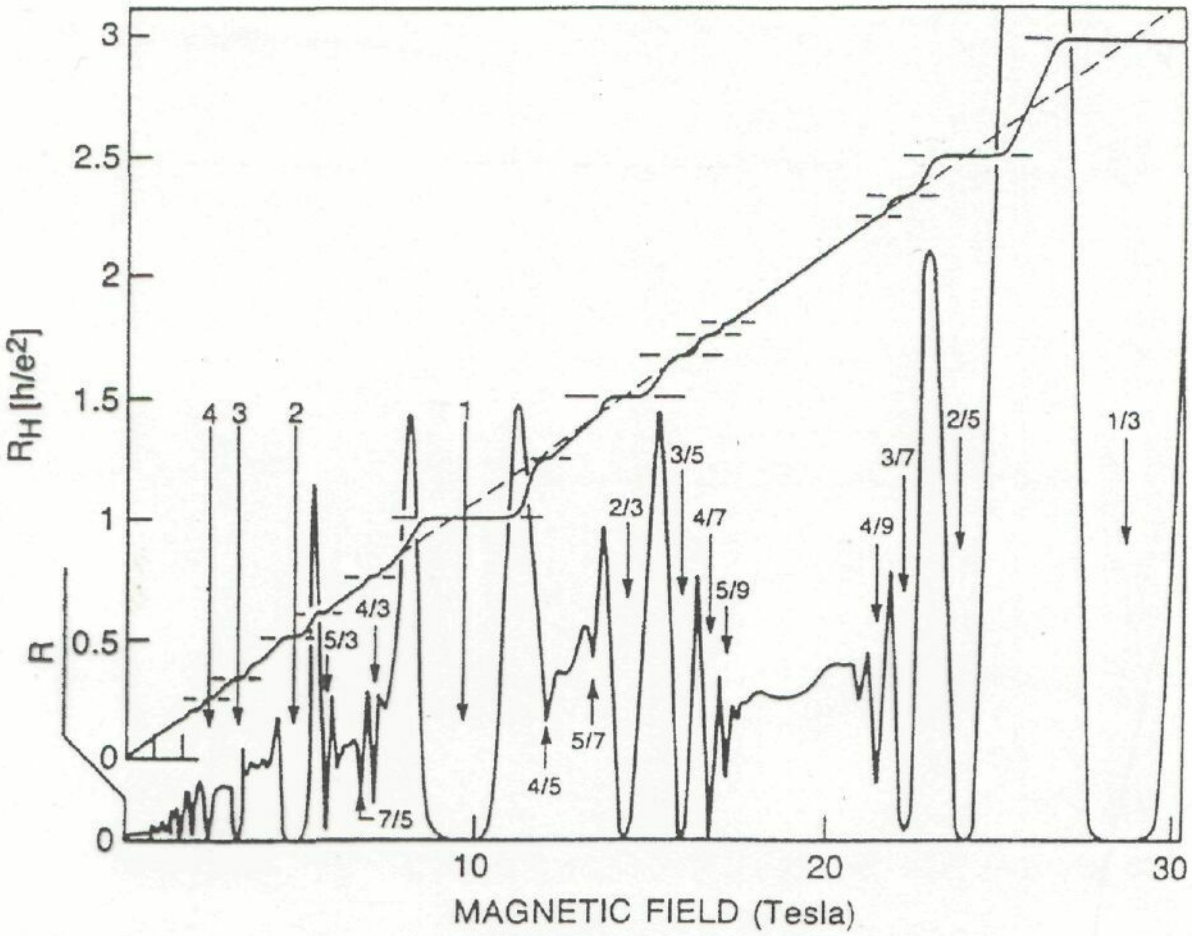

Figure 1.2: Integer and fractional quantum Hall transport data showing the plateau regions in the Hall resistance R_{H} and associated dips in the dissipative resistance R. The numbers indicate the Landau level filling factors at which various features occur. After ref. [15].

We see that including odd K essentially improve classification of fractions