# Transport in Granular Arrays


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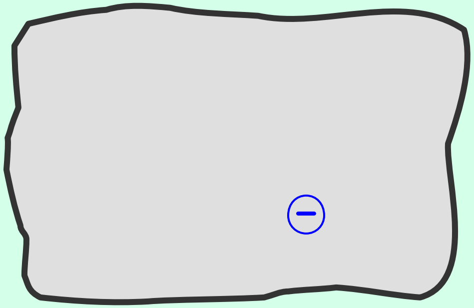
# the Coulomb blockade



- a quantum dot

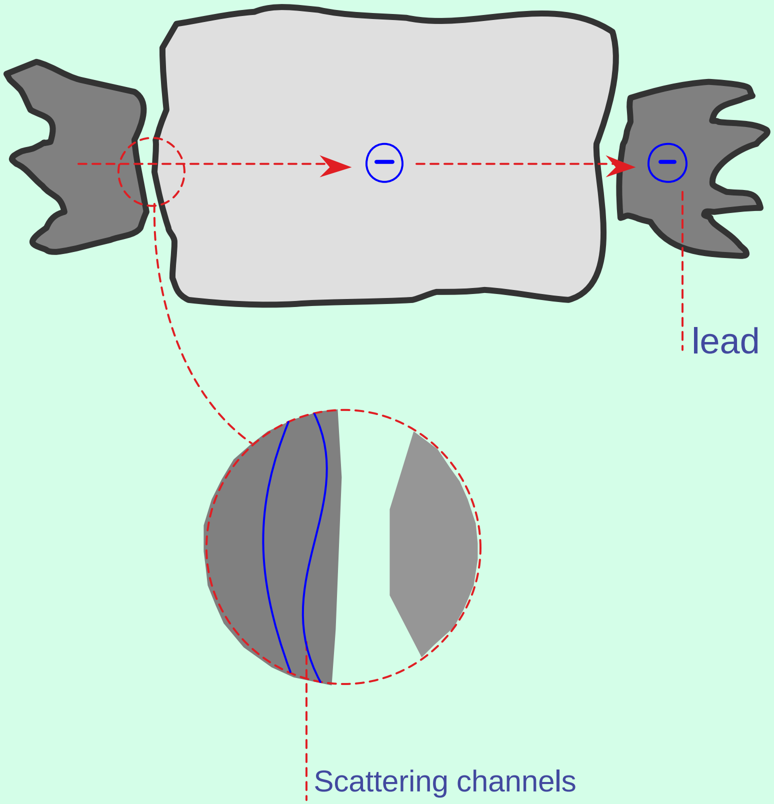
- accommodation of a single electron costs energy

$$E_c = \frac{e^2}{2C} >> T$$

- capacitance

# the Coulomb blockade

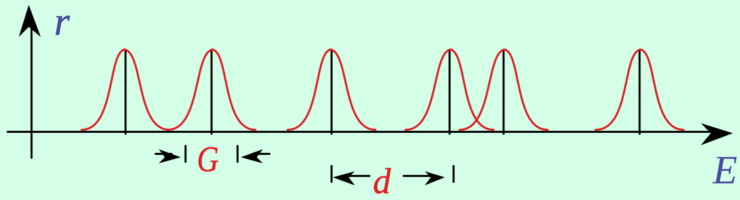
- coupling to external leads




- dot coupled to leads by $M$ channels with transmission coefficients: $0 < t_s < 1$.

