

# Stationary properties of Josephson junctions with ferromagnetic interlayer

*M. Yu. Kupriyanov,*

**Nuclear Physics Institute, Moscow State University**

*A.A. Golubov, M.M. Khapaev,*

**Department of Applied Physics, University of Twente**

**Computer Science Department, Moscow State University**

*and*

*M. Siegel*

**Universität Karlsruhe (TH) ,**

**Institut für Mikro- und Nanoelektronische Systeme**

# Outline

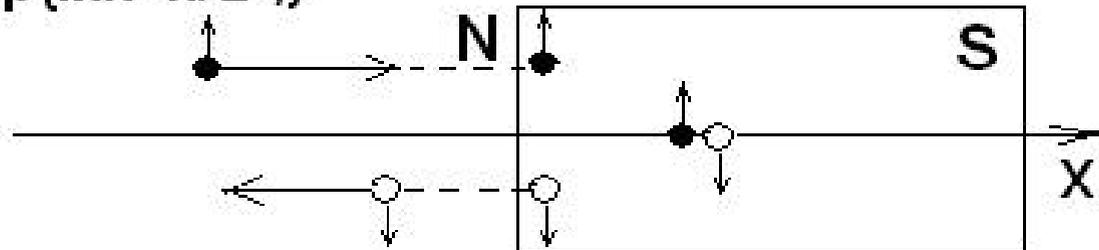
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- **Proximity effect in ferromagnet-superconductor (FS) structures:**  
oscillating nature of the order parameter in F
- **Current-phase relationships in Josephson in SFS –junctions.**
- Towards the engineering of SFS structures with predetermined properties
- Towards the fabrication of SFS structures with reliable parameters

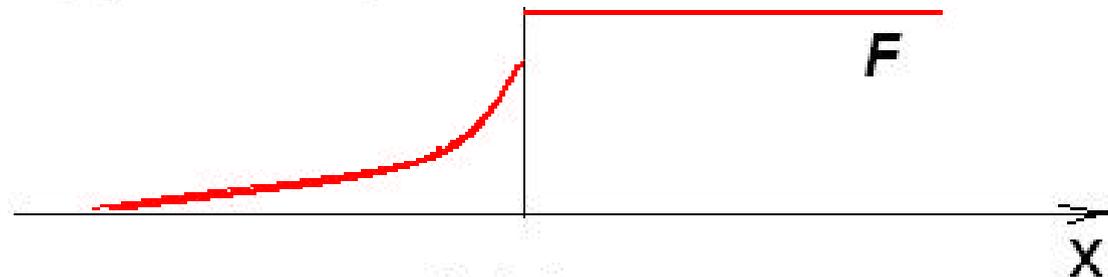
# Proximity effect in normal metal - superconductor (NS) structures

## Andreev reflection at SN interface

$$u \sim \exp\{ikx + x/2\xi\}$$



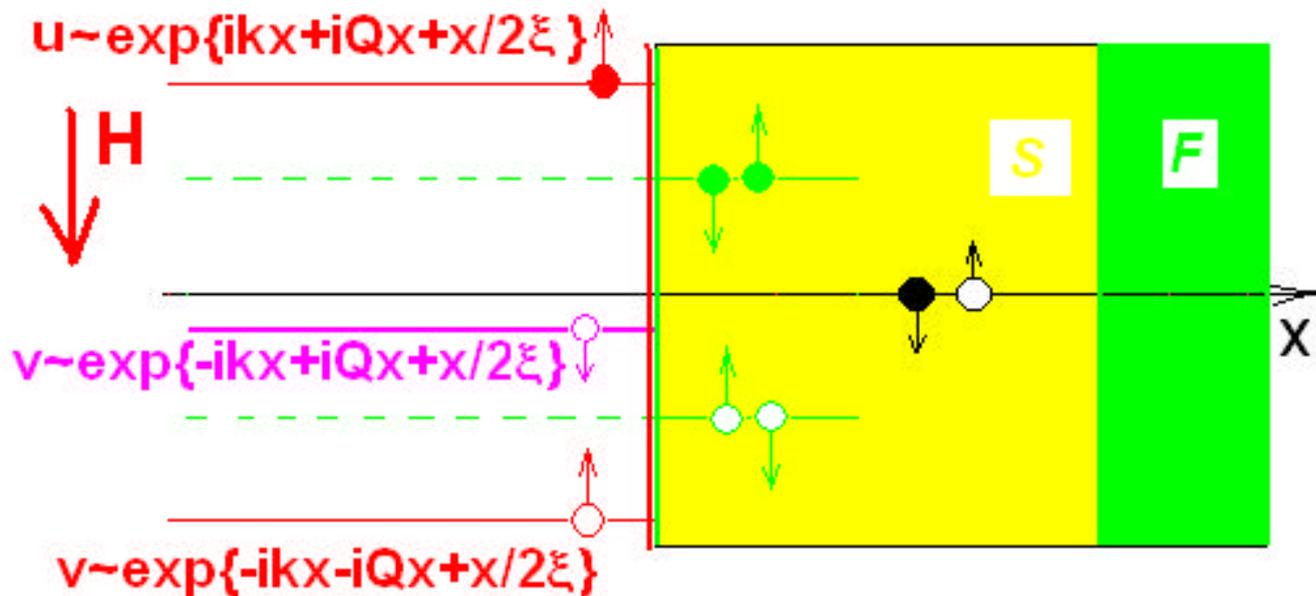
$$v \sim \exp\{-ikx + x/2\xi\}$$



$$\Delta \sim F \sim uv \sim \exp\{x/\xi\}$$

# Proximity effect in ferromagnetic - superconductor (FS) structures

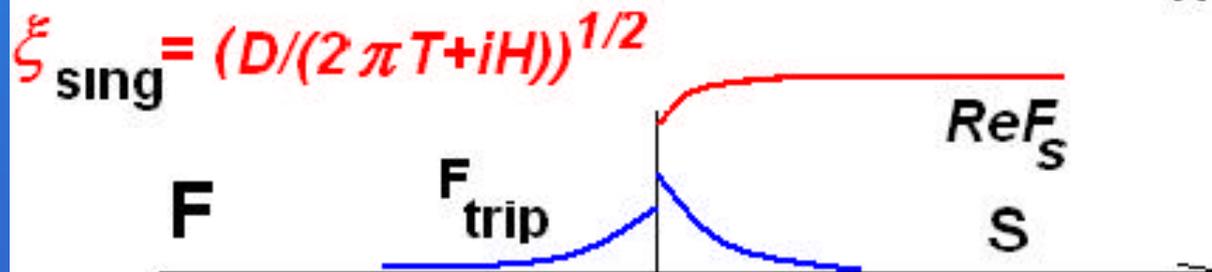
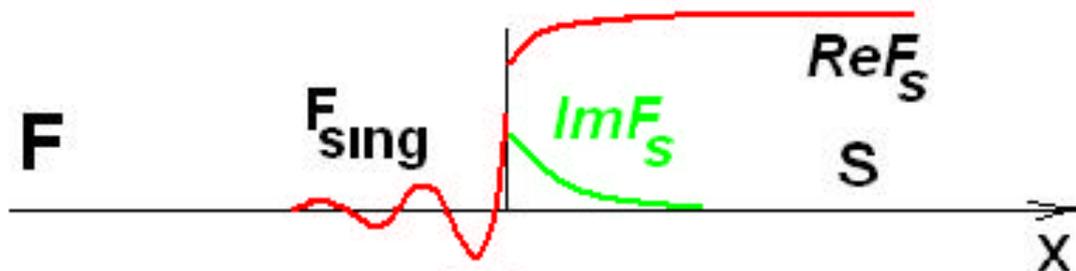
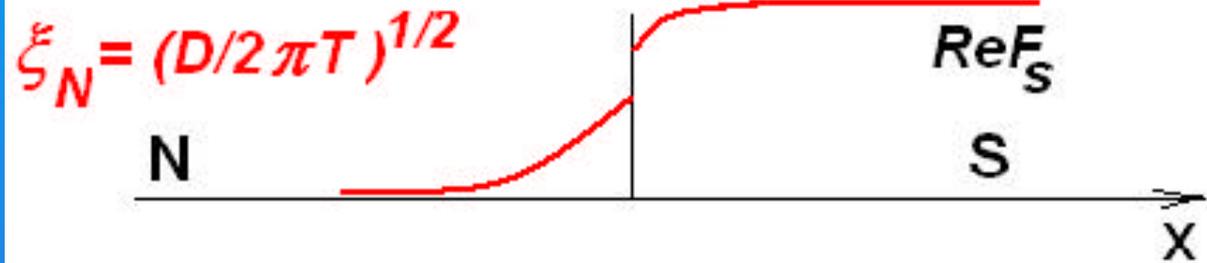
## Andreev reflection at SF interface



$$F_{\text{sing}} \sim v u \sim \exp\{x/\xi + 2iQx\}$$

$$F_{\text{trip}} \sim v u \sim \exp\{x/\xi\}$$

# Proximity effect in ferromagnetic - superconductor (FS) structures



$\xi_{\text{trip}} = (D/2\pi T)^{1/2}$

# Phase jump at SF interface $d_F \ll \xi_{\text{sing}}$

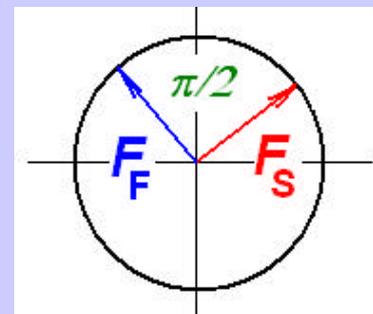
$$\chi = \frac{1}{2} \arctan \frac{q}{p} + \frac{\pi}{4} (1 - \text{sgn } p) \text{sgn } H,$$

$$p = 1 + \frac{\omega^2 - H^2}{(\pi T_C)^2} \gamma_{BM}^2 + 2 \frac{\omega^2 \gamma_{BM}}{\pi T_C \sqrt{\omega^2 + \Delta_0^2}},$$

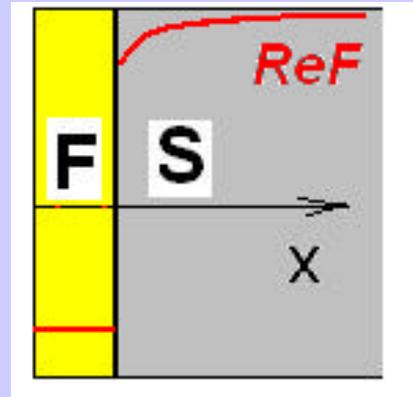
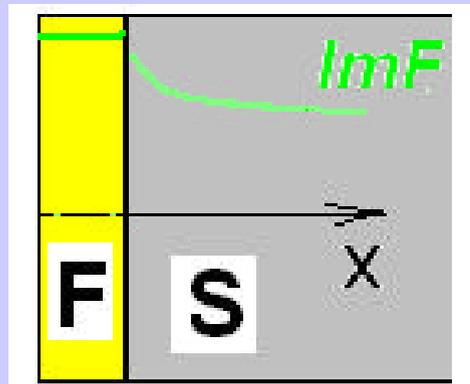
$$q = 2 \gamma_{BM} \frac{H \omega}{\pi T_C} \left( \frac{\gamma_{BM}}{\pi T_C} + \frac{1}{\sqrt{\omega^2 + \Delta_0^2}} \right).$$

At low  $\omega$  the phase shift  $\chi$  monotonously increases with  $H$  achieving maximum value

$$\chi_{\text{max}} = \frac{\pi}{2}$$



at  $H^* \sim \pi T_C / \gamma_{BM}$



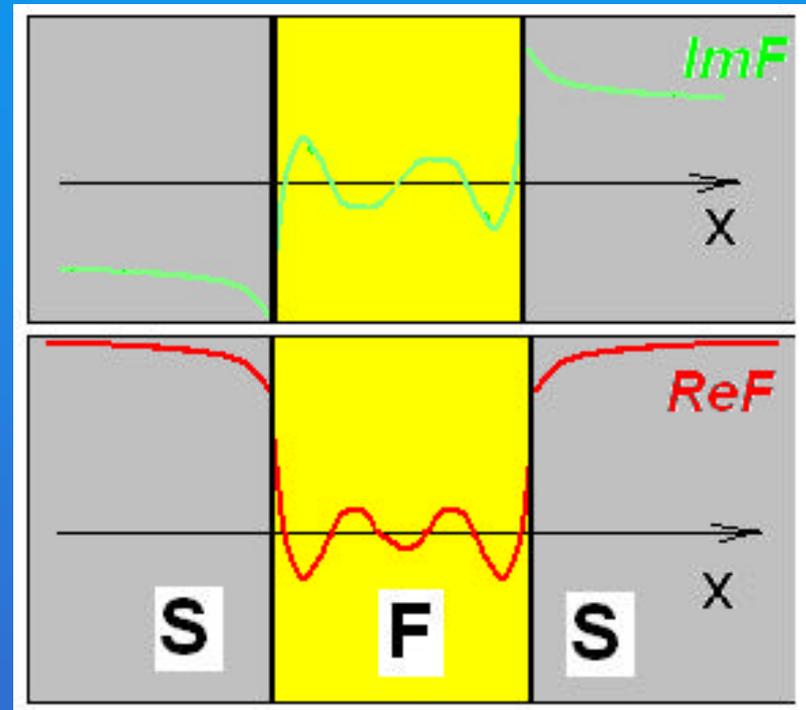
# 0 - $\pi$ transition due to thickness oscillations in F

$$\xi_{\text{sing}} = (D/(2\pi T + iH))^{1/2}$$

$$I_C R_N = \frac{\pi \Delta^2}{4eT_C} y \frac{\sinh y \cos y + \cosh y \sin y}{\sinh^2 y \cos^2 y + \cosh^2 y \sin^2 y}$$

$$y = \frac{d_F}{\xi_F} \sqrt{\frac{H}{2\pi T_C}} \quad y \gg 1$$

$$I_C R_N = 32\sqrt{2} \frac{\Delta}{e} \mathcal{F}(\Delta/T) y \exp(-y) \sin(y + \pi/4)$$

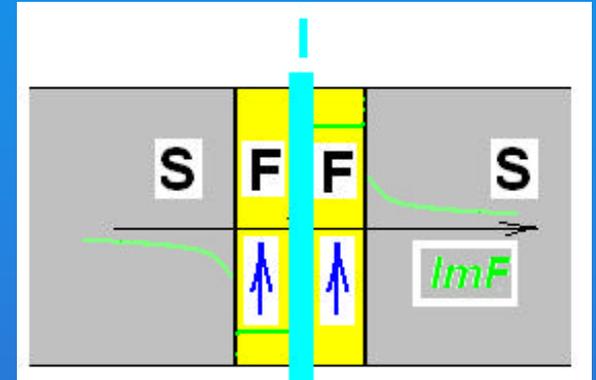
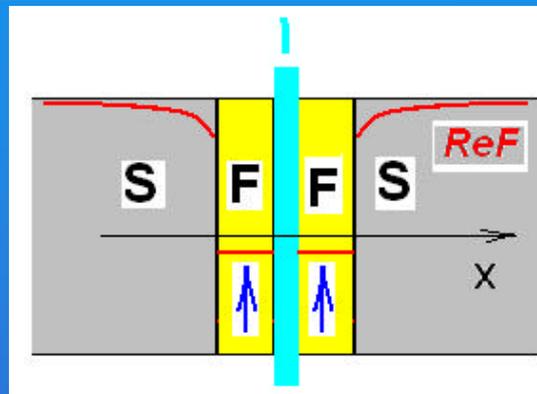
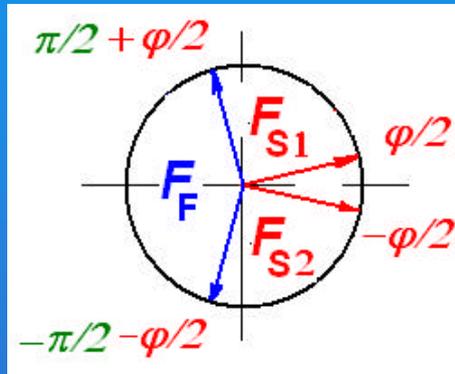


# 0 - $\pi$ transition due to phase jump at SF interface

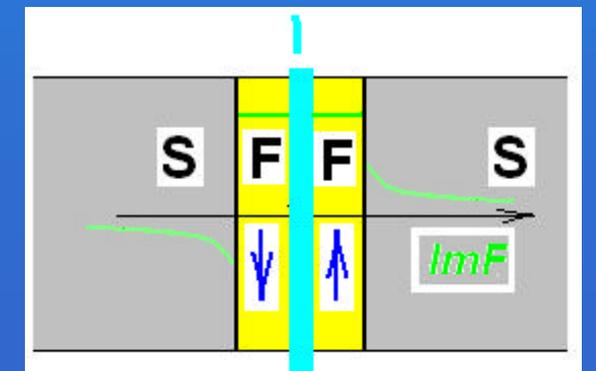
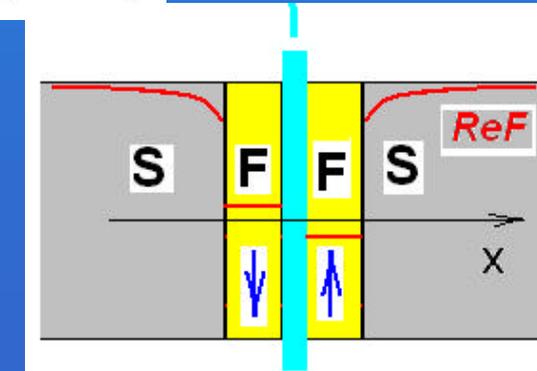
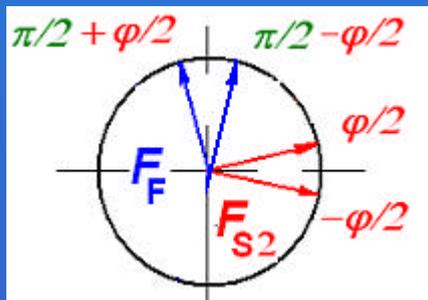
$$I_C = \frac{\pi T}{eR_{B,I}} \sum_{\omega} \frac{|\Phi_S|^2}{\omega^2 + |\Phi_S|^2} \frac{\cos \Psi}{(p^2 + q^2)^{1/2}}$$