dimensionless conductance: $g = \sum_{s=1}^{M} |t_s|^2$

- alternative interpretation: $g = G/d$

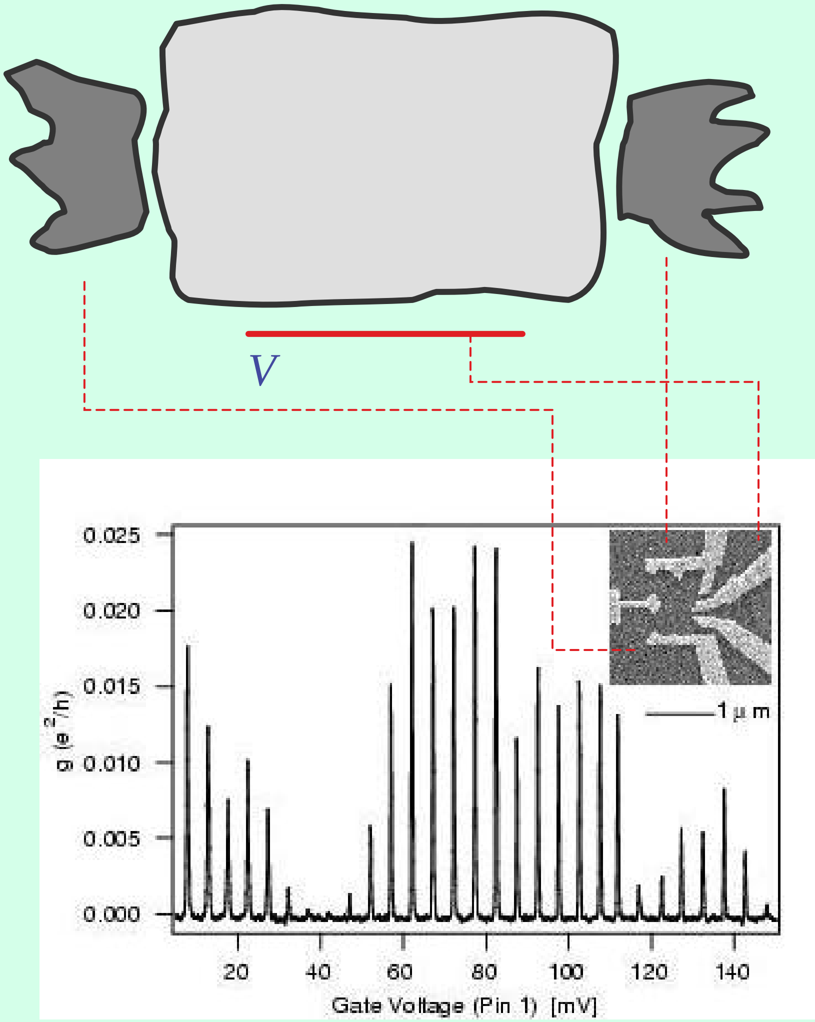



- for $g << 1$, the dot is in a state of 'Coulomb blockade': total conductance

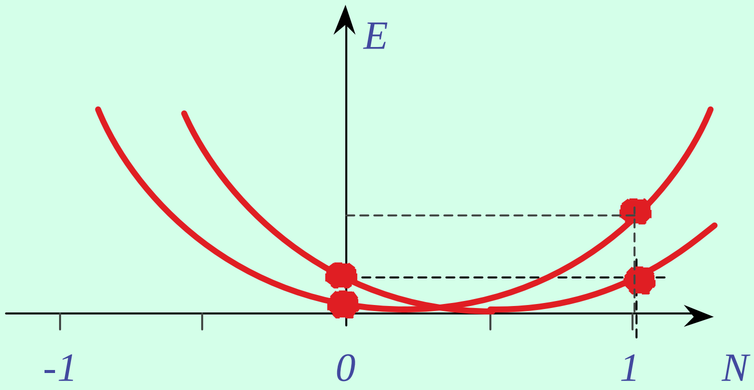
$$g_{tot} \sim \exp(-E_c/T)$$

# the Coulomb blockade

- gate voltage as an external probe




- gate electrode controls electrostatically preferred charge on the dot.




- generic values of $V$: transport blocked

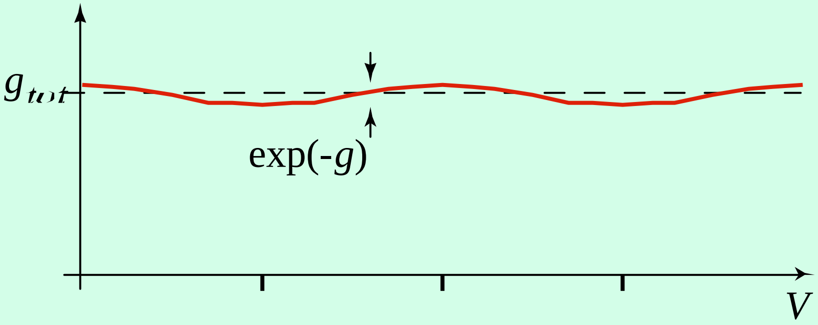
- $V=E_c/2$

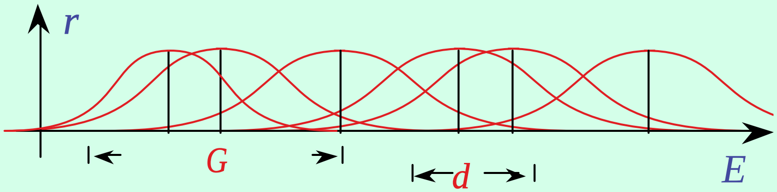
  free current flow (Coulomb blockade 'peak')

# ... however,

- the phenomenon is extremely susceptible to the tunneling conductance, $g$.
  - for $g \ll 1$, exponential suppression of the conductance.