$$\Psi = \frac{1}{2} (\chi_R(H_R) - \chi_L(-H_L))$$

$$\Psi_p = \arctan \frac{q}{p} + \frac{\pi}{2} (1 - \text{sgn } p)$$



$$\Delta\varphi = \pi/2 + \varphi/2 - (-\pi/2 - \varphi/2) = \pi + \varphi$$

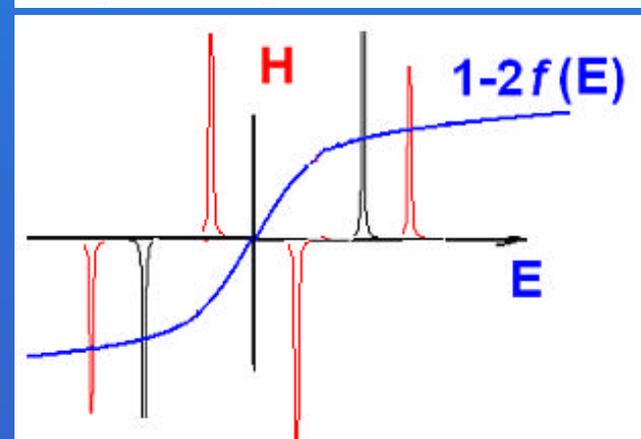
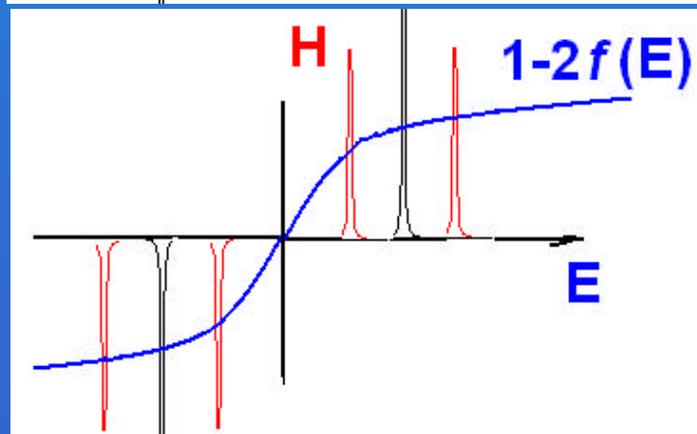
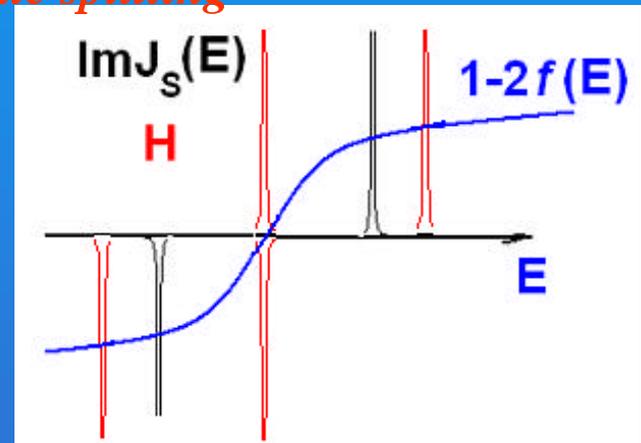
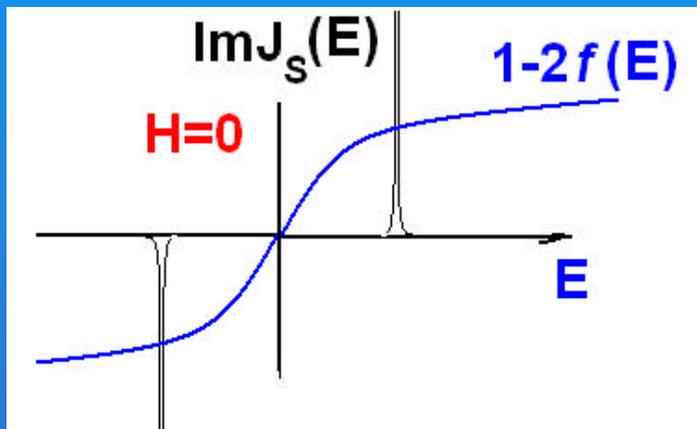


$$\Delta\varphi = \pi/2 + \varphi/2 - (\pi/2 - \varphi/2) = \varphi$$

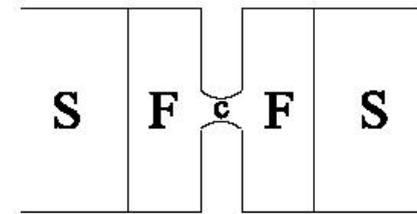
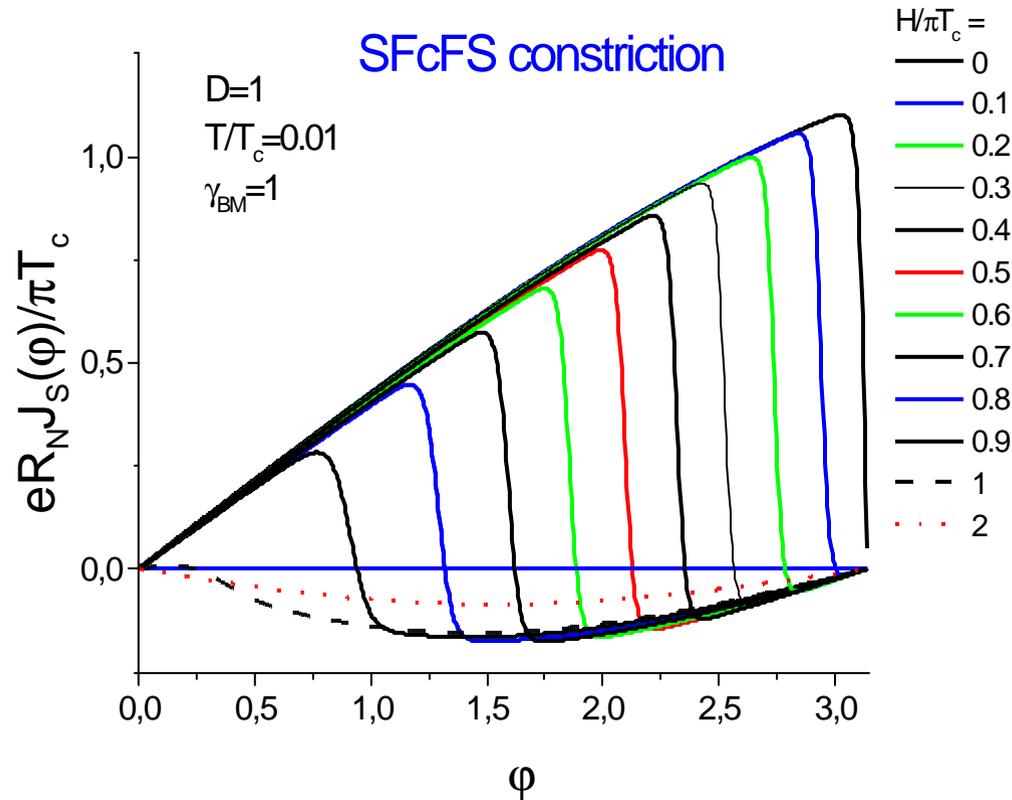
# General expression for the dc supercurrent

$$I_S(\varphi) \propto \int_{-\infty}^{\infty} dE [1 - 2f(E)] \text{Im}\{I_E(\varphi)\}$$

*Andreev bound state splitting*

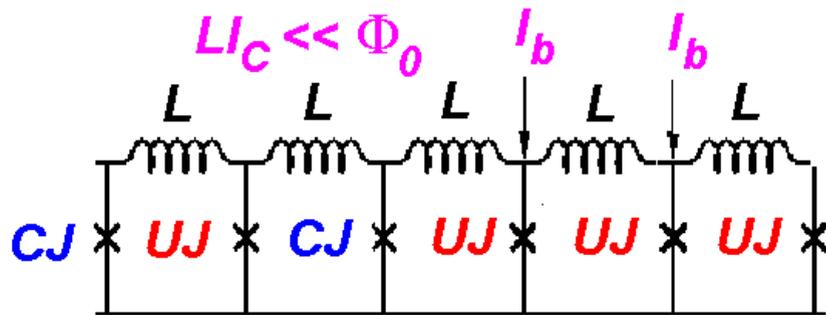


# Current-phase relations in more complex geometry: *ballistic SFcFS junction*



# Towards the engineering of SFS structures with predetermined properties

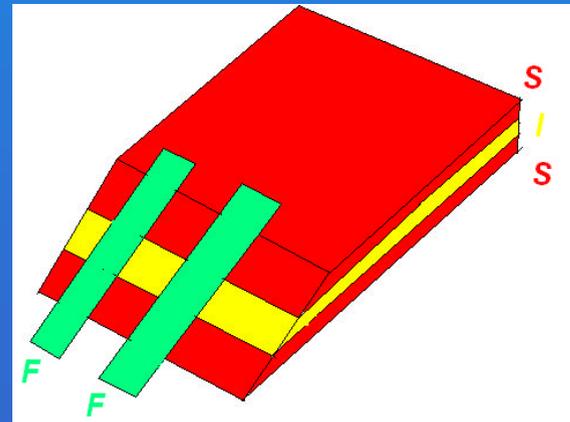
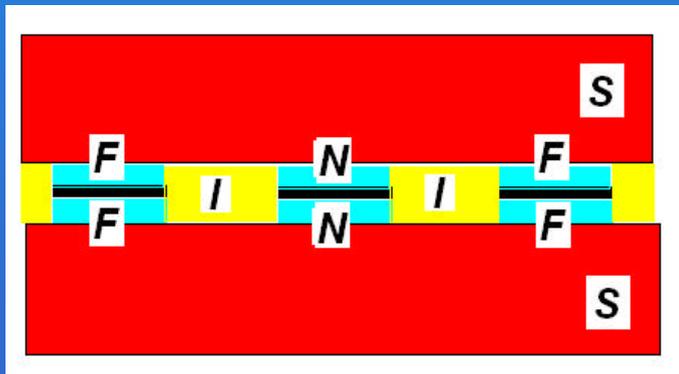
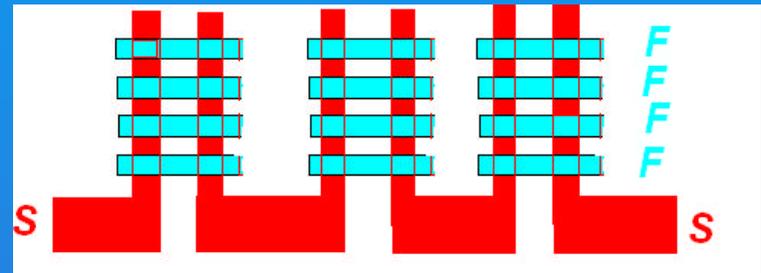
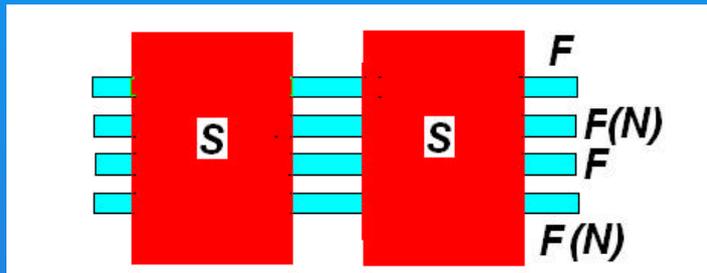
- The existence of “pi” and “0”+ “pi” Josephson junctions provides the way for engineering of Josephson structures with predetermined properties.



**CJ** - conventional "0" Josephson junctions

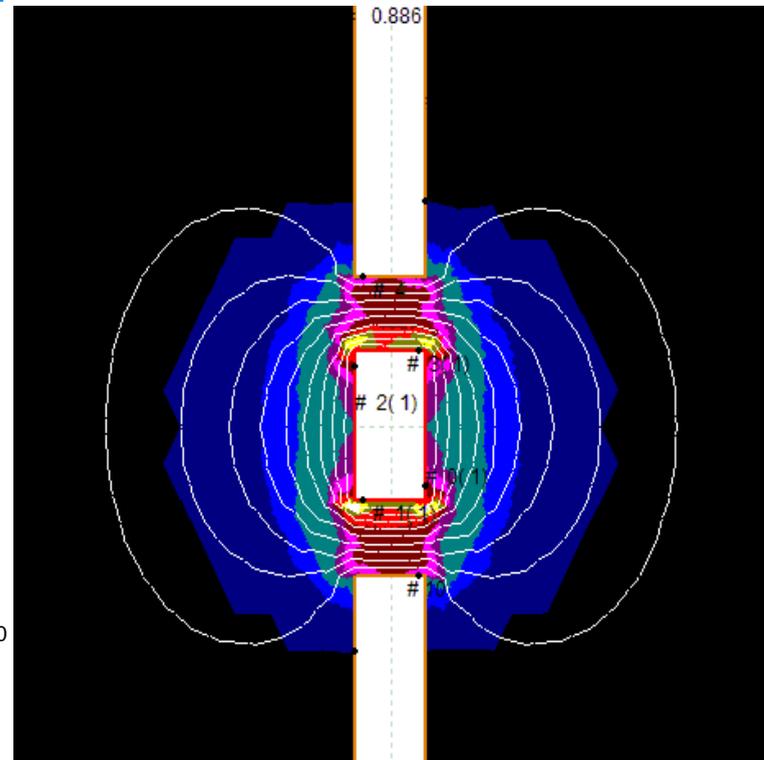
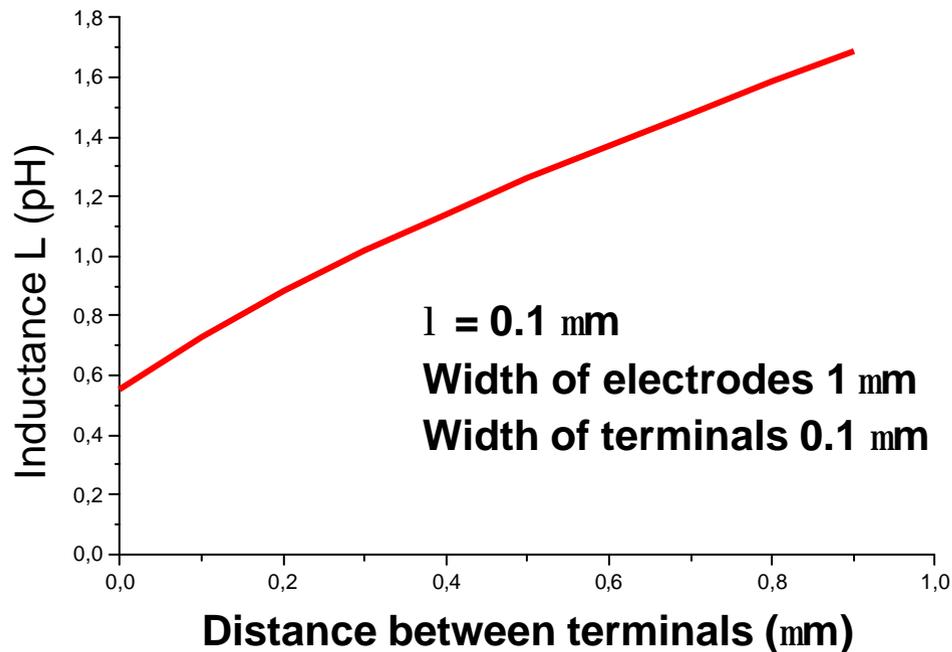
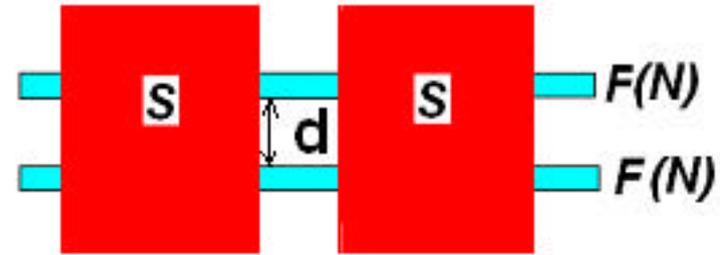
**UJ** - unconventional "pi" or "0"+ "pi" Josephson junctions