  - however, for $g \gg 1$, the Coulomb blockade diminishes down to a small correction: $dg \sim exp(-g)$

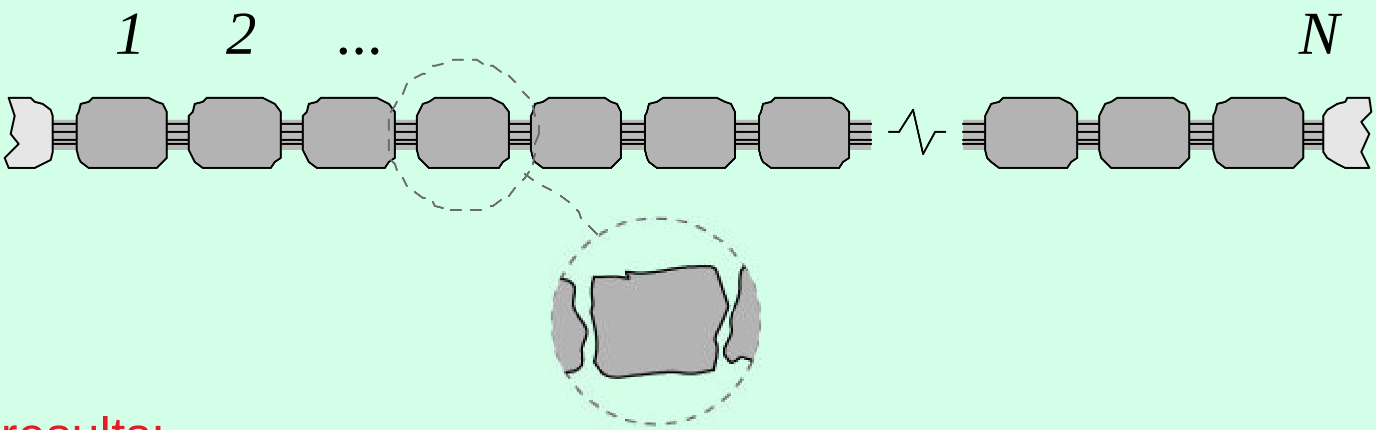



This contribution exists in parallel to all sorts of other quantum corrections (Altshuler-Aronov, weak localization ...) and is, therefore, nearly invisible.

question addressed in this talk:

- what happens if we consider an array of many strongly coupled $(g>>1)$ dots ?




main results:

- the Coulomb blockade drives the system into an insulating phase.

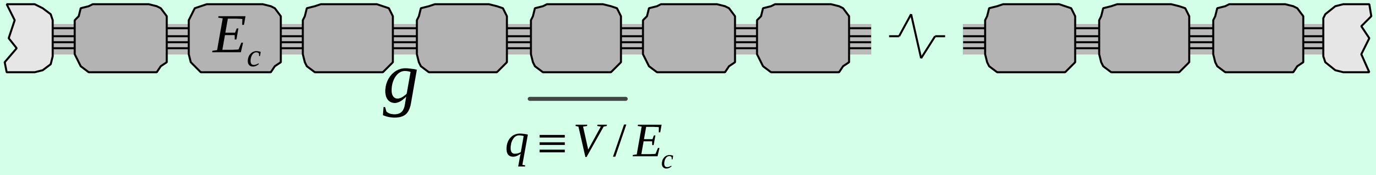
- the corresponding charge gap is given by

$$\Delta \equiv E_c \exp(-g/4)$$

- at temperatures $T<D$, both the conductance, and diff. capacitance show activated behavior:

$$g_{tot}, \partial_\mu N \sim \exp(-\Delta/T)$$

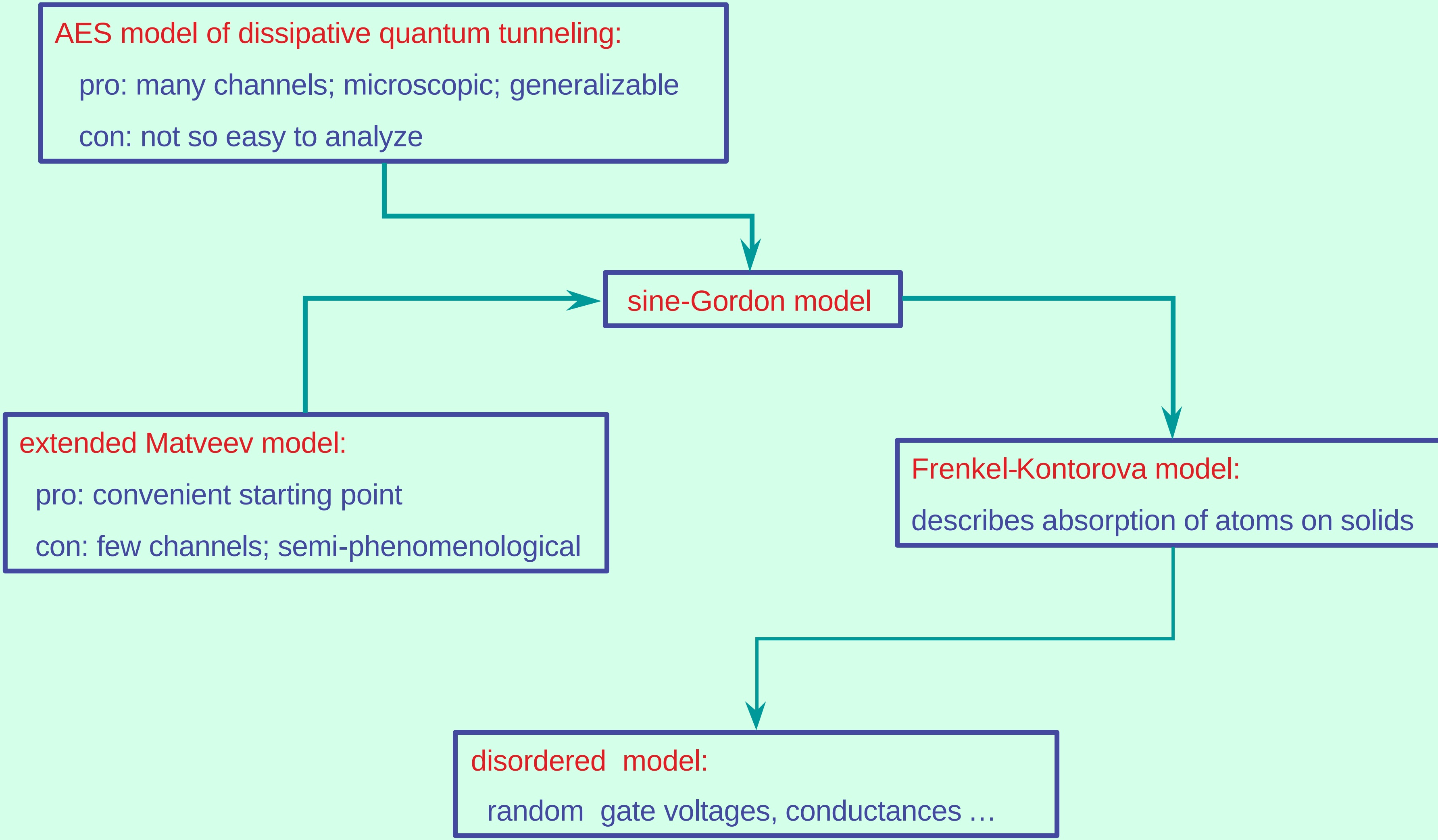
# the system




- tunneling incoherent (effects of quantum interference negligible) for $T > g\ d.$

- mechanisms relevant to the physics of the system:
  - charging: $E_c$ ; and gate voltage: $q = V / E_c$
  - interface scattering: $g$

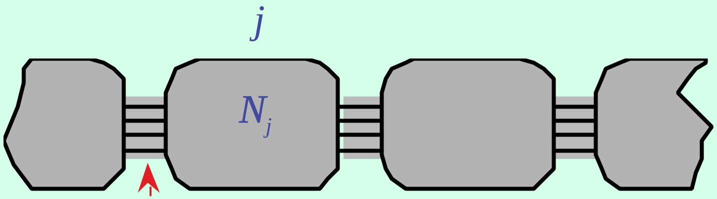
# strategy


AES model of dissipative quantum tunneling:
pro: many channels; microscopic; generalizable
con: not so easy to analyze
sine-Gordon model
extended Matveev model:
pro: convenient starting point
con: few channels; semi-phenomenological
Frenkel-Kontorova model:
describes absorption of atoms on solids
disordered model:
random gate voltages, conductances …

# extended Matveev model

- Flensberg 93, Matveev 94: semi-phenomenological model of the Coulomb blockade in few channel quantum dots.

- generalization to an array:




- charge displacement field: $Q_j(t)$. Physical meaning: $Q_{j+1}(t)$ - $Q_j(t)$ = $N_j(t)$ = charge sitting on grain no. $j$.

$$S[\theta] = S_c[\theta] + S_{scatt}[\theta]$$

$$S_c[\theta] = \frac{1}{T}\sum_{j=1}^{N}\sum_{m}\left[E_c(\theta_{j+1,m} - \theta_{j,m} - q)^2 + |\omega_m||\theta_{j,m}|^2\right]$$

$$S_{scatt}[\theta] = D\, r \sum_{j=1}^{N}\int_0^{\beta} d\tau\, \cos(\theta_j(\tau))$$

- reflection coefficient;

- for many channels: $r \longrightarrow \prod_{s=1}^{M} r_s << 1$

- high energy cutoff

# analysis of Matveev model

- a major simplification: physics controlled by temporal zero mode $Q_{m=0}$ . Dynamic modes give rise to inessential renormalization factors :

$$\langle q_j^2(t)\rangle = \frac{1}{N}\sum_k \sum_{m\neq 0}^{E_c/T} \frac{1}{E_C k^2 + |w_m|} = O(1)$$

- IR convergent !