# Possible configurations of DJJ



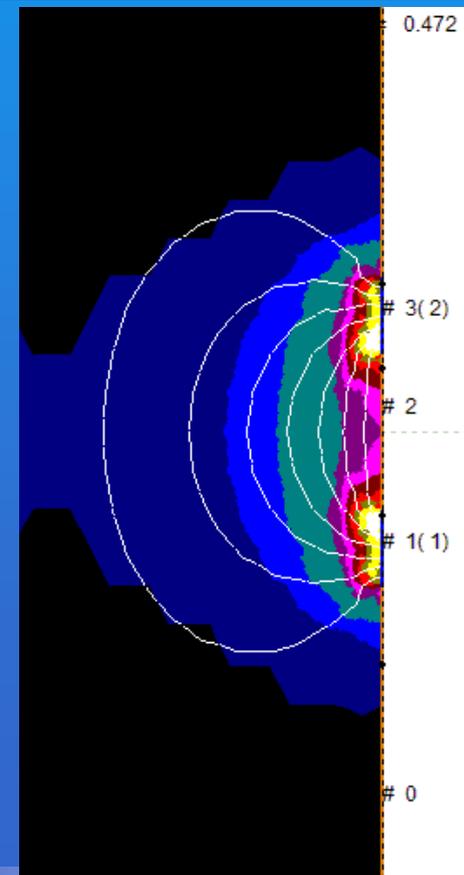
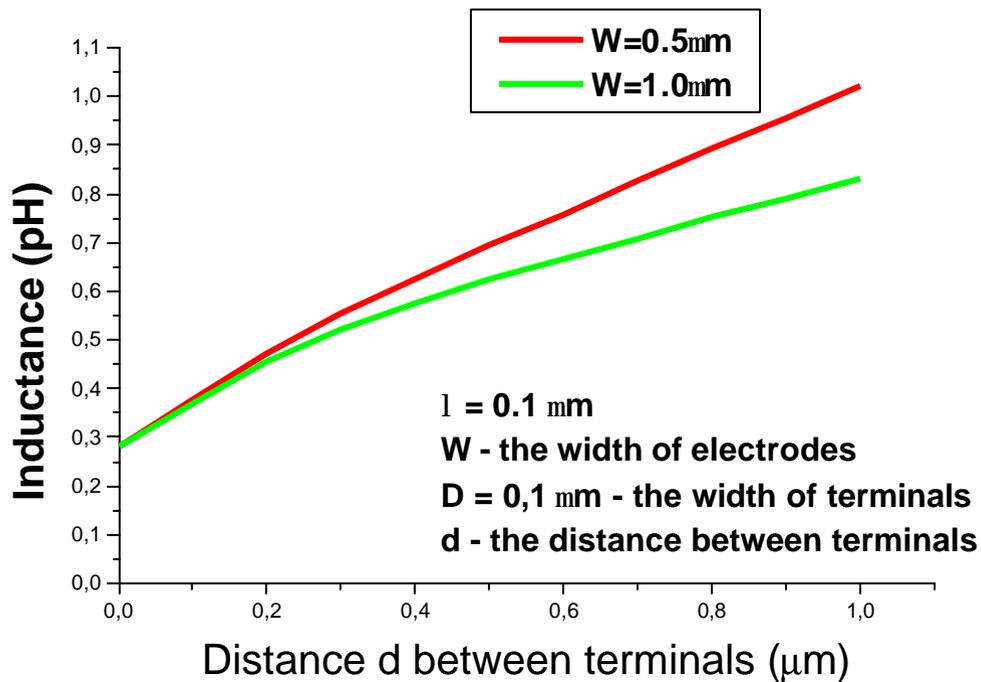
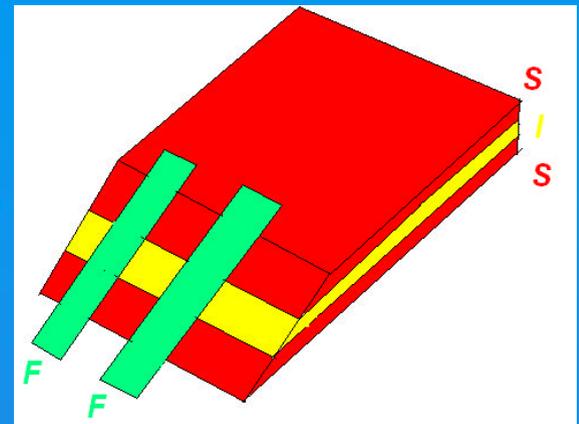
# Inductance estimations

- Variable thickness bridges



# Inductance estimations

- Variable thickness bridges



# $I_c$ estimations

- $L I_c \ll \Phi_0$
- $I_c \approx 0.05 F_0/L, L \gg 1 \text{ pH}$ 
  - $I_c \approx 0.05 \text{ mA}$
  - $j_c \approx 5 \text{ kA/cm}^2$

# Towards the fabrication of SFS structures with reliable parameters

- Problem N 1.

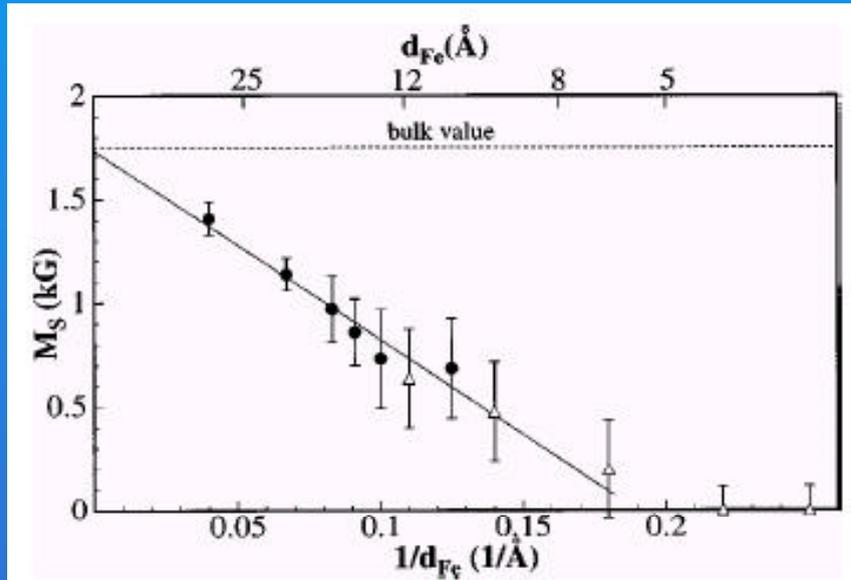
Small decay length

- Problem N 2.

Magnetically dead layer  $d_{\text{dead}}$  = the scale of interface roughness or mutual diffusion of S and F materials

$$\xi_{\text{sing}} = (D/(2\pi T + iH))^{1/2}$$

# Nb/Fe multilayers



- Th. Mühge et al., Phys. Rev. B 57, 6029 (1998)
- The roughness parameters of 3–4  $\text{\AA}$ . The magnetically “dead” layer arises due to an intermixing of Nb and Fe at the interface
- $x_{Fe} \gg 12 \text{\AA}$ .

# Nb/Co multilayers

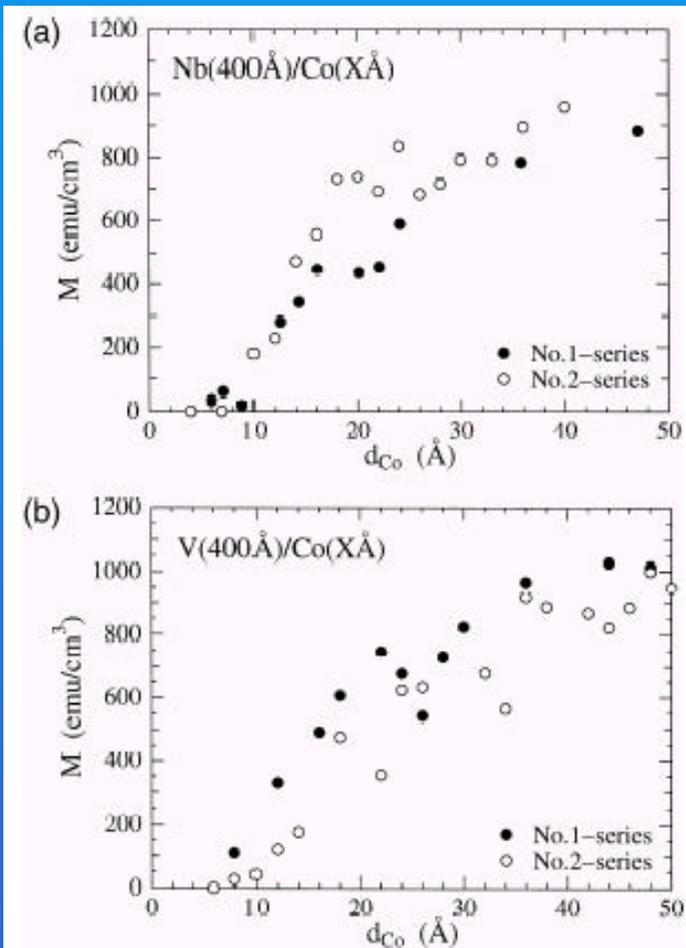


Fig. 1. Magnetization as a function of  $d_{\text{Co}}$  for (a) Nb/Co and (b) V/Co multilayers. Different symbols correspond to different sample series.