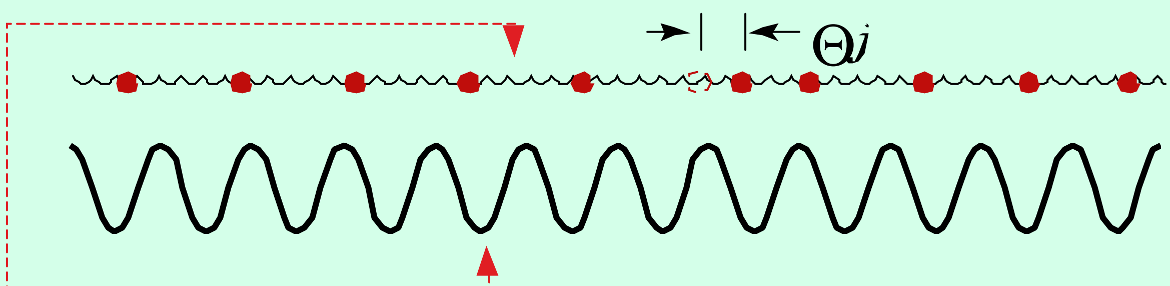
- action of the static sector:

$$S[\theta] = \frac{E_c}{T}\sum_{j=1}^{N}\left[(\theta_{j+1}-\theta_j-q)^2 + r\cos(\theta_j)\right]$$

- interpretation I: lattice version of the classical sin-Gordon model

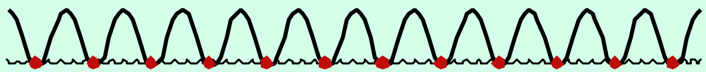
- interpetation II: action of Frenkel-Kontorova (1932) model of atomic absorption on substrates

# Frenkel-Kontorova model



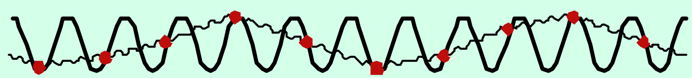

$$S[\theta] = \frac{E_c}{T} \sum_{j=1}^{N} \left[ (\theta_{j+1} - \theta_j)^2 + r \cos(\theta_j + qj) \right]$$

- atoms follow substrate, $Q_j = -qj$ , energy:



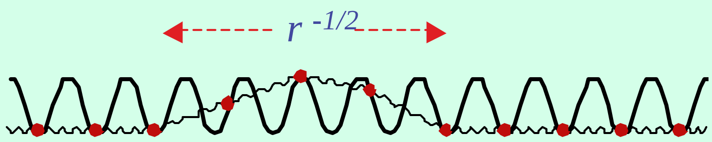
$$F[\theta] = \frac{N E_c}{T} \left[ q^2 - r \right]$$

- ground state of the chain, $Q_j = 0$ , energy:



$$F[\theta] = 0$$

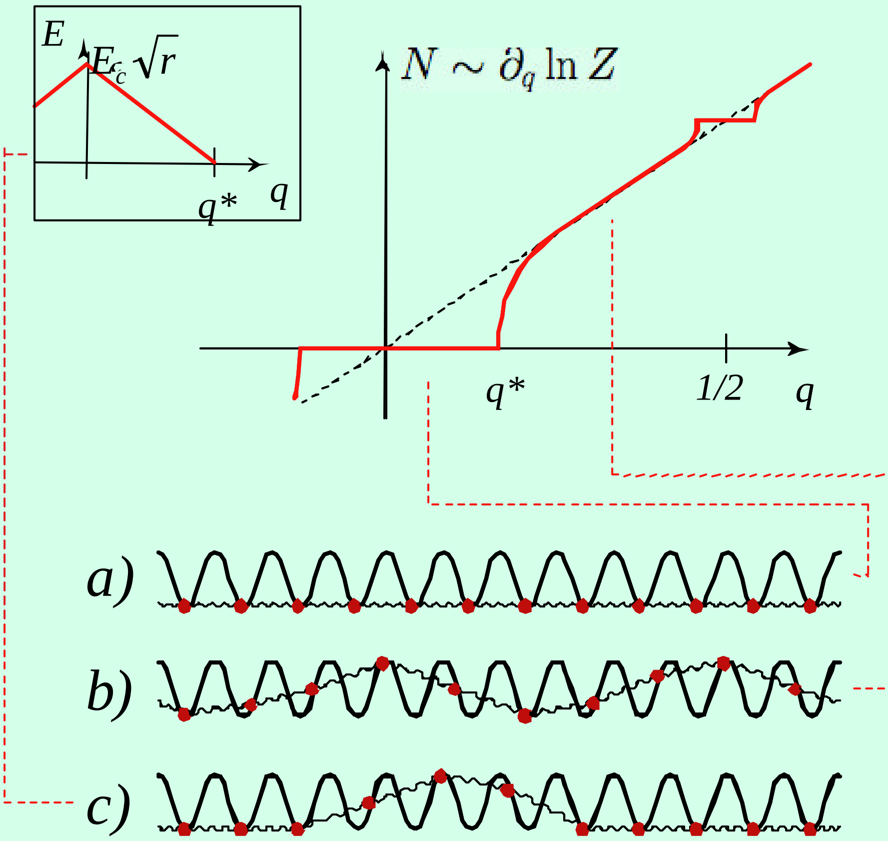
- phase transition at critical value: $q^* \sim r^{1/2}$.




- excitations of the system: long solitons.

# implications

- in an interval of width ~$q^*$, centered around $q=0$, the ground state of the system is $q$-independent (a)).
- for $q>q^*$, reentrance into $q$-dependent state (b)). However, plateau formation around other rational values of $q$.
- thermal fluctuations create $q$-dependent excited states (c)) that cost energy $D \sim E_c\, r^{1/2}$




# translation to the metallic context

- for zero gate voltage (and other rational values of $q$), the system is in an insulating state.
- the charge gap is given by: $D \sim E_c r^{1/2}$
- one can show that the insulating state survives generalization to random values of $q$, however, with a lower gap: $D \sim E_c\, r$ .

# the real thing

- shortcomings of the previous discussion
  - limitation to few channels.
  - unclear how quantum interference (localization, dephasing, etc.) can be built in.
  - connections  to other approaches are unclear.

- alternative approach: for $g$>>1, large charge fluctuations. Description in terms of the phase $f_i$ conjugate to the charge $N_i$ $([f_i, N_j] = -i\, d_{ij})$ is favorable.

- effective action: Ambegaokar, Eckern, Shoen 1984

$$S[\phi] = S_c[\phi] + S_{scatt}[\phi]$$

gate voltage

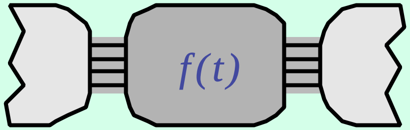
$$S_c[\phi] = \sum_j \int_0^\beta d\tau \left[ \frac{\dot{\phi}_j^2}{4E_c} - iq\dot{\phi}_j \right]$$

conductance

$$S_{scatt}[\phi] = \frac{gT^2}{2} \sum_j \int_0^\beta d\tau d\tau' \, \frac{\sin^2(\delta\phi_j(\tau) - \delta\phi_j(\tau'))}{\sin^2(\pi T(\tau - \tau'))}$$

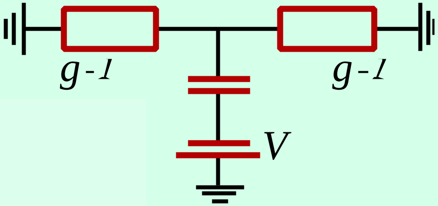
$$df_j \equiv (f_{j+1} - f_j)/2$$

# warmup: single grain




$$S_c[\phi] = \frac{1}{4E_c} \int_0^\beta d\tau \left[ \dot{\phi}^2 + 4iE_c q \dot{\phi} \right]$$

$$S_{scatt}[\phi] = g\, T^2 \int_0^\beta d\tau d\tau' \, \frac{\sin^2((\phi(\tau) - \phi(\tau'))/2)}{\sin^2(\pi T(\tau - \tau'))}$$




- for $g >> 1$, quadratic expansion:

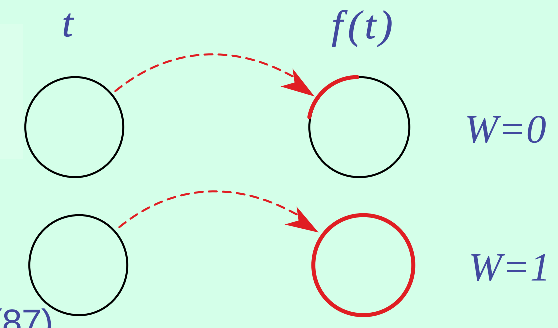
$$S[\phi] \approx \frac{1}{T} \sum_m \phi_m \left( \frac{\omega_m^2}{4E_c} + g|\omega_m| \right) \phi_{-m} - \frac{4E_c}{T} q^2$$

- anharmonic fluctuations lead to logarithmic corrections: $g_{tot} = \frac{g}{2}\left[ 1 - \frac{1}{g} \ln \frac{E_c}{T} \right]$

(Fazio & Schoen, 91, Golubev & Zaikin, 96, Efetov & Tschersich, 02, ...) small for $T > E_c exp(-g)$.

# instantons

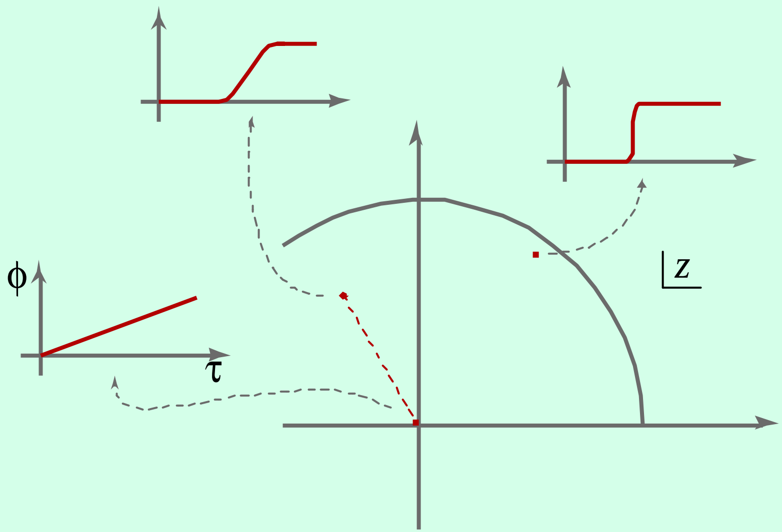
- mathematically: $\phi : S^1 \to S^1, \quad \tau \mapsto \phi(\tau)$