- For Nb/Co and for V/Co, MS continues to drop toward  $d_{\text{Co}} \gg 7 \text{\AA}$ , where it becomes zero in both systems. *The threshold value  $d_{\text{Co}} \gg 7 \text{\AA}$  may correspond to the formation of the nonmagnetic (magnetically "dead") interface layer.* (Y. Obi et al. Czech. J. Phys. 46 721 (1996), Physica C 317-318 149 (1999))
- $d_{\text{Co}} \gg 3 \text{\AA}$ , S. F. Lee et al., J.Appl.Phys., 87, 5564 (2000)
- $d_{\text{Co}} \gg 6 \text{\AA}$ , S. F. Lee et al., J.Appl.Phys., 89, 6364 (2001)
- $d_{\text{Co}} \gg 24 \text{\AA}$ , A. Ajan et al., J.Appl.Phys. 91, 1444 (2002).

# Nb/ Gd multilayers

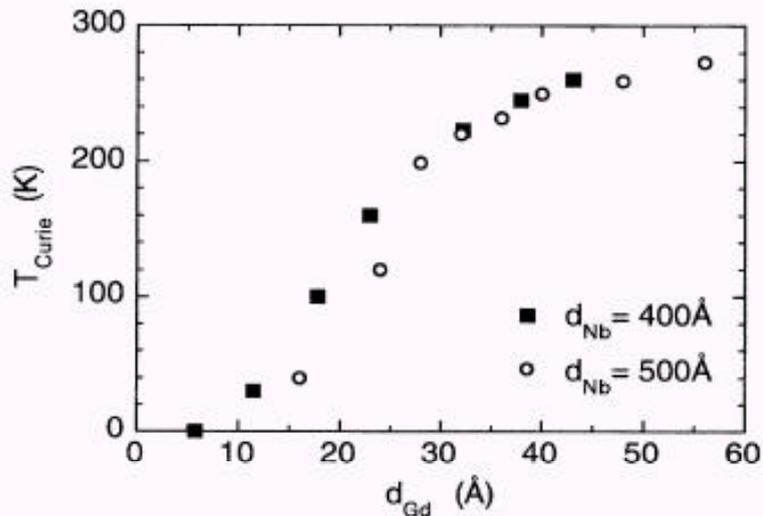


FIG. 2. Ferromagnetic ordering temperature  $T_{\text{Curie}}$  vs Gd layer thickness  $d_{\text{Gd}}$  for two series of Nb/Gd multilayers with  $d_{\text{Nb}} = 400$  and  $500 \text{ \AA}$ .

- C. Strunk et al., Phys. Rev. 49 (1994), p. 4053. J.S. Jiang et al., Phys. Rev. 64 (1996), p. 6119
- $x_{\text{Gd}} \approx 10 \text{ \AA}$ .  $d_{\text{Gd}} \gg 15 \div 20 \text{ \AA}$

# Strategy

Interface roughness, magnetically dead layer thickness and decay length in ferromagnetic are in the same scale!!! Therefore we have to

- fabricate interfaces as flat as it is possible and avoid mutual diffusion;
- use ferromagnetic with large decay length - small exchange energy - weak ferromagnetics

$$\xi_{\text{sing}} = (D/(2\pi T + iH))^{1/2}$$

# Existing solutions

- V. Ryazanov et al, 2001a,b  $Nb/Cu_{1-x}Ni_x/Nb$ ,  $x \gg 54\%$
- T. Kontos et al., 2001,2002;  $Pd_{1-x}Ni_x$  alloy with  $x \gg 10\%$
- Y. Blum et al., 2002;  $Ni$  in  $NbCuNiCuNb JJ$
- C. Surgerset al., 2002.  $Pd_{1-x}Fe_x$  alloy with  $x \gg 10-20\%$

# Materials with intermediate atomic concentration Nb/Cu<sub>1-x</sub>Ni<sub>x</sub>/Nb

- The Nb/Cu<sub>1-x</sub>Ni<sub>x</sub>/Nb structures was historically the first in which transition from 0-phase to *p* phase state was demonstrated on the temperature dependencies of the Josephson junction critical current( $x = 0.54$ )
- $\xi_{\text{CuNi}} = 7.6 \text{ nm}$ ,
- $\gamma \approx 0.15$
- $H \approx 130 \text{ K}$
- $\gamma_B \approx 0.3$

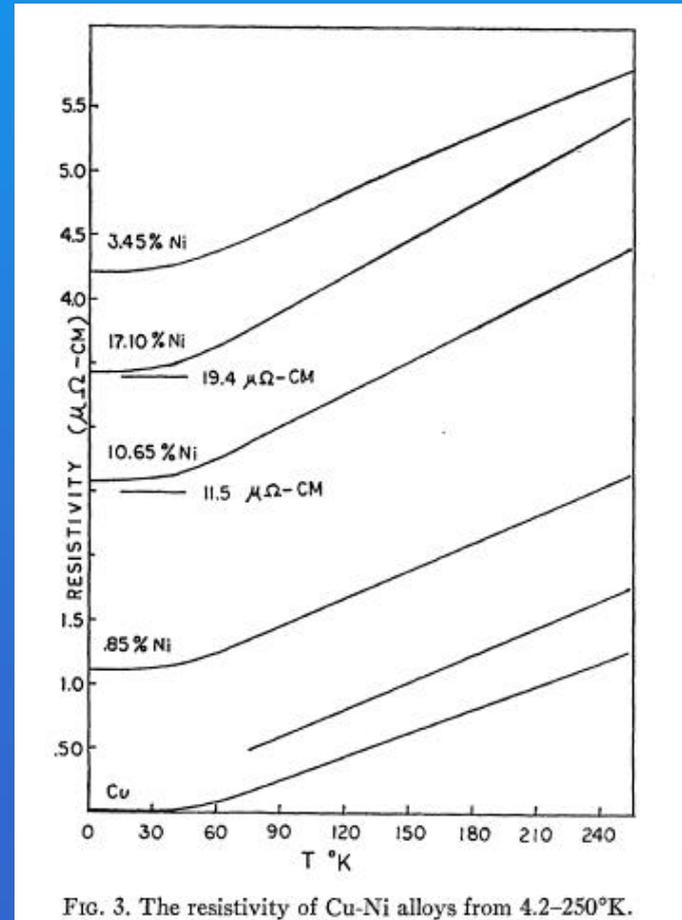
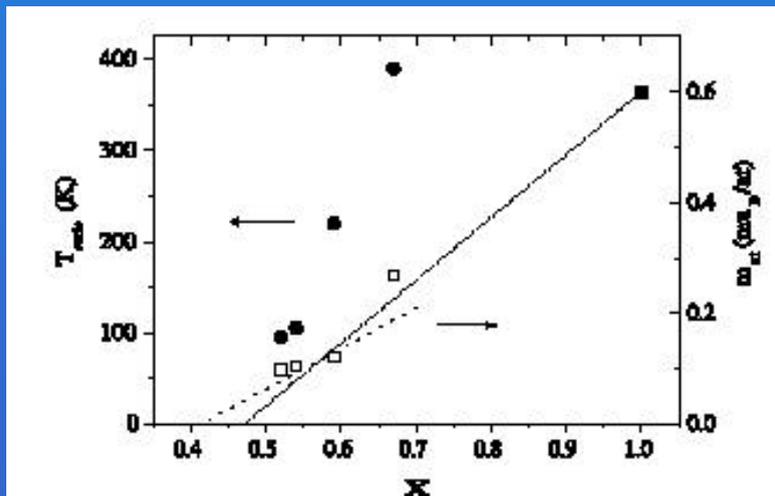


FIG. 3. The resistivity of Cu-Ni alloys from 4.2–250°K.

# $T_{\text{Curie}}$ of ferromagnets with relatively small concentration of F material Pd/Ni

$Pd_{1-x} Ni_x$  alloy with  $x = 10\%$   
 $H \approx 15 \text{ meV}$      $\xi_F \approx 45 \text{ \AA}$   
 T. Kontos et al., PRL 2002