- topologically non-trivial excitations - instantons: Korshunov (87)

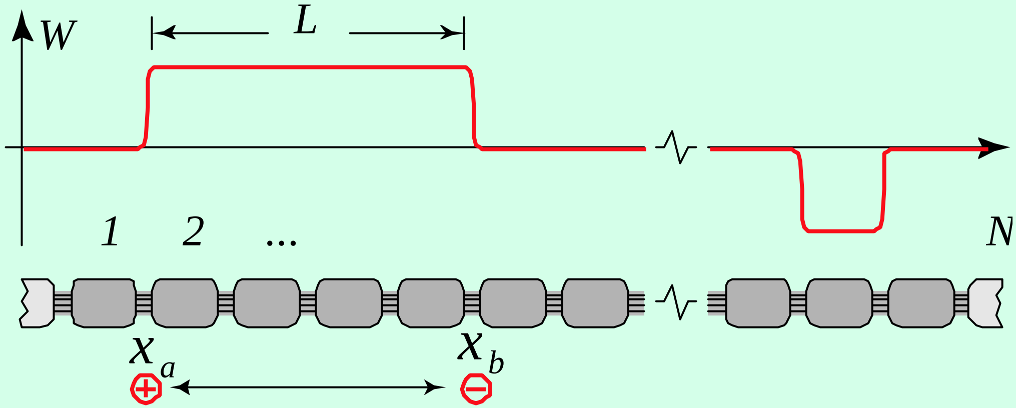
$$e^{i\phi(\tau)} = \frac{e^{2\pi i\tau T} - z}{1 - \bar{z}e^{2\pi i\tau T}}, \qquad |z| \leq 1$$

- instantons extremize scattering action
- responsible for gate voltage dependence
- action: $S = g + \frac{T}{E_c} + 2\,p\,iq$



# instanton formation in the array

- consider phase fluctuation of the type:




- action: $S = g + \frac{T}{E_c} | L | + 2piqL$
- re-interpret instanton configuration as a dipole of two opposite charges:
  - with the fugacity (core energy): *exp(-g/2)*,
  - interacting by one-dimensional Coulomb interaction: $(T/E_c)\,|x_a - x_b|$ ;
  - in a uniform external field: $2piq$

$$\frac{Z}{Z_0} = \sum_{k=0}^{\infty} \frac{1}{(k!)^2} \left( e^{-g/2} \frac{E_c}{T} \right)^{2k} \sum_{x_1 \ldots x_{2k}}^{N} e^{-\frac{T}{E_c} \sum_{a,b}^{2k} (-)^{a+b} |x_a - x_b| - 2\pi i q \sum_{a}^{2k} (-)^a x_a}$$

- fluctuation determinant

# instantons in the array cont'd

- key to solving the problem is equivalence of the Coulomb gas to the sine-Gordon model:

$$S[\theta] = \frac{E_c}{T} \sum_{j=1}^{N} \left[ (\theta_{j+1} - \theta_j - q)^2 - e^{-g/2} \cos(\theta_j) \right]$$

- fugacity ( = pinning strength):

$$\prod_{s=1}^{M} r_s = e^{\frac{1}{2}\sum_{s=1}^{M} \ln(1-|t_s|^2)} \approx e^{-\frac{1}{2}\sum_{s=1}^{M} |t_s|^2} = e^{-g/2}$$

- cf. with the previous approach ! $\Delta \sim E_c e^{-g/4}$

# dynamics and conductivity

- real time classical Langevin dynamics, $q_j$ --> $q_j(t)$:

$$\frac{1}{g}\frac{d\theta_j}{dt} = E_c\left[\theta_{j+1} - 2\theta_j + \theta_{j-1} - e^{-g/2}\sin(\theta_j + jq)\right] + E + \xi_j(t)$$

- external field

the noise correlator: $$\langle x_j(t) x_{j'}(t')\rangle = \frac{T}{g} d_{j,j'} d(t-t')$$

- soliton - antisloliton creation as an “under-barrier” process: $n_s = l_s^{-1}\, exp(-D/T)$,