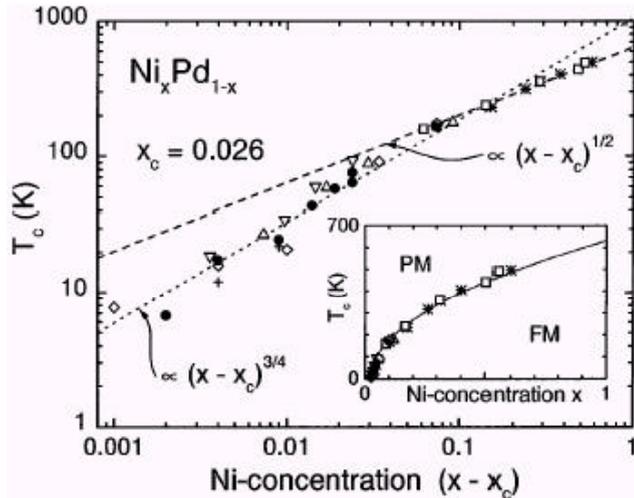
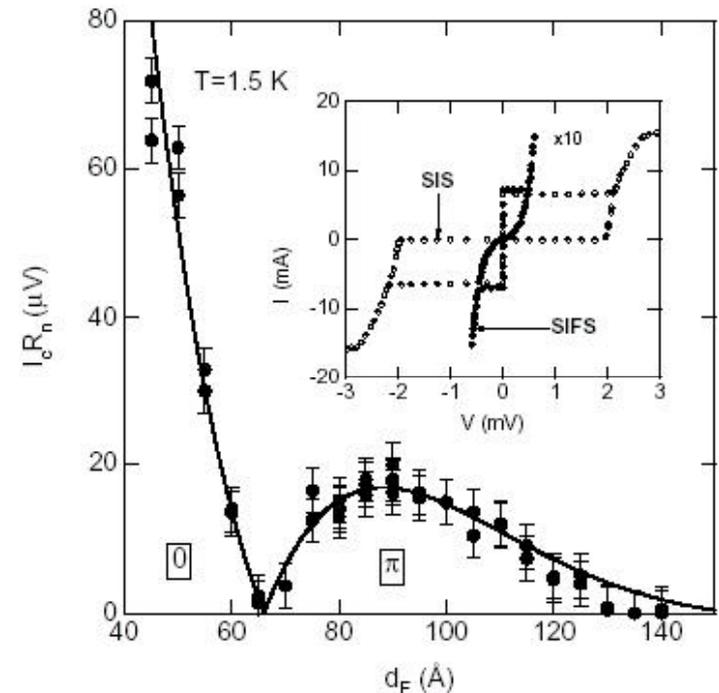
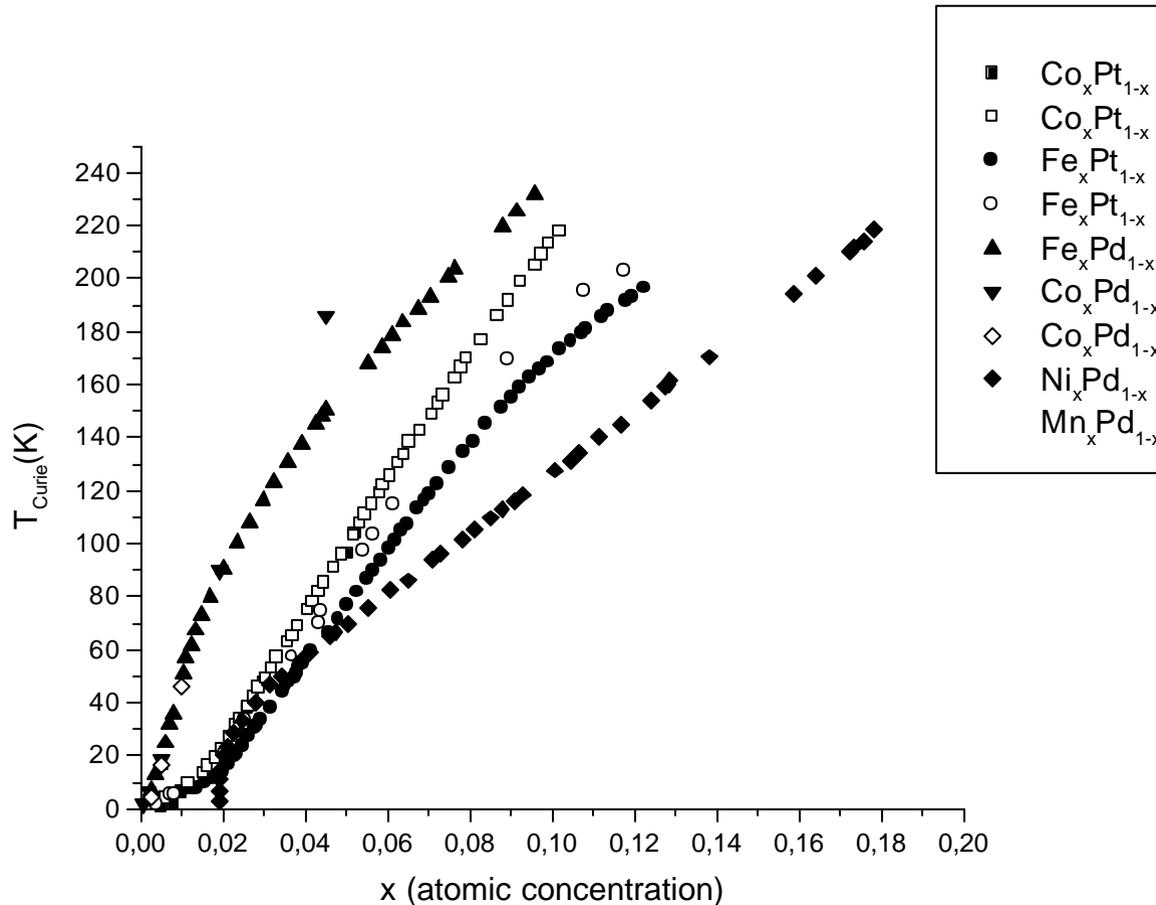


FIG. 1. Curie Temperature  $T_c$  as a function of  $x - x_c$  in a double logarithmic plot: ( $\square$ ) Marian (1937); ( $\Delta$ ) Crangle and Scott (1965); (+) Murani *et al.* (1974); ( $\ast$ ) Fujiwara *et al.* (1976); ( $\nabla$ ) Beille and Tournier (1976); ( $\diamond$ ) S.K. Burke *et al.* (1982); ( $\bullet$ ) present data [20,28,29]. The dashed line indicates  $(x - x_c)^{1/2}$ , dotted line  $(x - x_c)^{3/4}$ .  $x_c$  denotes the critical concentration. The inset shows  $T_c(x)$  on a linear scale following a square root behavior at high  $x$  (solid line).



M. Nicklas, M. Brando, G. Knebel, F. Mayr, W. Trinkl, and A. Loidl,  
 Non-Fermi-Liquid Behavior at a Ferromagnetic Quantum Critical Point in  $Ni_xPd_{1-x}$   
 Phys. Rev. Lett. 82, 4268 (1999)

# $T_{\text{Curie}}$ of ferromagnets with small concentration of F material

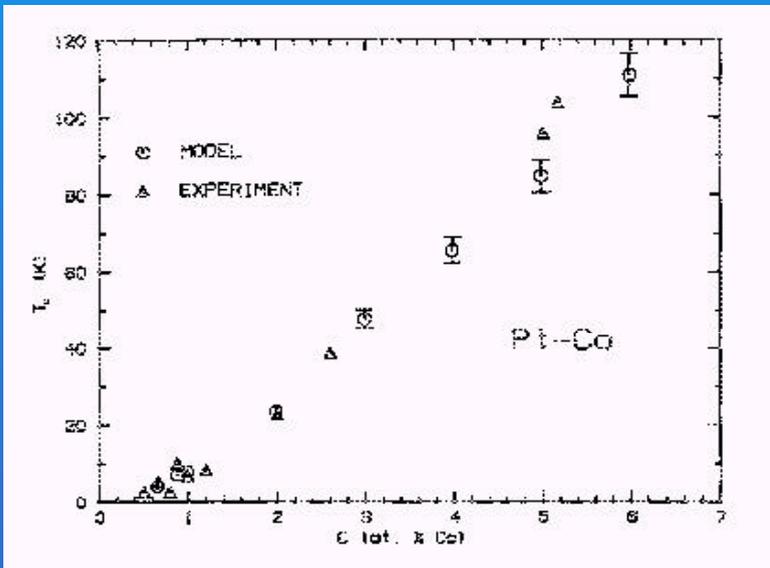


# $T_{\text{Curie}}$ of ferromagnets with small concentration of F material, Pt/Co

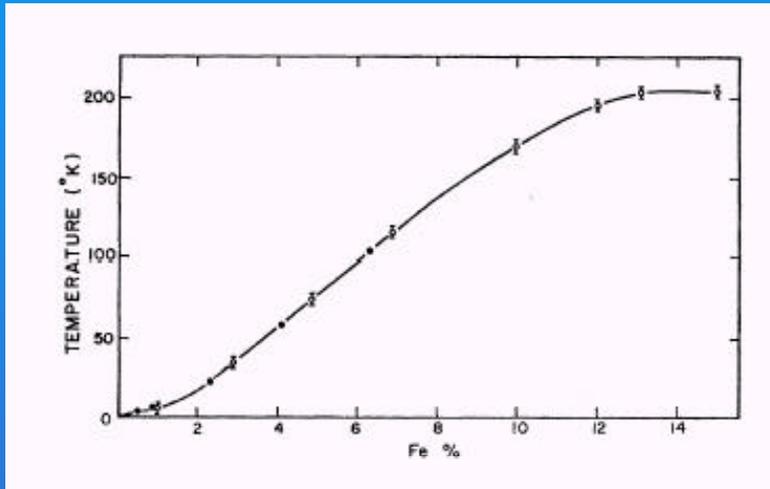
- $T_{\text{Curie}}$  is proportional to  $x^2$  for concentration

0.66 at % Co  $\leq x \leq$  1 at % Co  
and proportional to  $x$  for higher concentrations of Co.

- Critical concentration equals to 0,271 at % Co.
- At  $x < x_C$  PdCo is a paramagnetic, while at large Co concentrations PtCo films are hard magnetics with the magnetic moment oriented perpendicular to a substrate

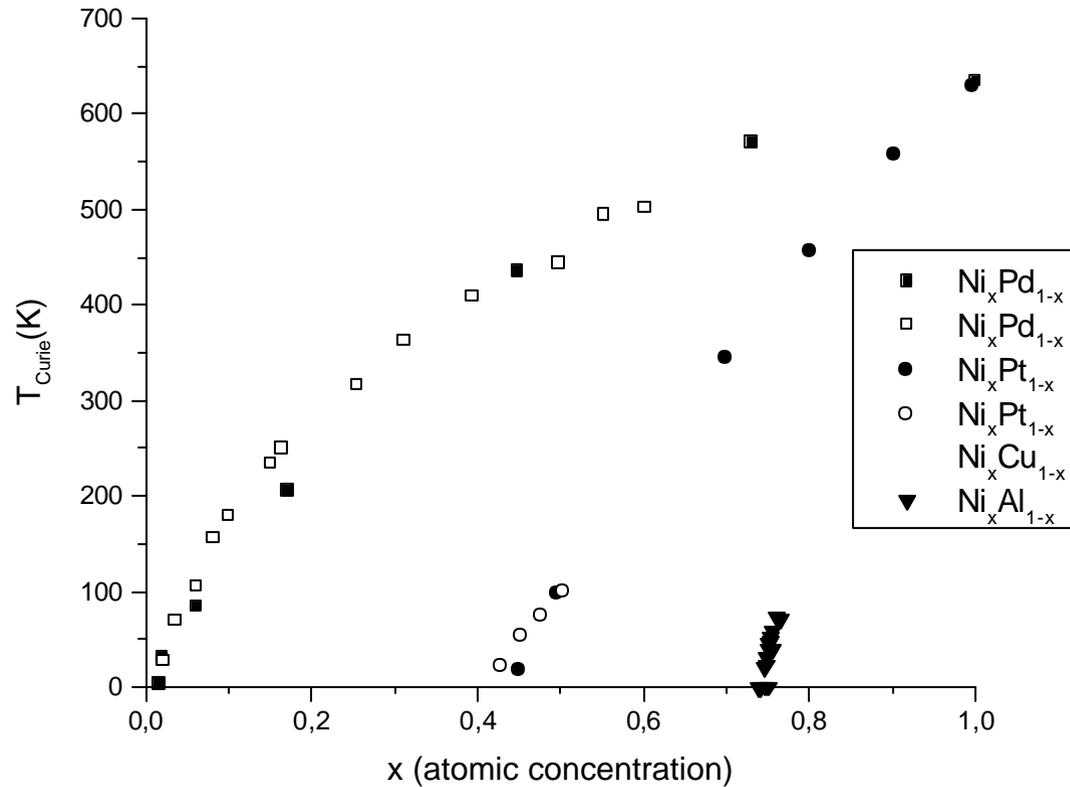


# $T_{\text{Curie}}$ of ferromagnets with small concentration of F material Pt/Fe



- In small concentration region up to 1%  $T_{\text{Curie}}$  depends on atomic Fe concentration  $x$  as  $T_{\text{Curie}} \approx 1,6 \times 10^3(x - 0.076)$  and characterize by spin exchange integral  $H \approx 0,14$  eV.
- At larger concentrations (between 2% and 8%)  $T_{\text{Curie}}$  also depends linear on  $x$  as
- $T_{\text{Curie}} \approx 20(x - 0.01)$ .

# Materials with intermediate atomic concentration



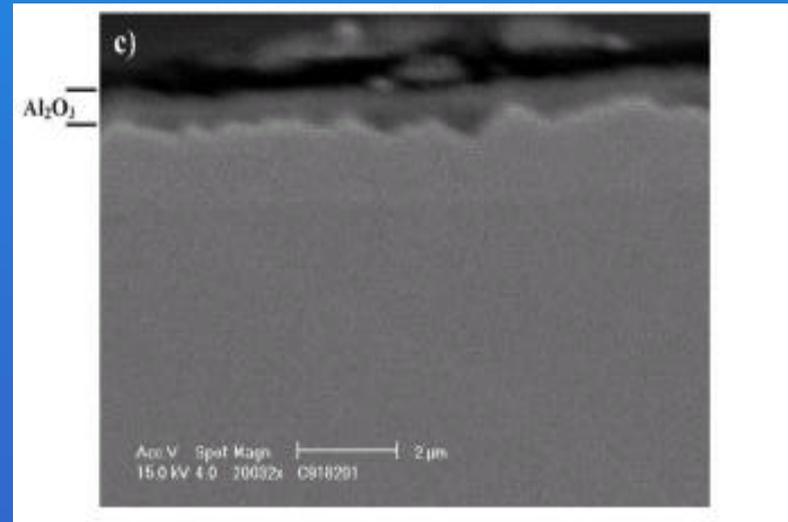


# Surface oxidation of $\text{Ni}_3\text{Al}$

- at low temperatures the film is amorphous (locally ordered), at higher temperatures (about 1300 K) it becomes globally ordered and takes on the structure of two O-Al bilayers, terminated with an Al layer on the interface and with an O layer from vacuum.
- The NiAl- $\text{Al}_2\text{O}_3$  interface is atomically sharp without any intermediate phases.

TABLE II. The 1 ML oxygen coverage: point defects present [23] and the composition of the upper NiAl layer (including oxygen) for different configurations and their surface energy  $\Delta\gamma$  (in eV/cell) relative to the energy of configuration A.

Configuration	Point defects	Upper layer	$\Delta\gamma$		
			Ni-rich	Stoichiometric	Al-rich
A	none	$\text{Ni}_2\text{Al}_2\text{O}_3$	0	0	0
B	$\text{V}_{\text{Ni}}^{(2)}$	$\text{Ni}_2\text{Al}_2\text{O}_3$	+0.53 (+0.58)	+0.18 (+0.31)	-0.51 (-0.22)
D	$\text{V}_{\text{Ni}}^{(2)} + \text{Ni}_{\text{Al}}^{(2)} + \text{Al}_{\text{Ni}}^{(1)}$	$\text{Ni}_1\text{Al}_3\text{O}_3$	-0.61 (-0.28)	-0.96 (-0.54)	-1.65 (-1.07)
F	$\text{V}_{\text{Al}}^{(2)} + \text{Al}_{\text{Ni}}^{(1)}$	$\text{Ni}_1\text{Al}_3\text{O}_3$	-0.72 (-0.78)	-1.07 (-1.05)	-1.76 (-1.58)
G	$\text{Ni}_{\text{Al}}^{(2)} + \text{Al}_{\text{Ni}}^{(1)}$	$\text{Ni}_1\text{Al}_3\text{O}_3$	-1.87 (-1.39)	-1.87 (-1.39)	-1.87 (-1.39)
C	$\text{V}_{\text{Ni}}^{(1)}$	$\text{Ni}_1\text{Al}_2\text{O}_3$	-1.92 (-1.73)	-2.27 (-2.00)	-2.96 (-2.53)
E	$\text{V}_{\text{Ni}}^{(1)} + \text{Ni}_{\text{Al}}^{(2)} + \text{Al}_{\text{Ni}}^{(1)}$	$\text{Al}_3\text{O}_3$	-2.72 (-2.42)	-3.07 (-2.69)	-3.76 (-3.22)



# Deposition of Ni<sub>3</sub>Al on Nb substrate

- The nucleation mode is induced by a positive surface energy balance, when  $\Delta\gamma_n = \gamma_{fn} + \gamma_{in} - \gamma_s > 0$ , where  $\gamma_f \approx 2.08 \text{ J/m}^2$  is the Ni<sub>3</sub>Al thin film surface energy for a monolayer,  $\gamma_{in} \approx 1.2 \text{ J/m}^2$  is the interface energy and  $\gamma_s \approx 3 \text{ J/m}^2$  is the Nb substrate surface energy. A three-dimensional epitaxial island growth corresponding to a Volmer-Weber should be achieved during the deposition process.
- The surface energy mismatch,  $\Gamma_{sf} = 2|(\gamma_s - \gamma_f)/(\gamma_s + \gamma_f)|$ , is equal to 0.36. The critical value defined for the formation of a superlattice structure is  $\Gamma_{sf} = 0.5$ . Therefore, the growth of a superlattice structure is energetically favored.

# Summary

**Physics of unconventional JJ :**

- oscillating order parameter in a ferromagnet
- $\pi/2$  phase shifts at the SF interfaces in SFS junctions
- generation of triplet superconductivity

are well understood to predict the mode of operation of SFS devices.

This knowledge permits to pose the problem of engineering of Josephson junctions with predetermined properties.

**Ni<sub>3</sub>Al** looks very promising for SF/FS tunnel junction fabrication.

**Pt based ferromagnetic alloys** can be used for weak link Josephson junction of a constriction or variable bridges types.