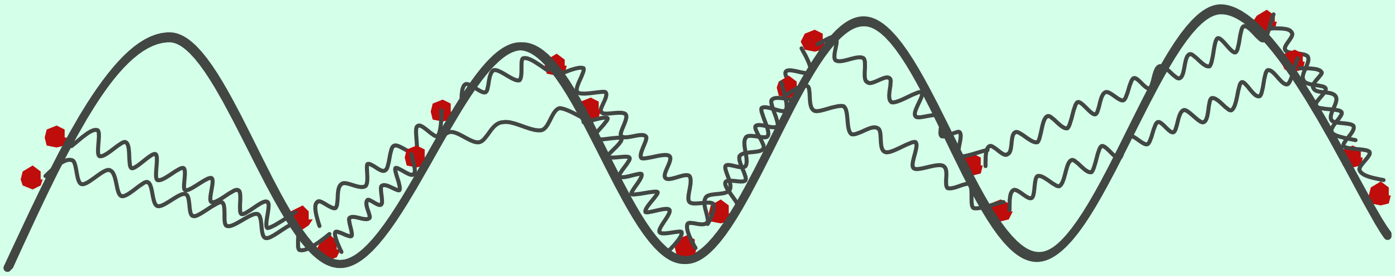
where $l_s = exp(+g/4)$ is the soliton length and $D = E_c\, exp(-g/4)$ is the charge gap,

- moving solitons, $q_j(t) = q(j-v_s t)$, where the soliton velocity: $v_s = l_s g E$

- current density: $J = e\, n_s v_s$; conductivity: $s = g\, exp\{-D/T\}$.

# disorder

- random gate voltages: $q$ --> $q_j$

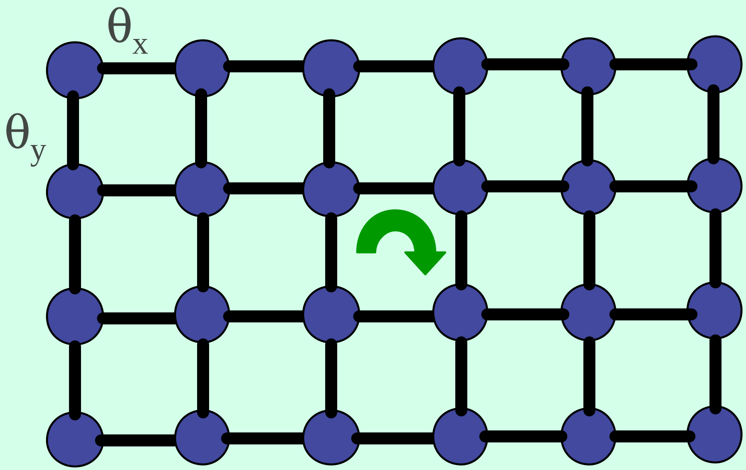


- pinning energy: $$E_{pin} = E_c \left(e^{-g/2}\right)^2$$

- charge gap: $$\Delta_{random} = \sqrt{E_c E_{pin}} \approx E_c e^{-g/2}$$

- role of rare events and relation to Burgulence, Feigelman 1980.

# 2D case




- for N dots -- 2N links:

N soft modes (placket rotations)

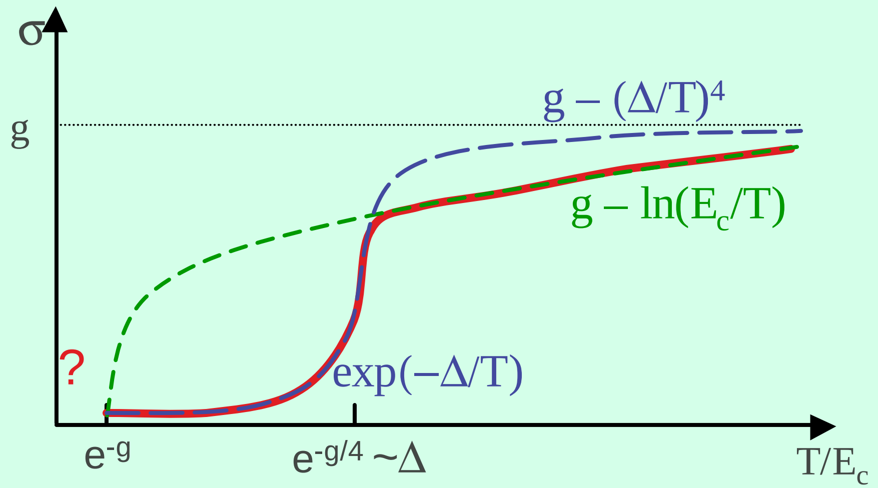
$$S = \frac{E_c}{T}\left(\text{div}\vec{\theta}\right)^2 - \sqrt{\frac{E_c}{T}}e^{-\frac{g}{2}}\left(\cos\theta_x + \cos\theta_y\right)$$

- single charge soliton energy: $$\Delta = \sqrt{E_c T}e^{-g/2}\log\left(\frac{E_c}{T}e^g\right)$$

- conductivity $g_{tot} = \exp(-\Delta/T)$

## conclusions:

- physics of arrays is equivalent to a classical pinned charge density wave.
- activation behavior with the charge gap $\Delta \sim exp(-g/4)$.




- partition function is dominated by the instanton configurations.

## open questions:

- inclusion of quantum interference, i.e. how does this mechanism compete/cooperate with effects of localization ?
- role of disorder and rare events.